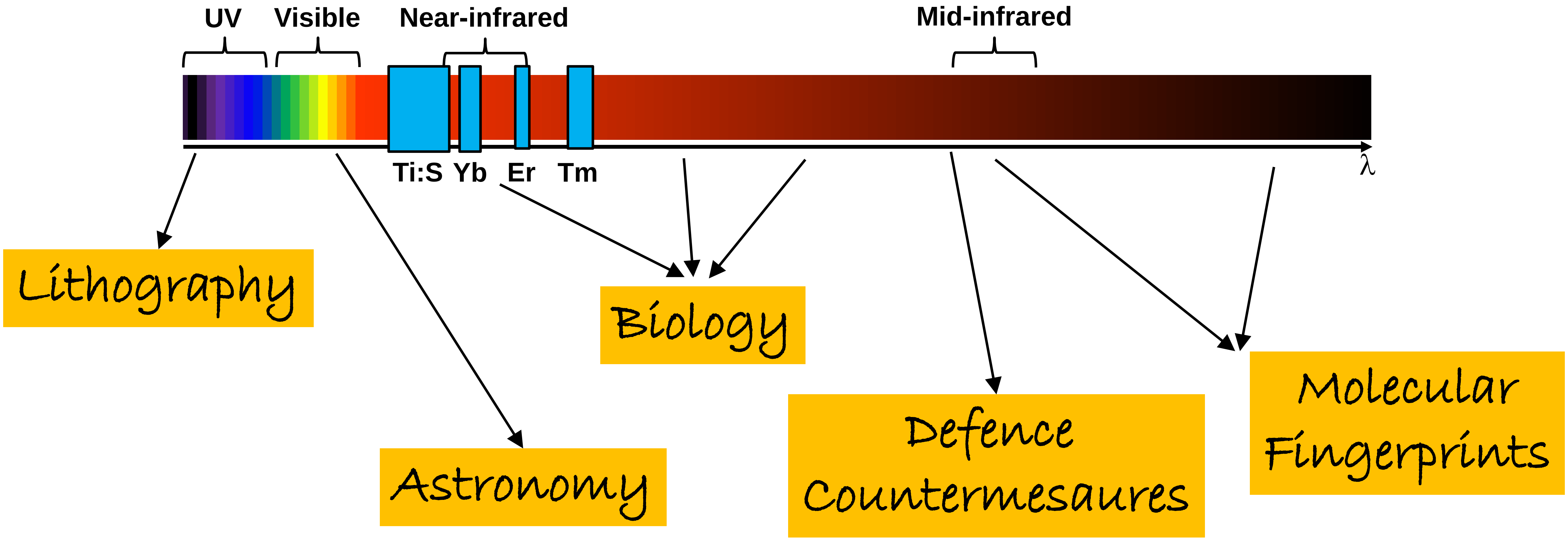
Multimode Nonlinear Fiber Optics

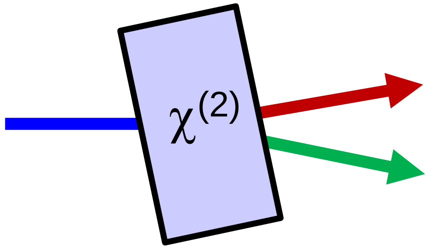
Siddharth Ramachandran

High Dimensional Photonics Lab, Boston University

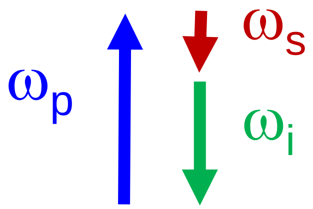
sidr@bu.edu; http://sites.bu.edu/ramachandranlab

Laser Sources – OPOs/OPAs

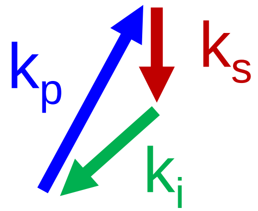




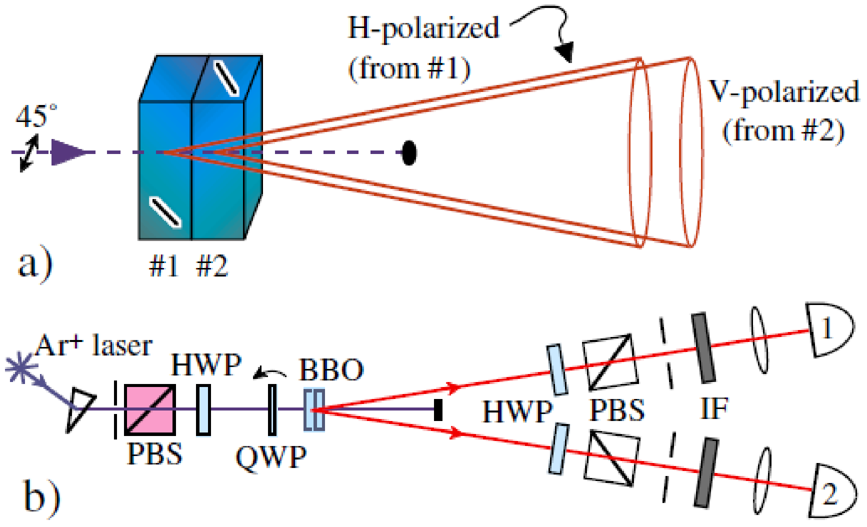
Energy conservation

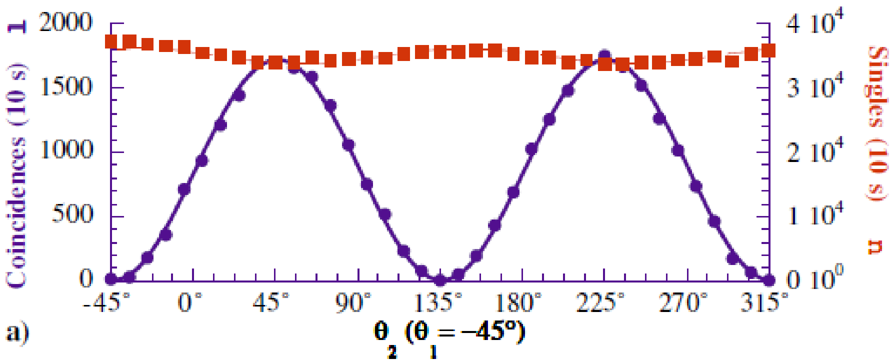


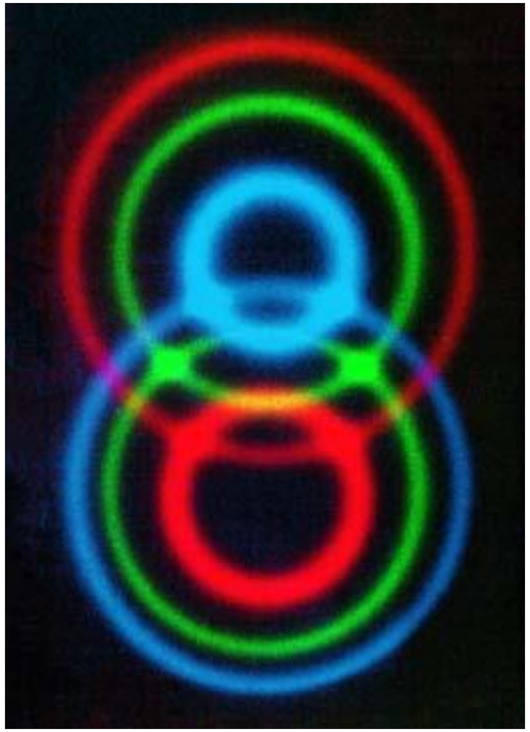
Momentum conservation



Sources for Quantum Optics





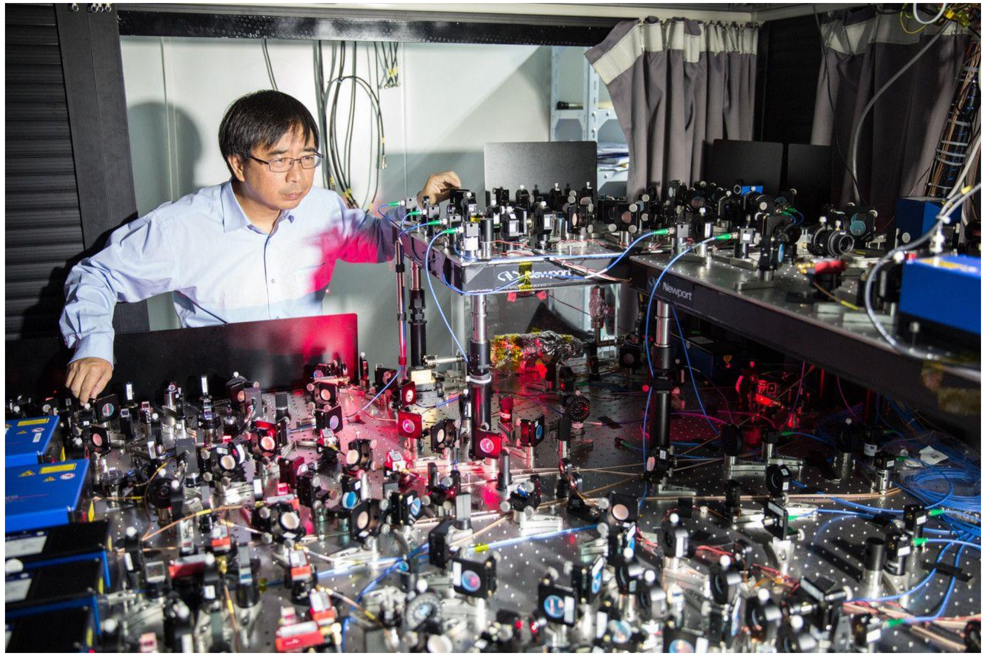


Kwiat et al, Phys. Rev. Lett. 75, 4337–4341 (1995)

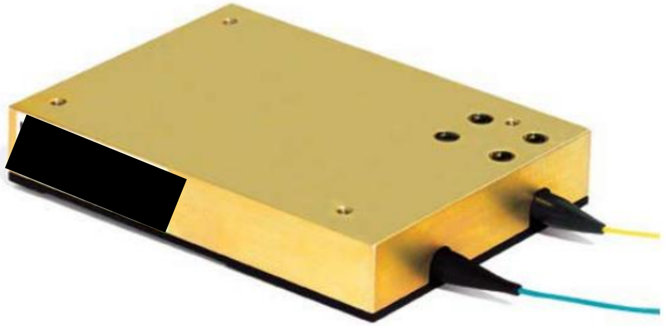
Why (multimode) fiber NLO?

Optical Parametric Oscillator

A recent QKD experiment



Replace with?



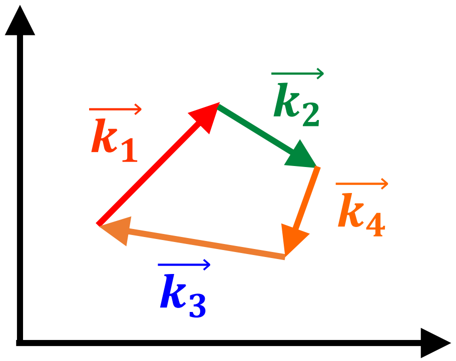
Outline

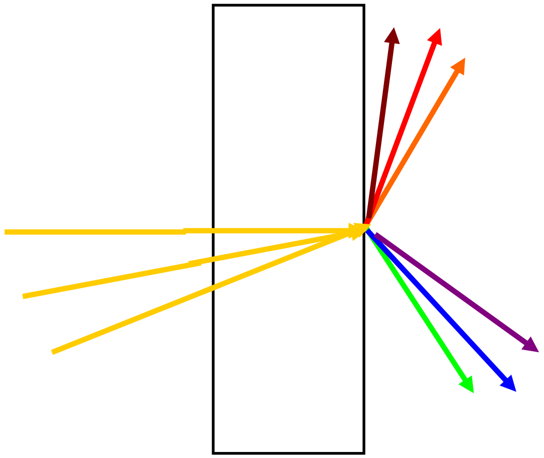
- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

Nonlinearities in Fibers

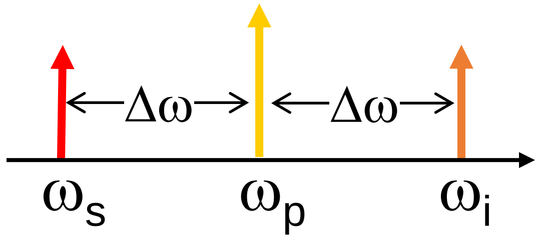
Group velocity — Dispersion — 3rd order Disp — SPM — Self-steepening — Intra-pulse Raman

$$\frac{\partial}{\partial z}A + \beta_1 \frac{\partial}{\partial t}A + \frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2}A - \frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}A = i\gamma\left[|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial t}\left(|A|^2 A\right) - T_R A \frac{\partial}{\partial t}|A|^2\right]$$





but in a fiber, there is only one direction!



four-wave mixing (FWM)

$$\Delta\beta = \beta_s(\omega_s) + \beta_i(\omega_i) - 2\beta_p(\omega_p) + 2\gamma P$$
$$= \beta_2 \cdot \Delta\omega^2 + \cdots + 2\gamma P = 0$$

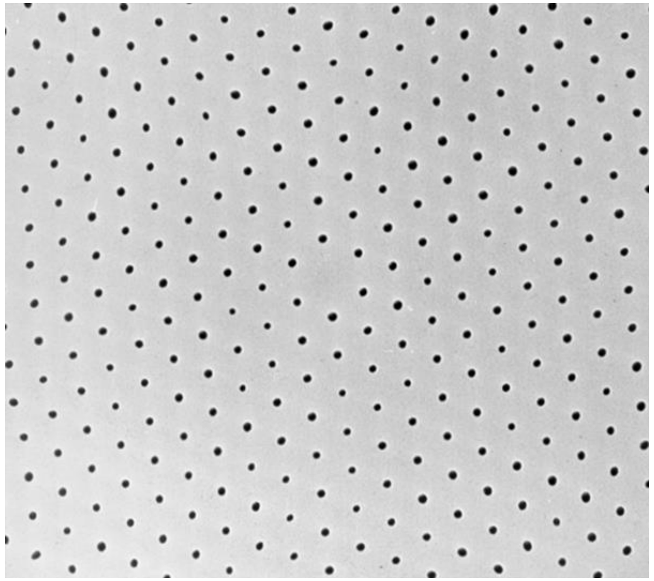
$$\text{need dispersion}, D = -\frac{2\pi c}{\lambda^2}\beta_2 > 0$$

Boyd, Nonlinear Optics, 3rd Ed.

Agrawal, Nonlinear Fiber Optics, 4th Ed.

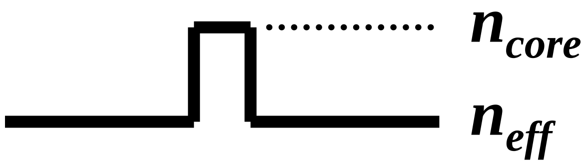
Index Guiding Photonic "Crystal" Fiber (PCF)

•eg the core is a missing hole among an array of holes



• cladding = glass + holes, core = glass only

• <u>effective</u> cladding index is less than core index ⇒ total internal reflection possible

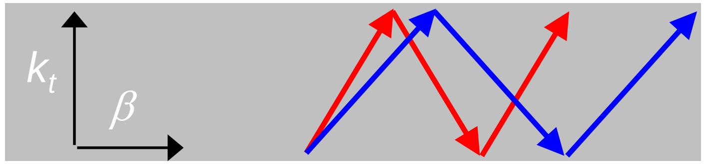


<u>effective</u> refractive index profile

Courtesy Tim Birks

JC Knight et al, Opt Lett 21 (1996) 1547

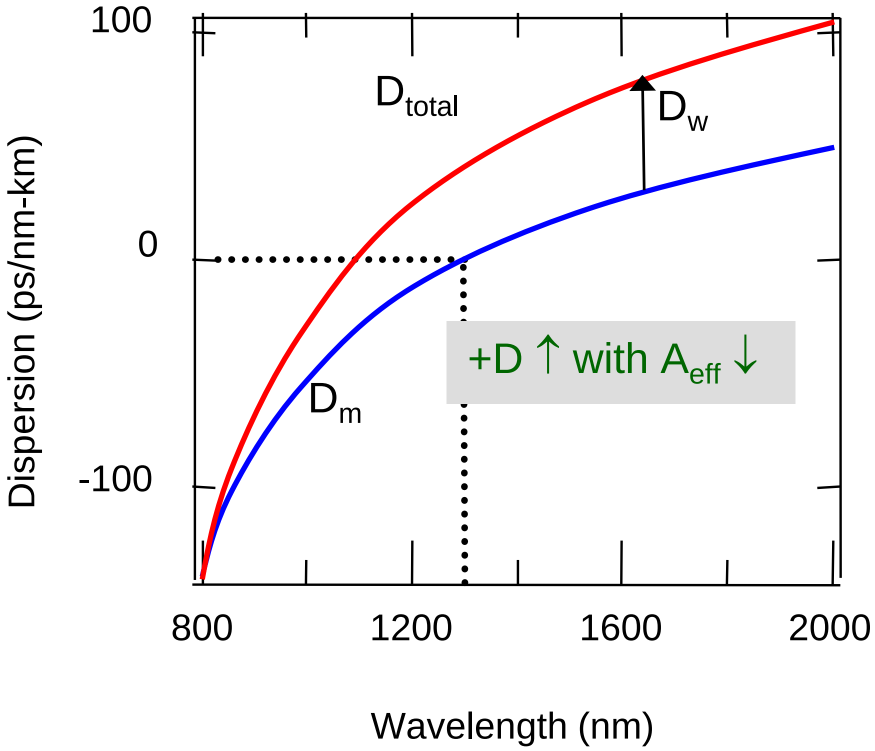
Dispersion in PCFs

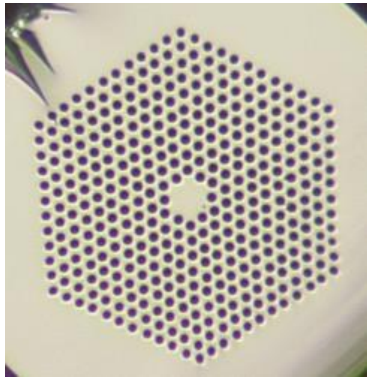


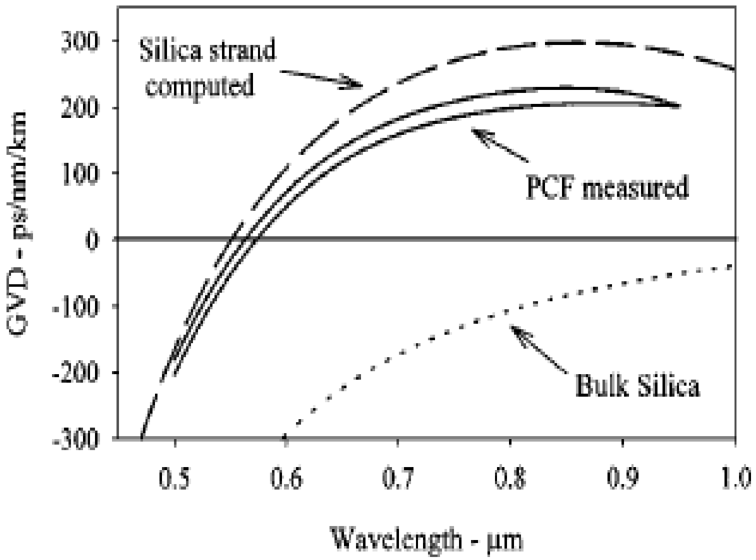
$$\beta^2 = \left(\frac{2\pi}{\lambda} n\right)^2 - k_t^2; \qquad k_t \cdot a = m\pi$$

$$\lambda \uparrow \Rightarrow \tau \uparrow$$
$$\therefore D_w = \frac{d\tau}{d\lambda} > 0$$

$$D_{total} = D_m + D_w$$







J.C. Knight *et al*, PTL, v12, p807, 2000

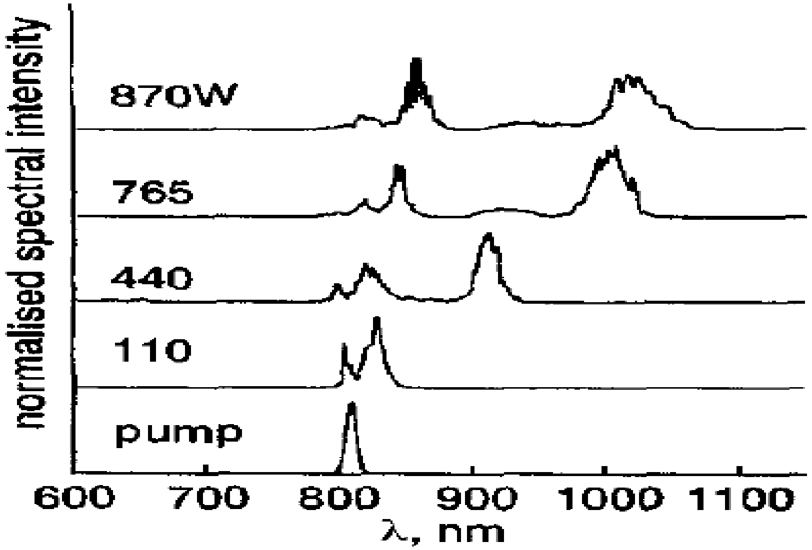
Low Power Nonlinear Optics with PCFs

J.K. Ranka **et.** al, Optics Lett., v25, p25 , 2000

$$E_{soliton} \propto D \cdot A_{eff}$$

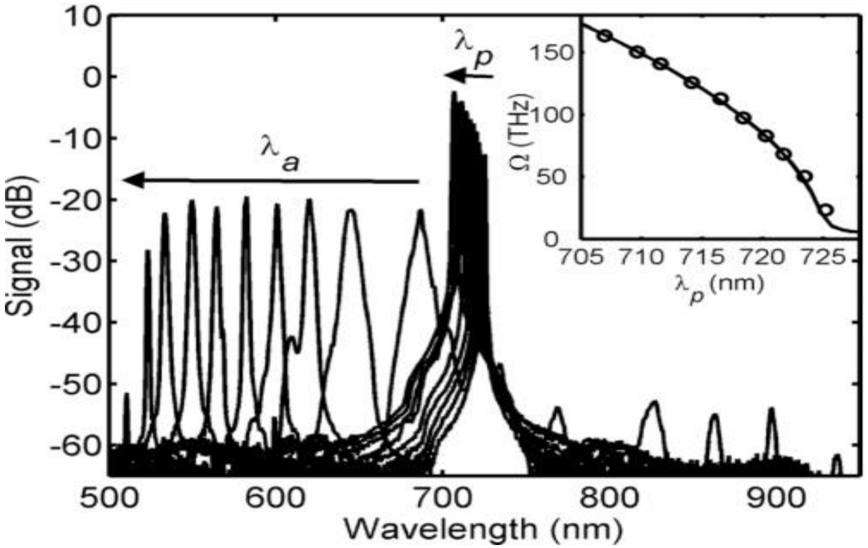
sub-nJ pulse energies with PCFs

Raman Soliton Shifting (fs)



B.R. Washburn **et.** al, Electron Lett., v37, p1510, 2001

ps fiber OPO

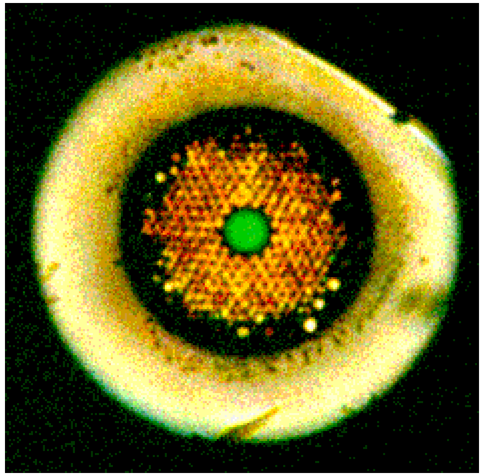


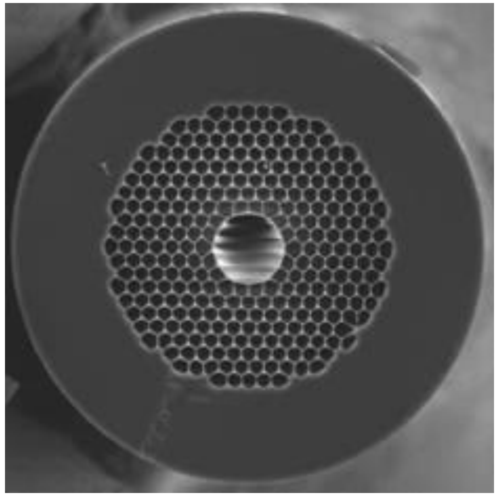
Y.Q. Xu **et.** al, Optics Lett., v33, p1351, 2008

Photonic Crystal Fibers

Bandgap guidance:

- Engineer cladding modes & guidance
- “Designer material”
- Free to use low-index materials:
- Hollow guidance possible!



