

Information Technology & Returns to Scale

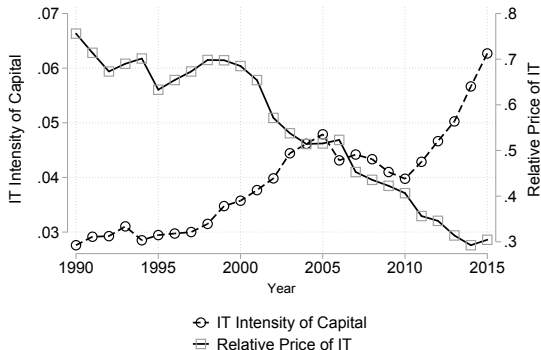
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September 12, 2020

Can the Rise of IT Explain Recent Macro Trends?

- Document French macro trends & micro facts about IT intensity
- Build theory that fits facts connecting IT and returns to scale
- Estimation + calibration $\Rightarrow$ IT explains $\sim$ half of the trends



Our Core Insight: IT Mitigates Organizational Complexity

- IT improves coordination across many different tasks
⇒ IT's marginal product higher for larger firms



Data: Sources

- Annual Survey of Firms (EAE):

- ▶ Information on **software** investments

Froutè (2001)

Details

- Administrative Data (BRN/RSI):

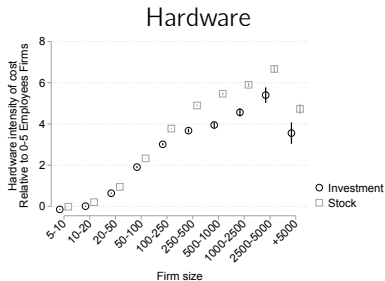
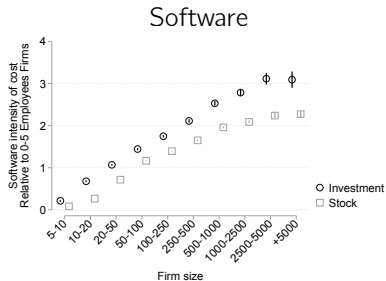
- ▶ Sales, value added, total capital & investment
- ▶ **BRN**: **hardware** (office furn. and computer equip.) investments
Barbesol, Heckel & Simon Quantin (2008); Luciani & Chevalier (2017)
- ▶ **RSI**: Labor share and concentration patterns

Summary

Construction of Stock Measures

Micro Fact: Size & IT Intensity

- Intensity of Costs: IT to total expenditure/factor payments



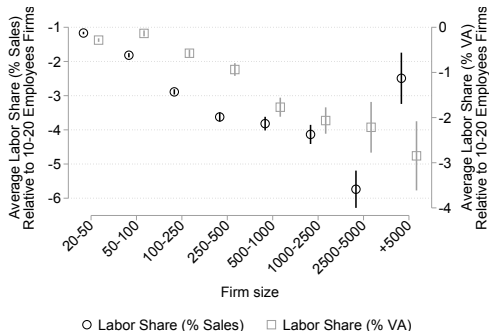
Robustness

$$IT\ Intensity_{firm,t} = FE_{Size_{firm,t}} + FE_{ind,t}$$

Micro Fact: Size & Labor Share

- Fact links rise in concentration to change in aggregate labor share

Autor et al. (2017)



$$Labor\ Share_{firm,t} = FE_{Size_{firm,t}} + FE_{ind,t}$$

Macro Facts: Trends in France (1990-2007)

1. Market Concentration:

- ▶ Rise in market concentration (proxies averaged across industries)

[Details](#)

2. Factor Payment Shares:

- ▶ Stable shares of labor and capital in aggregate income
- ▶ Rise in within-firm labor share, reallocation to low-labor share firms

[Details](#)

3. Business Dynamism:

- ▶ Fall in the number of firms (relative to number of workers)

[Details](#)

Firm-Level Production & Organizational Limits to Scale

- Nested implicit production function: $Y = \mathcal{F}(X_N, X_I)$

- ▶ Bundle of non-IT inputs: $X_N \equiv K^\alpha L^{1-\alpha}$

- ▶ Bundle of IT inputs: $X_I \equiv S^\beta H^{1-\beta}$

- Construction of $\mathcal{F}(\cdot, \cdot)$:

$$Y^\gamma = \left(O^{\frac{\sigma-1}{\sigma}} + X_N^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

with endogenous **organizational inputs** O :

$$O = X_I \times Y^{-\epsilon}$$

- Nonhomothetic IT demand with $W \equiv$ Relative price of IT:

$$\frac{X_I}{X_N} = W^{-\sigma} \times Y^{\epsilon(1-\sigma)}$$

Model in a Nutshell

- Nonhomothetic CES (nhCES) PF: $Y = \mathcal{F}(e^\theta X_N, e^{\theta+\phi} X_I; \epsilon, \sigma, \gamma)$

Hanoch (1975); Sato (1977); Comin, Lashkari & Mestieri (2015)

- ▶ Nonhomotheticity parameter $\epsilon \neq 0$
- ▶ Cost elasticity parameter γ and elasticity of substitution σ
- Melitz model: CES demand
 - ▶ Entry cost to draw $(\theta, \phi) \sim F$
 - ▶ AR(1) Markov evolution for productivities (θ, ϕ)
 - ▶ Fixed cost & endog. exit + exog. death shock

