# NEURAL DYNAMICS OF 3-D SURFACE PERCEPTION: FIGURE-GROUND SEPARATION AND LIGHTNESS PERCEPTION

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## A1 General Introduction

This section describes the FACADE model's BCS and FCS equations. These equations are similar to those in Grossberg and McLoughlin (1997) and Gove, Grossberg and Mingolla (1995) but there are several refinements: LGN, simple cell and complex stages better sharpen the responses of cells to boundaries. The boundary grouping process incorporates inhibitory feedback from bipoles at other positions and orientations that helps to break T-junctions. The remaining stages are unchanged, except again for minor parameter differences. See Grossberg and McLoughlin (1997) for a more complete discussion of the basic equations.

## A2 LGN ON and OFF Channels

The model LGN discounts the illuminant and computes Weber-law modulated and normalized estimates of image contrasts above an adaptation level. The LGN ON activities $x_{ij}^+$ and OFF activities $x_{ij}^-$ are described by on-center off-surround and off-center on-surround networks, respectively; that obey membrane, or shunting, equations (Grossberg, 1973, 1983; Hodgkin, 1964):

$$\frac{dx_{ij}{}^+}{dt} = -\alpha_1 x_{ij}{}^+ + (U_1 - x_{ij}{}^+)\,C_1 - (x_{ij}{}^+ + L_1)\,S_1 \qquad \text{(A1)}$$

and

$$\frac{dx_{ij}{}^-}{dt} = -\alpha_1 x_{ij}{}^- + (U_1 - x_{ij}{}^-)\,S_1 - (x_{ij}{}^- + L_1)\,C_1, \qquad \text{(A2)}$$

where the decay constant $\alpha_1 = 100$, the upper and lower activity bounds are $U_1 = L_1 = 50$, and the center $C_1$ and surround $S_1$ are defined by Gaussian kernels:

$$C_1 = \sum_{(p,q)} C_{pq} I_{i+p,j+q} \qquad \text{(A3)}$$

$$S_1 = \sum_{(p,q)} S_{pq} I_{i+p,j+q} \tag{A4}$$

where

$$C_{pq} = \frac{A_1}{2\pi{\sigma_c}^2} \exp\left(-\frac{p^2+q^2}{2{\sigma_c}^2}\right) \tag{A5}$$

and

$$S_{pq} = \frac{A_2}{2\pi{\sigma_s}^2} \exp\left(-\frac{p^2+q^2}{2{\sigma_s}^2}\right). \tag{A6}$$

To give the ON and OFF signals the same total strength, we use $A_1 = 1.0, A_2 = 1.03361$, where the width of center and surround are described by $\sigma_c = 0.5, \sigma_s = 1.5$. At equilibrium:

$$ {x_{ij}}^+ = \frac{\sum_{p,q} \left(U_1 C_{pq} - L_1 S_{pq}\right) I_{i+p,j+q}}{\alpha_1 + \sum_{p,q} \left(C_{pq} + S_{pq}\right) I_{i+p,j+q}} \tag{A7}$$

and

$$ {x_{ij}}^- = \frac{\sum_{p,q} \left(U_1 S_{pq} - L_1 C_{pq}\right) I_{i+p,j+q}}{\alpha_1 + \sum_{p,q} \left(C_{pq} + S_{pq}\right) I_{i+p,j+q}} \tag{A8}$$

The difference of these ON and OFF activities is computed to generate opponent output signals:

$$ {X_{ij}}^+ = [{x_{ij}}^+ - {x_{ij}}^-]^+ = \left[\frac{(U_1 + L_1)\left(\sum_{p,q} C_{pq} - S_{pq}\right) I_{i+p,j+q}}{\alpha_1 + \sum_{p,q} \left(C_{pq} + S_{pq}\right) I_{i+p,j+q}}\right]^+ \tag{A9}$$

$$ {X_{ij}}^- = [{x_{ij}}^- - {x_{ij}}^+]^+ = \left[\frac{(U_1 + L_1)\left(\sum_{p,q} S_{pq} - C_{pq}\right) I_{i+p,j+q}}{\alpha_1 + \sum_{p,q} \left(C_{pq} + S_{pq}\right) I_{i+p,j+q}}\right]^+ \tag{A10}$$