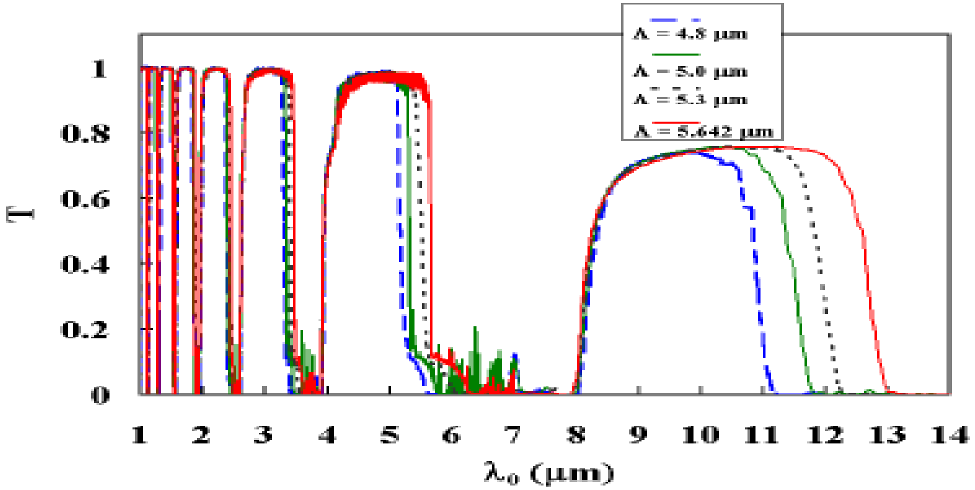


Cregan et al, *Science* 285, 1537 (1999)

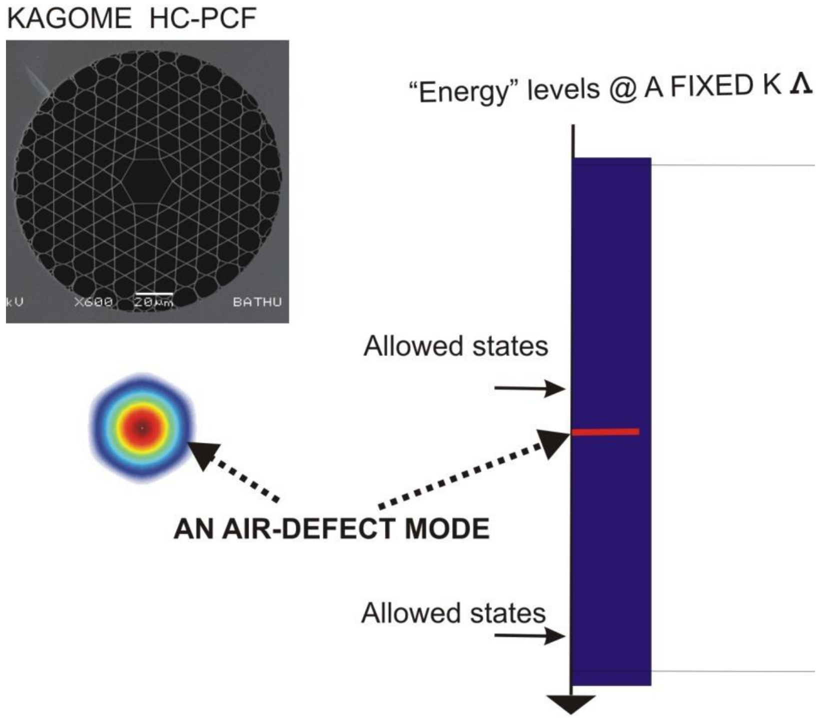
Gas Nonlinear Optics

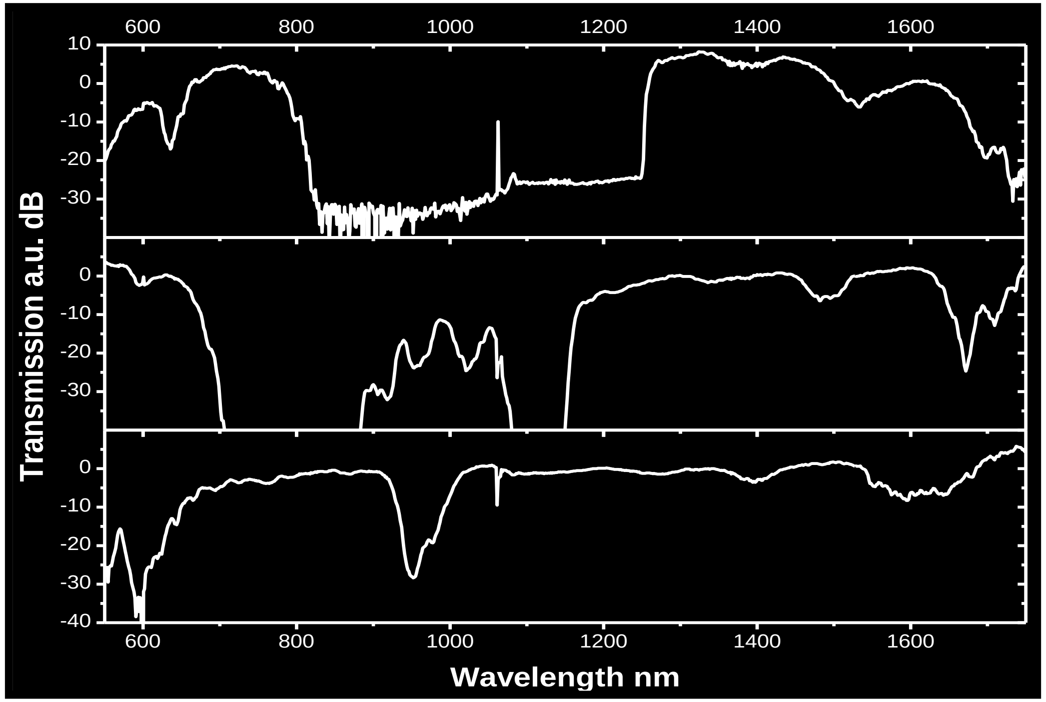
- Change Dispersion by gas pressure

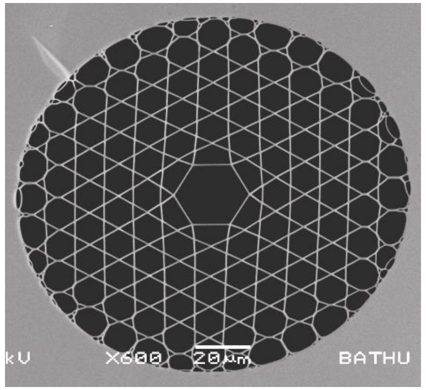
Bandwidth limitations

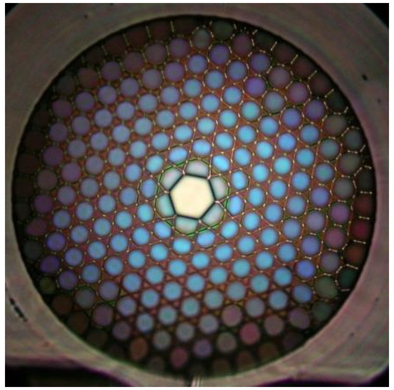


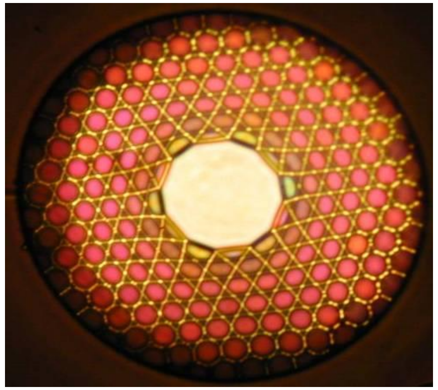
Inhibited Coupling/Anti-Resonant/ARROW Guidance

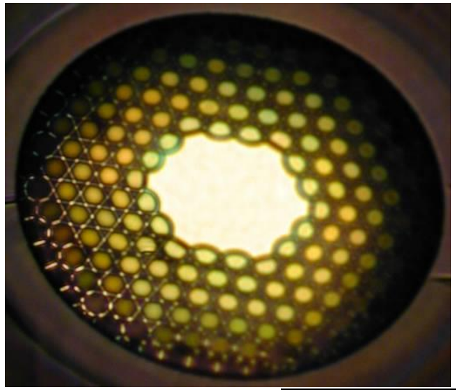












Courtesy
Fetah Benabid

GPPMM Gas-Phase Photonic & Microwave Materials

Benabid et *al.* Science, 298, 399 (2002)

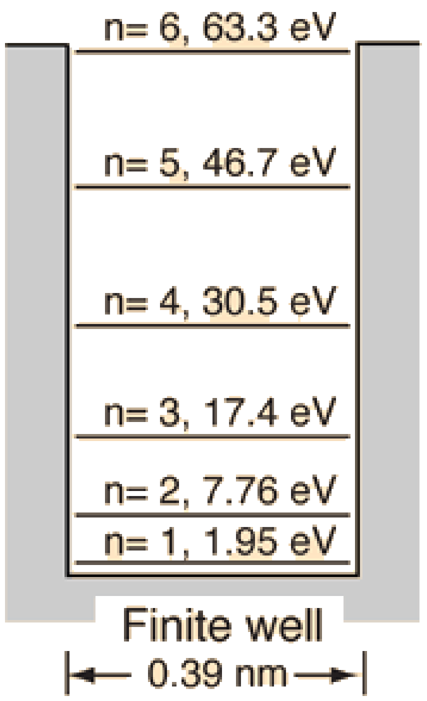
Outline

- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

Modes in Waveguides

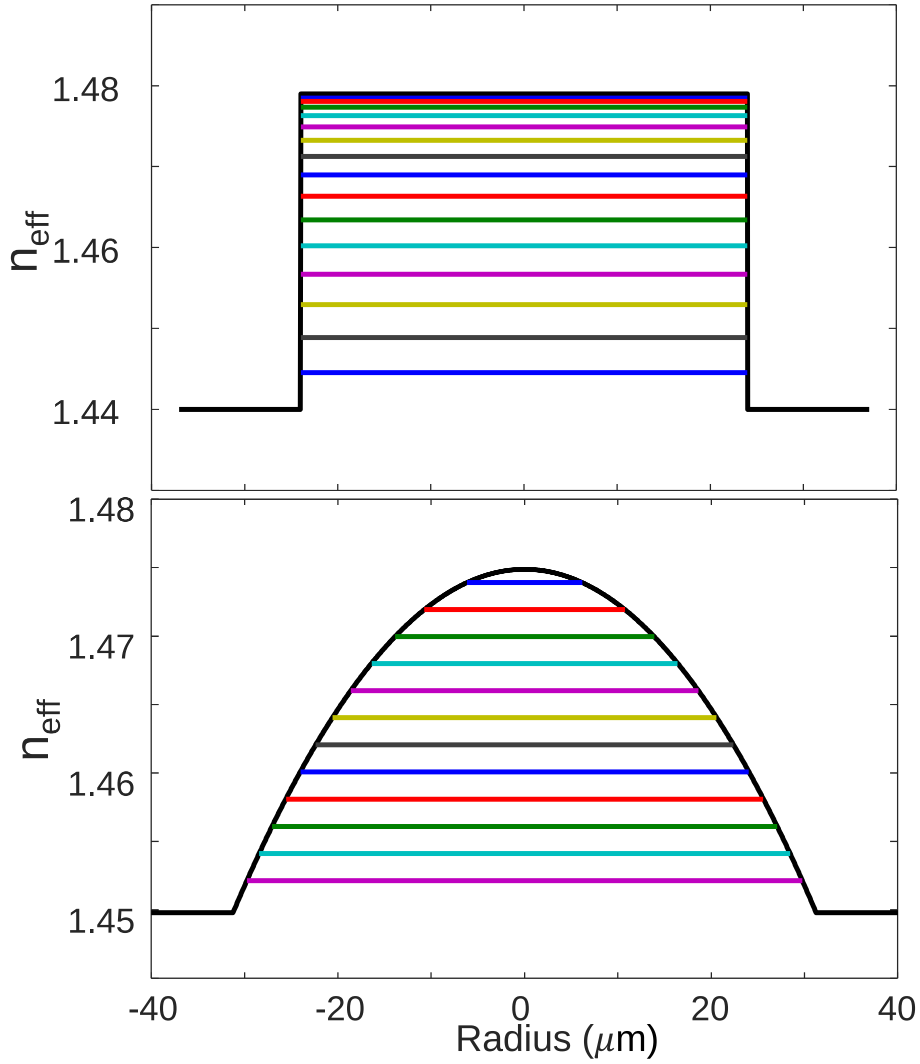
Particle in a Box

$$\hat{H}\psi(r) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(r) = E\cdot\psi(r)$$

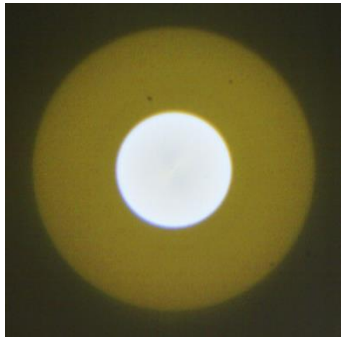


Optical Waveguide

$$\left[\nabla_t^2 + k^2 n(r)^2\right]\mathcal{E}(r) = k^2 n_{eff}^2 \cdot \mathcal{E}(r)$$



Modes in multimode fibers



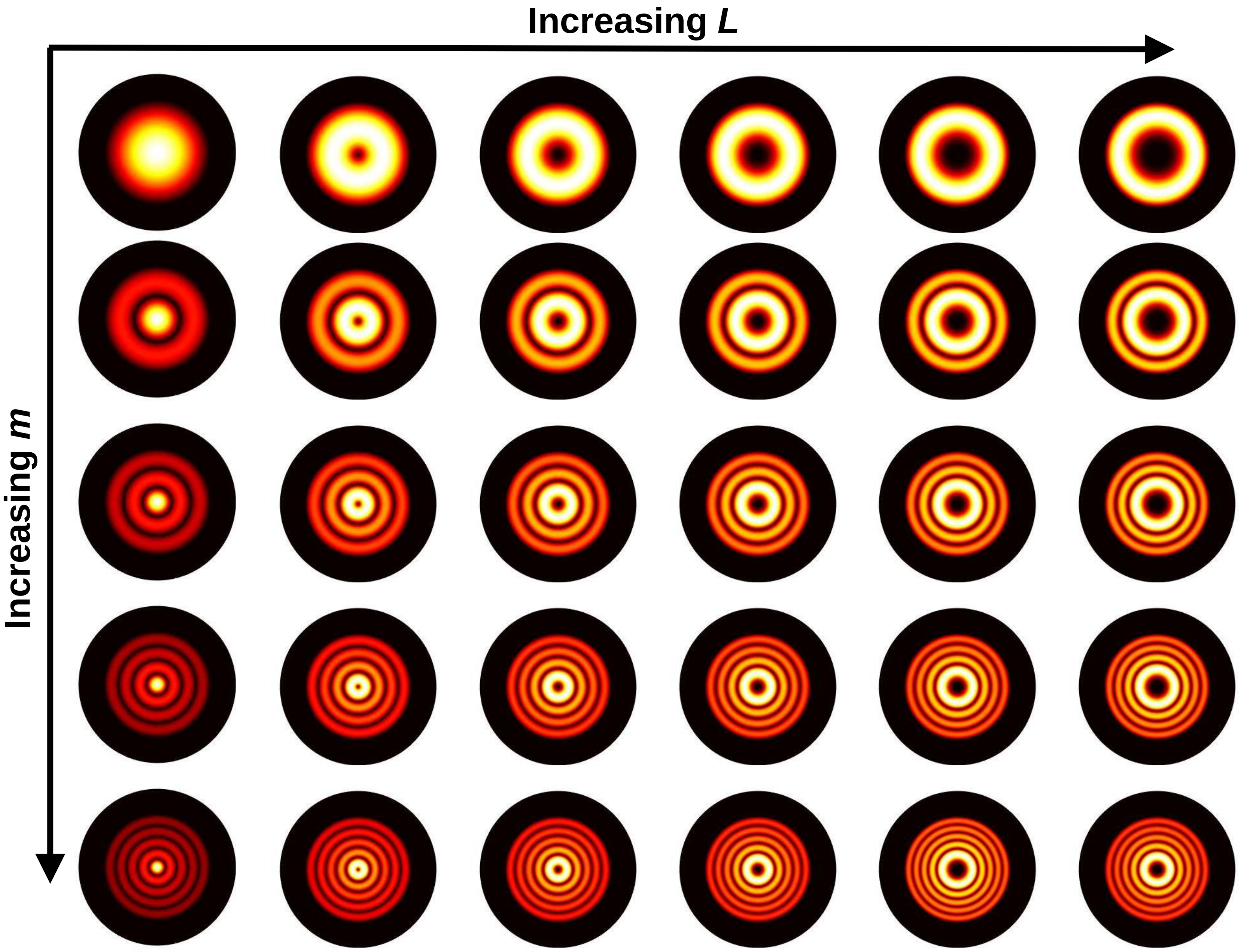
Linearly Polarized (LP) modes

$$\vec{E}_t \sim J_L(k_t \cdot r) \cdot \begin{Bmatrix} \cos(L\varphi) \\ \sin(L\varphi) \end{Bmatrix} \cdot \begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix}$$

OAM modes

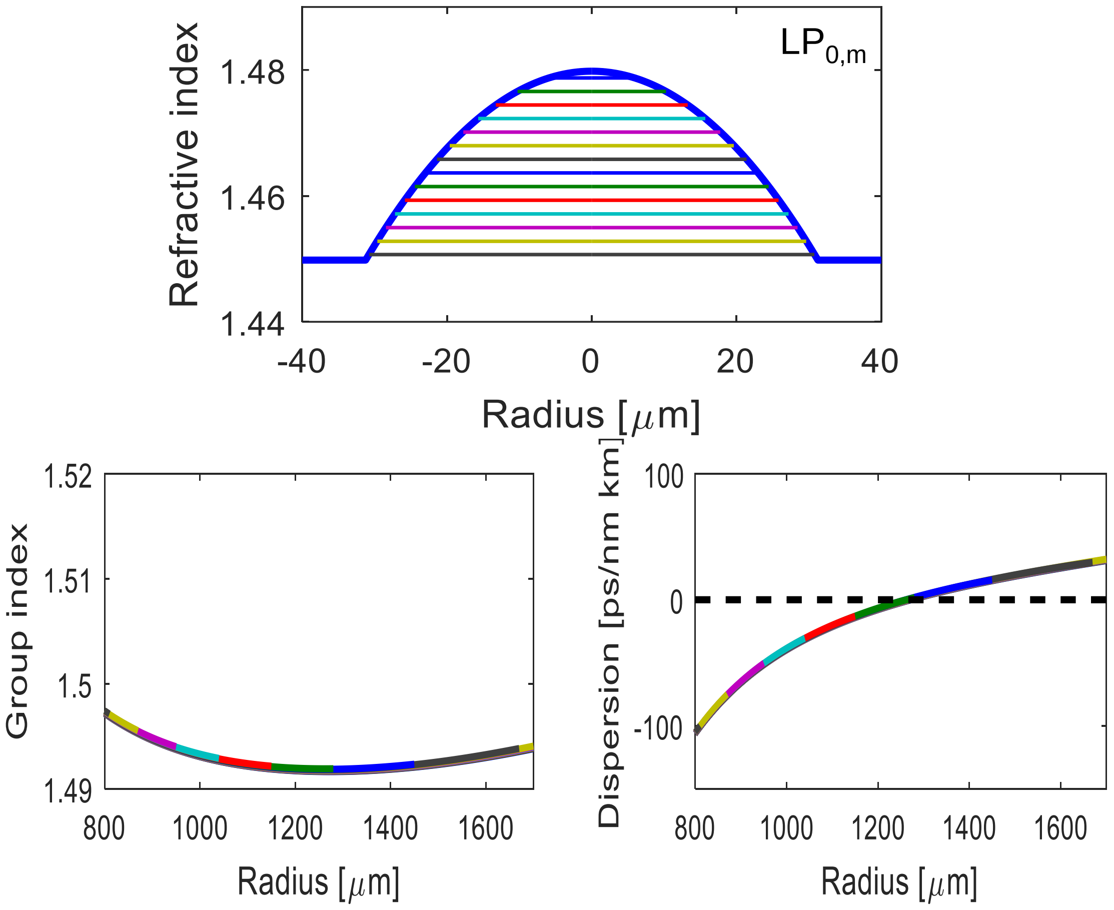
$$\vec{E}_t \sim J_L(k_t \cdot r) \cdot e^{\pm iL\varphi} \cdot \begin{Bmatrix} \hat{\sigma}^+ \\ \hat{\sigma}^- \end{Bmatrix}$$

L: Orbital Angular Momentum (OAM)



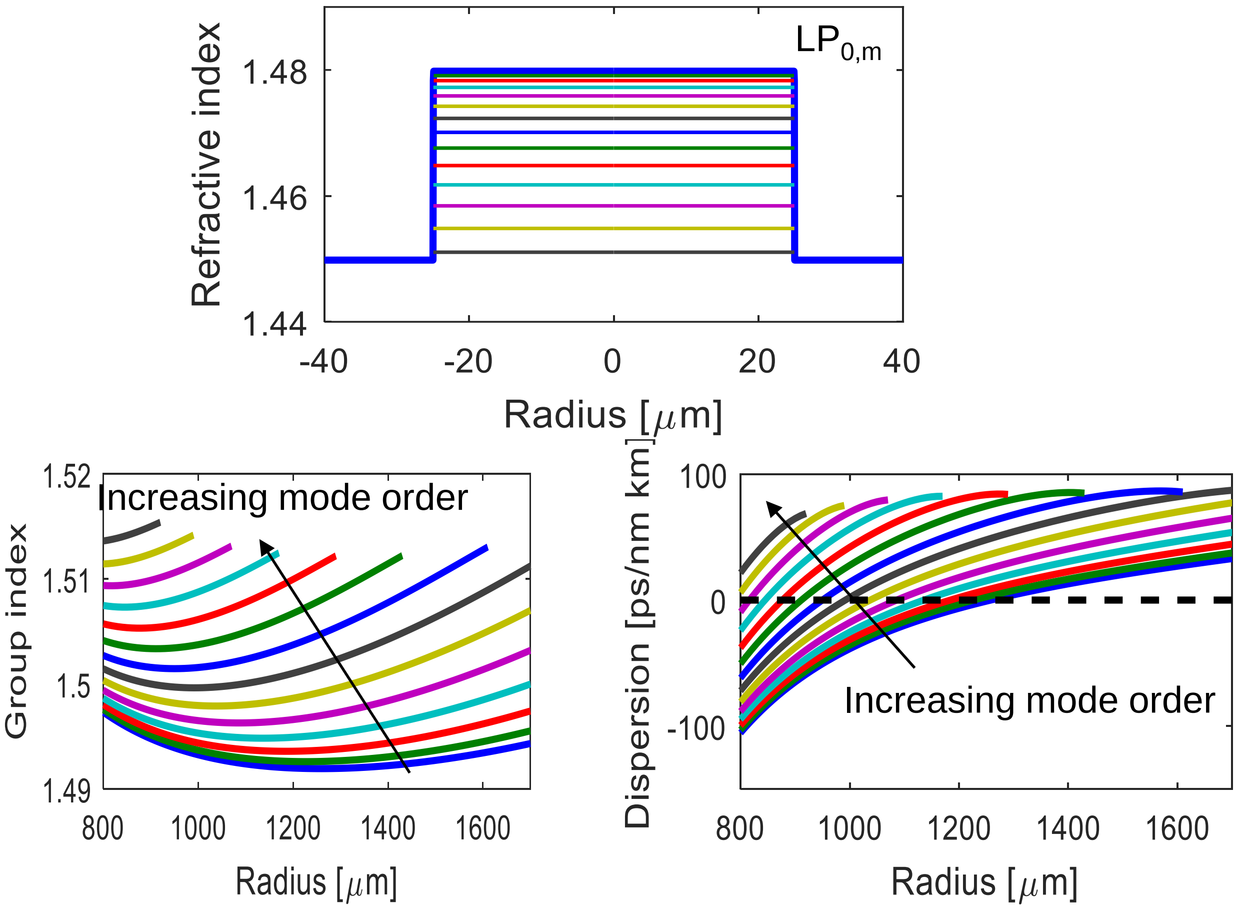
Dispersive properties of modes

Graded index fiber



- Modes have same group index and dispersion
  - ⇒ behaves like a bulk medium

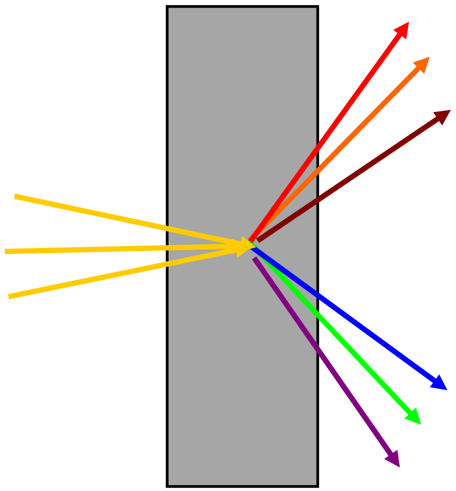
Step-index fiber



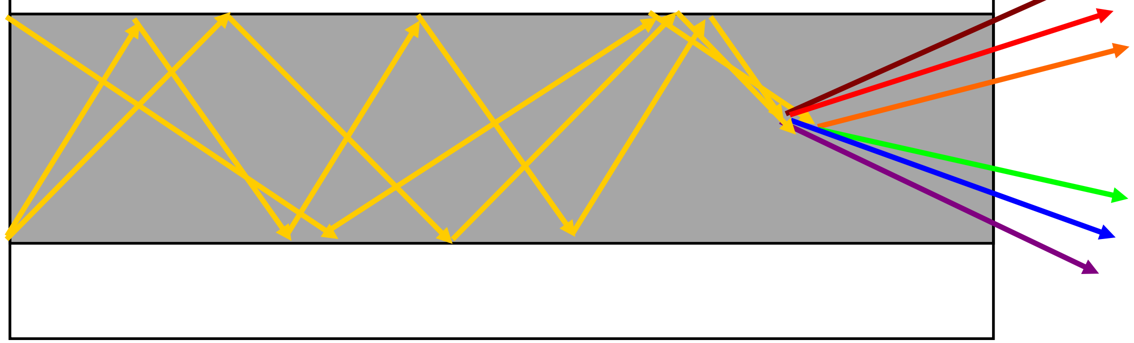
- Modes have unique group index and dispersion
  - ⇒ De-coupled area and dispersion
  - ⇒ Anomalous dispersion at 1 μm

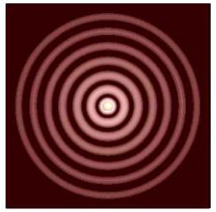
S. Ramachandran et al., LPR 2, 429 (2008)

Intermodal fiber NLO ⇔ Free Space Xtal NLO



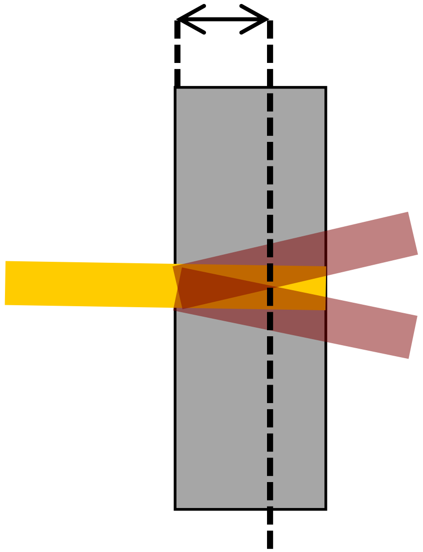
$$\Delta\beta = \beta_1 + \beta_2 - \beta_3 - \beta_4 = 0$$

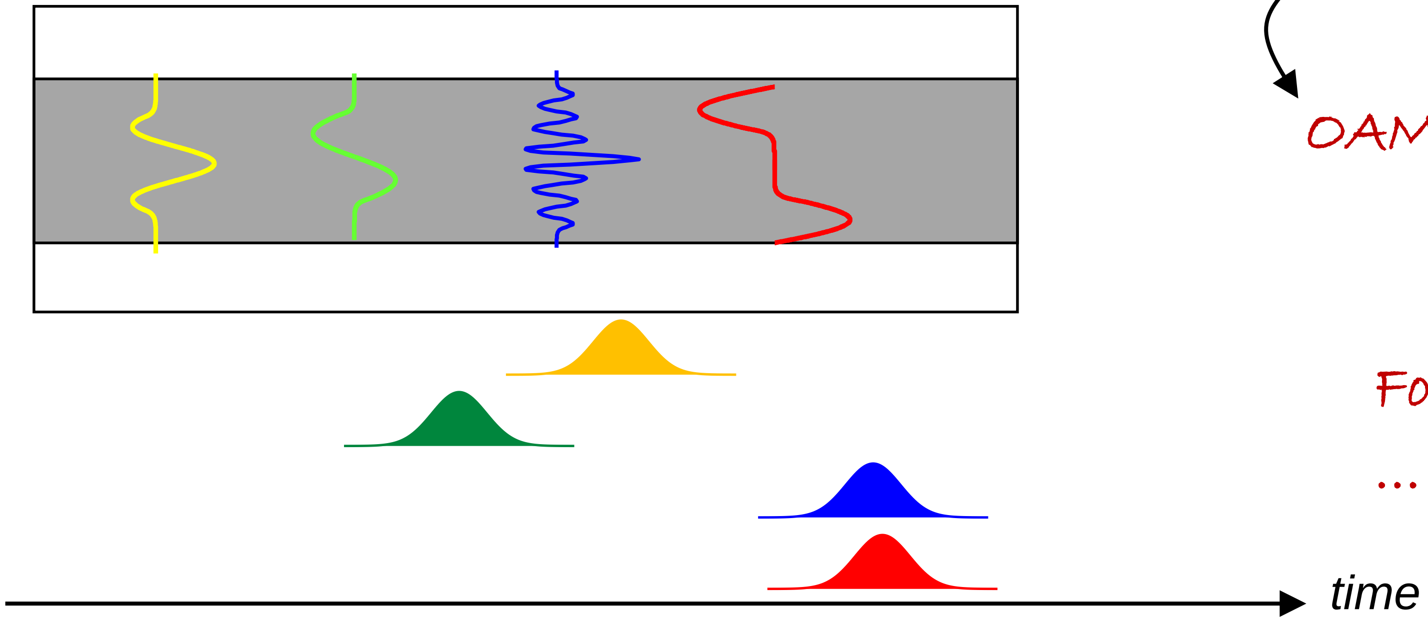




Interaction length ⇔ Overlap: $\int E_1 E_2 E_3^* E_4^* \cdot dA = \int F_1 F_2 F_3 F_4 \cdot exp(L_1 + L_2 - L_3 - L_4) \cdot dA$