Key Theoretical Results

- **Identity:** Returns to Scale (RtS) $\propto$ Share of Labor + Capital
1. Nonhomothetic IT demand $\Rightarrow$ RtS falls in size (if $\sigma < 1$)
 $\Rightarrow$ Labor share falls in size
 2. Nonhomothetic IT demand $\Rightarrow$ Rise of IT increases RtS
 $\Rightarrow$ Rise of IT increases within-firm labor share
 3. Nonhomothetic IT demand $\Rightarrow$ Rise of IT increases concentration
 $\Rightarrow$ Rise of IT reallocates toward low-labor share firms

Estimation of nhCES Production Function

- Rely on timing of firm decisions ($\sim$ literature on PF estimation)
- Exploit wage differences across local areas to identify σ

Oberfield & Raval (2014)

		All Industries			Manufacturing		
		Nonhomothetic CES	CES	Cobb-Douglas	Nonhomothetic CES	CES	Cobb-Douglas
IT Nonhomotheticity	ϵ	0.389 (0.011)			0.477 (0.019)		
Elasticity of substitution	σ	0.225 (0.012)	0.125 (0.016)		0.171 (0.009)	0.165 (0.013)	
Cost elasticity	γ	0.947 (0.004)	0.978 (0.004)	1.001 (0.004)	0.954 (0.006)	1.014 (0.005)	1.022 (0.006)
Observations	N	302318	302318	307227	145966	145966	147471

Full Table

Industry-Level Results

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[Full Table](#)
[Industry-Level Results](#)

Calibration Results: Aggregate Response

- Shock: fall in the IT price $\Delta \log W = \log(0.37)$
- Aggregate Response:

Aggregate Variable		CES	nhCES	Data
Change in Share of Top 1% of Firms in Sales		+3.0 p.p.	+5.4 p.p.	+8.1 p.p.
Change in Share of Top 5% of Firms in Sales		+1.4 p.p.	+2.7 p.p.	+6.4 p.p.
Change in Labor Share (Production)	LS^{prod}	+0.6 p.p.	-0.0 p.p.	-0.0 p.p.
<i>Within-Firm Contribution</i>		+1.8 p.p.	+2.1 p.p.	+3.8 p.p.
<i>Reallocation Contribution</i>		-1.2 p.p.	-2.1 p.p.	-3.9 p.p.
Change in Mass of Active Firms	N	+3.7%	-3.4%	-14.1%
Aggregate Elasticity of Substitution	$\bar{\sigma}$	0.750	1.007	—

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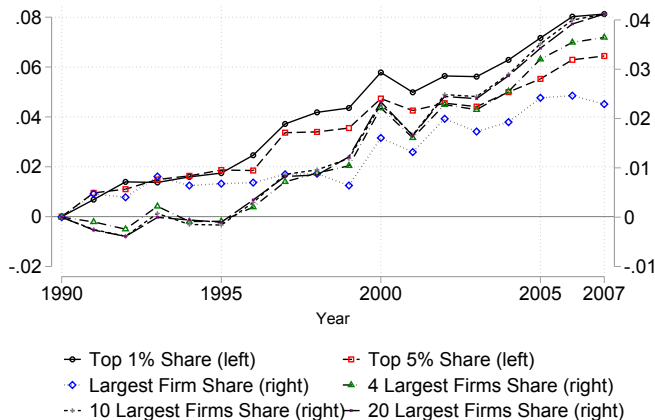
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Conclusion

- Using universe of French firms firm-level data
 - ▶ Documented relationship between IT intensity & firm size
 - ▶ Identified:
 - nonhomotheticity in demand for IT inputs
 - complementarity between IT and non-IT inputs
 - ⇒ MR of IT relative to non-IT grows with firm size
- GE model with nonhomothetic production function
 - ▶ IT price fall explains ~ half of trends in concentration & labor share

Macro Trends in France: Rise in Concentration

Cumulative Changes in Share of Top Firms in Sales (Across 3-Digit Industries)

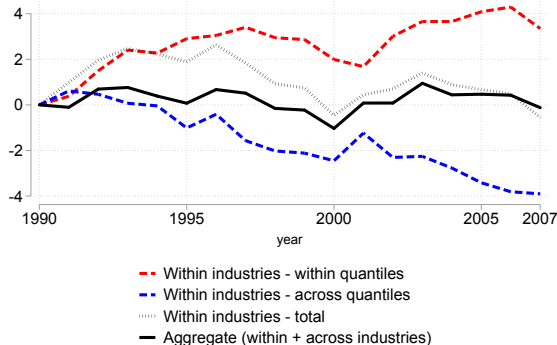


- Rise in industry concentration & dispersion also in US & OECD

Andrews et al. (2016); CEA (2016); Autor et al. (2017); & others

Macro Trends in France: Labor Share

Shift/Share Decomposition of Change in Labor Share (Across 3-Digit Industries)



- Diverging within and across-firm patterns of labor share also in US

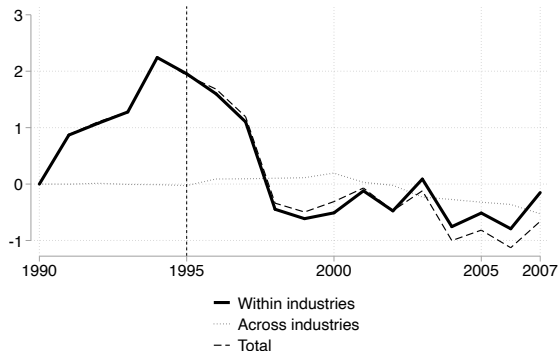
Kehrig & Vincent (2015); Autor et al. (2017)