The output is rectified using $[x]^+ = max(x, 0)$

## A3 Simple Cells

Model simple cells respond to oriented contrasts in the image. They respond to a prescribed contrast polarity. Even-symmetric and odd-symmetric simple cell receptive fields centered on the two-dimensional location $(i, j)$ and of orientation $k$ were defined using even and odd Gabor kernels:

$$ {S_{ijk}}^{even+/-} = \left[\sum_{(p,q)} {s_{pqk}}^{even+/-} {X_{i-p,j-q}}^{+/-} - \sum_{(p,q)} {s_{pqk}}^{even-/+} {X_{i-p,j-q}}^{-/+}\right]^+ \tag{A11}$$

and

$$ {S_{ijk}}^{odd+/-} = \left[\sum_{(p,q)} {s_{pqk}}^{odd+/-} {X_{i-p,j-q}}^{+/-} - \sum_{(p,q)} {s_{pqk}}^{odd-/+} {X_{i-p,j-q}}^{-/+}\right]^+, \tag{A12}$$

where

$$s_{pqk}{}^{odd} = \sin(2k) \exp\left[-\frac{1}{2}\left(\frac{p^2}{\sigma_{pk}{}^2} + \frac{q^2}{\sigma_{qk}^2}\right)\right] \tag{A13}$$

and

$$s_{pqk}{}^{even} = \cos(2k) \exp\left[-\frac{1}{2}\left(\frac{p^2}{\sigma_{pk}{}^2} + \frac{q^2}{\sigma_{qk}^2}\right)\right]. \tag{A14}$$

For vertical cells at Scale 1 : $\sigma_{pk} = 1.0, \sigma_{qk} = 0.75$. For horizontal cells at Scale 1 : $\sigma_{pk} = 0.75, \sigma_{qk} = 1.0$. For vertical cells at the larger Scale 2 : $\sigma_{pk} = 1.25, \sigma_{qk} = 1.0$. For horizontal cells at Scale 2 : $\sigma_{pk} = 1.0, \sigma_{qk} = 1.25$. These parameters are slightly smaller than those in the Grossberg and McLoughlin (1997) implementation to make edges slightly sharper. The extent of the receptive fields of the cells at each scale are as follows: For Scale 1 : $-4 \leq p, q \leq 4$, and for Scale 2 : $-6 \leq p, q \leq 6$.

## A4 Complex Cells

Model complex cells pool signals from like-oriented simple cells that are sensitive to opposite contraxt polarities. They also pool left and right eye input to compute binocular disparity. Different cell sizes, or scales, can compute different disparity ranges. The two scales are repeated at the complex cell stage. Scale 1 contains two pools of disparity-sensitive cells (near-zero disparity and a disparity of 3 pixels). Scale 2 contains three pools of such cells (near-zero disparity, a disparity of 3 pixels, and a disparity of 7 pixels). Two fields of monocular complex cells are used to represent the left and right images. The dynamics of the complex cell stage are defined mathematically as follows:

$$\frac{dc_{ijkd}}{dt} = -\alpha_2 c_{ijkd} + \beta \left(U_2 - c_{ijkd}\right) G_{ijkd} - \left(c_{ijkd} + L_2\right) \sum_{p,e} h\left(c_{i(j+p)ke}\right) D_{i(j+p)kde}, \tag{A15}$$

where: $\alpha_2 = 0.01, \beta = 15, U_2 = L_2 = 1.0$. The inhibitory signal function

$$h\left(c_{ijke}\right) = \begin{cases} c_{ijke} - ,_{e} & c_{ijke} > ,_{e} \\ 0 & otherwise \end{cases} \tag{A16}$$

where, $,_{e} = 0.2$ for all scales and disparities. $G_{ijkd}$ is the excitatory input formed by the binocular filter:

$$\begin{aligned} G_{ijkd} = &\left| \left(\sum_{l=1}^{L} W_{dlk}^{even} S_{i,j+l,k}^{even+} + \sum_{r=1}^{R} W_{drk}^{even} S_{i,j+r,k}^{even+}\right) \right. \\ &- \left. \left(\sum_{l=1}^{L} W_{dlk}^{even} S_{i,j+l,k}^{even-} + \sum_{r=1}^{R} W_{drk}^{even} S_{i,j+r,k}^{even-}\right) \right| \\ &+ \left| \left(\sum_{l=1}^{L} W_{dlk}^{odd} S_{i,j+l,k}^{odd+} + \sum_{r=1}^{R} W_{drk}^{odd} S_{i,j+r,k}^{odd+}\right) \right. \\ &- \left. \left(\sum_{l=1}^{L} W_{dlk}^{odd} S_{i,j+l,k}^{odd-} + \sum_{r=1}^{R} W_{drk}^{odd} S_{i,j+r,k}^{odd-}\right) \right|. \end{aligned}$$