OAM Conservation

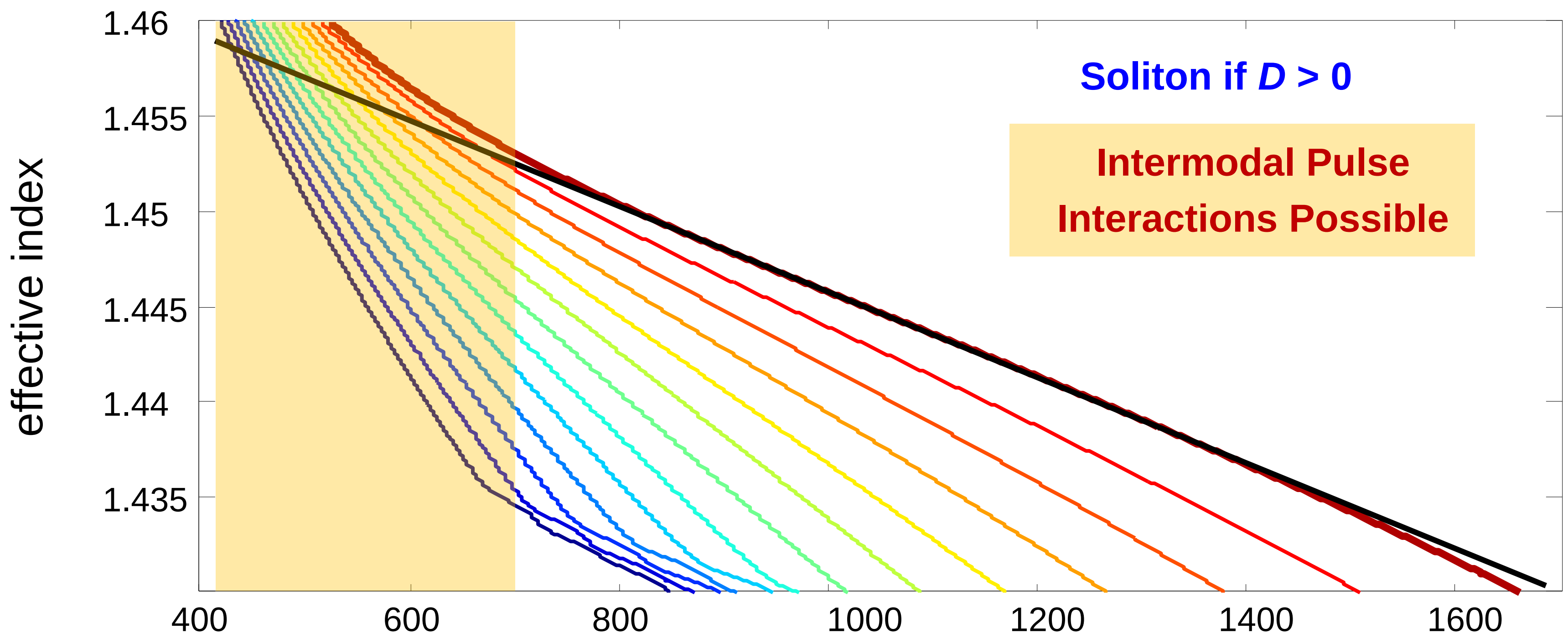




For pulses…

… match n_g

Phase Matching in Nonlinear Optics



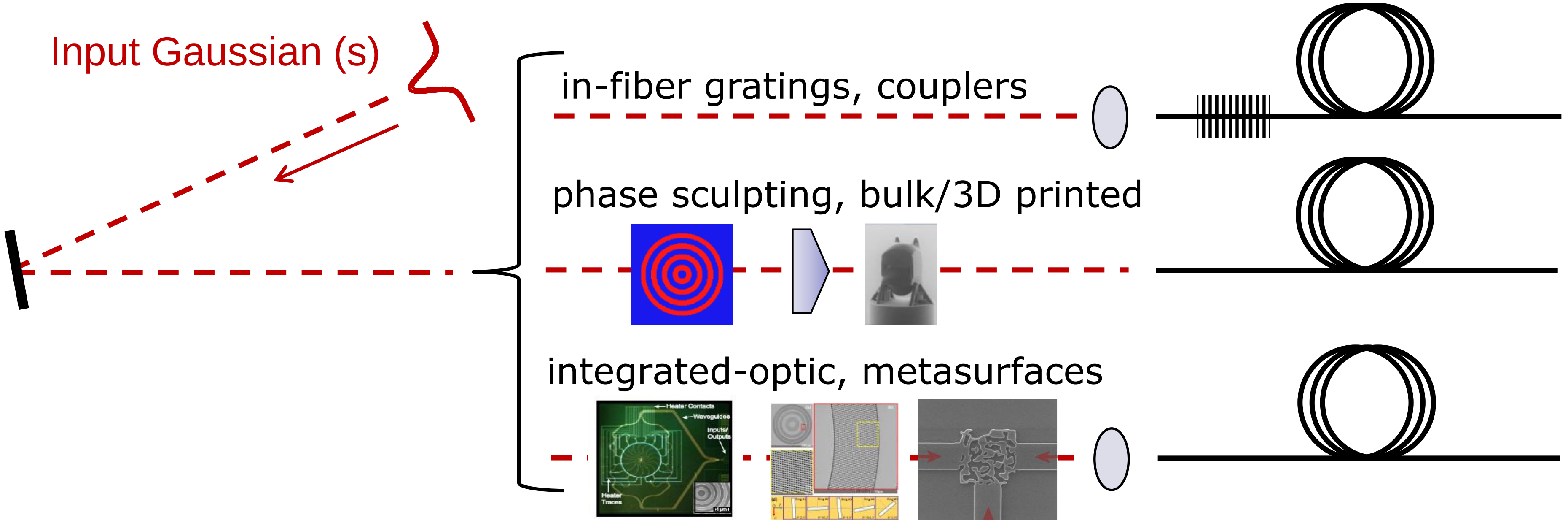
$$n(\lambda) \sim linear \Rightarrow \frac{d^2 n}{d\lambda^2} \sim D \sim 0$$

Multiple modes
=> multiple new phase matching possibilities (FWM, THG, etc.)

=> Phase matched: $\frac{n(\lambda_1)}{\lambda_1} \pm \frac{n(\lambda_2)}{\lambda_2} \pm \frac{n(\lambda_3)}{\lambda_3} \pm \frac{n(\lambda_4)}{\lambda_4} = 0$

if energy matched: $\frac{1}{\lambda_1} \pm \frac{1}{\lambda_2} \pm \frac{1}{\lambda_3} \pm \frac{1}{\lambda_4} = 0$

Mode Shaping & Transformations



- **Long-period gratings (LPGs)**
- **Acousto-optic fiber gratings**
- **Fused couplers**
- **Phase plates & Axicons**
- **Algorithmic phase sculpting**
- **Log-polar transformations**
- **Multiplane Holography**
- **MMI couplers**
- **On-chip multiplexers**
- **Metasurfaces**

A.M. Vengsarkar et. al, JLT 14, p. 58 (1996)
K. Lai et. al, OL 32, p. 328 (2007)
Y.O. Yilmaz et. al, OL 32, p. 3170 (2007)
J-F. Morizur et al, JOSA A 27, 2524 (2010)
Berkhout et al, PRL 105,153601 (2010)
A. Sridharan et. al, OE 20, p. 28792 (2012)
T. Su et al, OpEx 20, 9396 (2012)
X. Cai et al, Science 338, 363, (2012)
M. Mirhosseini, Nat. Comm. 4, 2781 (2013)
Yu & Capasso, Nat. Mat. 13, 139 (2014)
J. Demas et al., OE 23, 28531 (2015)
S. Wang et al, OL 40, 4711 (2015)
S. Pidishetty et al, OL 42, 4347 (2017)
Y. Wen et al, Optica 7, 254 (2020)
S. Lightman et al, Opt. Lett. 47, 3491-3494 (2022)
A.D. White et al, *ACS Photonics* 10, 803 (2023)

Outline

- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

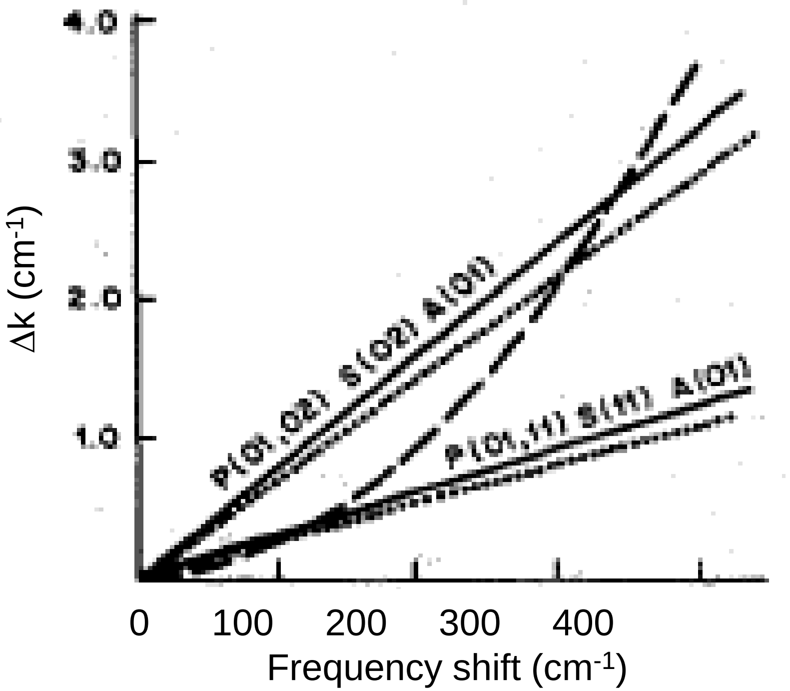
Birth of fiber (*not just multimode*) NLO

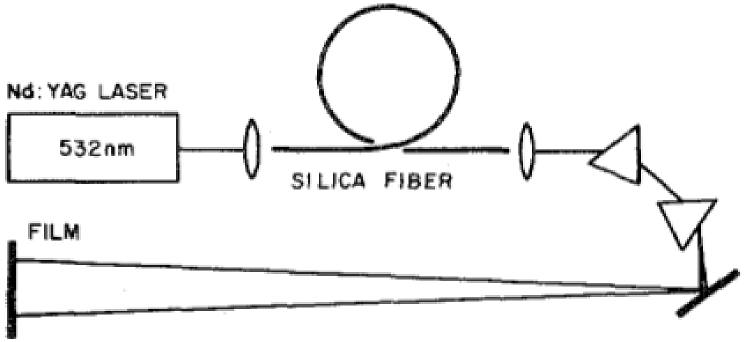
Appl. Phys. Lett. 24, 308 (1974)

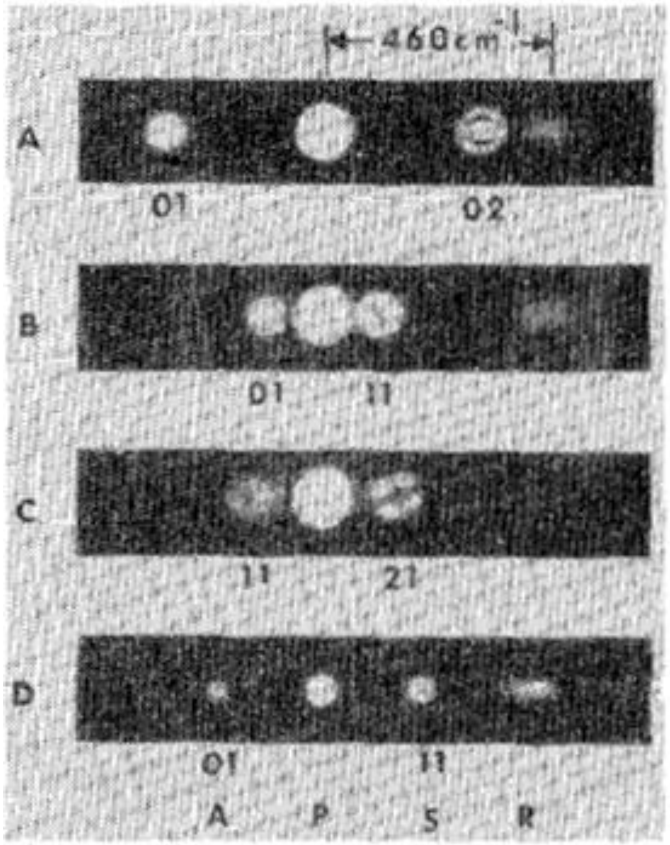
Phase-matched three-wave mixing in silica fiber optical waveguides

R. H. Stolen, J. E. Bjorkholm, and A. Ashkin

Bell Telephone Laboratories, Holmdel, New Jersey 07733
(Received 6 December 1973)







Intermodal Four-Wave Mixing in Telecom

BOSTON UNIVERSITY

IEEE PHOTONICS TECHNOLOGY LETTERS, VOL. 25, NO. 6, MARCH 15, 2013 539

Experimental Investigation of Inter-Modal Four-Wave Mixing in Few-Mode Fibers

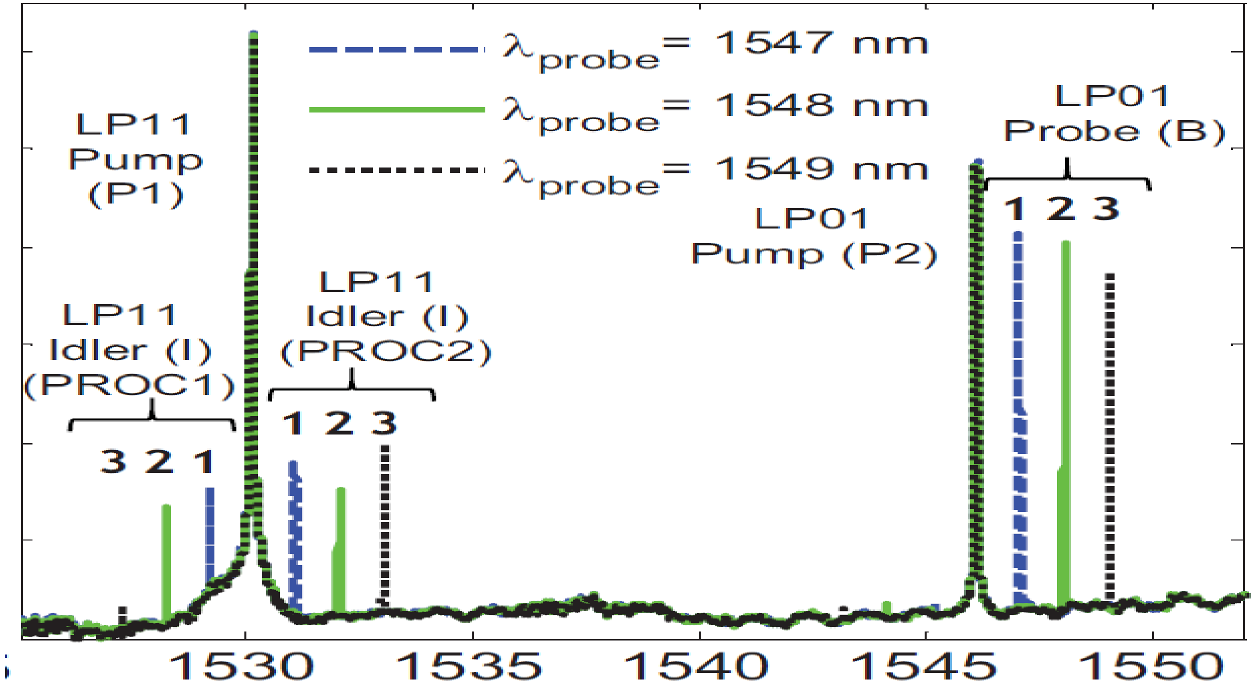
Rene-Jean Essiambre, *Fellow, IEEE,* Miquel A. Mestre, Roland Ryf, *Member, IEEE,* Alan H. Gnauck, *Fellow, IEEE,* Robert W. Tkach, *Fellow, IEEE,* Andrew R. Chraplyvy, *Fellow, IEEE,* Yi Sun, Xinli Jiang, and Robert Lingle, Jr.

Research Article Vol. 24, No. 26 | 26 Dec 2016 | OPTICS EXPRESS 30338

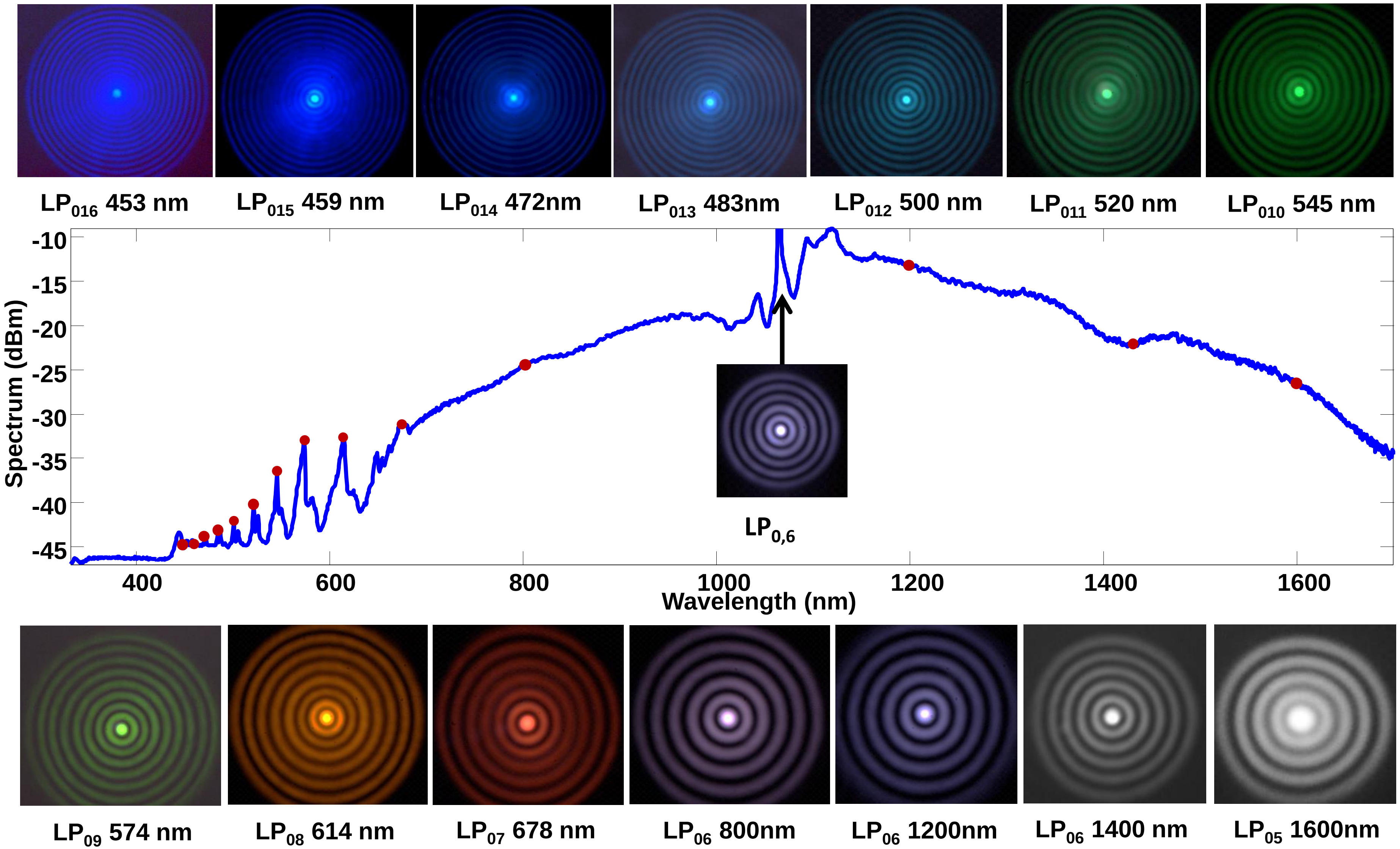
Optics EXPRESS

Inter-modal four-wave mixing study in a two-mode fiber

S. M. M. FRIIS,[1,2,*] I. BEGLERIS,[1] Y. JUNG,[1] K. ROTTWITT,[2] P. PETROPOULOS,[1] D. J. RICHARDSON,[1] P. HORAK,[1] AND F. PARMIGIANI[1,3]



Four-Wave Mixing over 2 Octaves



J. Demas et al, *Optica* v2, p14, 2015

New nanosecond continuum for excited-state spectroscopy

Chinlon Lin and R. H. Stolen

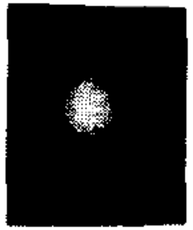
Bell Telephone Laboratories, Holmdel, New Jersey 07733

(Received 20 October 1975; in final form 1 December 1975)

Appl. Phys. Lett. **28**, 216 (1976)

441.6 nm

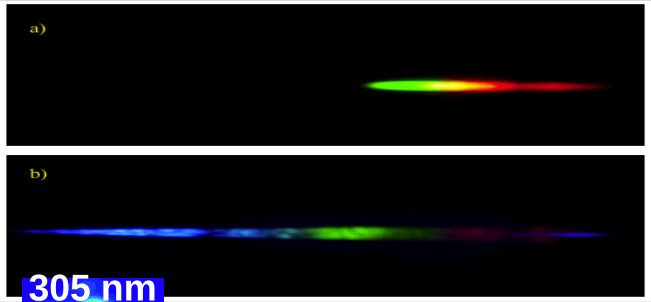
632.8 nm

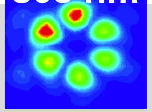


Multimode Supercontinuum in PCF

Center Launch

Offset Launch

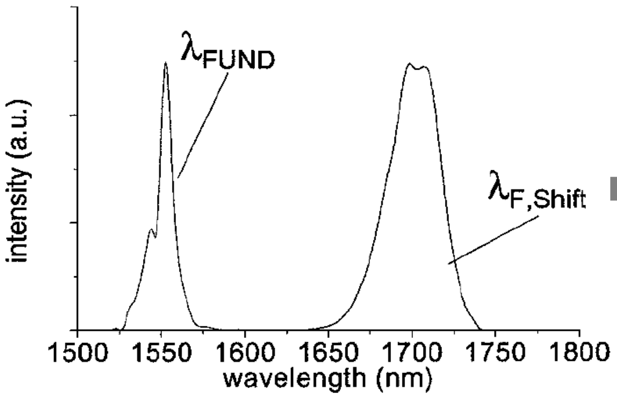


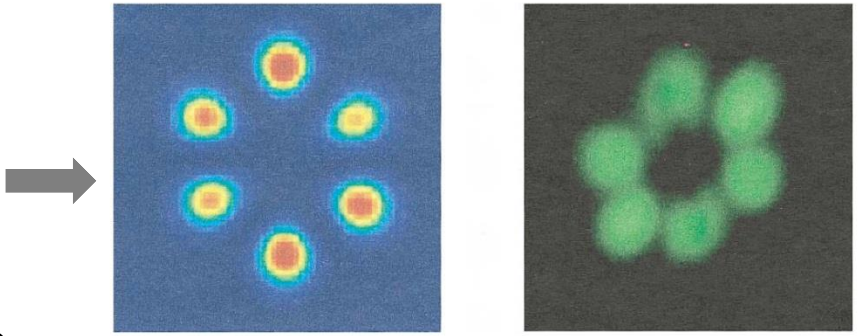


A. Efimov et al, Opt. Exp. 11, 910, 2003

Multimode Harmonics

F.G. Omenetto et al, "Simultaneous generation of spectrally distinct 3[RD] harmonics in a PCF"
OL 26, 1158, 2001

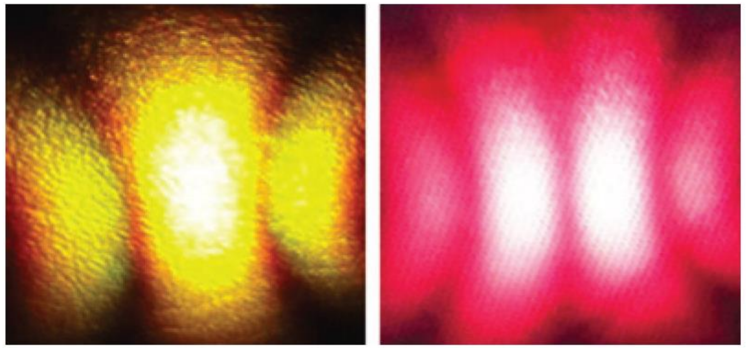




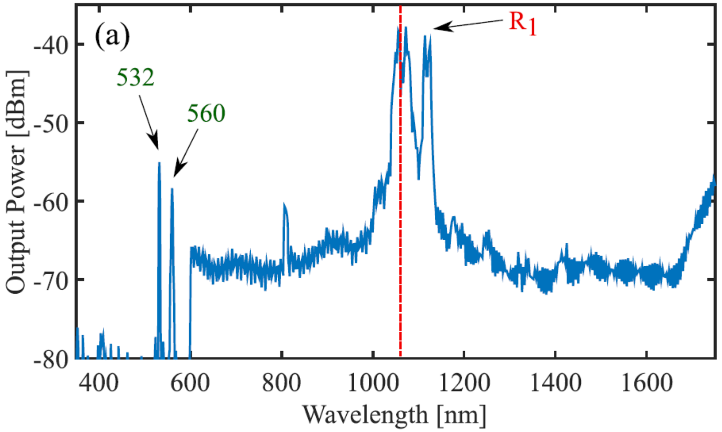
Simultaneous generation of second and third harmonics by red-shifted solitons in photonic crystal fibre

Jinhui Yuan, Xinzhu Sang, Chongxiu Yu, Xiangwei Shen, Kuiru Wang, Binbin Yan, Ying Han, Guiyao Zhou and Lantian Hou

ELECTRONICS LETTERS 30th August 2012 Vol. 48 No. 18



M. A. Eftekhar et al, "Instant and efficient second-harmonic generation and downconversion in unprepared graded-index multimode fibers,"
Opt. Lett. 42, 3478-3481 (2017)



Multimode Solitons

Theoretical Prediction
1980

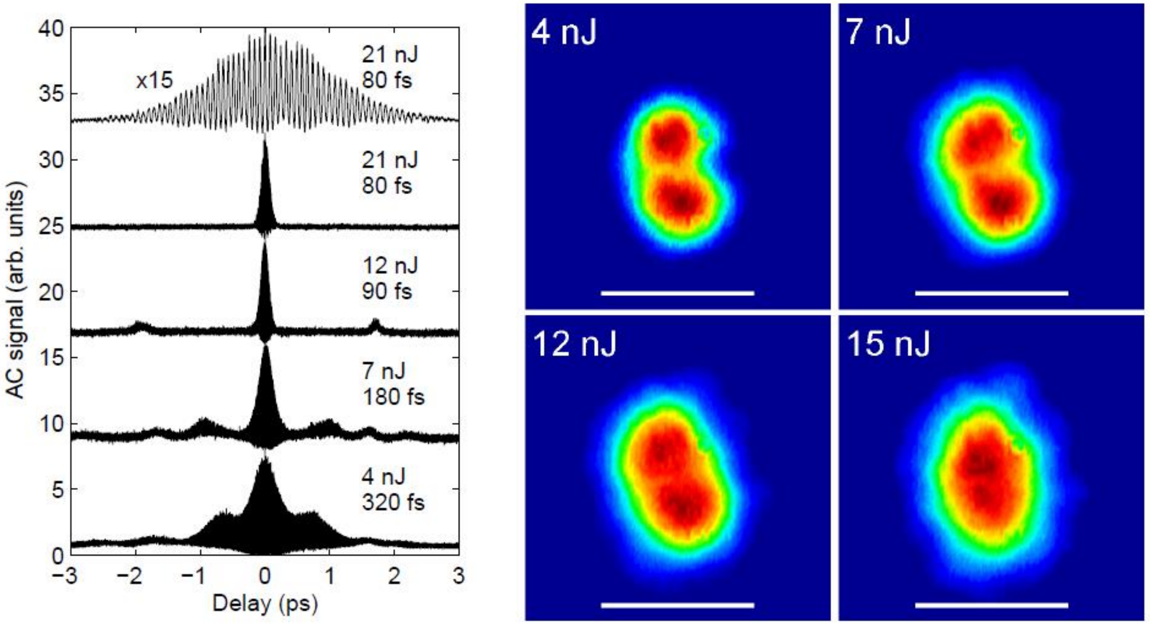
416 OPTICS LETTERS / Vol. 5, No. 10 / October 1980

Self-confinement of multimode optical pulse in a glass fiber

Akira Hasegawa

Bell Laboratories, Murray Hill, New Jersey 07974

Experiment
2015



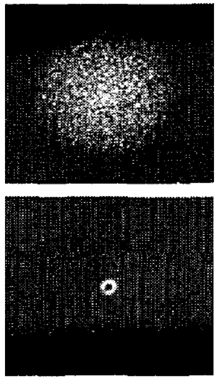
L.G. Wright et al, Nat. Photon. 9, 310 (2015)

Nonlinear Beam Cleanup

588 OPTICS LETTERS / Vol. 12, No. 8 / August 1987

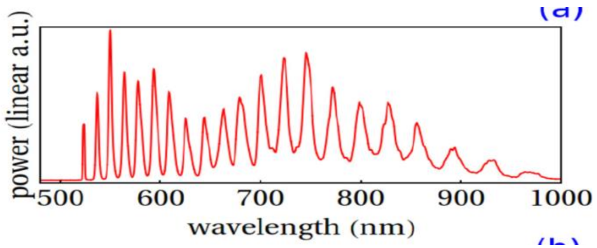
Observation of self-focusing in optical fibers with picosecond pulses

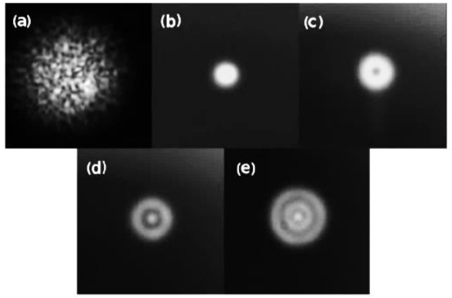
P. L. Baldeck, F. Raccah, and R. R. Alfano