[Details of the Decomposition](#)

[Back](#)

Macro Trends in France: Capital Share

Cumulative Changes in Capital Share (Across 3-Digit Industries)



- Similar pattern in the US

Barkai (2020)

[Explaining the Early Hump Shape](#)

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Macro Trends in France: Relative Number of Firms



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Decompositions of Labor Share

- Notation:

Λ $\equiv$ Labor share

S_k $\equiv$ Industry- k share in tot. VA

$\Delta_T Z_t \equiv Z_T - Z_0$

$\Lambda(q)$ $\equiv$ Avg. labor share in quantile q

$S_k(q)$ $\equiv$ Share of quantile q in ind. VA

$\bar{X}_t \equiv \frac{1}{2} (X_t + X_{t-1})$

- Within & Across Industries:

$$\Delta_T \Lambda \equiv \underbrace{\sum_{t=1}^T \sum_k \bar{S}_{kt} \Delta \Lambda_{kt}}_{\text{within industry}} + \underbrace{\sum_{t=1}^T \sum_k \Delta S_{kt} \bar{\Lambda}_{kt}}_{\text{across industries}}. \quad (1)$$

- Within & Across Quantiles of Labor Share:

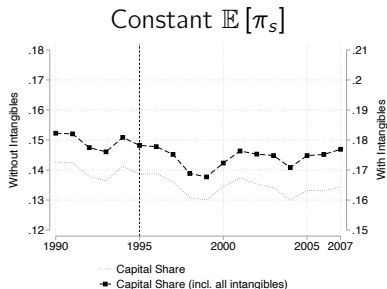
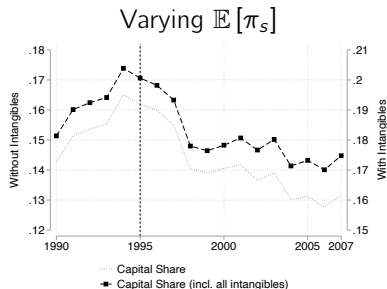
$$\Delta \Lambda_{kt} \equiv \underbrace{\int_0^1 \bar{S}_{kt}(q) \Delta \Lambda_{kt}(q) dq}_{\text{Within quantiles}} + \underbrace{\int_0^1 \Delta S_{kt}(q) \bar{\Lambda}_{kt}(q) dq}_{\text{Cross quantiles (concentration)}}. \quad (2)$$

More on Capital Share

- Return on Capital of type s :

Barkai (2020)

$$R_s = \left(\left(\frac{D}{D+E} i^D (1-\tau) + \frac{E}{D+E} i^E \right) - \mathbb{E}[\pi_s] + \delta_s \right) \frac{1 - d_s \tau}{1 - \tau}$$



Back

Data: IT Investment in EAE

- EAE variables I460-I465: Software (Investissements en Logiciels)
 - ▶ Decomposition to purchased vs. in-house creation
 - Only for manufacturing
 - ▶ Component of investment in update/improvement of software
 - Only for retail/services
- Expenses on the purchases of software services (D329):
 - ▶ Available only for manufacturing and agri-food firms

Data: Software Investment in EAE

- Sample of the source of EAE information for variable I460 (Logiciel)

4 - Analyse économique des investissements matériels et immatériels

cf.notice

04A0

A - Dépenses informatiques de l'exercice

● Dépenses en logiciels

- dépenses en immobilisations incorporelles

achats de logiciels (acquisitions y compris de logiciels réalisés sur mesure à l'extérieur) et créations en interne

- dépenses non immobilisées

achats de logiciels (acquisitions y compris de logiciels réalisés sur mesure à l'extérieur) et créations en interne

● Matériel informatique

Il s'agit ici d'équipements en ordinateurs, serveurs et périphériques (hors logiciels), à l'exclusion du matériel intervenant directement dans votre production (système de contrôle, commande de machines, ...)

- montant des acquisitions de matériels inscrites en immobilisations corporelles

- montant des acquisitions et locations de matériels enregistrés en charges (y compris les redevances de crédit-bail)

● Achats de services informatiques auprès d'un partenaire extérieur :

Informatique sous-traitée, conseil-assistance, informatique-ingénierie, contrat de maintenance ...

Data: Software Investment in EAE

Cleaned I460 Summary Statistics (1996)

1996	Number of Firms				Average values		Median values
	All	EAE	No software investment	Some software investment	EAE	Some software investment	Some software investment
0-5	206,385	10,551	10,517	34	0.01	2.58	2.21
5-10	131,300	13,538	13,266	272	0.08	4.17	1.83
10-20	56,554	11,658	10,788	870	0.51	6.81	2.90
20-50	47,232	32,496	26,155	6,341	1.64	8.38	3.51
50-100	10,991	9,406	6,404	3,002	4.71	14.77	6.10
100-250	6,956	6,158	3,602	2,556	11.95	28.80	10.98
250-500	2,111	1,960	966	994	31.71	62.53	31.86
500-1000	916	880	362	518	91.07	154.71	70.51
1000-2500	445	427	161	266	237.96	381.98	158.01
2500-5000	119	112	42	70	517.22	827.55	401.09
+5000	55	51	18	33	5741.85	8873.76	759.20

Cleaned I460 Summary Statistics (2006)