The binocular complex cell receptive fields are as follows: For scale 1, $L, R = 20$; for scale 2, $L, R = 28$. Kernels satisfy:

$$W_{drk}^{odd/even} = \frac{S_{i,j+r,k}^{odd/even}}{\sum_{r=1}^{R} S_{i,j+r,k}^{odd/even}} \tag{A17}$$

and

$$W_{dlk}^{odd/even} = \frac{S_{i,j+l,k}^{odd/even}}{\sum_{l=1}^{L} S_{i,j+l,k}^{odd/even}}, \tag{A18}$$

The inhibitory connections between complex cells tuned to different disparities (possibly at different scales) obey:

$$D_{i(j+p)kde} = A_{de} \exp\left(-\mu_{de}(p + S_{de})^2\right) \tag{A19}$$

for $d \neq e$. The strength of the inhibitory connections between complex cells depends on the scale and disparity of the pool of cells in question. For cells at Scale 1 whose disparity is $d$: $A_{dd} = 2.5, A_{de} = 1.0, A_{d,mon} = 0.25$, $\mu_{dd} = 0.05, \mu_{de} = 0.05, \mu_{d,mon} = 0.075$. For cells at Scale 2 with disparity $d$: $A_{dd} = 2.5, A_{de} = 1.0, A_{d,mon} = 0.15$, $\mu_{dd} = 0.05, \mu_{de} = 0.05, \mu_{d,mon} = 0.075$. In (A19), $S_{de}$ determines the shift in the center of the Gaussians between disparities $d$ and $e$. Also

$$D_{i(j+p)kdd} = A_{dd}\left[\exp\left(-\mu_d^c p^2\right) - \exp\left(-\mu_d^s p^2\right)\right], \tag{A20}$$

where for Scale 1, $\mu_{dd}^c = 0.015, \mu_{dd}^s = 0.5$, whereas for Scale 2: $\mu_{dd}^c = 0.015, \mu_{dd}^s = 0.5$,

### A4.1 Horizontal Complex Cells

Cells sensitive to horizontal boundaries are spatially sharpened using an on-center off-surround network:

$$\frac{dc_{ijk}}{dt} = -\alpha_3 c_{ijk} + (U_3 - c_{ijk})\sum_p C_{i+p,j} G_{i+p,jk} - (c_{ijk} + L_3)\sum_{p,e} E_{i+p,j} G_{i+p,jk}, \tag{A21}$$

where $\alpha_3 = 0.1$ and $U_3 = L_3 = 1.0$. The Gaussian kernels are:

$$C_{i+p,j} = A_1 \exp\left[-\mu^c p^2\right] \tag{A22}$$

and

$$E_{i+p,j} = A_2 \exp\left[-\mu^s p^2\right], \tag{A23}$$

where $A_1 = A_2 = 1.0$. For Scale 1, $\mu_c = 1.5, \mu_s = 0.06$. For Scale 2 : $\mu_c = 0.5, \mu_s = 0.06$. At equilibrium, after rectification to generate output signals, we find:

$$C_{ijk} = \left[\frac{\sum_p \left(U_3 C_{i+p,j} - L_3 E_{i+p,j}\right) G_{i+p,jk}}{\alpha_3 + \sum_{p,q}\left(C_{i+p,jd} + E_{i+p,jd}\right) G_{i+p,jk}}\right]^+ \tag{A24}$$

## A5 Hypercomplex Cells : Spatial Competition

Hypercomplex cells carry out spatial and orientational competition in response to complex cell inputs. The spatial competition models the neural process of endstopping. The spatial competition uses an on-center off-surround network that includes both excitatory and inhibitory shunting feedback from the bipole cells, rather than just the additive excitatory feedback used by Grossberg and McLoughlin (1997):

$$\frac{dy_{ijkd}}{dt} = -\alpha_4 y_{ijkd} + (U_4 - y_{ijkd})\left[C_4 + T + f\left(C_7\right)\right] - (y_{ijkd} + L_4)\left[E_4 + g\left(E_7\right)\right]. \tag{A25}$$