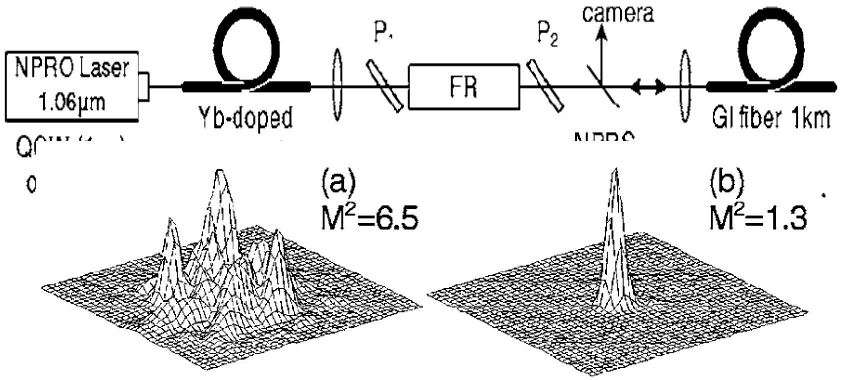


SBS: L. Lombard et al, OL, 31, 158, 2006

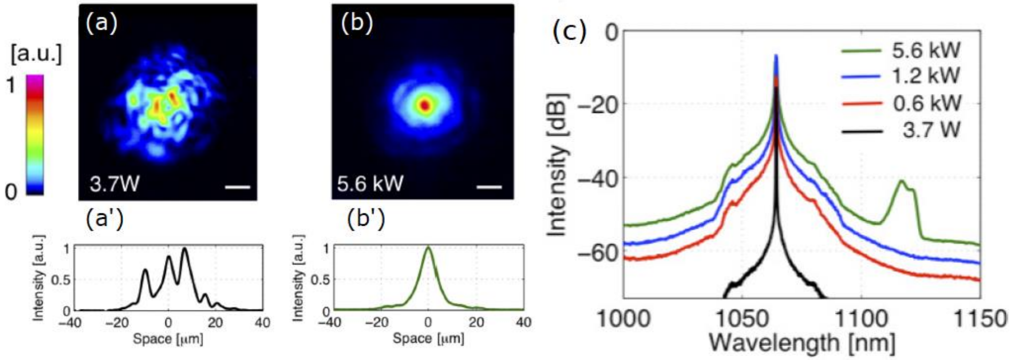
SRS: H. Pourbeyram, et al, APL 102, 201107 (2013)







Kerr: K. Krupa et al, Nat. Photon. 11, 237 (2017)

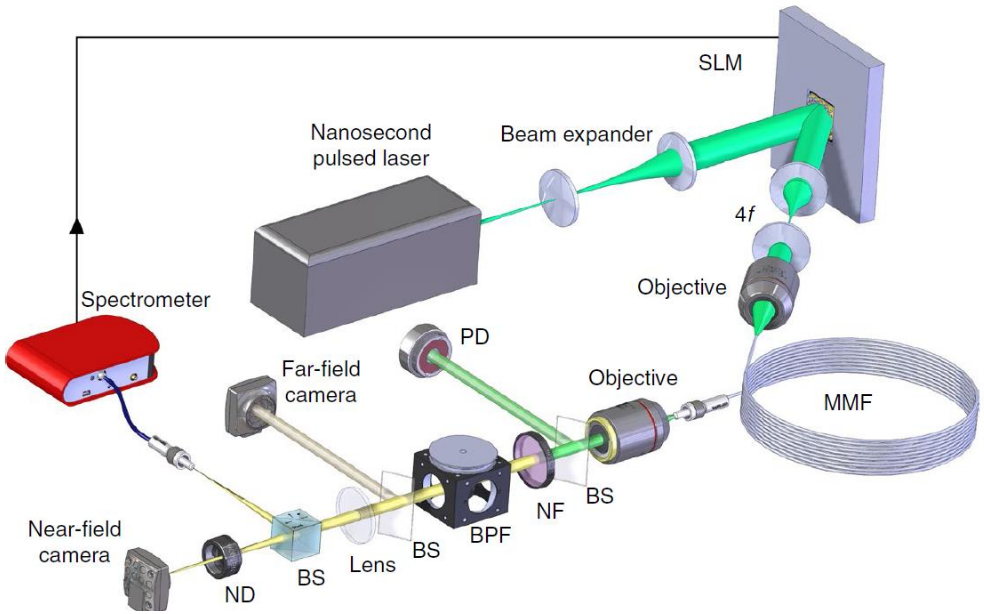


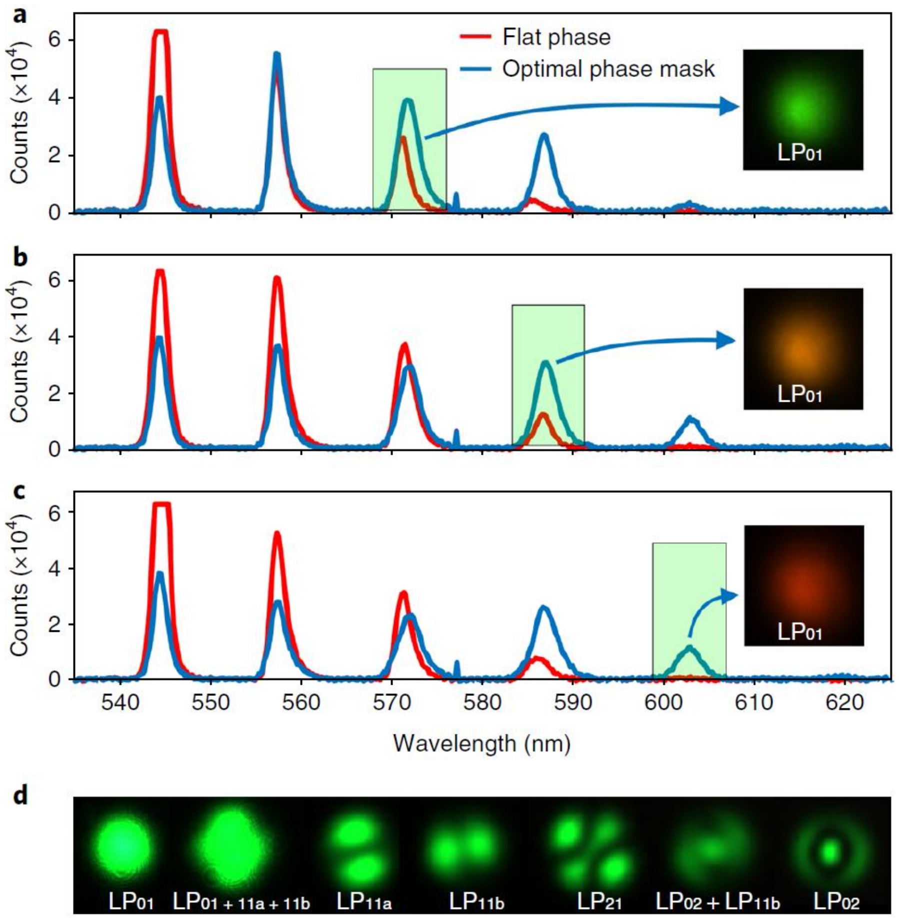
Adaptive Nonlinearity Control

ARTICLES
https://doi.org/10.1038/s41566-018-0167-7
NATURE PHOTONICS | VOL 12 | JUNE 2018 | 368-374
nature photonics

Adaptive wavefront shaping for controlling nonlinear multimode interactions in optical fibres

O. Tzang, A.M. Caravaca-Aguirre, K. Wagner, R. Piestun

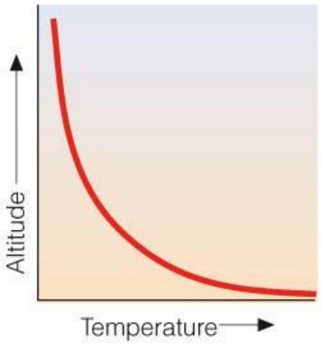




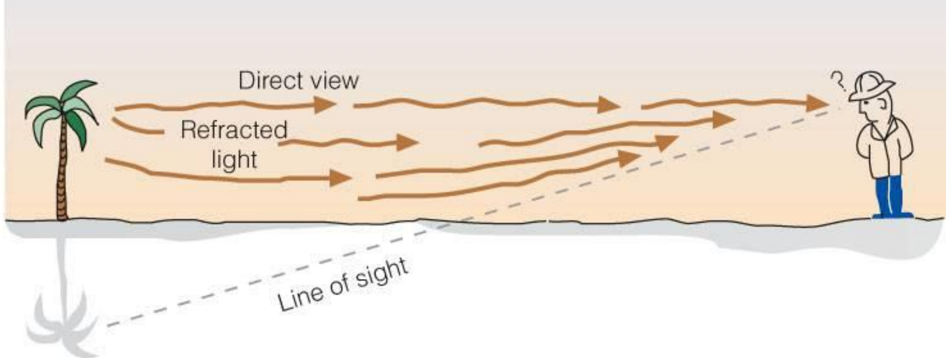
Outline

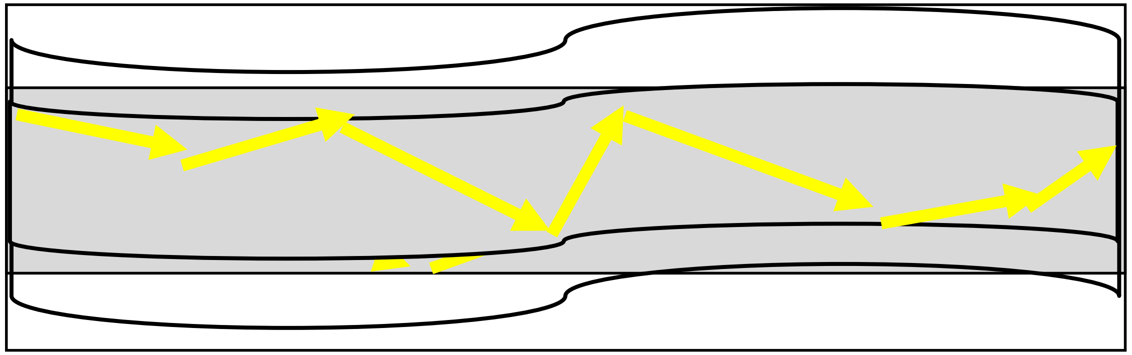
- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

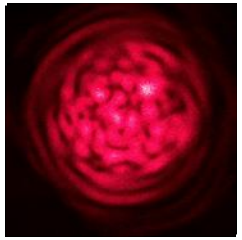
Optical Mirages & Mode coupling



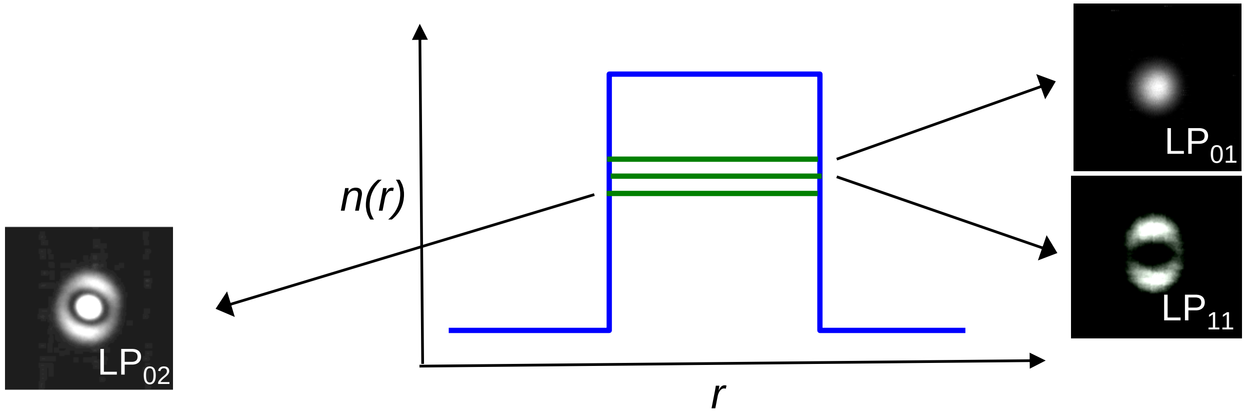
Mirages… randomize light's $\vec{k}$







Bent fibers… randomize $\vec{k}$

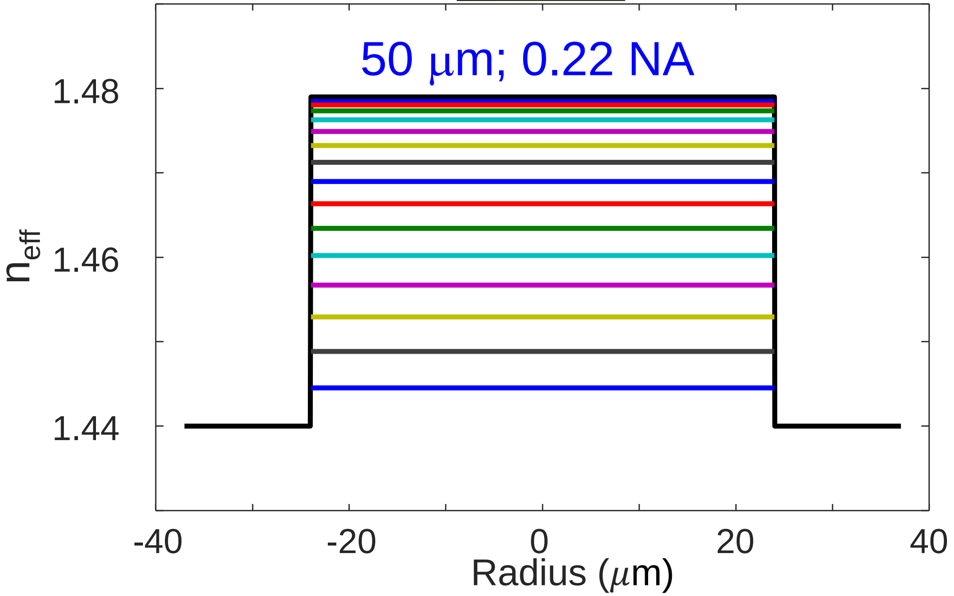


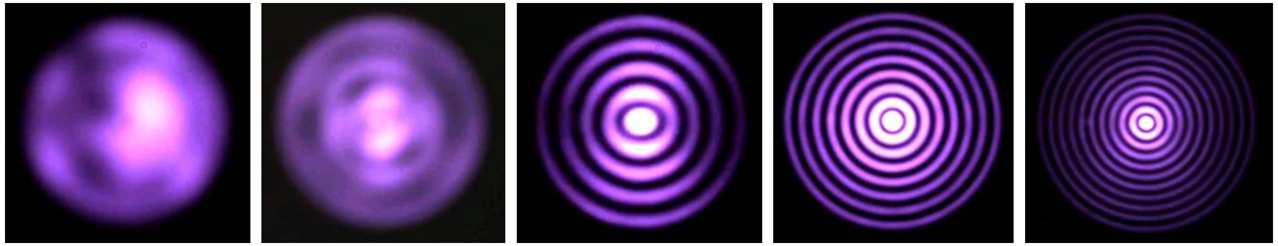
Coupling

$$\int E_1 \cdot P_{pert}(r,\varphi,z) \cdot E_2^* \cdot e^{-i\frac{2\pi}{\lambda}\Delta n_{eff}} \cdot dA\,dz$$

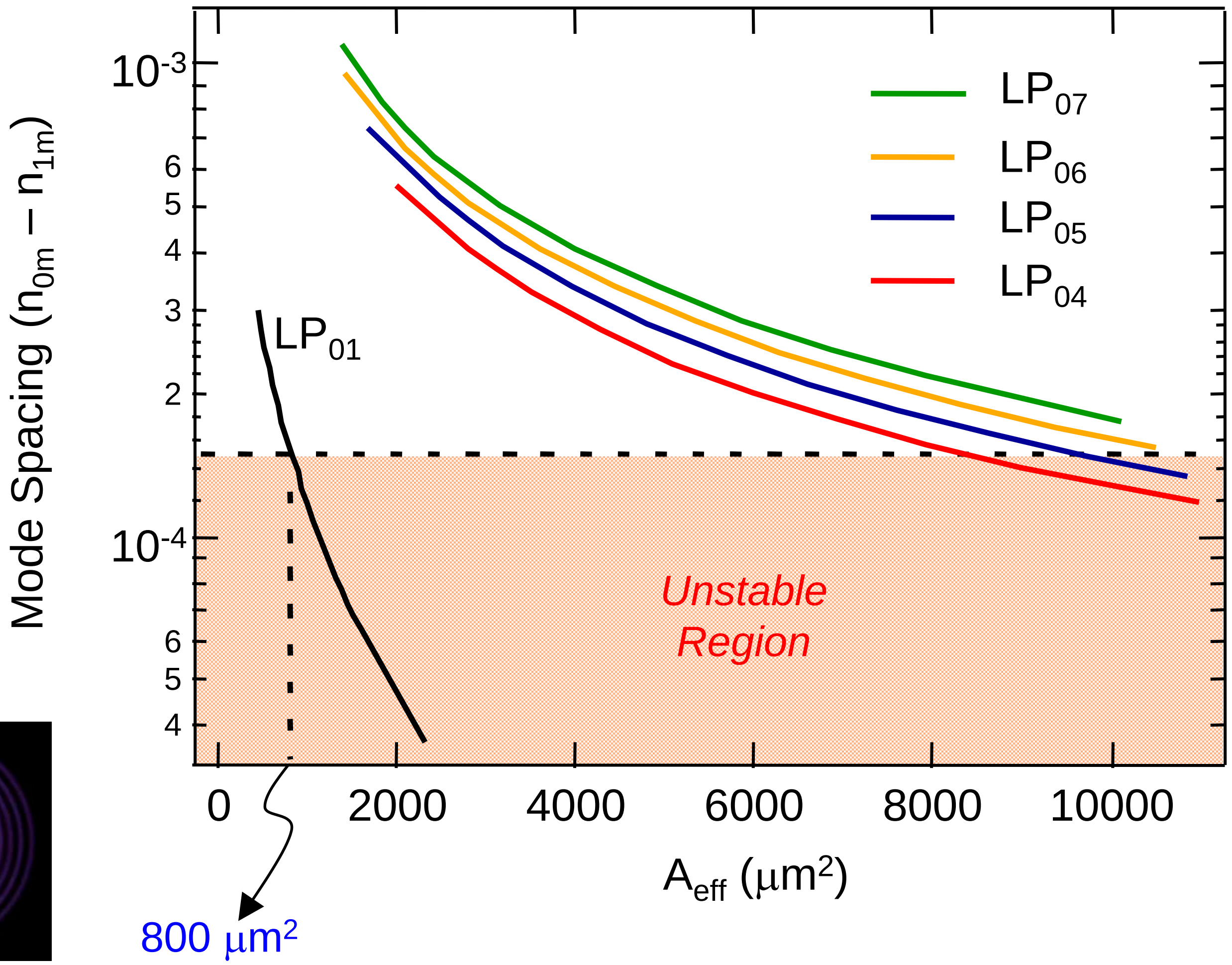
To minimise <u>linear</u> coupling, ↑↑ Δn_{eff}

Natural stability of LP_{0m} modes (Bessel beams)





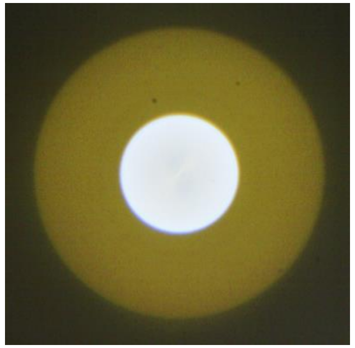
Large Mode Area Fibers



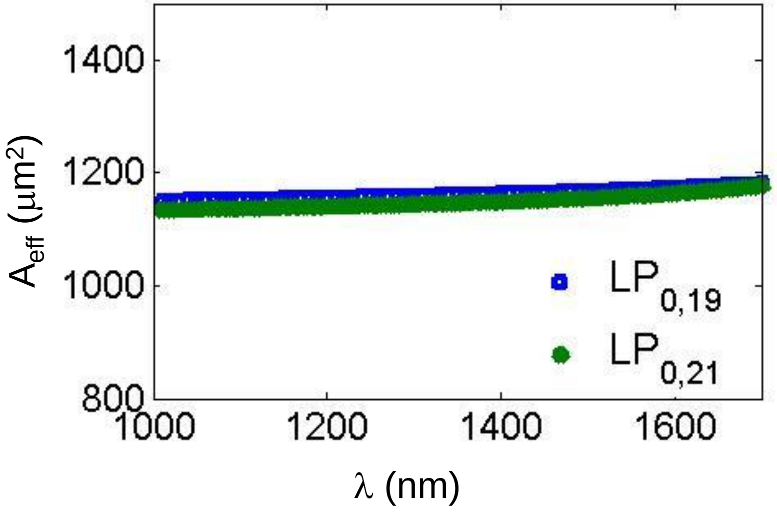
S. Ramachandran et al, OL, v31, p1797 , 2006

S. Ramachandran et al, Laser & Photon. Rev. , v2, p. 429, 2008

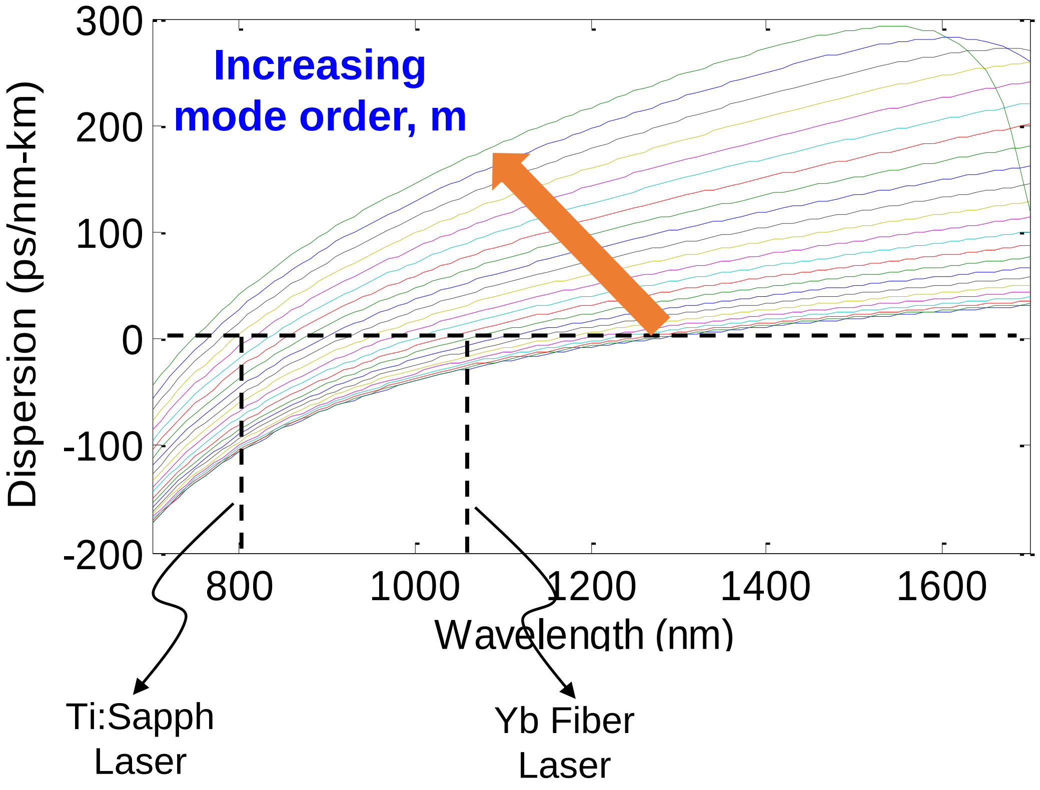
Nonlinear Figure of Merit – Step Index Multimode Fibers



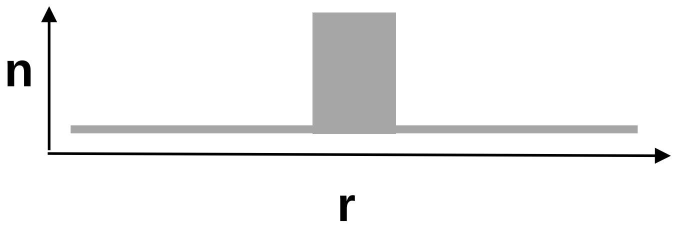
50 – 100 μm; 0.2 – 0.3 NA



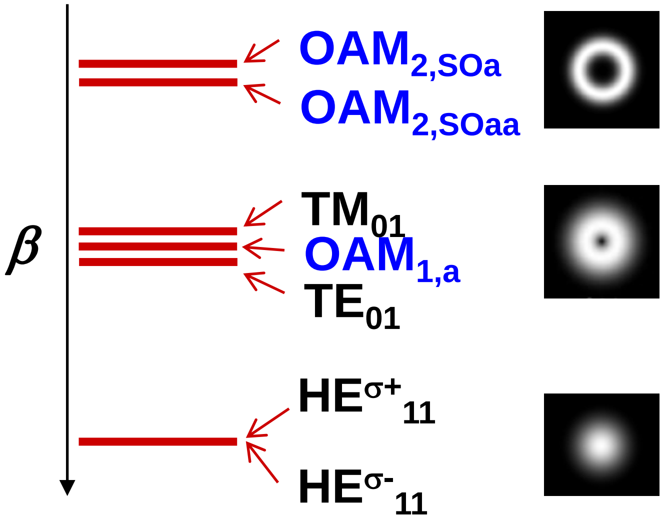
$D \bullet A_{eff}$ ~ 10^3–10^4 more than PCF



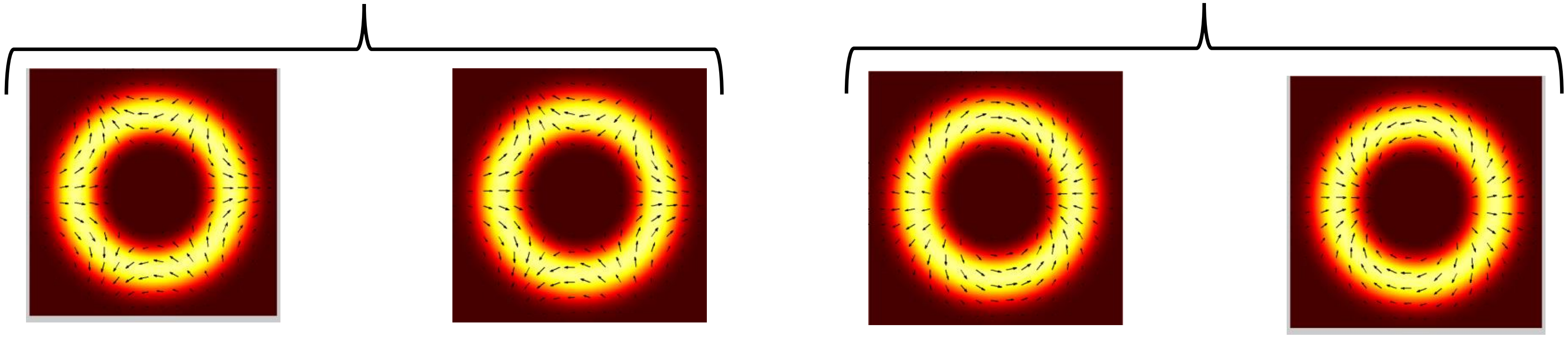
OAM modes in fibers



$$\vec{E_t} \sim J_L(k_t \cdot r) \cdot e^{\pm iL\varphi} \cdot \begin{Bmatrix} \hat{\sigma}^+ \\ \hat{\sigma}^- \end{Bmatrix}$$