2006	Number of Firms				Average values		Median values
	All	EAE	No software investment	Some software investment	EAE	Some software investment	Some software investment
0-5	221,513	11,945	11,927	18	0.01	4.39	4.00
5-10	152,626	16,098	15,869	229	0.08	5.85	3.00
10-20	83,251	16,743	15,649	1,094	0.55	8.38	4.00
20-50	54,374	35,908	29,712	6,195	1.94	11.22	5.00
50-100	13,209	10,247	7,371	2,876	5.74	20.46	8.00
100-250	8,758	7,158	4,459	2,699	14.18	37.60	15.00
250-500	2,490	2,123	1,131	992	43.40	92.88	32.00
500-1000	1,056	931	460	471	108.65	214.77	80.00
1000-2500	558	485	212	273	235.52	418.41	164.00
2500-5000	146	130	58	72	884.48	1596.99	788.00
+5000	96	76	32	44	5515.61	9526.95	743.50

Data: Summary Statistics

- Period 1995-2007: 67% of agg. emp. & 58% of agg. VA

	Source	Obs. (Nb)	Mean	Median	Sd
Sales	BRN + RSI	15,202,967	2,498.8	265	85,056.8
Value-Added	BRN + RSI	15,202,967	708.3	106	33,071.4
Number of Employees	BRN + RSI	15,202,967	13.8	3	480.7
Wage Bill	BRN + RSI	15,202,967	472.4	74	18,404.5
Labor Share (%)	BRN + RSI	15,202,967	74.0	73.0	33.9
Total Investment	BRN	6,166,342	143.1	5	9,880.3
Total Capital Stock	BRN	6,166,342	1,202.2	88.0	92,297.9
Total Cost	BRN	6,166,341	898.1	181.0	33,623.7
IT Measures					
Software Investment	EAE	2,435,356	5.9	0	528.2
Software Stock	EAE	2,435,356	15.8	0	1,216.2
Hardware Investment	BRN	6,166,342	6.1	0	405.2
Hardware Stock	BRN	6,166,342	24.5	0	1,857.4

Values in K euro

Data: Construction of PIM Measures of Capital

- **Perpetual Inventory Method:** for type j capital for firm i at time t

$$K_{j,it} = K_{j,it-1} (1 - \delta_j) + \frac{I_{j,t}}{W_{j,t}}$$

with time-0 values imputed based on aggregate values $\bar{K}_{j,0}$:

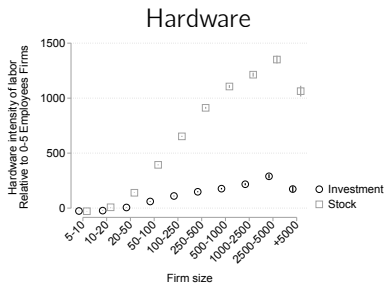
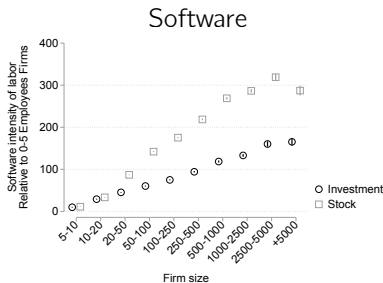
$$K_{j,i0} = \frac{\frac{1}{T_i} \sum_t I_{j,it}}{\sum_{i'} \frac{1}{T_{i'}} \sum_{t'} I_{j,i't'}} \times \bar{K}_{j,0}$$

where T_i the horizon of time we see the firm in our data

- Time-0 is year 1989 (6 years before the first year for the analysis)
- For all industries the depreciation rate of software is 0.315

Micro Fact: Size & IT Intensity of Labor

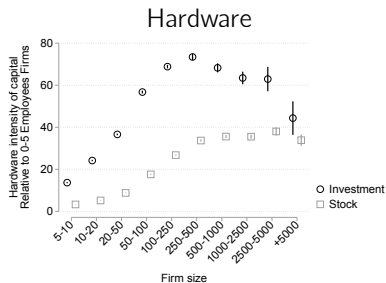
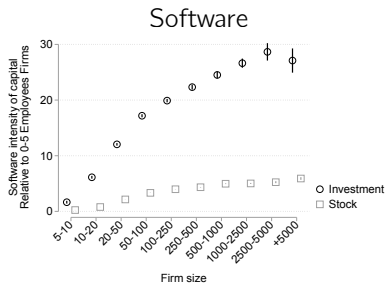
- Ratio of IT investment/stock to number of workers



$$IT\ Intensity_{firm,t} = FE_{Size_{firm,t}} + FE_{ind,t}$$

Micro Fact: Size & IT Intensity of Capital

- Ratio of IT to non-IT capital investment/stock



$$IT\ Intensity_{firm,t} = FE_{Size_{firm,t}} + FE_{ind,t}$$

Facts: IT Intensity and Employment By Size Bracket

IT Intensity of Cost										
	[1; 50[	[50; 100[	[100; 250[	[250; 1000[	≥ 1000	[1; 50[	[50; 100[	[100; 250[	[250; 1000[	≥ 1000
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.2871 (0.0057)	0.2528 (0.0136)	0.2199 (0.0140)	0.2861 (0.0185)	0.2761 (0.0370)					
Size (proxied by VA)						0.2159 (0.0065)	0.2202 (0.0167)	0.1959 (0.0170)	0.2906 (0.0213)	0.3024 (0.0395)
Observations	379,543	91,406	66,022	30,925	6,375	379,603	91,430	66,030	30,933	6,375
R2	0.2506	0.2132	0.2216	0.2716	0.3715	0.2478	0.2117	0.2202	0.2705	0.3718
Panel 2: Hardware (Stock)										
Size (proxied by sales)	0.0812 (0.0009)	0.7282 (0.0061)	0.5073 (0.0058)	0.2934 (0.0070)	0.1400 (0.0127)					
Size (proxied by VA)						-0.0343 (0.0010)	0.5140 (0.0074)	0.3700 (0.0070)	0.2135 (0.0082)	0.0911 (0.0132)
Observations	2,563,488	132,537	93,307	41,372	8,295	2,563,741	132,577	93,338	41,393	8,295
R2	0.3752	0.3428	0.3847	0.3817	0.4572	0.3735	0.2960	0.3525	0.3647	0.4514