At equilibrium:

$$y_{ijkd} = \frac{U_4\left[C_4 + T + f\left(C_7\right)\right] - L_4\left[E_4 + g\left(E_7\right)\right]}{\alpha_4 + \left[C_4 + E_4 + T + f\left(C_7\right) + g\left(E_7\right)\right]}, \tag{A26}$$

where $U_4 = L4 = 1.0, \alpha_4 = 0.01$, $T = 0.0000189$, and the feedback terms $C_7, E_7$ are defined as follows:

$$C_7 = Z_{ijkd} \tag{A27}$$

and

$$E_7 = \sum_{h,v,k} E_{hvk} Z_{i+h,j+v,kd}, \tag{A28}$$

where $-9 \leq h, v \leq 9$ for both scales. Term $C_7$ provides positive feedback from like-oriented bipole cells (see Section A6) to help complete boundaries, whereas $E_7$ sums bipole cell outputs across orientations and space to provide negative feedback with which to supress weaker boundaries. This orientational competition works across space to break the stems of T-junctions from their tops. The feedback signal functions $f$ and $g$ are linear, with gains that depend upon scale. Thus for Scale 1, $f(x) = f^{(1)}(x) = 0.1x$, and $g(x) = g^{(1)}x = 0.35x$, whereas for Scale 2, $f(x) = f^{(2)}(x) = 0.1x$, and $g(x) = g^{(2)}x = 0.2x$. The feedforward excitatory term is:

$$C_4 = \sum_{h,v} \Phi_{hv} C_{i+h,j+v,kd}, \tag{A29}$$

with Gaussian kernels

$$\Phi_{hv} = \frac{\exp\left[-\frac{1}{2}\left(\frac{h^2+v^2}{\sigma_c^2}\right)\right]}{\sum_{h,v}\exp\left[-\frac{1}{2}\left(\frac{h^2+v^2}{\sigma_c^2}\right)\right]}. \tag{A30}$$

The inhibitory term is:

$$E_4 = \sum_{h,v} E_{hv} C_{i+h,j+v,kd}, \tag{A31}$$

where

$$E_{hv} = \frac{\exp\left[-\frac{1}{2}\left(\frac{h^2+v^2}{\sigma_s^2}\right)\right]}{\sum_{h,v}\exp\left[-\frac{1}{2}\left(\frac{h^2+v^2}{\sigma_s^2}\right)\right]}. \tag{A32}$$

For Scale 1 and Scale 2, $\sigma_c = 1.0, \sigma_s = 2.0$. The size of the kernels is $-4 \leq h, v \leq 4$ for both Scales 1 and 2. The output signals from this stage are $Y_{ijkd} = max\left(y_{ijkd}, 0\right)$.

### A5.1 Hypercomplex Cells : Orientational Competition

The hypercomplex competition between orientations obeys:

$$\frac{dn_{ijkd}}{dt} = -\alpha_5 n_{ijkd} + (U_5 - n_{ijkd})\, C_5 - (n_{ijkd} + L_5)\, E_5 - \sum_{e<d} F\,(v_{ijke})\,, \qquad \text{(A33)}$$

where $\alpha_5 = 1.0, U_5 = L_5 = 1.0$, and $F(x) = 20x$. This inhibitory feedback term $F(x)$ comes from those monocular FIDO cells which represent nearer depths (i.e., smaller disparities) as defined by equation (A64). The excitatory input is

$$C_5 = \sum_{pqr} C_{kr} Y_{ijrd}, \qquad \text{(A34)}$$

where the amount of inhibition between orientations $(k, r)$ is

$$C_{kr} = \frac{c}{2\pi{\sigma_c}^2} \exp\left[-\frac{1}{2}\left(\frac{r-k}{{\sigma_c}^2}\right)^2\right]. \qquad \text{(A35)}$$