Spin-Orbit Aligned (SOa) β_{SOa} | Spin-Orbit anti-Aligned (SOaa) β_{SOaa}

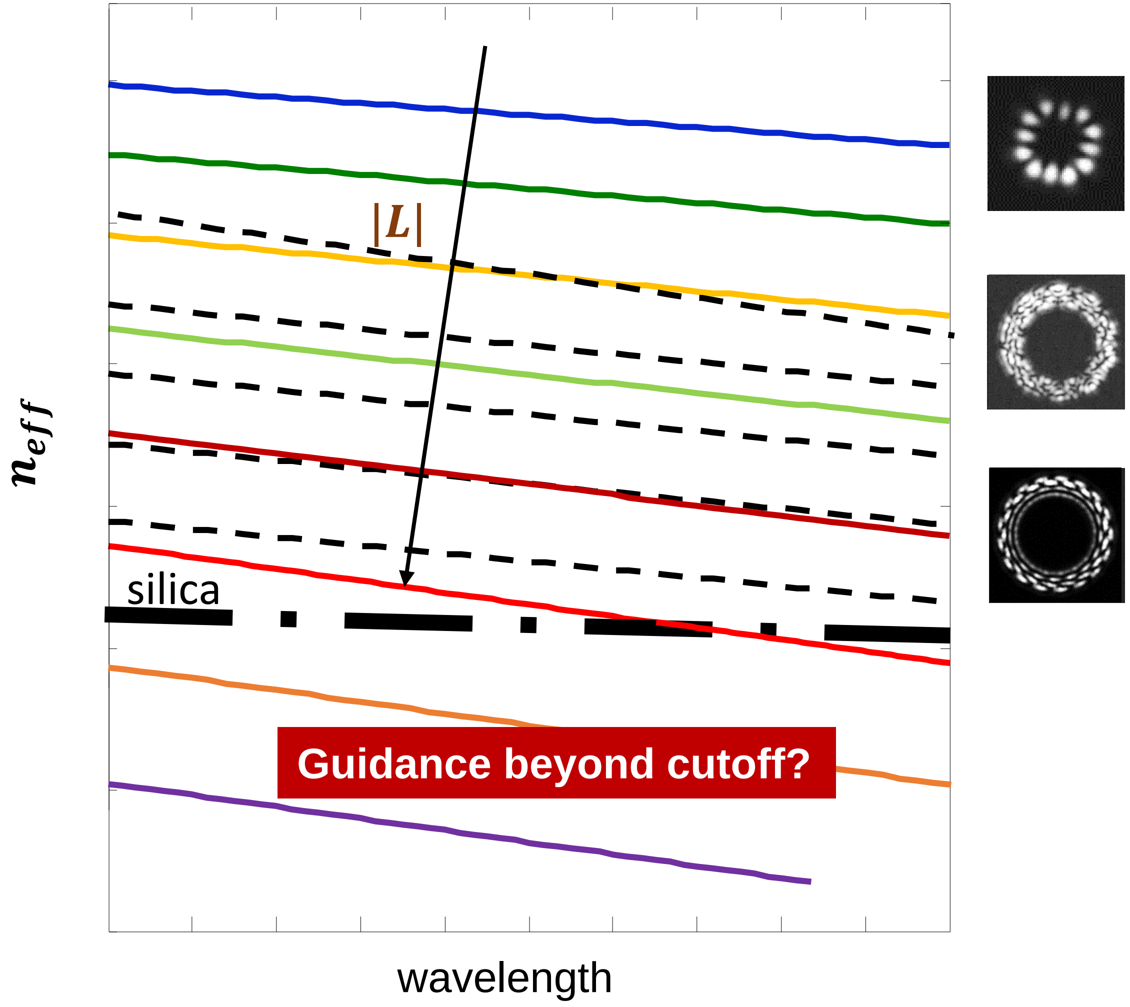


$\hat{\sigma}^+; L = +2$ | $\hat{\sigma}^-; L = -2$ | $\hat{\sigma}^-; L = +2$ | $\hat{\sigma}^+; L = -2$

Z. Ma, S. Ramachandran, *Nanophotonics* 10, 209 (2021)

Scaling channel count with OAM modes & ring core fibers

$\hat{\sigma}^{\pm} e^{\pm i|L|\varphi}$ each $|L|$ contains four stable modes

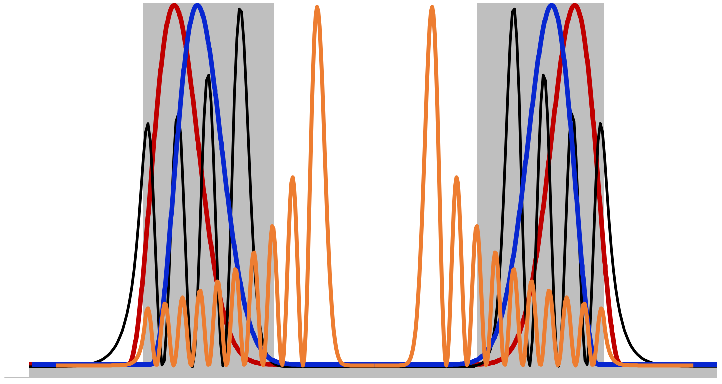


Spin-Orbit Interaction

$$\Delta n_{eff} \propto L \int E^2(r) \cdot \frac{d\Delta n(r)}{dr} \cdot dr$$

Intensity

Index gradient



Uncoupled OAM modes

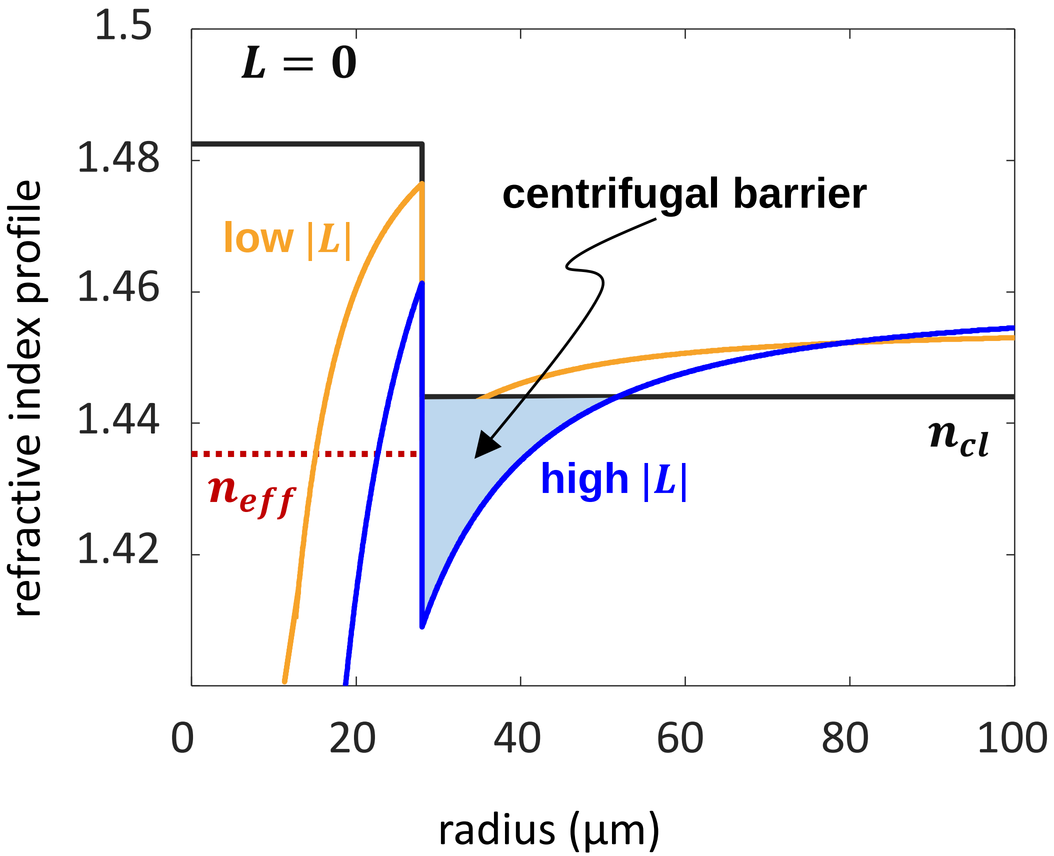
- High enough $|L|$
- Ring-core reduced high-m modes

Mode-count bottleneck?

- 12 for ~km fiber

S. Ramachandran et al, Opt. Exp. 23, 3721 (2015)

Topologically confined modes (TCMs) & Centrifugal barriers



Total internal reflection not satisfied

- $n_{eff} < n_{cl}$

With OAM present?

$$\frac{d^2F(r)}{dr^2} + \frac{1}{r}\frac{dF(r)}{dr} + \left[k_0^2\left(n^2(r) - \frac{L^2}{k_0^2 r^2}\right) - \beta^2\right]F(r) = 0$$

A. Ghatak et al, *Intro. to Fiber Optics* (1998)

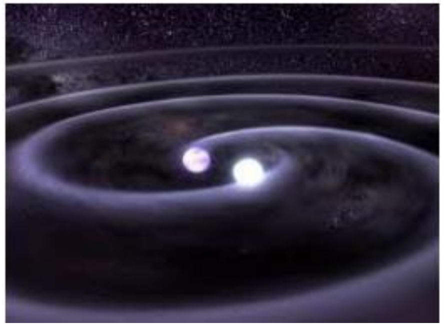
$$n_{OAM}^2(r) = n^2(r) - \frac{L^2}{k_0^2 r^2}$$

Centrifugal barrier effect

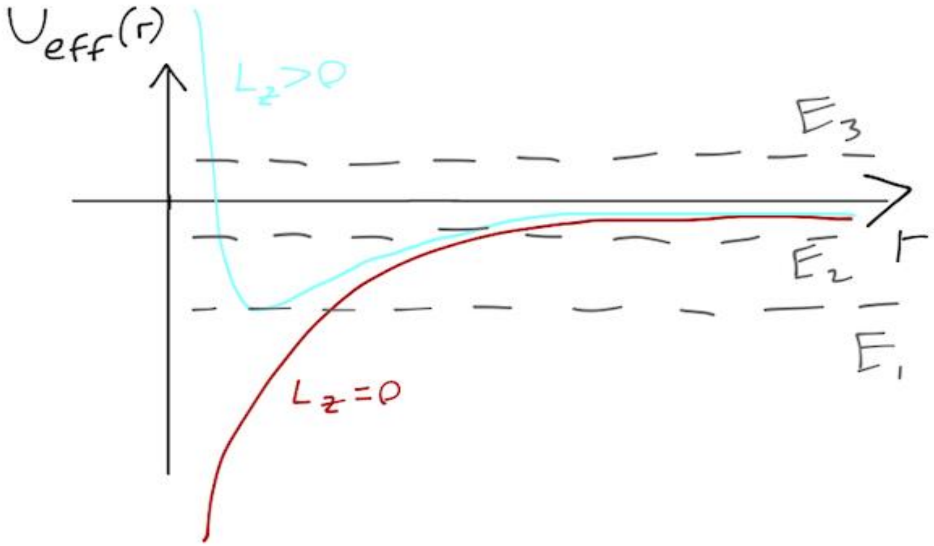
- OAM-induced confinement

Centrifugal Barriers in other fields of physics

Binary Stars

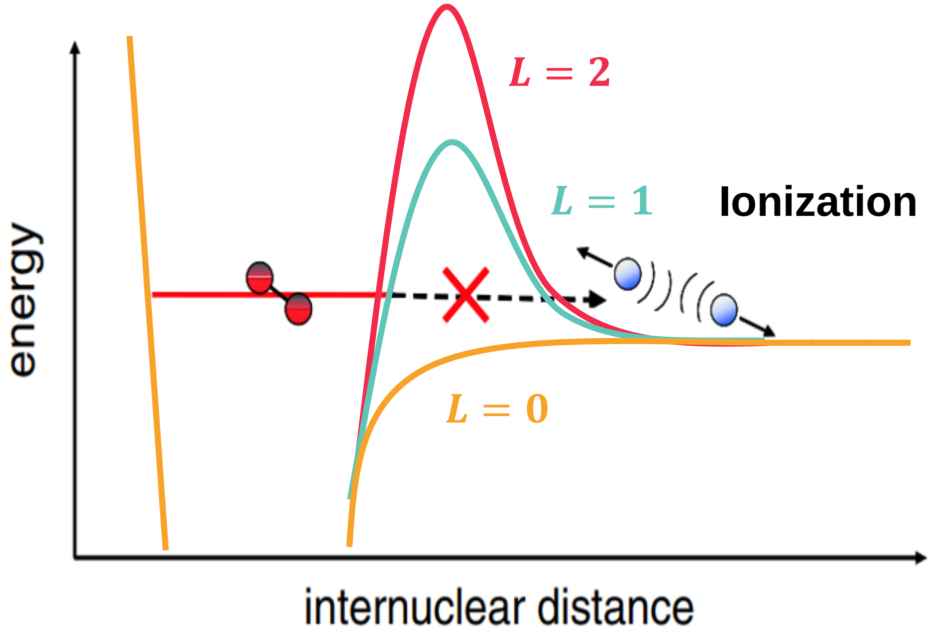


$$U_{eff} = \frac{\ell^2}{2\mu r^2} - \frac{GMm}{r}$$



physicscourses.colorado.edu

Feshbach Molecules (short range potentials)



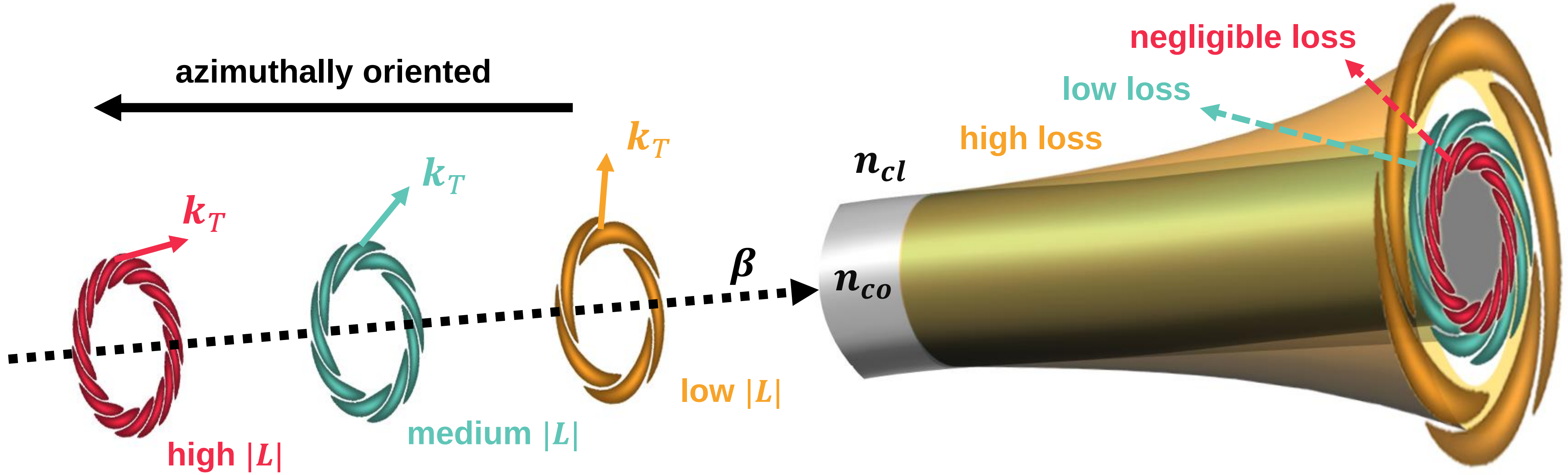
S. Knoop et al, PRL 100, 083002 (2008)

P. Bucksbaum et al, PRL 56, 2590 (1986)

Also in Nuclear Physics (short range potentials)

J. M. Blatt et al, *Theoretical Nuclear Physics* (1952)

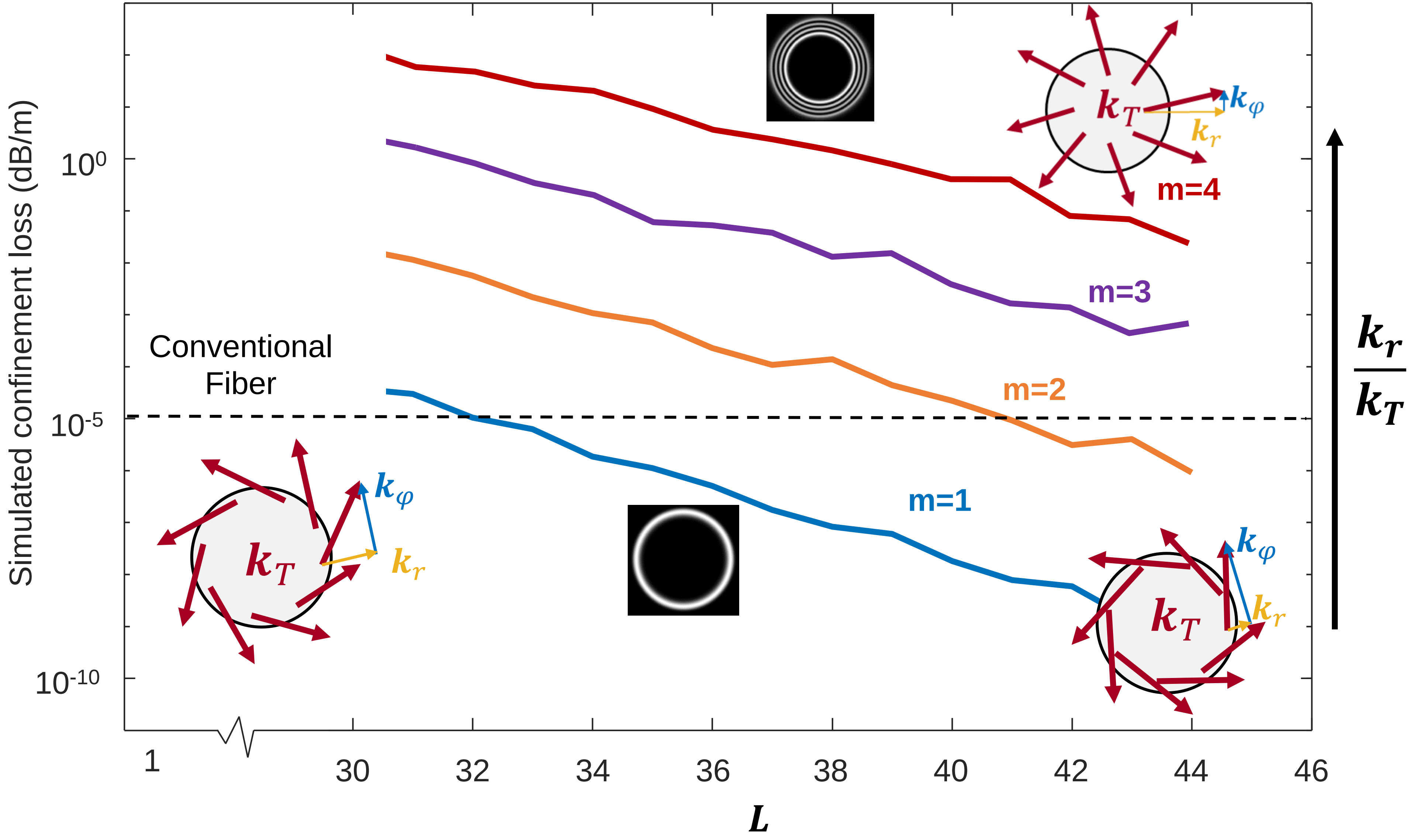
Topological Confinement & the Ray Picture



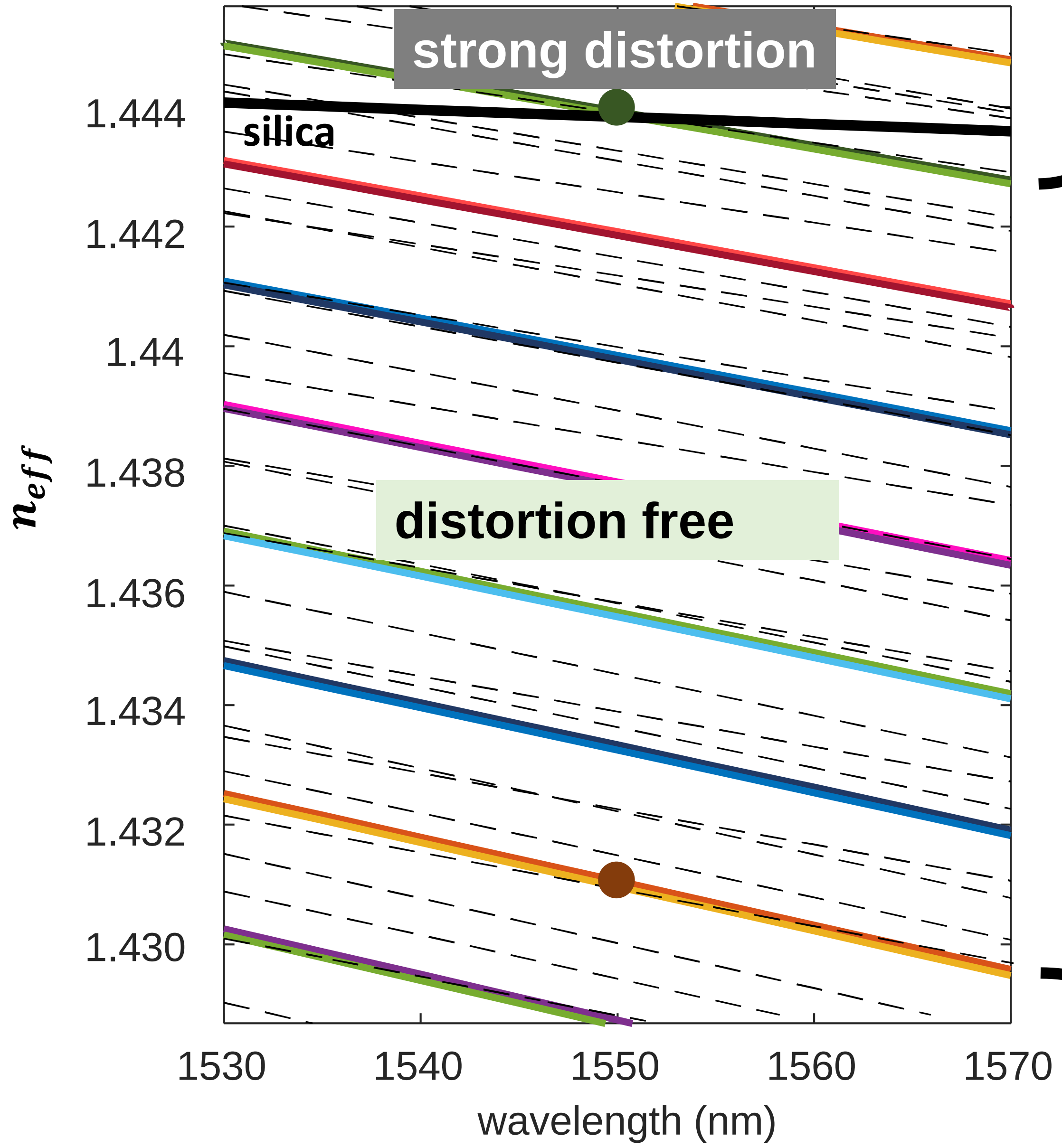
Radial Dependence of Topological Confinement

increases with radial order m

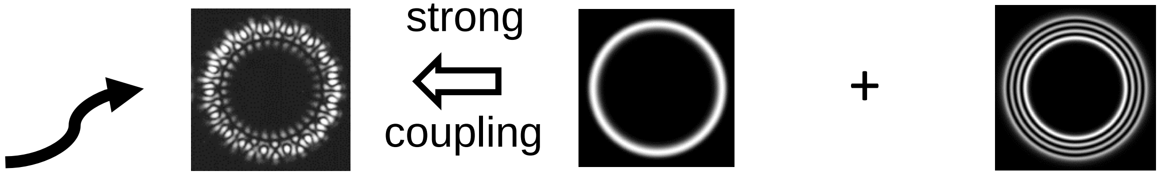
Loss decreases with azimuthal order L



Frustrated coupling of TCMs



L = 33 | L = 33, m = 1 | L = 16, m = 4

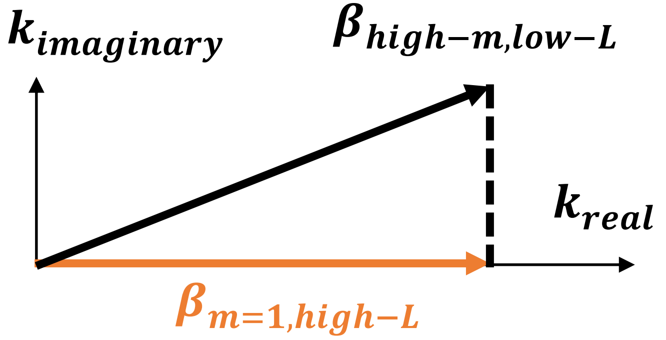


zero loss | zero loss

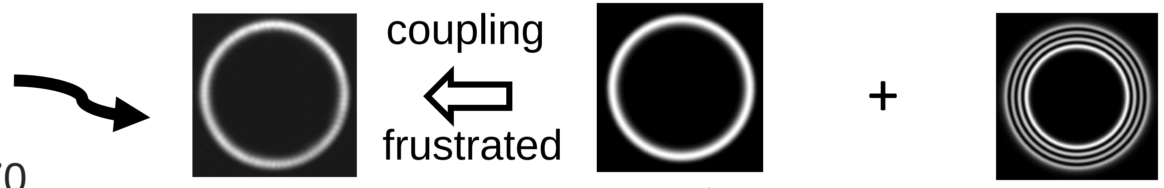
experiment | **simulation**

Frustrated coupling

- modes with very different losses don't couple



L = 39 | L = 39, m = 1 | L = 25, m = 4



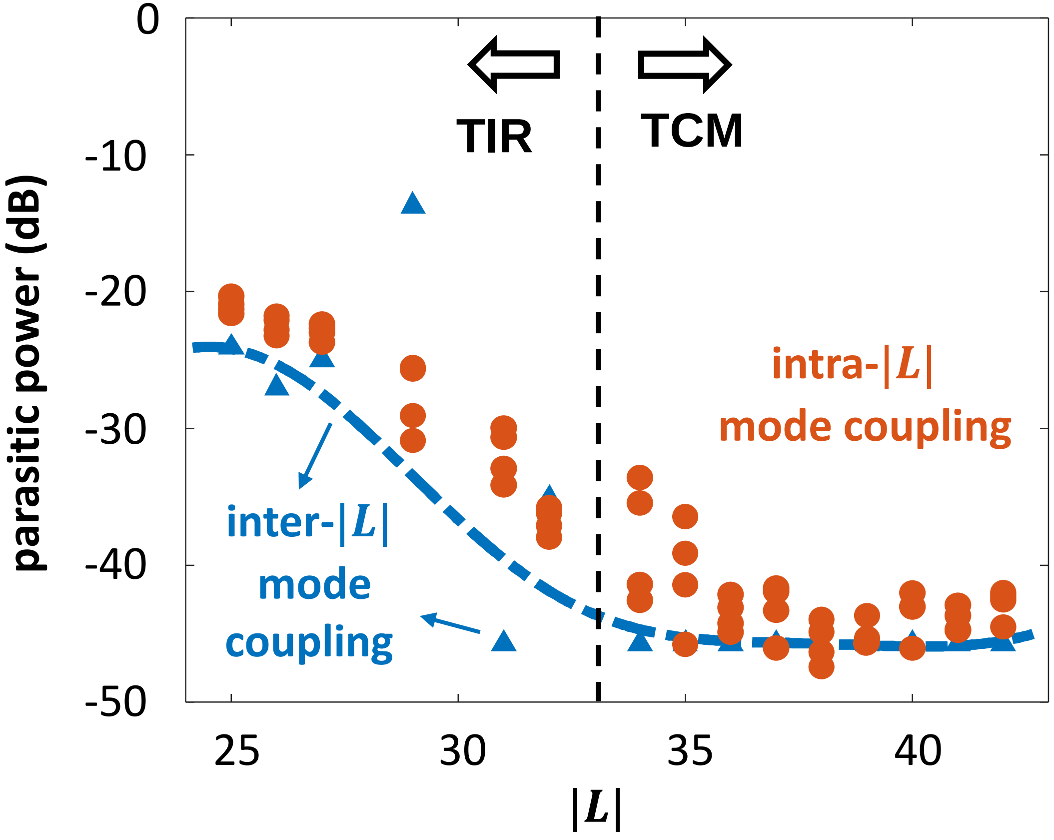
loss ~10^{-9} dB/m | loss ~10^{3} dB/m

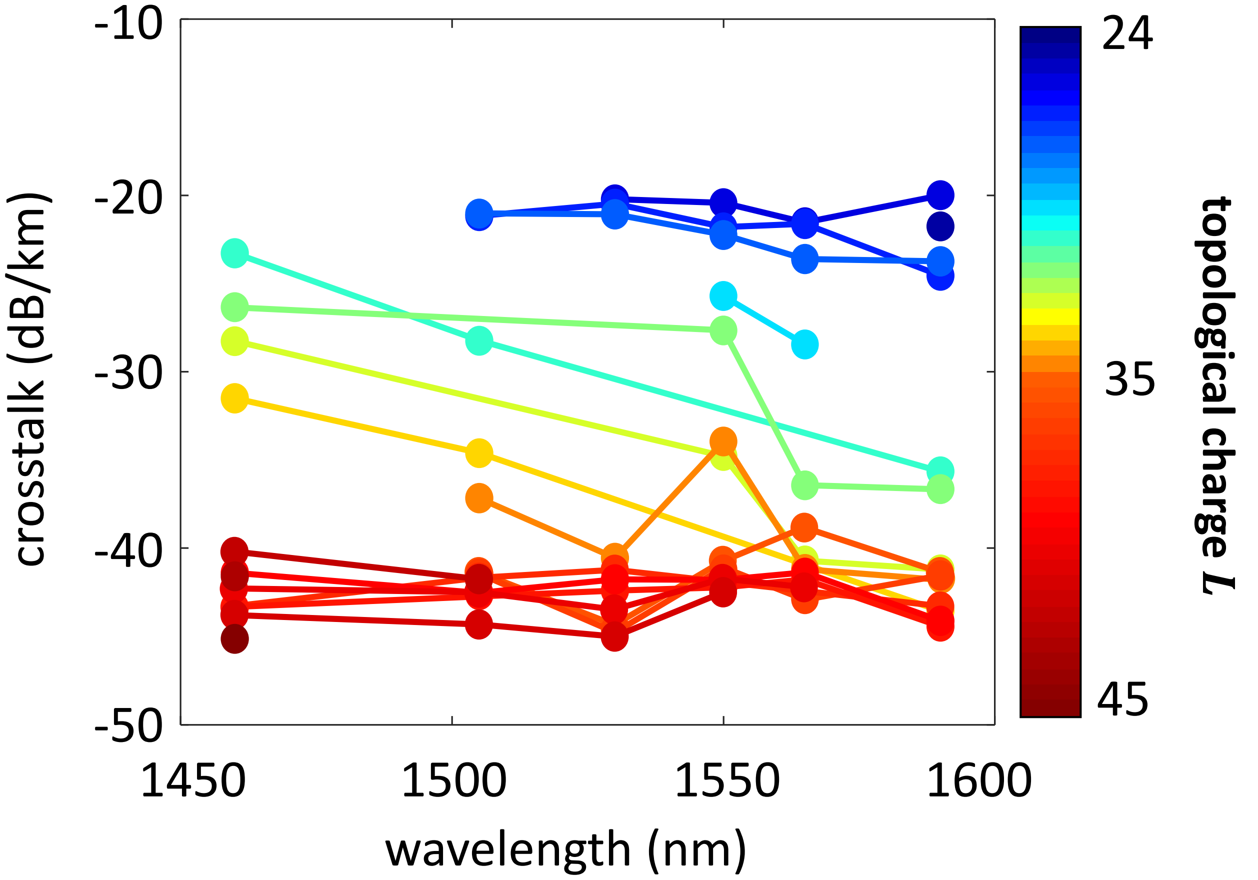
experiment | **simulation**

Z. Ma et al, CLEO, SM1F.4 (2021)

D. B. Stegall et al, Photon. Tech. Lett. 11, 343 (1999)

Crosstalk: TIR modes vs. TCMs

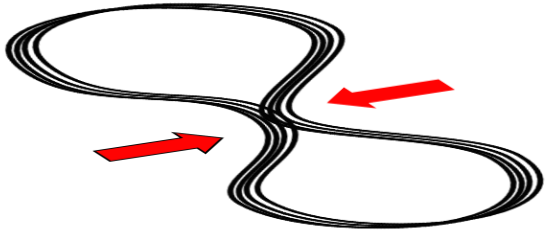




Same xtalk with
6-mm-radius bend!