Facts: IT Intensity and Size

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.3650 (0.0031)		0.3115 (0.0031)		0.2779 (0.0032)		0.2842 (0.0032)		0.2996 (0.0030)	
Size (proxied by VA)		0.3458 (0.0033)		0.2933 (0.0033)		0.2899 (0.0034)		0.2980 (0.0035)		0.2834 (0.0033)
Observations	575,594	575,686	575,579	575,676	530,334	530,395	529,045	529,104	575,655	575,755
R2	0.2396	0.2356	0.2281	0.2249	0.2346	0.2341	0.2350	0.2346	0.2314	0.2286
Panel 2: Software (Investment)										
Size (proxied by sales)	20.5010 (0.1070)		0.5034 (0.0029)		4.4246 (0.0277)		5.2615 (0.0348)		0.4286 (0.0025)	
Size (proxied by VA)		21.1063 (0.1131)		0.5191 (0.0030)		4.7405 (0.0293)		5.6491 (0.0368)		0.4409 (0.0026)
Observations	1,145,874	1,146,068	1,145,917	1,146,115	1,127,629	1,127,816	1,117,482	1,117,639	1,146,512	1,146,707
R2	0.0911	0.0896	0.0840	0.0829	0.0820	0.0826	0.0756	0.0762	0.0829	0.0817
Panel 3 : Hardware (Stock)										
Size (proxied by sales)	0.2630 (0.0007)		0.2031 (0.0007)		0.2134 (0.0008)		0.2279 (0.0008)		0.1993 (0.0007)	
Size (proxied by VA)		0.1991 (0.0008)		0.1289 (0.0008)		0.1705 (0.0009)		0.1884 (0.0009)		0.1279 (0.0008)
Observations	2,839,365	2,839,569	2,839,373	2,839,754	2,755,218	2,755,436	2,756,088	2,756,211	2,840,459	2,840,804
R2	0.4188	0.4068	0.3823	0.3718	0.4163	0.4104	0.4491	0.4435	0.3474	0.3367
Panel 4 : Hardware (Investment)										
Size (proxied by sales)	41.1954 (0.1841)		0.8812 (0.0052)		17.1734 (0.0571)		19.7820 (0.0712)		0.7486 (0.0037)	
Size (proxied by VA)		32.2761 (0.1894)		0.5930 (0.0054)		15.7131 (0.0587)		18.2151 (0.0732)		0.5411 (0.0038)
Observations	4,340,454	4,341,159	4,340,014	4,340,853	4,366,163	4,366,860	4,302,290	4,302,802	4,344,803	4,345,554
R2	0.1647	0.1607	0.1386	0.1353	0.1860	0.1826	0.2409	0.2384	0.1478	0.1438

Facts: IT Intensity and Scale

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Cost	Cost
Panel 1 : Software (Stock)										
Number of plants	0.0015 (0.0002)		0.0014 (0.0002)		0.0014 (0.0002)		0.0016 (0.0002)		0.0013 (0.0002)	
Number of occupational layers		0.2623 (0.0047)		0.2230 (0.0047)		0.2558 (0.0049)		0.2603 (0.0050)		0.2250 (0.0047)
Observations	562,858	562,858	562,997	562,997	518,716	518,716	517,470	517,470	563,027	563,027
R2	0.2214	0.2255	0.2143	0.2173	0.2237	0.2276	0.2238	0.2278	0.2186	0.2217
Number of destination countries	0.0276 (0.0004)		0.0243 (0.0004)		0.0225 (0.0004)		0.0232 (0.0004)		0.0238 (0.0004)	
Number of products		0.0065 (0.0002)		0.0059 (0.0002)		0.0054 (0.0002)		0.0056 (0.0002)		0.0057 (0.0002)
Observations	278,803	278,803	279,590	279,590	261,609	261,609	261,144	261,144	279,902	279,902
R2	0.1958	0.1871	0.1887	0.1817	0.1867	0.1806	0.1875	0.1811	0.1921	0.1854
Panel 2: Hardware (Stock)										
Number of plants	0.0040 (0.0001)		0.0040 (0.0001)		0.0036 (0.0001)		0.0040 (0.0001)		0.0039 (0.0001)	
Number of occupational layers		0.0986 (0.0009)		0.0700 (0.0009)		0.1124 (0.0010)		0.1251 (0.0010)		0.0735 (0.0009)
Observations	2,696,655	2,696,655	2,698,300	2,698,300	2,622,236	2,622,236	2,622,998	2,622,998	2,698,872	2,698,872
R2	0.3913	0.3933	0.3632	0.3640	0.4003	0.4027	0.4319	0.4347	0.3275	0.3286
Number of destination countries	0.0337 (0.0002)		0.0299 (0.0002)		0.0258 (0.0002)		0.0272 (0.0002)		0.0291 (0.0002)	
Number of products		0.0084 (0.0001)		0.0078 (0.0001)		0.0068 (0.0001)		0.0071 (0.0001)		0.0076 (0.0001)
Observations	553,427	553,427	555,879	555,879	546,058	546,058	546,478	546,478	555,847	555,847
R2	0.2831	0.2628	0.2533	0.2366	0.3129	0.3033	0.3339	0.3237	0.2226	0.2058

Stationary GE

Definition.