The inhibitory input is

$$E_5 = \sum_{pqr} E_{kr} Y_{ijrd} \qquad \text{(A36)}$$

with

$$E_{kr} = \frac{s}{2\pi{\sigma_s}^2} \exp\left[-\frac{1}{2}\left(\frac{r-k}{{\sigma_s}^2}\right)^2\right]. \qquad \text{(A37)}$$

For both Scales: $c = 1.0, s = 1.5, \sigma_c = 0.5$, and $\sigma_s = 0.75$. Solving at equilibrium and rectifying yields the output signals

$$N_{ijkd} = \left[\frac{\sum_r (U_5 C_{kr} - L_5 E_{kr})\, Y_{ijkd} - \sum_{e<d} FB\,(v_{ijke})}{\alpha_5 + \sum_r (C_{kr} + E_{kr})\, Y_{ijkd}}\right]^+. \qquad \text{(A38)}$$

## A6 Cooperative Bipole Cells

The bipole cells initiate boundary grouping by collecting oriented signals from two oriented branches of their receptive fields. Bipole cell activity satisfies:

$$\frac{dZ_{ijkd}}{dt} = -Z_{ijkd} + h\,[g\,(A_{ijkd}) + g\,(B_{ijkd})]\,, \qquad \text{(A39)}$$

where $g\,(x)$ bounds each branch's activity:

$$g\,(x) = \frac{[x]^+}{D + [x]^+} \qquad \text{(A40)}$$

and $D = 0.1$. The output threshold, , , in $h$ helps ensure that both lobes are active before a bipole cell fires :

$$h\,(x) = [x - ,\,]^+. \qquad \text{(A41)}$$

Here , $= 0.1$. Terms $A_d^b$ and $B_d^b$ in (A39) correspond to two oriented branches of the bipole cell receptive field:

$$A_{ijkd} = \sum_{p,q,r} \left[ (aN_{i+p,j+q,rd} - bN_{i+p,j+q,Rd}) \left[ z_{pgkr} \right]^+ \right] \tag{A42}$$

and

$$B_{ijkd} = \sum_{p,q,r} \left[ (aN_{i+p,j+q,rd} - bN_{i+p,j+q,Rd}) \left[ -z_{pgkr} \right]^+ \right], \tag{A43}$$

where orientation $R$ is perpendicular to orientation $r$, $a = 1.0, b = 2.0$. The subfraction of mutually perpendicular input prevents colinear grouping from crossing regions that contain other, non-colinear contrasts. For Scale 1 and Scale 2: $-P \leq p, q \leq P$ . The bipole cell's receptive fields are implemented as in Gove *et al.* (1995). In particular, the branches of the bipole cell receptive field obey a Gaussian weighting operator:

$$z_{ijkr} = sgn\,[i] \exp\,[T_g + T_k + T_r]. \tag{A44}$$

Term $T_g$ modulates filter values based on their distance from the bipole's center, where $\rho$ is the optimal distance from the center:

$$T_g = \frac{-\left(\sqrt{i^2 + j^2} + \rho^2\right)^2}{2\sigma_g^2}. \tag{A45}$$

In (A45), we set $\rho = 0.0$ and $\sigma_g = 7.0$. In the current implementation the input images were 164 x 204 pixels in size and $\sigma_g$ was chosen to make the bipoles 40 pixels in length. Term $T_k$ favors tangent values closer to the orientation of the bipole's main axis:

$$T_k = \frac{-\left(\tan^{-1}(i/j)\right)^2}{2\sigma_k^2}, \tag{A46}$$

where $\sigma_k = 0.15$. Term $T_r$ measures the similarity of the orientation of a point $(p, q, r)$ and the angle formed by the tangent at that point. The tangent defines the optimal orientation for that point and filter element orientations closer to this optimal value will have greater strength than those at larger angular separations:

$$T_r = \frac{-\left(\frac{r\pi}{T} - tan^{-1}(i/j)\right)^2}{2\sigma_r^2}, \tag{A47}$$

with $\sigma_r = 0.15$.

### A6.1 Bipole Cell Feedback to Hypercomplex Cells : Orientational Competition

Unlike in Gove *et al.* (1995), the bipole-to-hypercomplex cell feedback is calculated directly from the bipole cell output, as in equations (A25)–(A28).