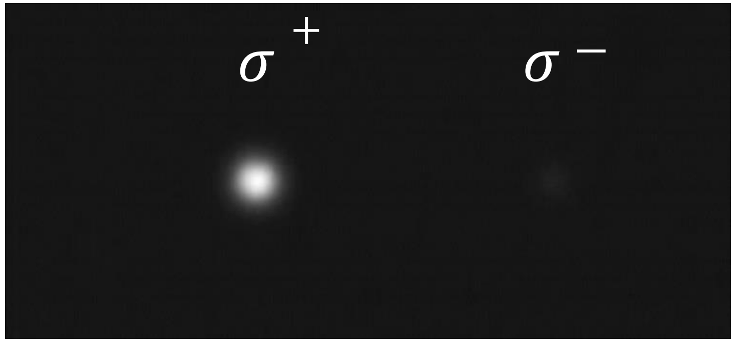
Z. Ma et al, CLEO, SM4L.1 (2023)

OAM conservation – Bend Insensitivity & Naturally PM

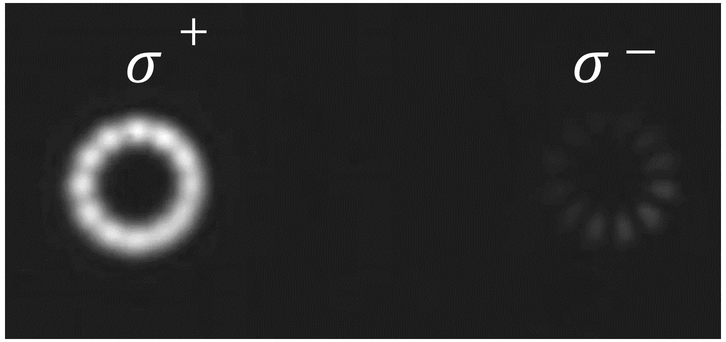


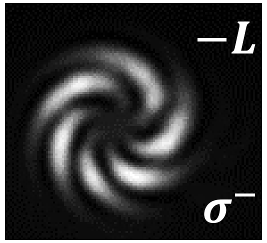
- **OAM conserved**
- **Modes more stable for higher *L***

***L* = 0**



|*L*| = 6





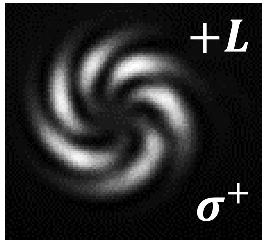
Momentum Exchange

$\Delta L = \pm 2|L|$

↔

$|\Delta\sigma| = \pm 2$

(birefringence)

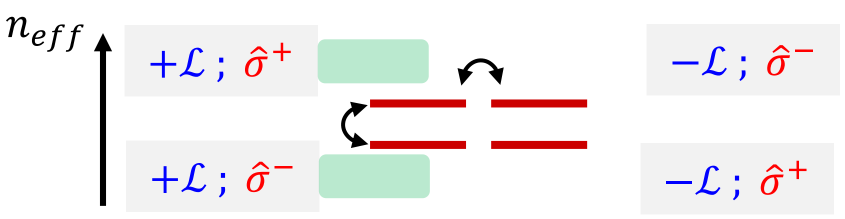


N. Bozinovic et al, Opt. Lett. v37p2451 (2012)
C. N. Alexeyev, J. Optics, 14 085702 (2012)
P. Gregg et al, *Optica* v2, p267, 2015

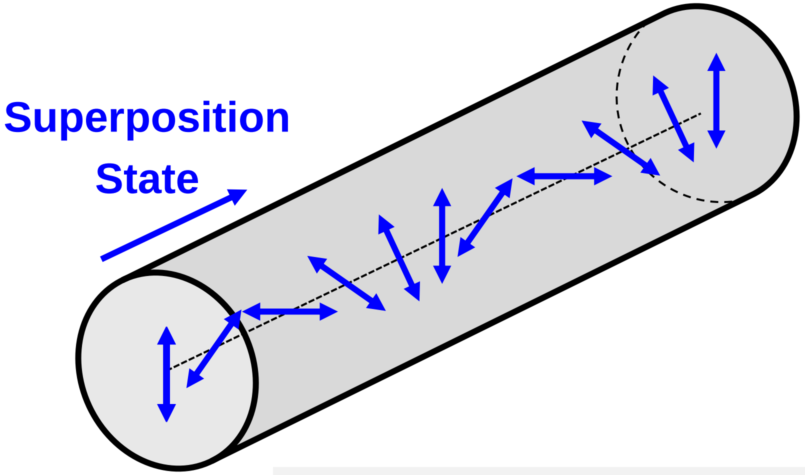
Engineered Optical Activity

BOSTON UNIVERSITY

D.L.P. Vitullo *et al*., *PRL.* 118, 083601 (2017)



A.P. Greenberg et al. *Nature Comm.*, 11, 5257 (2020)



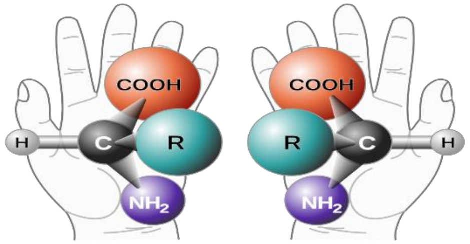
Controlled Superposition Possible

$$\vec{E}(r,\varphi) = F(r)\cdot\exp(i\mathcal{L}\varphi)\cdot[\hat{\boldsymbol{\sigma}}^{+} + \hat{\boldsymbol{\sigma}}^{-}]exp\left(i\frac{2\pi}{\lambda}\Delta n_{eff}.z\right)]$$

Optical Activity

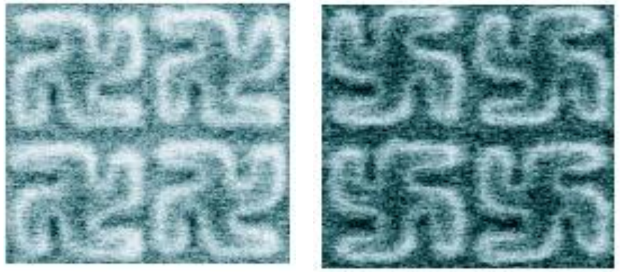
- Obtained in purely isotropic medium
- Engineerable inherent chirality

Chiral Molecules

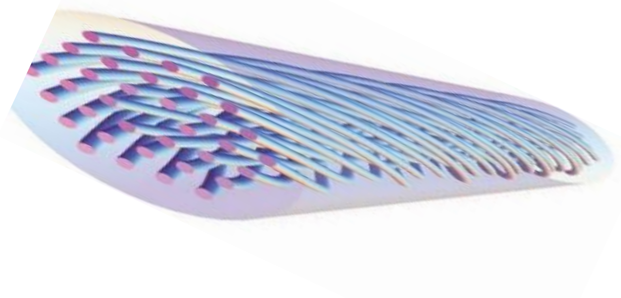


Amino acids
Source: NASA

Nanostructured Metasurfaces



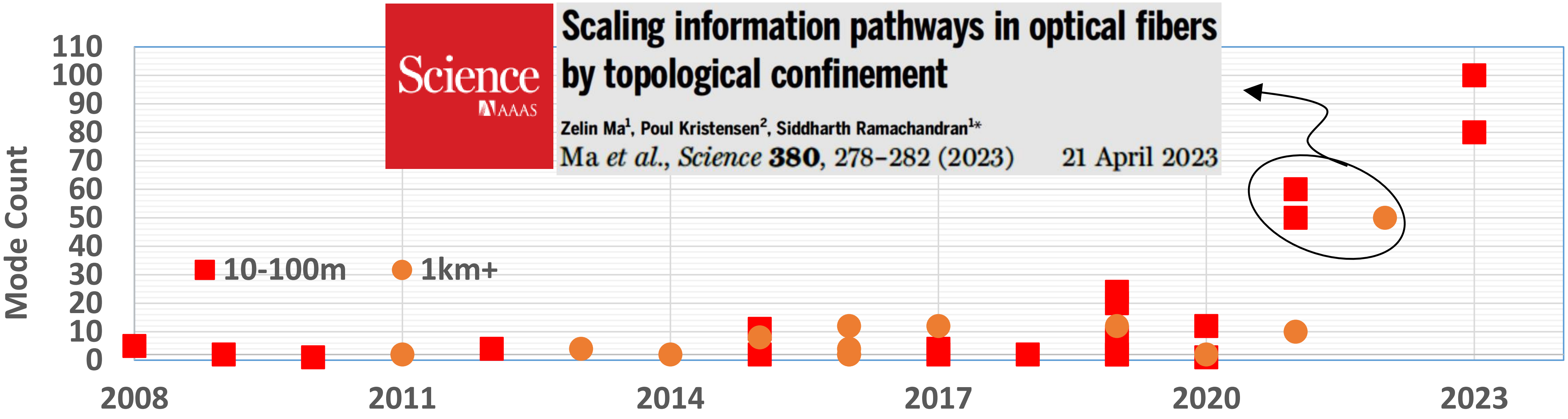
Twisted Fibers



V. Kopp *et al*, Science 305, 5680 (2004)
M. Kuwata-Gonokami *et al*, PRL 95, 227401 (2005)
P.St.J Russell *et al*, Phil. Trans. R. Soc. A 375, 20150440 (2017)

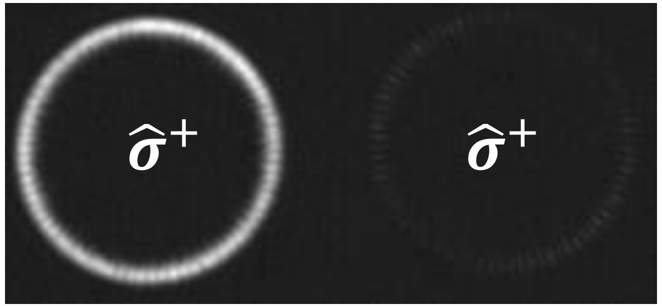
Linear Properties of Multimode Fibers

BOSTON UNIVERSITY



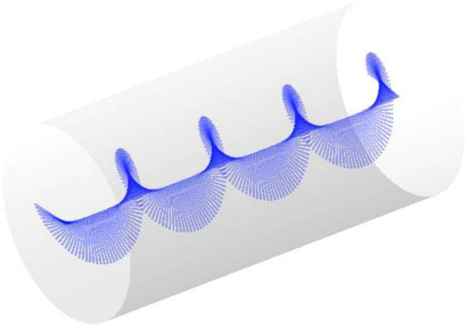
~20 dB PER after ~km

even in a strictly circular fiber



$L =$ 40 SOaa

Behaves like a Chiral Medium



Outline

- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

Geometric Parametric Instabilities

PRL 116, 183901 (2016) PHYSICAL REVIEW LETTERS

K. Krupa, A. Tonello, A. Barthélémy, V. Couderc, B.M. Shalaby, A. Bendahmane, G. Millot and S. Wabnitz

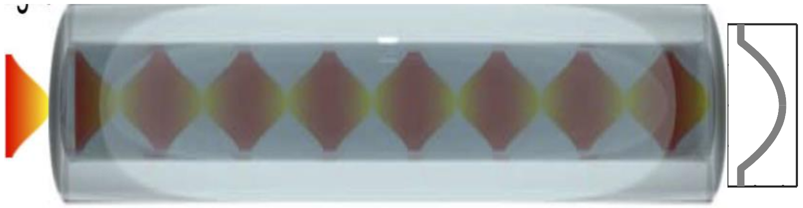
Observation of Geometric Parametric Instability Induced by the Periodic Spatial Self-Imaging of Multimode Waves

A systematic analysis of parametric instabilities in nonlinear parabolic multimode fibers

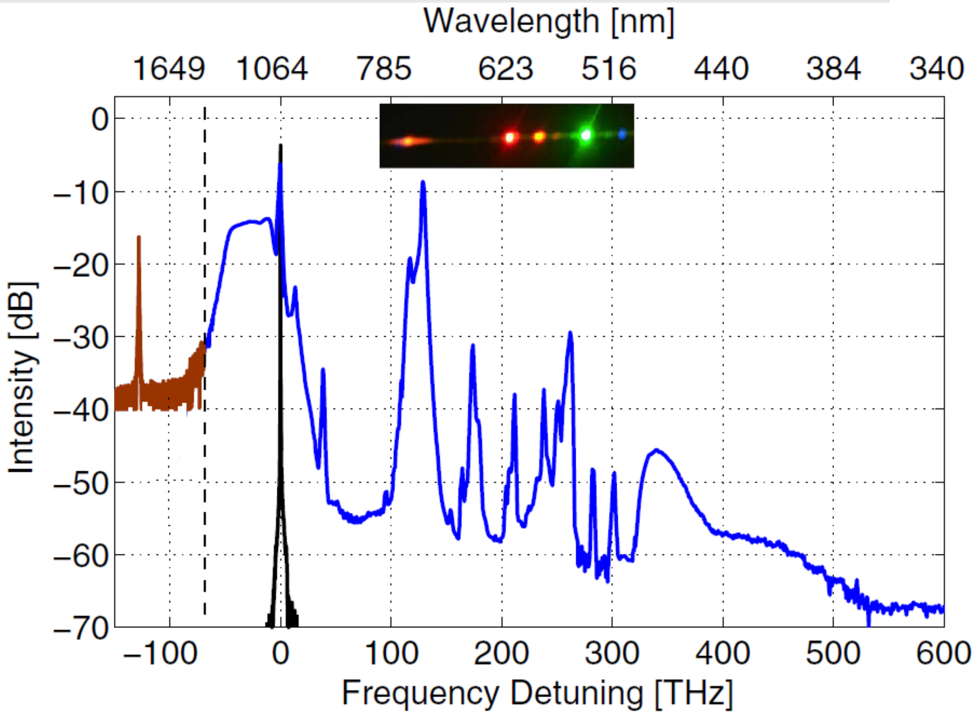
Cite as: APL Photonics 4, 022803 (2019); doi: 10.1063/1.5044659
Submitted: 14 June 2018 • Accepted: 2 October 2018 •
Published Online: 13 December 2018

View Online Export Citation

H. E. Lopez-Aviles,[1,a)] F. O. Wu,[1] Z. Sanjabi Eznaveh,[1] M. A. Eftekhar,[1] F. Wise,[2] R. Amezcua Correa,[1] and D. N. Christodoulides[1]

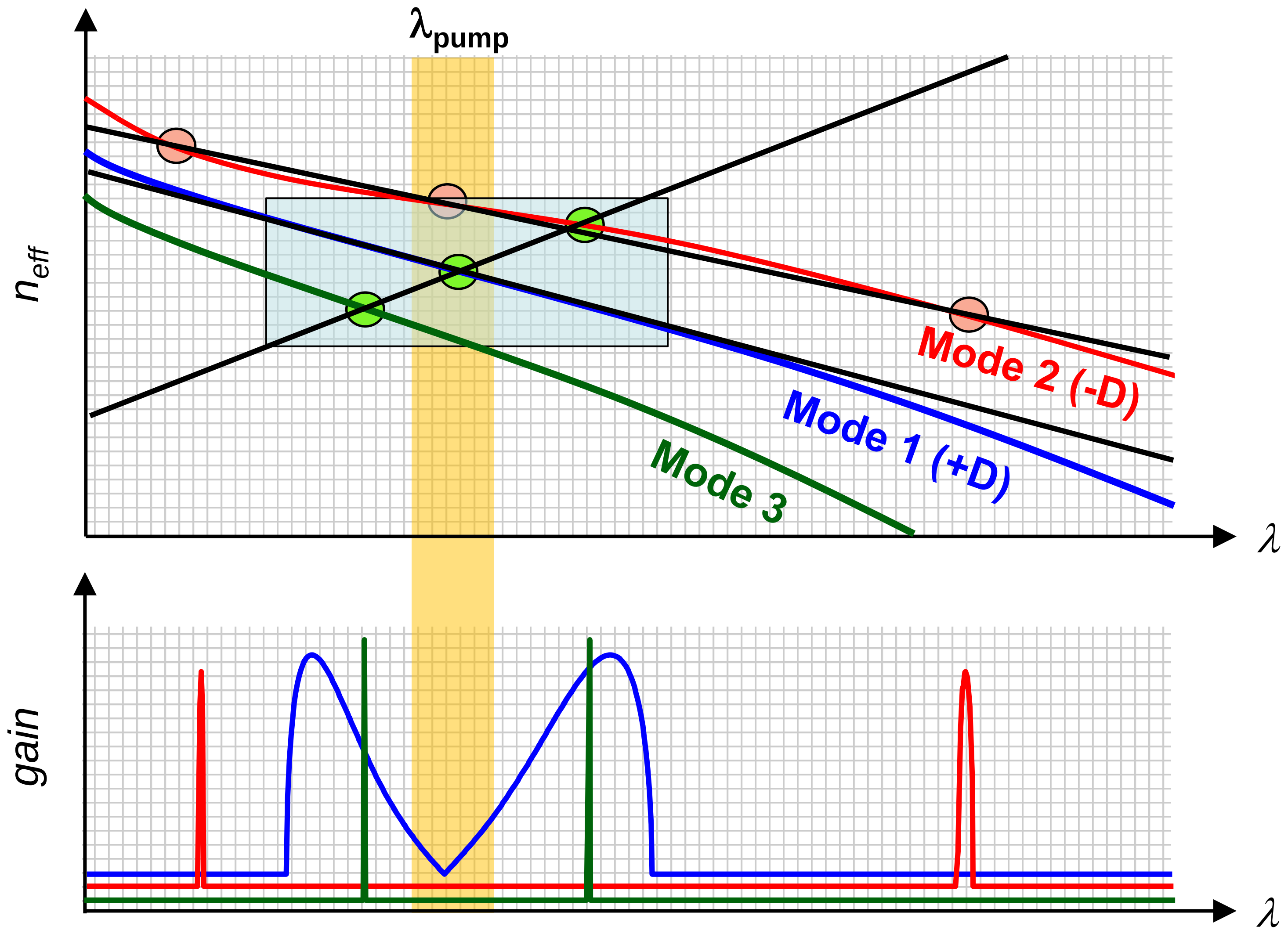


Periodic focus
⇒ Periodic Intensity
⇒ Nonlinear Grating
⇒ Phase Matching
• No need for +D in fiber



Group velocity ⇔ Phase Matching Gradients ⇔ FWM bandwidth

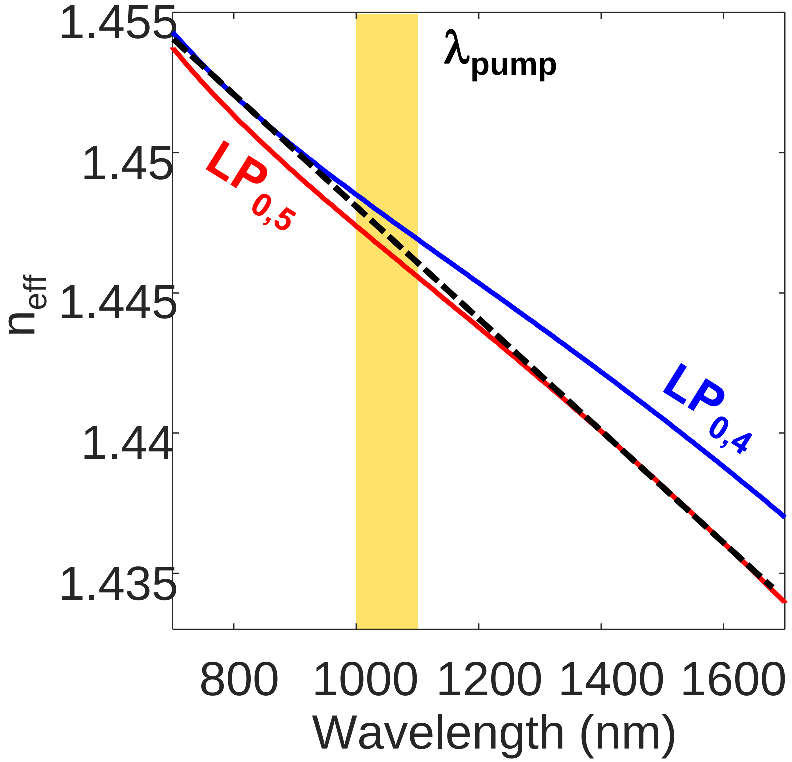
Straight line in n_{eff} + energy matching → phase matching



Group-velocity *and* Phase-velocity Matched FWM

Pump with **$LP_{0,4}$** + **$LP_{0,5}$**

- ➢ Phase matching line falls between n_{eff} curves at λ_{pump}

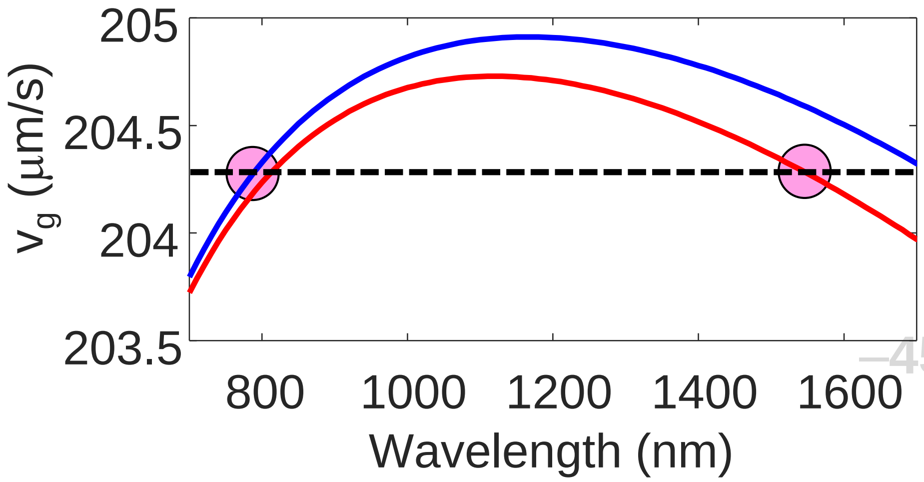


Phase matching line intersects and lies tangent

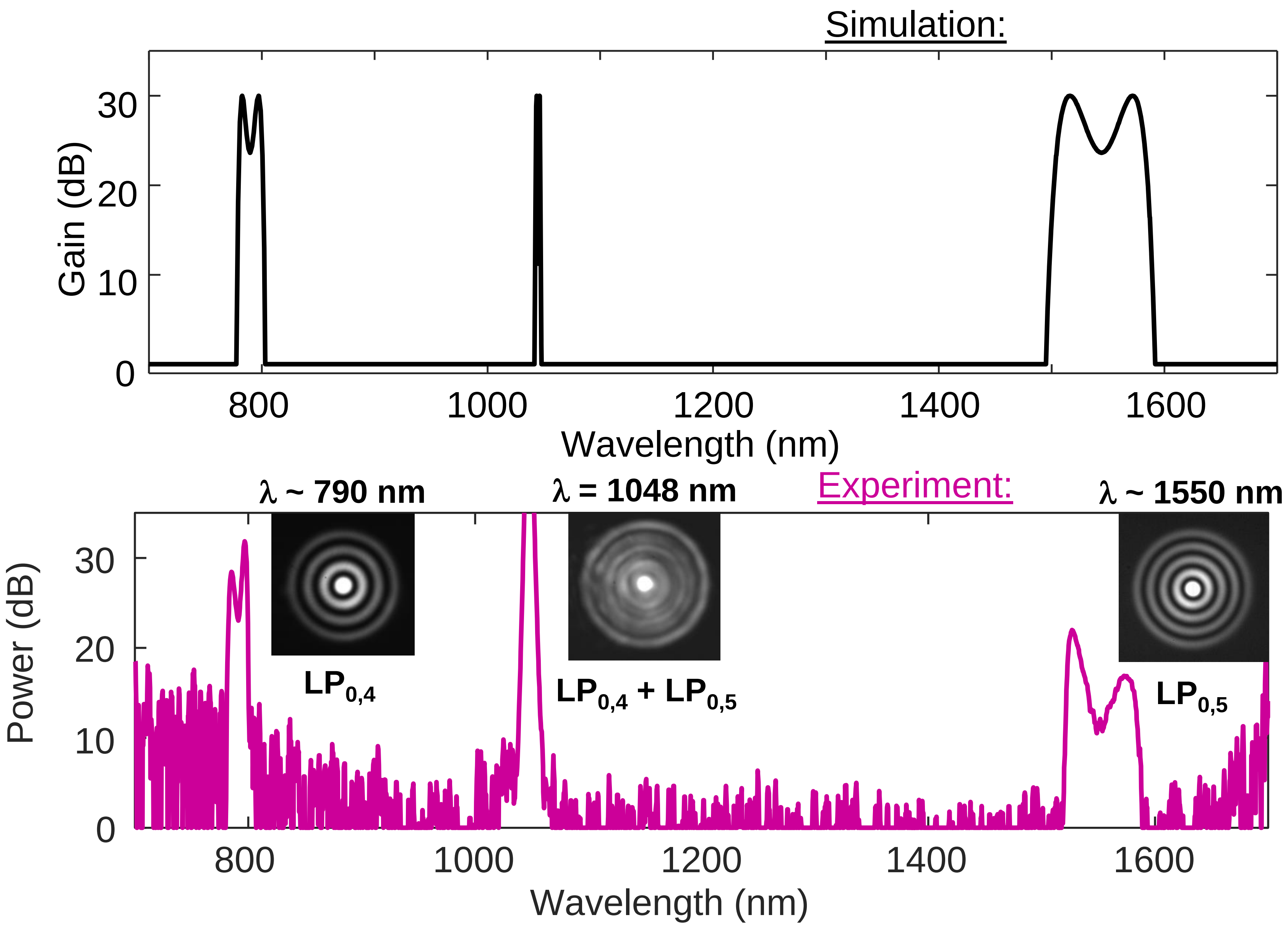
- ➢ Large intersection range → **bandwidth ↑↑↑**
- ➢ Implies group-velocity-matching:

$$\beta_4(\lambda_{pump}) + \beta_5(\lambda_{pump}) - \beta_4(\lambda_{as}) - \beta_5(\lambda_s) \approx 0$$

$$\Delta\omega \frac{d\beta_4}{d\omega} = \Delta\omega \frac{d\beta_5}{d\omega}$$