Tuple (W, Y, P, N) & stationary distribution g that satisfy:

1. Stationarity:

$$g_o(\vartheta) = \delta f(\vartheta) + (1 - \delta) \int \int \mathbb{P}(\vartheta | \tilde{\vartheta}) g_o(\tilde{\vartheta}) d^2 \tilde{\vartheta}.$$

$$g(\vartheta) = \frac{g_o(\vartheta)}{\int_{\mathcal{J}} g_o(\vartheta) d^2 \vartheta}, \quad (\theta, \phi) \in \mathcal{J}$$

2. CES aggregation:

$$1 = N \int \int \tilde{P}(\vartheta)^{1-\lambda} g(\vartheta) d^2 \vartheta$$

3. Market cleaning:

$$N \left[\psi + \int \int (1 - \tilde{\Omega}(\vartheta)) \tilde{C}(\vartheta) g(\vartheta) d^2 \vartheta \right] + N_o \delta \chi = \bar{X}_N \quad (3)$$

$$N \int \int \tilde{\Omega}(\vartheta) \tilde{C}(\vartheta) g(\vartheta) d^2 \vartheta = W \bar{X}_I \quad (4)$$

4. Free entry condition:

$$\chi = \int \int V(\vartheta) f(\vartheta) d^2 \vartheta \quad (5)$$

Allocations Across Firms

- Let $\vartheta \equiv (\theta, \phi)$: output $Y_i = Y \tilde{P}(\vartheta_i)^{-\lambda}$ & cost, IT cost shares, and cost elasticity:

$$C_i = \tilde{C}(\vartheta_i) \equiv e^{-\theta_i} \mathcal{C}\left(Y \tilde{P}(\vartheta_i)^{-\lambda}; e^{-\phi_i} W\right)$$

$$\Omega_i = \tilde{\Omega}(\vartheta_i) \equiv \Omega\left(Y \tilde{P}(\vartheta_i)^{-\lambda}; e^{-\phi_i} W\right)$$

$$\mathcal{E}_i = \tilde{\mathcal{E}}(\vartheta_i) \equiv \mathcal{E}\left(Y \tilde{P}(\vartheta_i)^{-\lambda}; e^{-\phi_i} W\right) = \gamma + \epsilon \tilde{\Omega}(\vartheta_i)$$

- Firm revenue and profits:

$$\begin{aligned} R_i &= \tilde{R}(\vartheta_i) \equiv \frac{\lambda}{\lambda - 1} \tilde{\mathcal{E}}(\vartheta_i) \tilde{C}(\vartheta_i) \\ \Pi_i &= \tilde{\Pi}(\vartheta_i) \equiv \left(\frac{\lambda}{\lambda - 1} \tilde{\mathcal{E}}(\vartheta_i) - 1 \right) \tilde{C}(\vartheta_i) \end{aligned} \quad (6)$$

- GE Conditions:

- Fixed point problems for value func. $V(\vartheta_i)$ and stat. dist. $g(\vartheta_i)$
- Market clearing pins down (P, Y, W) & mass of active firms N

Aggregate Predictions: Fall in W & Cost Elasticity

- Small relative change $\Delta w < 0$ in price of IT inputs
- Change in average cost elasticity with $\vartheta \equiv (\theta, \phi)$:

$$\Delta \bar{\mathcal{E}} \approx \underbrace{\int \int \Delta \tilde{\mathcal{E}}(\vartheta) \Lambda(\vartheta) d^2 \vartheta}_{\text{Within-firm Effect}} + \underbrace{\int \int \tilde{\mathcal{E}}(\vartheta) \Delta \Lambda(\vartheta) d^2 \vartheta}_{\text{Reallocation Effect}}$$

1. Within-firm response in output elasticity of costs:

$$\Delta \mathcal{E}(\vartheta) \equiv (1 - \sigma) \epsilon (1 - \Omega(\vartheta)) \Omega(\vartheta) \left[\underbrace{\Delta w}_{\text{Direct Price effect}} + \underbrace{\epsilon \Delta \tilde{y}(\vartheta)}_{\text{Scale effect}} \right]$$

2. Reallocation effect:

$$\frac{\Delta \Lambda(\vartheta)}{\Lambda(\vartheta)} = \underbrace{(\gamma + \epsilon \Omega(\vartheta)) \Delta \tilde{y}(\vartheta)}_{\text{Scale effect}} + \underbrace{\Omega(\vartheta) \Delta w}_{\text{Direct Price effect}} - \Delta \bar{c}$$

Cross-sectional Relationships

- Covariance of IT intensity and firm size under g :

$$\text{Cov}(\log \Omega_i, \log Y_i) \equiv (1 - \sigma) \epsilon \text{Var}(\log Y_i) + (\sigma - 1) \text{Cov}(\phi_i, \log Y_i)$$

- Covariance of profit-to-cost ratios and firm size under g :

$$\text{Cov}\left(\frac{\Pi_i}{C_i}, \log Y_i\right) \equiv \frac{\lambda}{\lambda - 1} \epsilon \text{Cov}(\Omega_i, \log Y_i)$$

Fall in W & Scale Effect

- Small relative change $\Delta w < 0$ in price of IT inputs

Lemma.