## A7 Monocular Filling-in and Monocular FIDOs

The monocular FIDO ON cell activities $F^+_{ijd}$ and OFF cell activities $F^-_{ijd}$ diffuse the FCS outputs $X^+_{ijd}$ and $X^-_{ijd}$, respectively, from the monocular preprocessing stage. Boundary outputs create resistive barriers to the diffusion process. Filling-in obeys the following equations (Grossberg and Todorović, 1988):

$$\frac{dF_{ijd}{}^{+}}{dt} = -MF_{ijd}{}^{+} + \sum_{(p,q)\in N} \left(F_{pqd}{}^{+} - F_{ijd}{}^{+}\right) \Psi_{pqijd} + X^{+}_{ijd} \tag{A48}$$

and

$$\frac{dF_{ijd}{}^{-}}{dt} = -MF_{ijd}{}^{-} + \sum_{(p,q)\in N} \left(F_{pqd}{}^{-} - F_{ijd}{}^{-}\right) \Psi_{pqijd} + X^{-}_{ijd}, \tag{A49}$$

where $N$ consists of the four nearest neighbors to a cell and where the boundary-dependent diffusion coefficient obeys

$$\Psi_{pqijd} = \frac{\delta}{\kappa + \epsilon\left(Z_{pqd} + Z_{ijd}\right)} \tag{A50}$$

where $M = 0.1, \delta = 100,000, \kappa = 1$, and $\epsilon = 1000$ . The boundary term

$$Z_{ijd} = \sum_{k} Z_{ijkd}. \tag{A51}$$

Thus any large boundary value at the nearest neighbor positions reduces the diffusion coefficient and thereby blocks filling-in. At equilibrium:

$$F^{+}_{ijd} = \frac{X^{+}_{ijd} + \sum_{(p,q)\in N} F^{+}_{pqd} \Psi_{pqijd}}{M + \sum_{(p,q)\in N} \Psi_{pqijd}} \tag{A52}$$

and

$$F^{-}_{ijd} = \frac{X^{-}_{ijd} + \sum_{(p,q)\in N} F^{-}_{pqd} \Psi_{pqijd}}{M + \sum_{(p,q)\in N} \Psi_{pqijd}}. \tag{A53}$$

### A7.1 Output from Monocular FIDOs

Outputs from the monocular FIDOs generate both boundary pruning and surface pruning signals. In order to generate such signals at the contours of filled-in regions, the filled-in activities are processed by a contrast-sensitive on-center off-surround network as follows:

$$\frac{dr_{ijd}{}^{+}}{dt} = -\alpha_8 r_{ijd}{}^{+} + \left(U_8 - r_{ijd}{}^{+}\right) C_8 - \left(r_{ijd}{}^{+} + L_8\right) E_8 \tag{A54}$$

and

$$\frac{dr_{ijd}{}^{-}}{dt} = -\alpha_8 r_{ijd}{}^{-} + \left(U_8 - r_{ijd}{}^{-}\right) C_8 - \left(r_{ijd}{}^{-} + L_8\right) E_8, \tag{A55}$$

where $\alpha_8 = 0.01$, and $U_8 = L_8 = 1$. The excitatory input

$$C_8 = \sum_{(p,q)} C_{pq} F_{i+p,j+q,d}, \tag{A56}$$

has the on-center kernel

$$C_{pq} = \frac{C}{2\pi\sigma_c{}^2} \exp\left(-\frac{p^2 + q^2}{2\sigma_c{}^2}\right), \tag{A57}$$

where $C = 0.0398, \sigma_c = 2.0$. The inhibitory input

$$E_8 = \sum_{(p,q)} E_{pq} F_{i+p,j+q,d}, \tag{A58}$$

has the off-surround kernel

$$E_{pq} = \frac{S}{2\pi {\sigma_s}^2} \exp\left(-\frac{p^2+q^2}{2{\sigma_s}^2}\right), \tag{A59}$$

where $S = 0.181$ and $\sigma_s = 3.0$. At equilibrium:

$${r_{ijd}}^+ = \frac{\sum_{p,q} (U_8 C_{pq} - L_8 S_{pq})\, {F_{i+p,j+q,d}}^+}{\alpha_8 + \sum_{p,q} (U_8 C_{pq} + L_8 S_{pq})\, {F_{i+p,j+q}}^+} \tag{A60}$$

and

$${r_{ijd}}^- = \frac{\sum_{p,q} (U_8 C_{pq} - L_8 S_{pq})\, {F_{i+p,j+q,d}}^-}{\alpha_8 + \sum_{p,q} (U_8 C_{pq} + L_8 S_{pq})\, {F_{i+p,j+q}}^-}. \tag{A61}$$