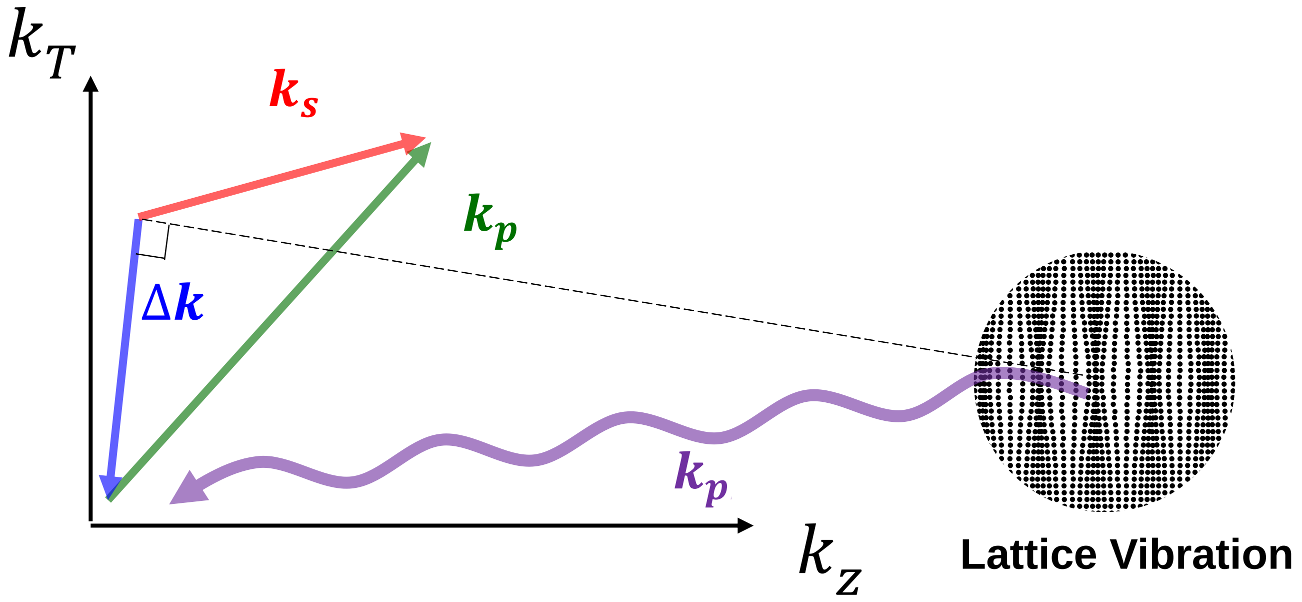
Wideband *and* broadband FWM



J. Demas et al, Photon. Res. 7, 1-7 (2019)

Role of Phase in Raman Scattering

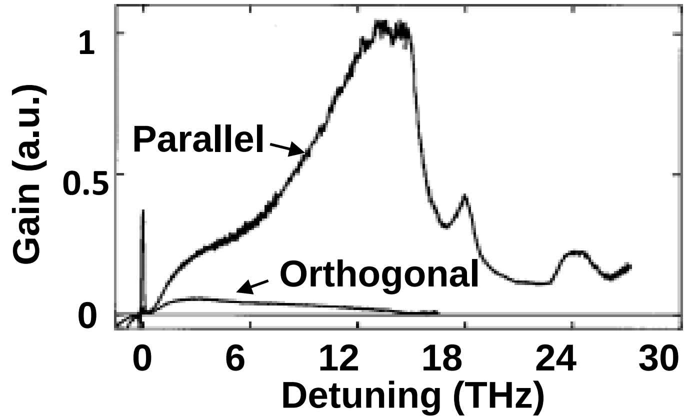
Raman Scattering



Phase-insensitive process

- Agnostic to wave-vectors/phases of the light
- Nonlinear response $g_R(\Omega)$ is material dependent

Raman response: $g_R(\Omega)$



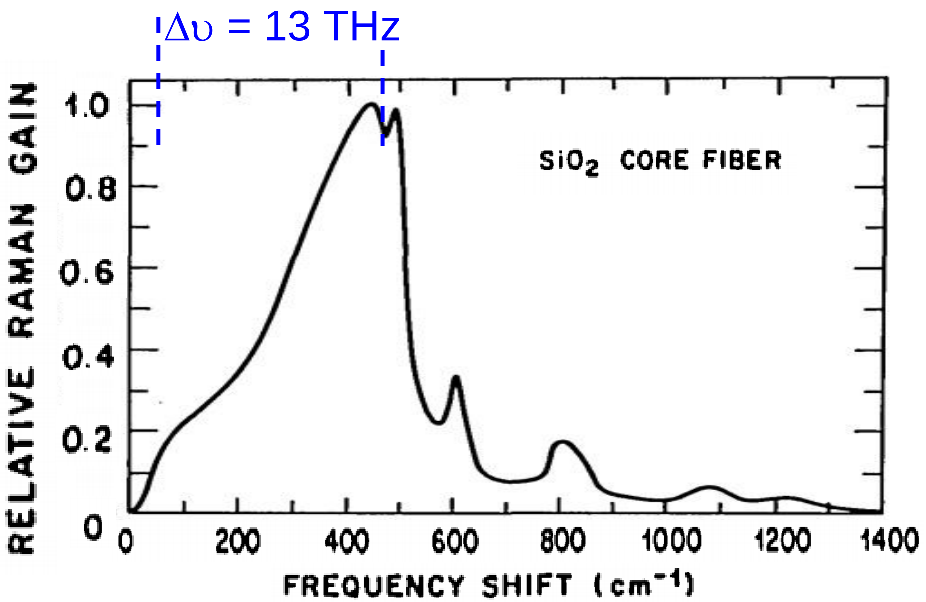
R. H. Stolen et al., *Appl. Phys. Lett.* 20, 62-64 (1972)

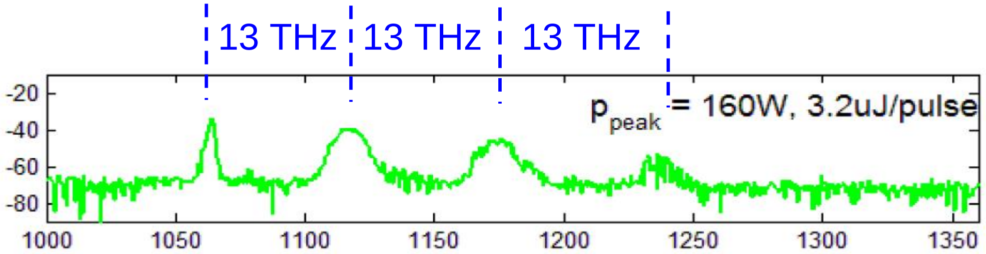
Overlap integral: η

$$\eta \propto \int I_p(r,\phi)\cdot I_s(r,\phi)\cdot dA$$

I_p	I_s
Normalized Pump intensity	Normalized Stokes intensity

CW or Long Pulses: Cascaded Raman

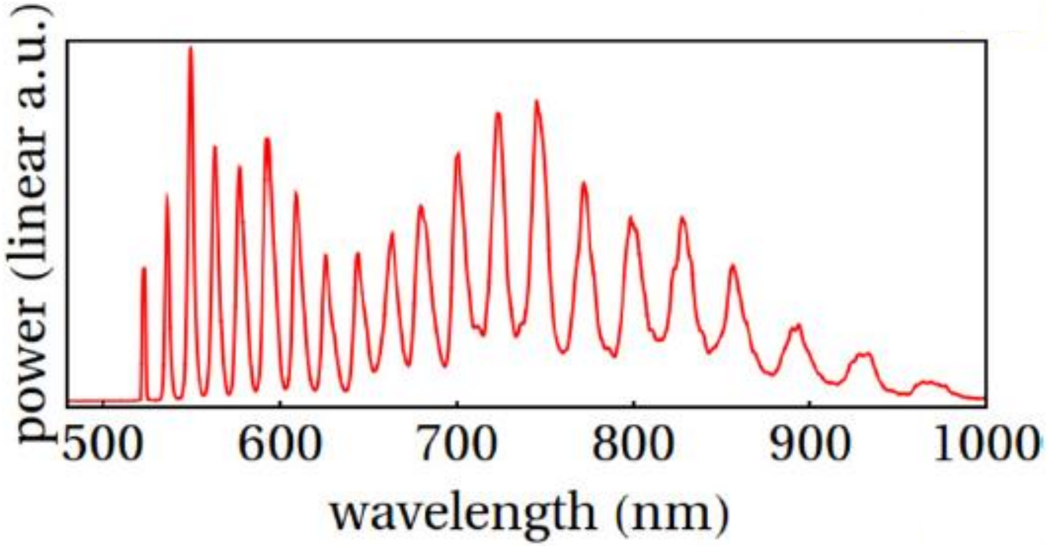


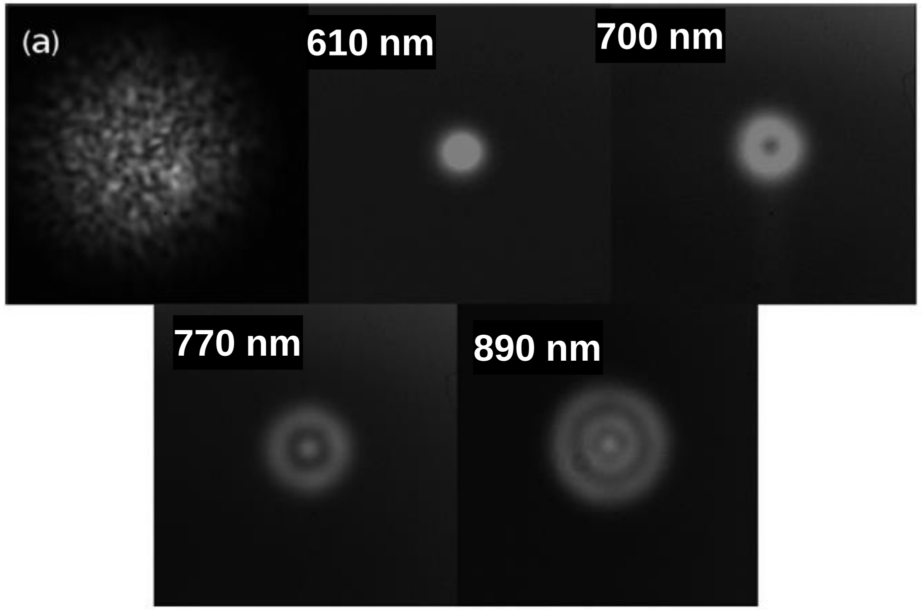


Long pulses → Negligible temporal walk-off → Discrete energy transfer

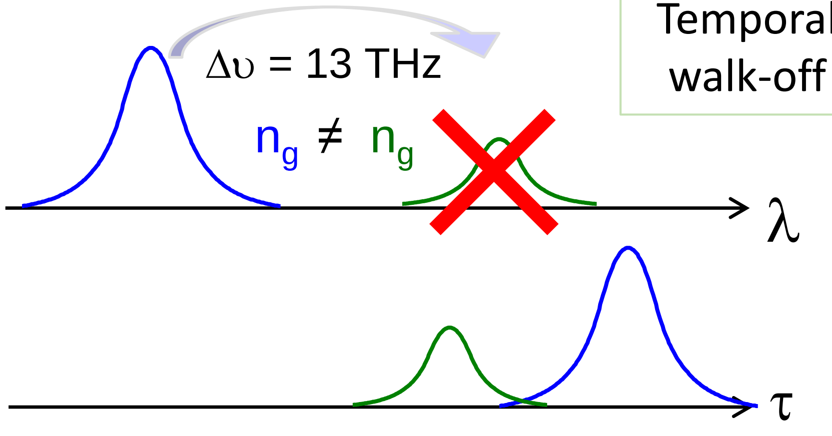
R. H. Stolen et al., JOSA B 1, 652 (1984)
S. Ramachandran et al., Opt. Exp. 18, 23212 (2010)

H. Pourbeyram, G. P. Agrawal, A. Mafi, Appl. Phys. Lett. 102, 201107 (2013)

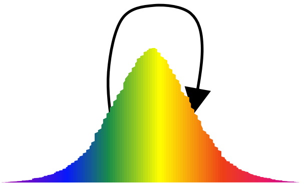




Ultrafast Pulses: Soliton Self-Frequency Shift (SSFS)

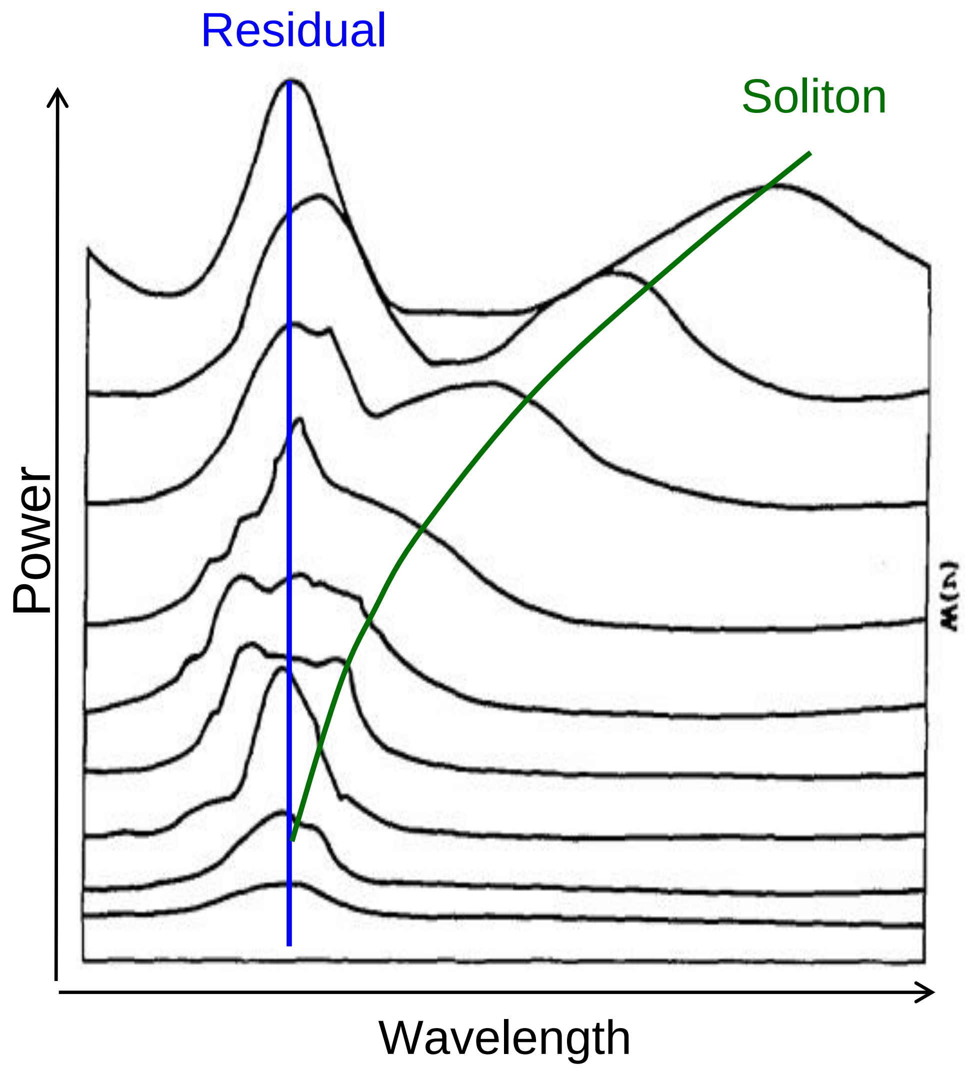


Temporal walk-off



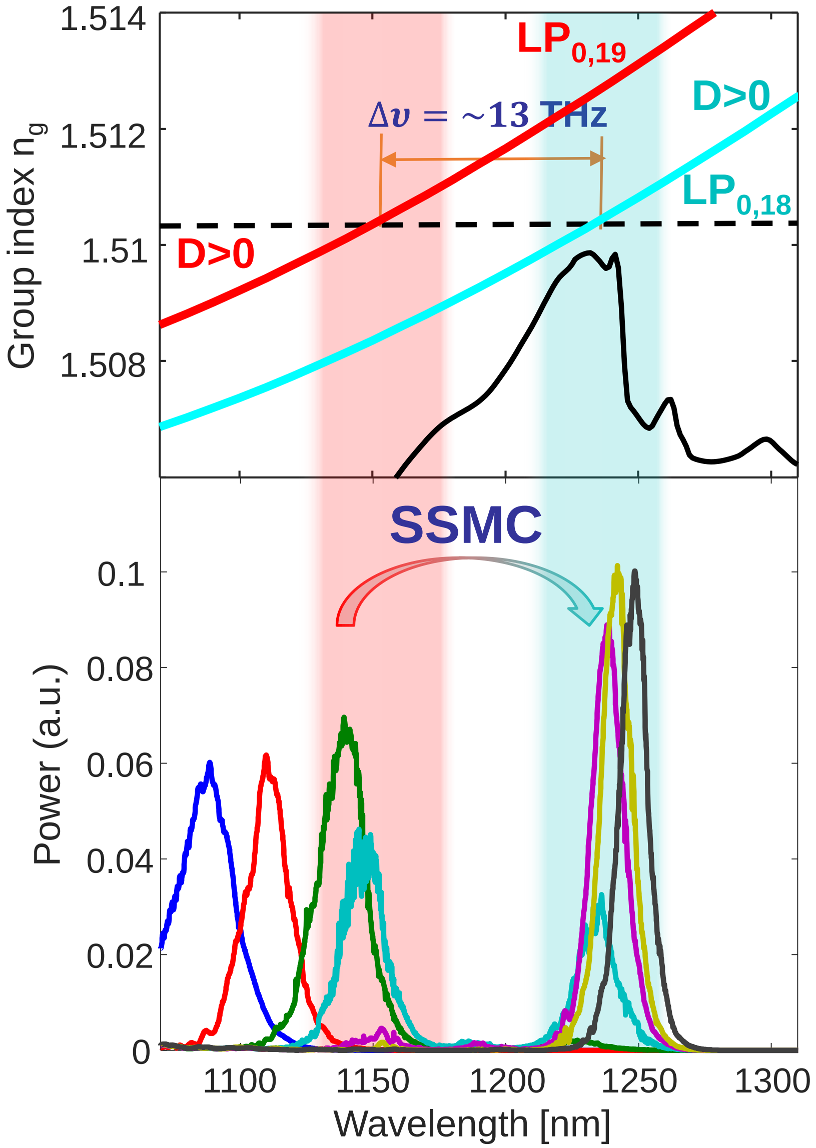
Continuous energy transfer (intra-pulse)

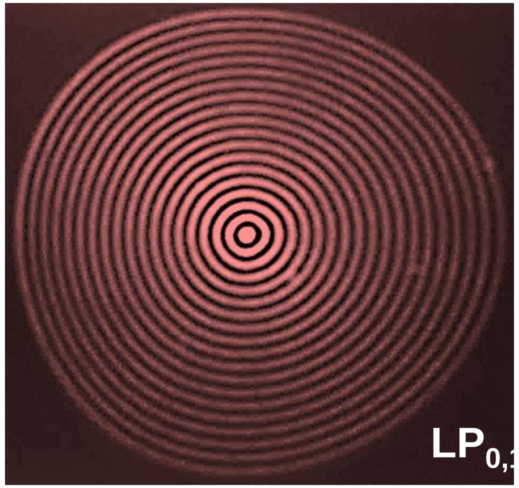
Self Raman Shift

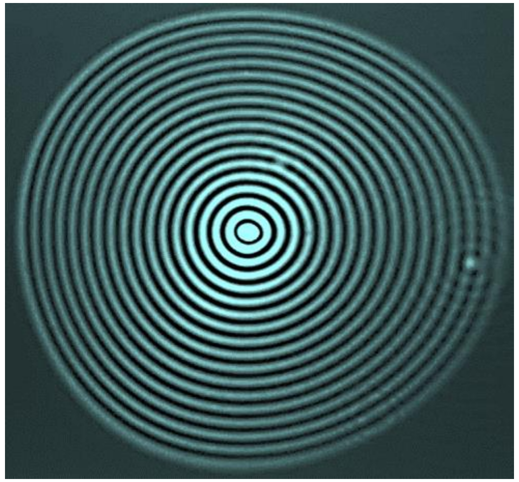


F. M. Mitschke & L. F. Mollenauer, Opt. Lett. 11, 10 (1986)

Influence of modal dispersion on Raman







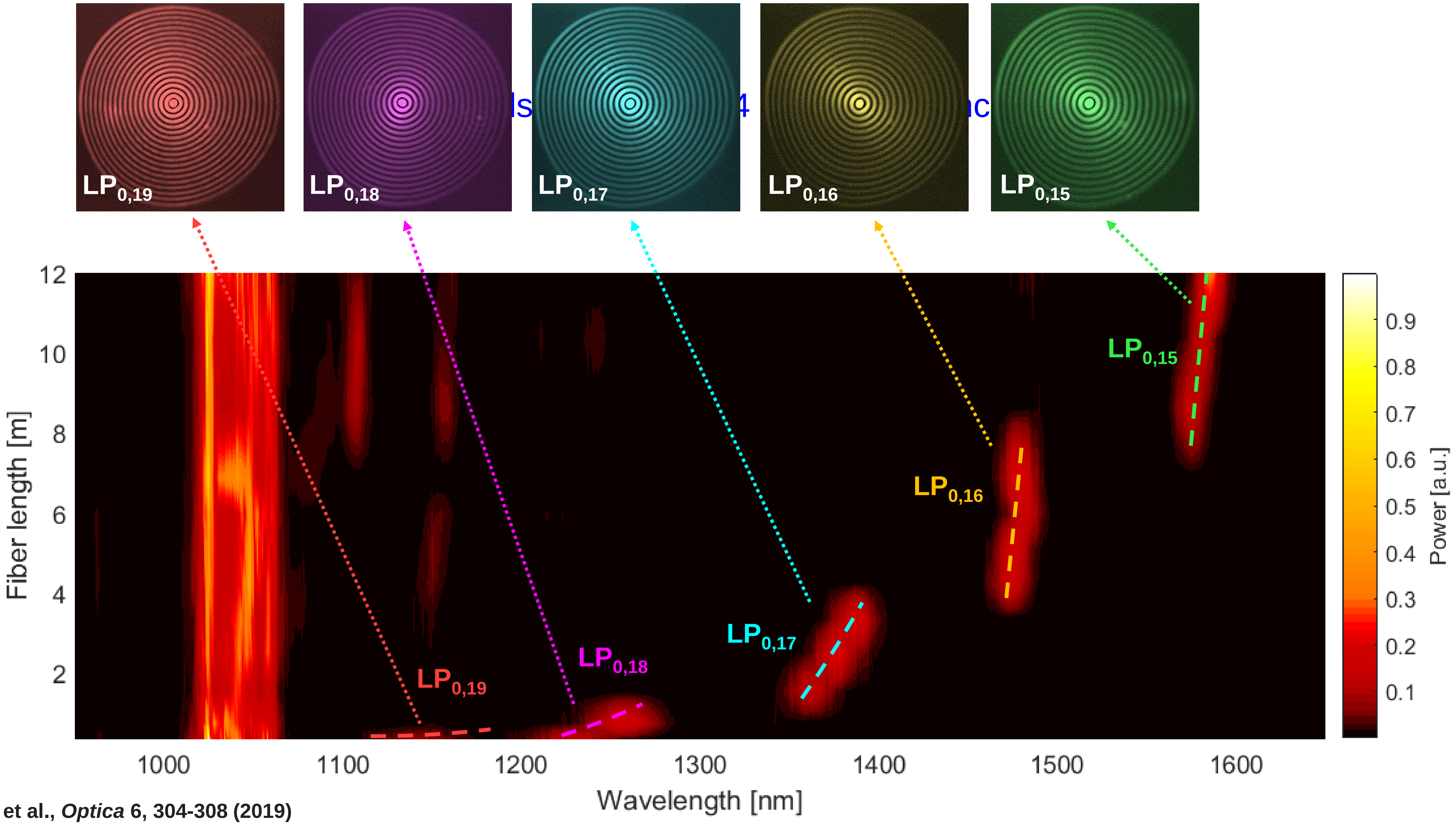
- n_g matching possible in multimode systems
- Raman gain peak is @ 13 THz

Ultrafast intermodal interaction

- Behave like in quasi-CW regime
- 100% Photon Conversion!