Change in the output of firms with productivity ϑ in response to

$$\Delta \tilde{y}(\vartheta) = \frac{1}{1 + \lambda \frac{\partial mc}{\partial y}(\vartheta)} \left[-\lambda \frac{\partial mc}{\partial w}(\vartheta) \Delta w + \Delta y + \lambda \Delta p \right]$$

where the IT price elasticity of marginal costs:

$$\frac{\partial mc}{\partial w}(\vartheta) \equiv \frac{\partial \log \mathcal{C}'}{\partial \log W} = \tilde{\Omega}(\vartheta) \left(1 + \frac{(1 - \sigma) \gamma \epsilon (1 - \tilde{\Omega}(\vartheta))}{1 + \gamma \epsilon \tilde{\Omega}(\vartheta)} \right)$$

and the output elasticity of marginal costs:

$$\frac{\partial mc}{\partial y}(\vartheta) \equiv \frac{\partial \log \mathcal{C}'}{\partial \log Y} = \gamma - 1 + \epsilon \tilde{\Omega}(\vartheta) \left(1 + \frac{(1 - \sigma) \gamma \epsilon (1 - \tilde{\Omega}(\vartheta))}{1 + \gamma \epsilon \tilde{\Omega}(\vartheta)} \right)$$

Moments Conditions

- Apply GMM with the following moment conditions
- Moment Conditions Based on Timing Assumption:

$$\mathbb{E} \left[\begin{pmatrix} \Theta(\mathbf{D}_{it+1}; \varsigma) \\ \Phi(\mathbf{D}_{it+1}; \varsigma) \end{pmatrix} - \begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Theta(\mathbf{D}_{it}; \varsigma) \\ \Phi(\mathbf{D}_{it}; \varsigma) \end{pmatrix} - \begin{pmatrix} \eta_{\theta} + \mu_{\theta} t \\ \eta_{\phi} + \mu_{\phi} t \end{pmatrix} \middle| \mathcal{I}_t \right] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

- Moment Conditions Based on Relative Factor Price IV:

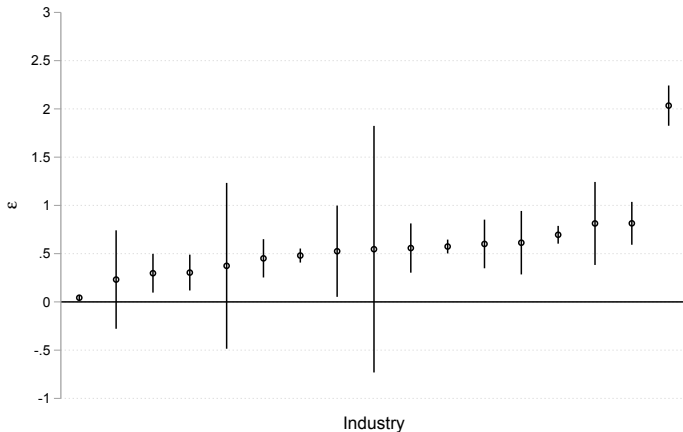
$$\mathbb{E} \left[\Phi(\mathbf{D}_{it+1}; \varsigma) - \begin{pmatrix} \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Theta(\mathbf{D}_{it}; \varsigma) \\ \Phi(\mathbf{D}_{it}; \varsigma) \end{pmatrix} - (\eta_{\phi} + \mu_{\phi} t) \middle| z_{it+1} \right] = 0$$

Complete Estimation Results

		All Industries			Manufacturing		
		Nonhomothetic CES	CES	Cobb-Douglas	Nonhomothetic CES	CES	Cobb-Douglas
IT Nonhomotheticity	ϵ	0.389 (0.011)			0.477 (0.019)		
Elasticity of substitution	σ	0.225 (0.012)	0.125 (0.016)		0.171 (0.009)	0.165 (0.013)	
Cost elasticity	γ	0.947 (0.004)	0.978 (0.004)	1.001 (0.004)	0.954 (0.006)	1.014 (0.005)	1.022 (0.006)
Capital elasticity	α	0.074 (0.005)	0.068 (0.005)	0.070 (0.005)	0.182 (0.009)	0.166 (0.008)	0.167 (0.008)
Software elasticity	β	0.113 (0.029)	0.185 (0.023)	0.015 (0.038)	0.120 (0.034)	0.148 (0.028)	0.303 (0.042)
Persistence of θ	$\rho_{\theta\theta}$	0.830 (0.003)	0.825 (0.003)	0.808 (0.003)	0.834 (0.005)	0.804 (0.005)	0.802 (0.005)
Persistence of θ wrt ϕ	$\rho_{\theta\phi}$	-0.010 (0.000)	-0.012 (0.001)		-0.008 (0.001)	-0.014 (0.001)	
Persistence of ϕ wrt θ	$\rho_{\phi\theta}$	-0.072 (0.003)	-0.047 (0.003)		-0.066 (0.004)	-0.048 (0.004)	
Persistence of ϕ	$\rho_{\phi\phi}$	0.898 (0.001)	0.908 (0.001)		0.901 (0.002)	0.919 (0.001)	
Trend for θ	μ_{θ}	0.002 (0.000)	0.002 (0.000)	-0.001 (0.001)	0.004 (0.000)	0.004 (0.000)	0.003 (0.000)
Trend for ϕ	μ_{ϕ}	0.008 (0.001)	0.008 (0.001)		-0.007 (0.001)	-0.003 (0.001)	
Shifter for θ	η_{θ}	0.590 (0.014)	0.626 (0.015)	0.718 (0.013)	0.479 (0.019)	0.658 (0.021)	0.662 (0.018)
Shifter for ϕ	η_{ϕ}	0.677 (0.021)	0.185 (0.016)		0.795 (0.024)	0.306 (0.017)	
Observations	N	302318	302318	307227	145966	145966	147471