These contrast-sensitive signals were then subtracted and rectified to generate double-opponent output signals:

$$R^+_{ijd} = \left[r^+_{ijd} - r^-_{ijd}\right]^+ \tag{A62}$$

and

$$R^-_{ijd} = \left[r^-_{ijd} - r^+_{ijd}\right]^+. \tag{A63}$$

## A8 Boundary Pruning Signals to Hypercomplex Cells

In order to transform unoriented FIDO activities into boundary pruning signals at oriented hypercomplex cell activities in equation (A33), they are processed by oriented filters:

$$v_{ijkd} = b^{odd+}_{ijkd} + b^{odd-}_{ijkd} + b^{even+}_{ijkd} + b^{even-}_{ijkd}, \tag{A64}$$

where

$$b^{odd+/-}_{ijk} = \left[\sum_{(p,q)} B^{odd+/-}_{pqk} R^{+/-}_{i-p,j-q} - \sum_{(p,q)} B^{odd-/+}_{pqk} R^{-/+}_{i-p,j-q}\right]^+ \tag{A65}$$

and

$$b^{even+/-}_{ijk} = \left[\sum_{(p,q)} B^{even+/-}_{pqk} R^{+/-}_{i-p,j-q} - \sum_{(p,q)} B^{even-/+}_{pqk} R^{-/+}_{i-p,j-q}\right]^+, \tag{A66}$$

with odd-symmetric kernels

$$B^{odd}_{pqk} = A \sin\left(\frac{2\pi k}{T}\right) \exp\left[-\frac{1}{2}\left(\frac{p^2}{{\sigma_{pk}}^2} + \frac{q^2}{\sigma^2_{qk}}\right)\right] \tag{A67}$$

and even-symmetric kernels

$$B^{even}_{pqk} = A \cos\left(\frac{2\pi k}{T}\right) \exp\left[-\frac{1}{2}\left(\frac{p^2}{\sigma^2_{pk}} + \frac{q^2}{\sigma^2_{qk}}\right)\right]. \tag{A68}$$

All parameters for these FCS simple cells are the same as for the BCS simple cells in Section A3.

## A9 Surface Pruning Signals to the Binocular FIDOs

The binocular FIDOs receive inputs from both the left and right eye monocular FIDOs and the monocular preprocessing stage. Inputs from the monocular preprocessing stage are excitatory and are binocularly matched at the binocular FIDOs. Inputs from the monocular FIDOs are inhibitory surface pruning signals. Both excitatory and inhibitory signals are combined binocularly via the following equations:

$$\frac{da_{ijd}{}^{+}}{dt} = -\alpha_9 a_{ijd}{}^{+} + \left(U_9 - a_{ijd}{}^{+}\right) E^{+}_{ijd} - \left(a_{ijd}{}^{+} + L_9\right) I^{+}_{ijd} \tag{A69}$$

and

$$\frac{da_{ijd}{}^{-}}{dt} = -\alpha_9 a_{ijd}{}^{-} + \left(U_9 - a_{ijd}{}^{-}\right) E^{-}_{ijd} - \left(a_{ijd}{}^{-} + L_9\right) I^{-}_{ijd}, \tag{A70}$$

where $\alpha_9 = 0.01, U_9 = 0.5$, and $L_9 = 50$. The excitatory term $E_{ijd}$ matches left and right monocular preprocessing signals:

$$E^{+}_{ijd} = X^{L+}_{ijd} + X^{R+}_{ijd} \tag{A71}$$

and

$$E^{-}_{ijd} = X^{L-}_{ijd} + X^{R-}_{ijd}. \tag{A72}$$

The Binocular FIDOs also receive inhibitory surface pruning inputs from monocular FIDO cells that represent smaller disparities:

$$I^{+}_{ijd} = \sum_{e<d} R^{+}_{ije} \tag{A73}$$

and

$$I^{-}_{ijd} = \sum_{e<d} R^{-}_{ije}, \tag{A74}$$

where $R^{+}$ and $R^{-}$ are the monocular FIDO outputs in (A62) and (A63). The values of the saturation terms $U_9$ and $L_9$ are chosen to enable the monocular FIDO outputs to inhibit monocular preprocessing signals and thereby prevent filling-in of occluded regions at the Binocular FCS. Solving at steady state and rectifying yields:

$$A^{+}_{ijd} = \left[\frac{U_9 E^{+}_{ijd} - L_9 I^{+}_{ijd}}{\alpha_9 + E^{+}_{ijd} + I^{+}_{ijd}}\right] \tag{A75}$$

and

$$A^{-}_{ijd} = \left[\frac{U_9 E^{-}_{ijd} - L_9 I^{-}_{ijd}}{\alpha_9 + E^{-}_{ijd} + I^{-}_{ijd}}\right]. \tag{A76}$$

The diffusive spread of binocular FIDO activity is defined by the following equations:

$$\frac{d\Omega^{+}_{ijd}}{dt} = -M\Omega^{+}_{ijd} + \sum_{(p,q)\in N} \left(\Omega^{+}_{pqd} - \Omega^{+}_{ijd}\right) \Psi^{b}_{pqijd} + A^{+}_{ijd} \tag{A77}$$

and

$$\frac{d\Omega^{-}_{ijd}}{dt} = -M\Omega^{-}_{ijd} + \sum_{(p,q)\in N} \left(\Omega^{-}_{pqd} - \Omega^{-}_{ijd}\right) \Psi^{b}_{pqijd} + A^{-}_{ijd}, \tag{A78}$$

where $N$ is the set of nearest neighbors, $M = 0.1$, and the boundary gating term is defined by

$$\Psi_{pqijd} = \frac{\delta}{\kappa + \epsilon\left(Z^*_{pqd} + Z^*_{ijd}\right)}, \tag{A79}$$

where $\delta = 100,000, \kappa = 1.0$, and $\epsilon = 1000$. In (A79), the boundary signals $Z^*$ are enriched by adding the boundaries $Z$ of nearer objects to the boundaries of farther objects, thereby preventing occluded regions of the binocular FIDO from filling-in and giving a percept of transparency where none exists. Thus:

$$Z^*_{pqd} = \sum_{e \leq d} Z_{pge}. \tag{A80}$$

where $Z_{pge}$ is defined by (A51). Solving at equilibrium yields:

$$\Omega^+_{ijd} = \frac{A^+_{ijd} + \sum_{(p,q)\in N} \Omega^+_{pqd} \Psi^b_{pqijd}}{M + \sum_{(p,q)\in N} \Psi^b_{pqijd}} \tag{A81}$$

and

$$\Omega^-_{ijd} = \frac{A^-_{ijd} + \sum_{(p,q)\in N} \Omega^-_{pqd} \Psi^b_{pqijd}}{M + \sum_{(p,q)\in N} \Psi^b_{pqijd}}. \tag{A82}$$

These equations were solved at equilibrium using the Y12M package (Zlatev, Wasniewski & Schaumburg, 1981) because solving these equations using 4th-order Runge-Kutta (step size 0.0000025) or adaptive step Runge-Kutta was computationally intractable. The Y12M package uses an approximation algorithm to calculate final FCS values based on filling-in. Unfortunately one of the problems with this approximation is that it allows boundaries to leak color. This problem was solved by increasing the strength of boundaries by setting $\epsilon$ in (A79) to 100,000. In the Munker-White simulation, because of the sparsity of filling-in signals at the far depth binocular FCS (Figure 15f) relative to the area these signals must fill in, we set $\delta = 1,000,000, \kappa = 1.0$, and $\epsilon = 500,000$.

Equilibrium opponent ON−OFF and OFF−ON values are calculated as follows:

$$\Lambda^+_{ijd} = \frac{\Omega^+_{ijd} - \Omega^-_{ijd}}{\Omega^+_{ijd} + \Omega^-_{ijd}} \tag{A83}$$

and

$$\Lambda^-_{ijd} = \frac{\Omega^-_{ijd} - \Omega^+_{ijd}}{\Omega^+_{ijd} + \Omega^-_{ijd}} \tag{A84}$$

These are the binocular FIDO activities that are plotted in the simulations.