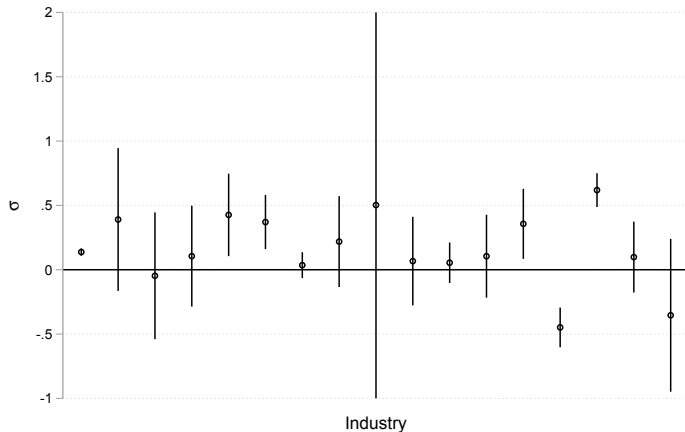
Industry Results (1/3): Nonhomotheticity Parameter ϵ

- Estimates across 17 industries (na38)
 - ▶ Mean = 0.86
 - ▶ Median = 0.57



Industry Results (2/3): Elasticity of Substitution σ

- Estimates across 17 industries (na38)
 - ▶ Mean = 0.18
 - ▶ Median = 0.14

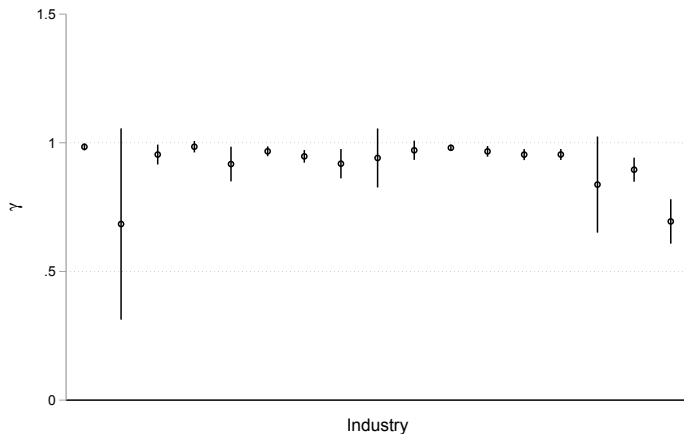


Industry Results (3/3): Cost Elasticity γ

- Estimates across 17 industries (na38)

Mean = 0.94

Median = 0.97



CES Model Parameters

Model Component	Parameter		Value	
Production Function	IT nonhomotheticity	ϵ	0.00	Estimated
	Cost Elasticity Parameter	γ	0.98	Estimated
	Elasticity of substitution	σ	0.13	Estimated
	Fixed Costs	ψ	0.05	Search
Productivity Process	Persistence of Shocks	$(\rho_{\theta\theta}, \rho_{\phi\phi})$	(0.83, 0.91)	Estimated
	Long-run Mean Productivities	$(\eta_{\theta}, \eta_{\phi})$	(0.63, 0.19)	Estimated
	Variances of Innovations	$(\kappa_{\theta}^2, \kappa_{\phi}^2)$	(0.09, 0.31)	Estimation
Entry & Exit	Distribution of Entry	$(\xi_o, \bar{\theta}_o)$	(2.77, 8.46)	Estimation
		κ_o^2	2.10	Estimation
		$\underline{\theta}_o$	2.78	Search
		$\bar{\phi}_o$	-2.59	Search
	Costs of Entry	χ	0.05	Calibrated
	Exogenous Probability of Exit	δ	0.03	Calibrated
	Demand	Elasticity of Substitution	λ	5.00
	Discount Factor	ϱ	0.95	Calibrated

CES Calibration Moments

	Moments	Source	Data	Model	Relevant Parameters
Targeted	Entrant Top %1 θ	EAE (Estimation)	3.72	4.44	ξ_o
	Entrant Top %0.1 θ	EAE (Estimation)	4.55	5.28	ξ_o
	Entrant Highest θ	EAE (Estimation)	8.46	8.46	$\bar{\theta}_o$
	Share of Top 1% of Firms in Sales	BRN+RSI	59.3%	60.1%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Share of Top 5% of Firms in Sales	BRN+RSI	77.4%	77.5%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Aggregate IT Intensity	INSEE	3.8%	3.8%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Mass of Firms (N)	BRN+RSI	0.073	0.073	χ
	Rate of Exit of Large Firms	BRN+RSI	0.031	0.031	δ
Untargeted	Aggregate Labor Share	BRN+RSI	66.2%	65.4%	—
	Unweighted Mean of Labor Share	BRN+RSI	73.5%	70.6%	—
	Unweighted Mean of IT Intensity	EAE	0.2%	2.2%	—
	S.D. of Log Sales	BRN+RSI	1.4	1.5	—
	S.D. of Log Employment	BRN+RSI	1.2	1.5	—