L. Rishøj et al., *Optica* 6, 304-308 (2019)

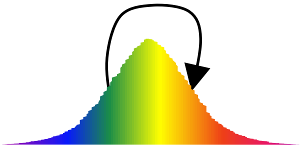
Soliton Self-Mode Conversion – a new Raman pathway in multimode



L. Rishøj et al., *Optica* 6, 304-308 (2019)

Spontaneous SSMC over Seeded SSFS

BOSTON UNIVERSITY

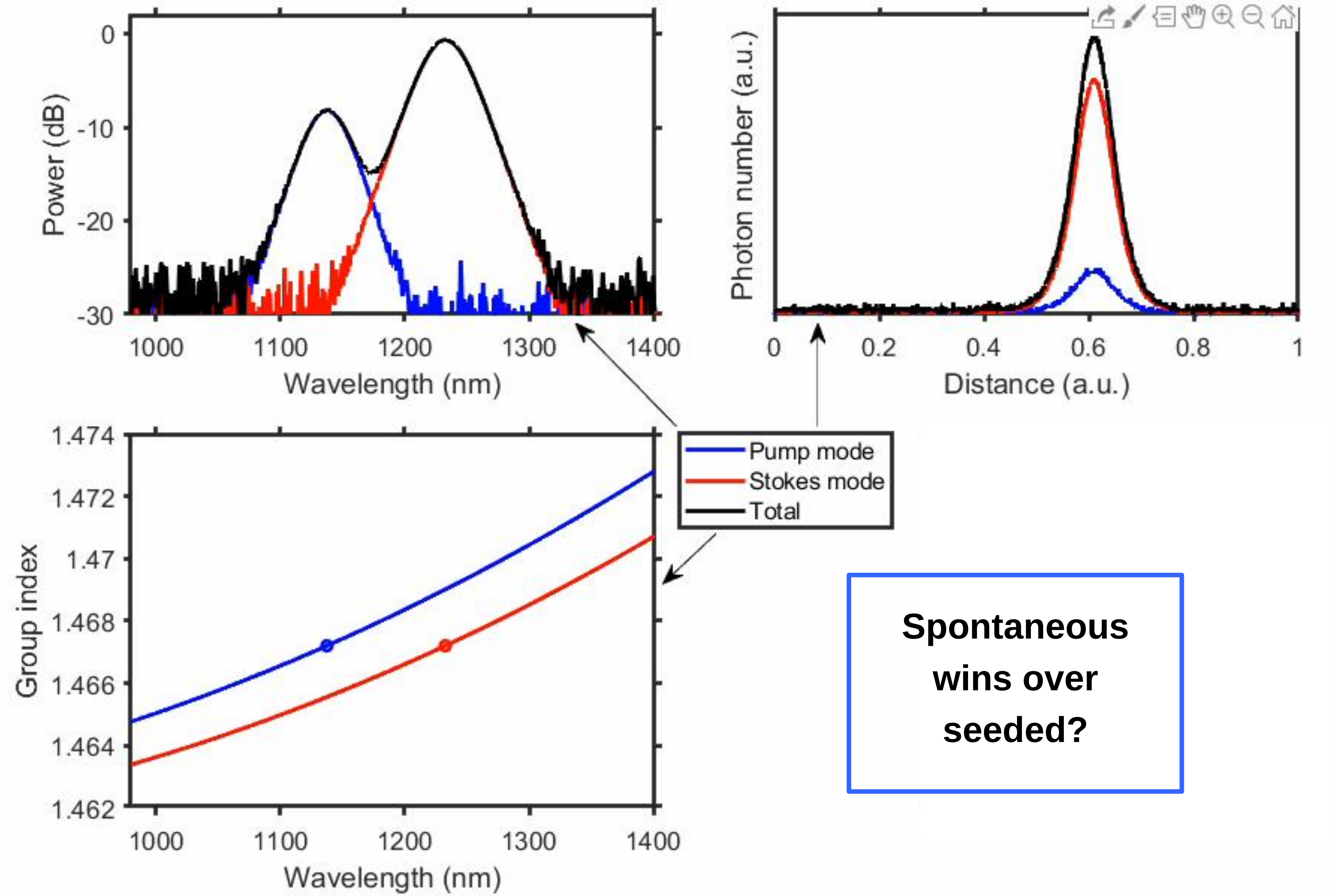


Self-seeded

Soliton Self-Frequency Shift (SSFS)

Group Index Matched
@ Raman gain peak

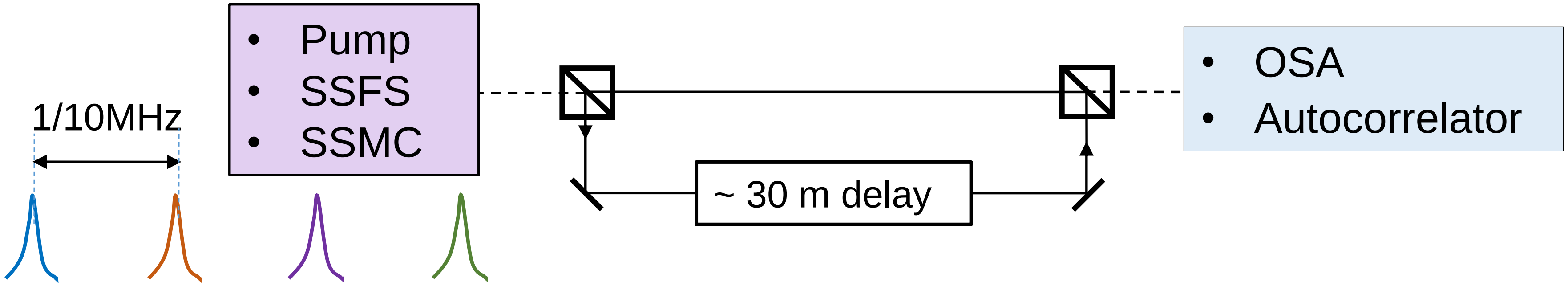
Soliton Self-Mode Conversion (SSMC)



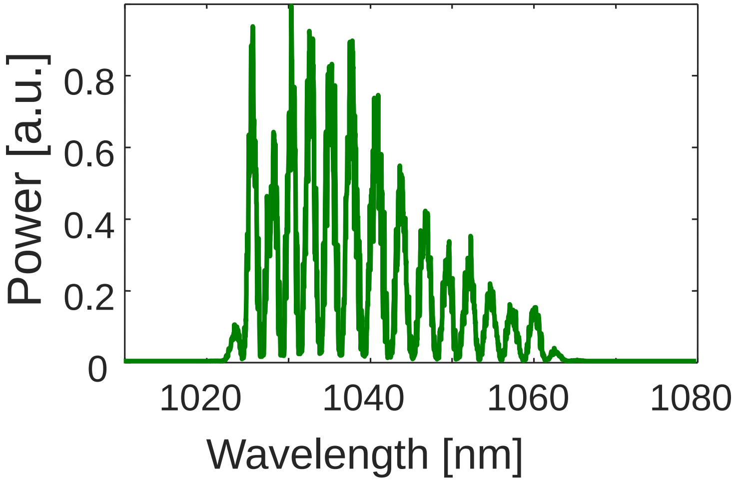
Spontaneous wins over seeded?

A. Antikainen et al., Phys. Rev. Lett. 122, 023901 (2019)

SSMC: Spontaneous



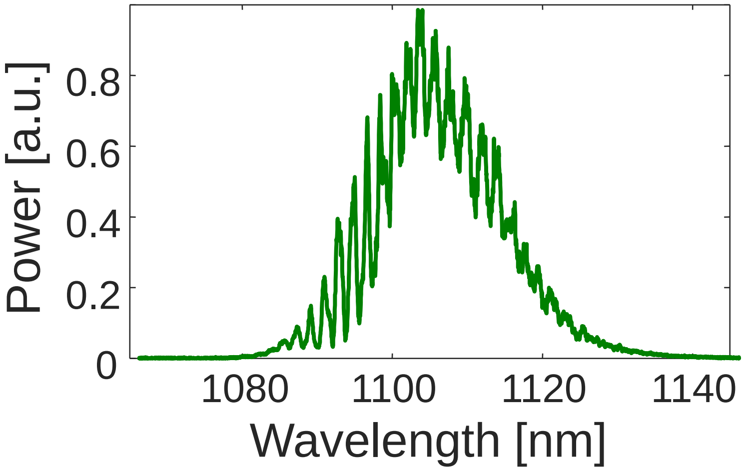
Pump



V ~ 95%

Shot-to-shot coherent → Mode-locked

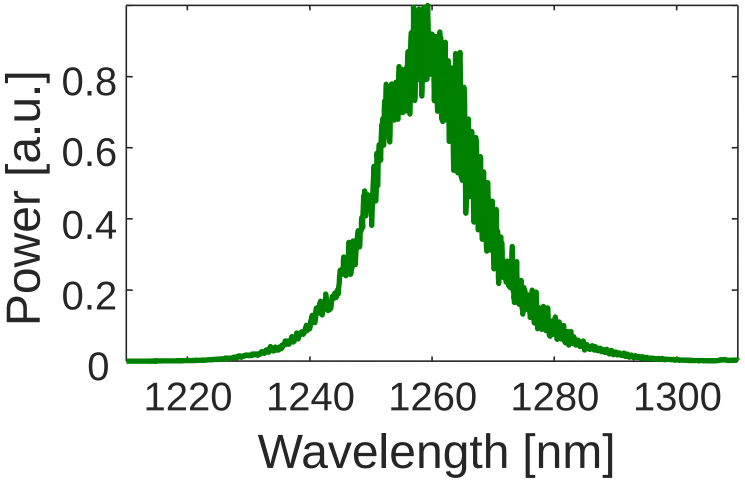
SSFS



V ~ 34%

Shot-to-shot coherent → Self-seeded

SSMC



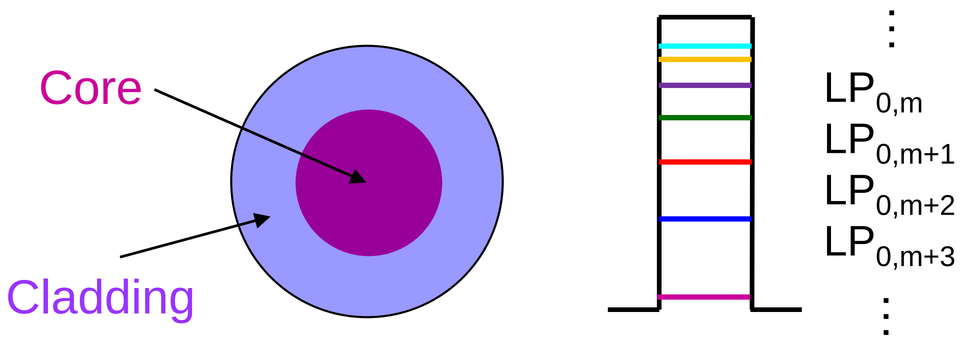
V < measurement noise

Shot-to-shot **incoherent** → **Noise initiated**

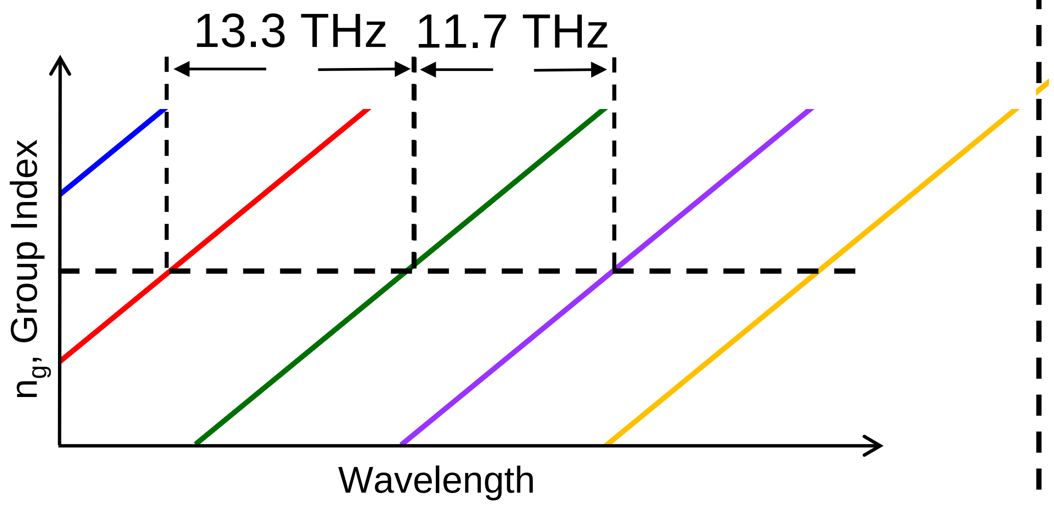
J.W. Nicholson and M. F. Yan, Opt. Exp. 12, 679 (2004)

H. B. Kabagöz, et al., Opt. Express 29, 18315 (2021)

Fiber Design ⇒ Process Control

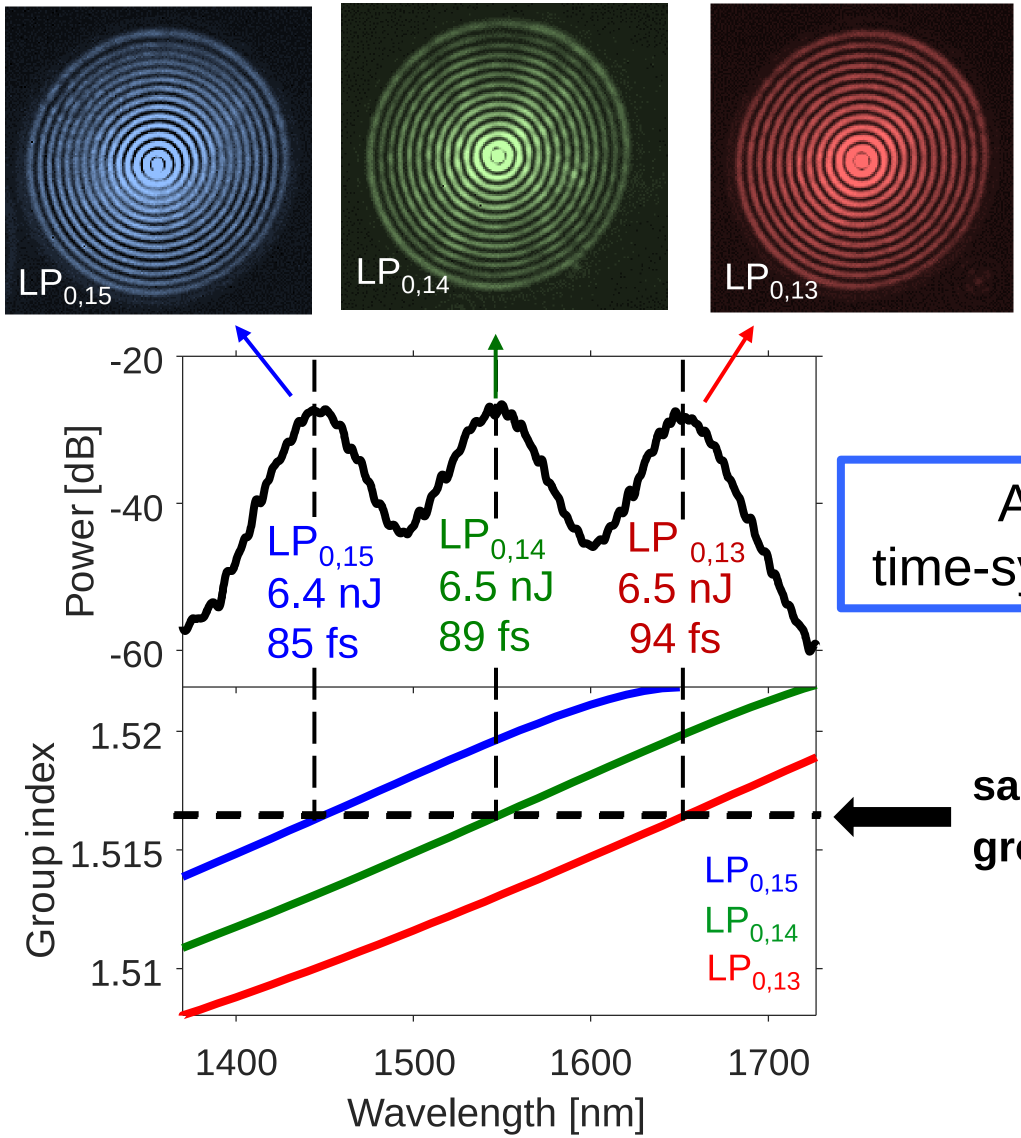


Core Ø → n_g spacing



Multiple Peaks with Equal Power

Core Ø = 70 µm



All three time-synchronized!

same group index

H. B. Kabagöz, et al., APL Photonics 7, 029901 (2022)

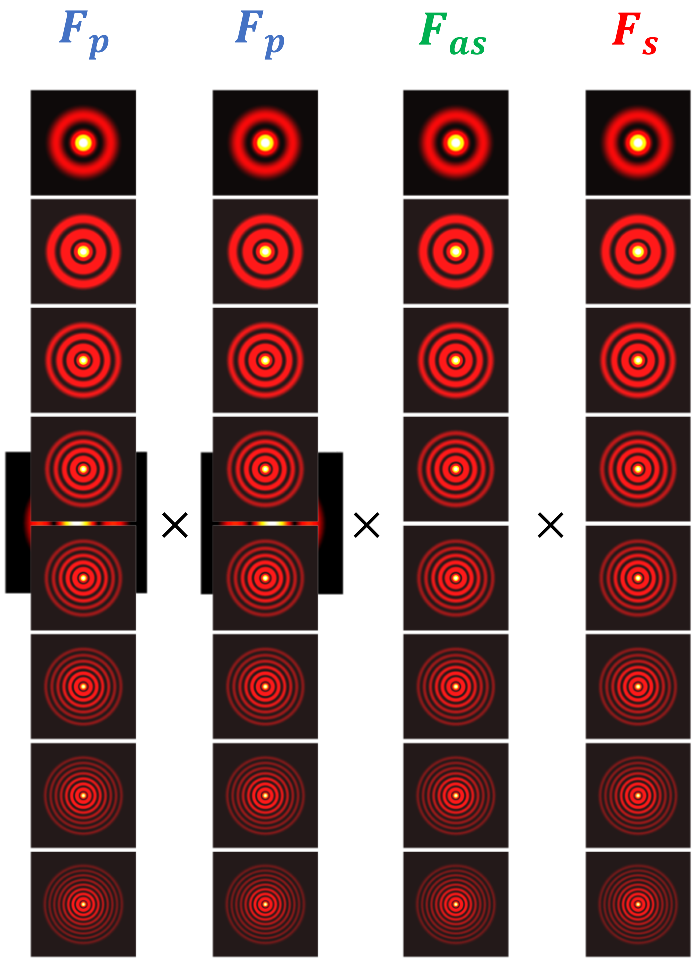
Outline

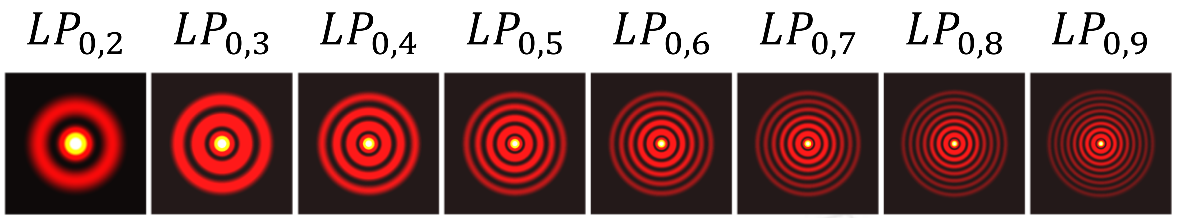
BOSTON UNIVERSITY

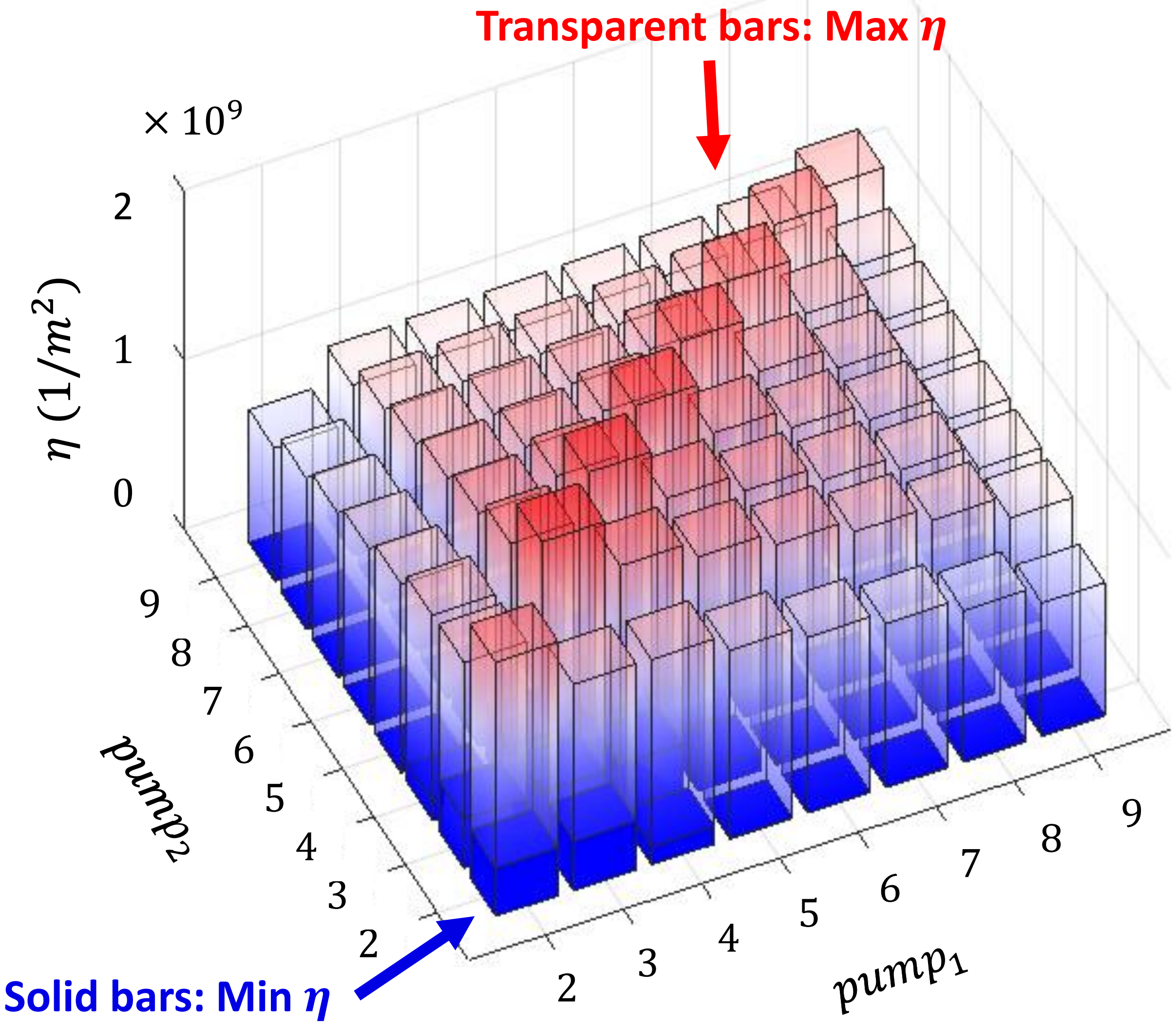
- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

OAM-FWM Unique Properties – Comparison with LP Modes

$$\eta \propto \int F_p \cdot F_q \cdot F_{as} \cdot F_s \cdot r\,dr$$

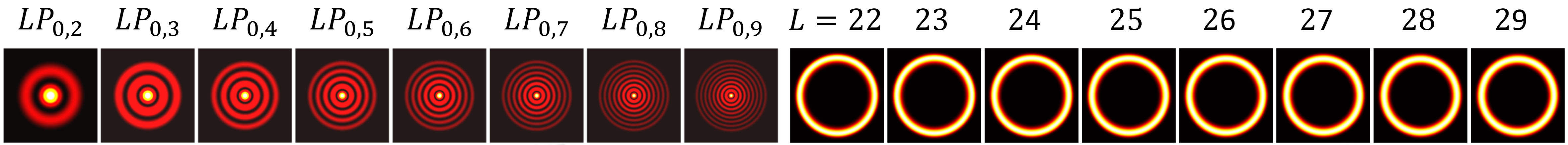


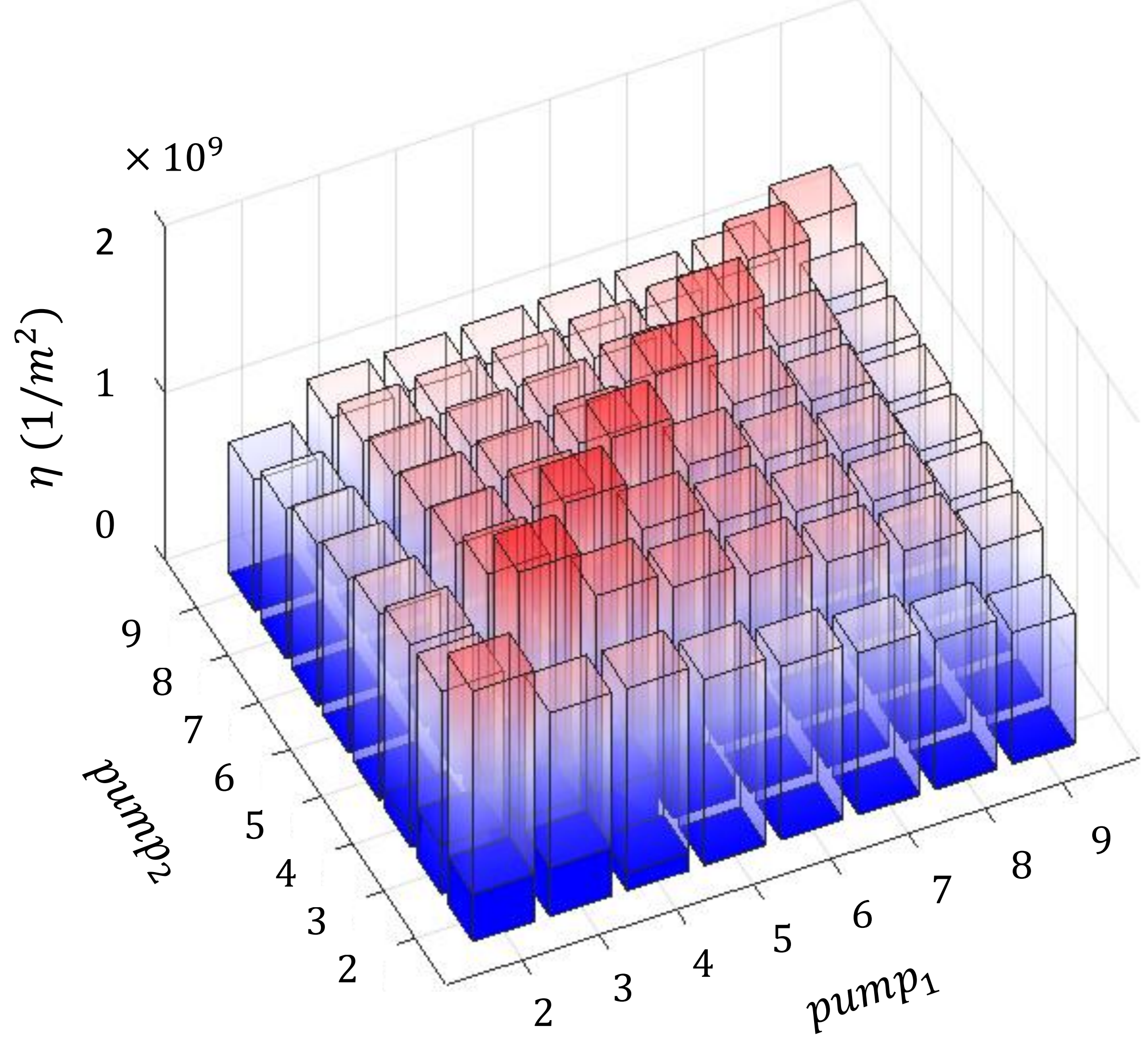


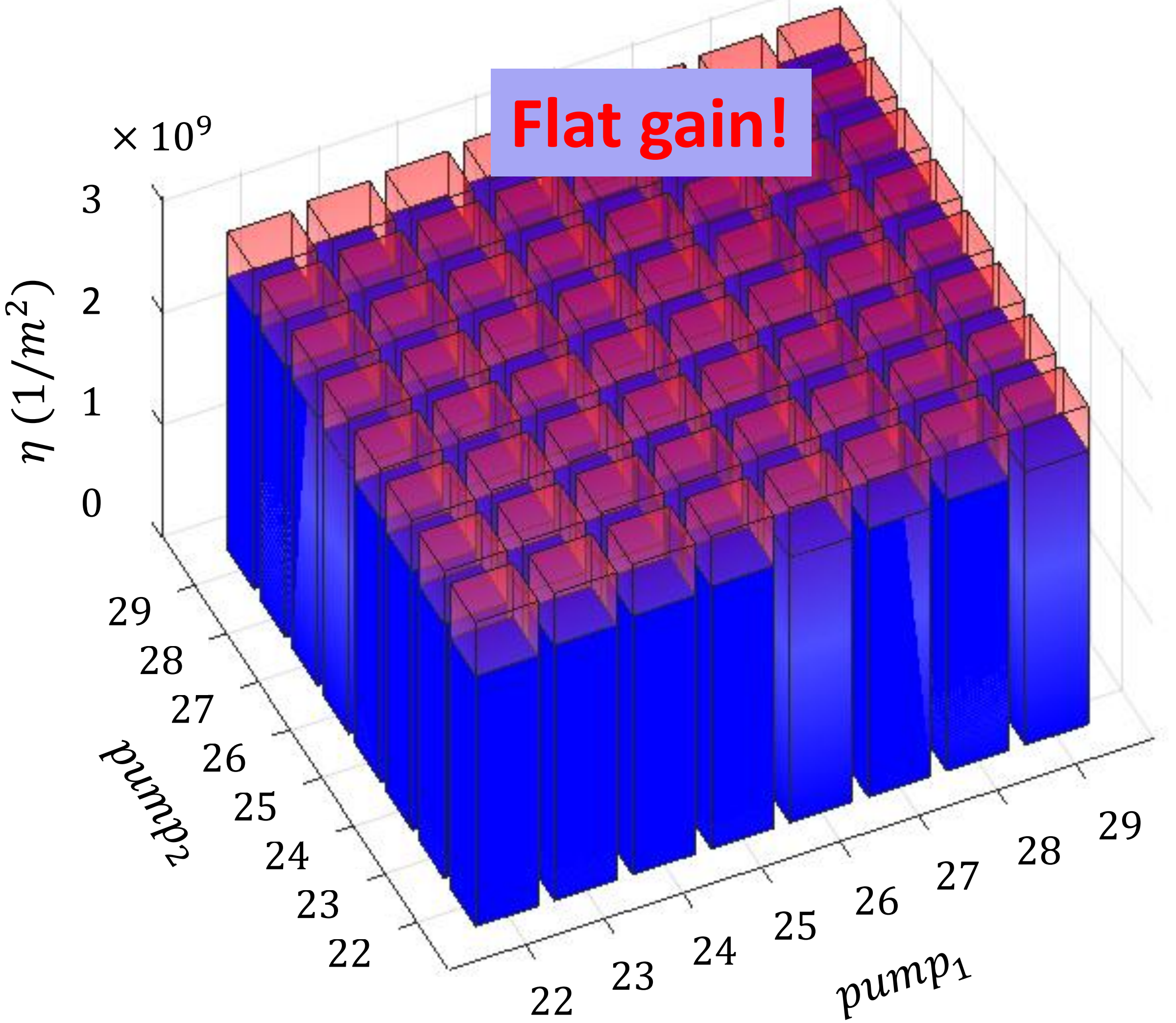


Assumption: mode profiles not ω-dependent

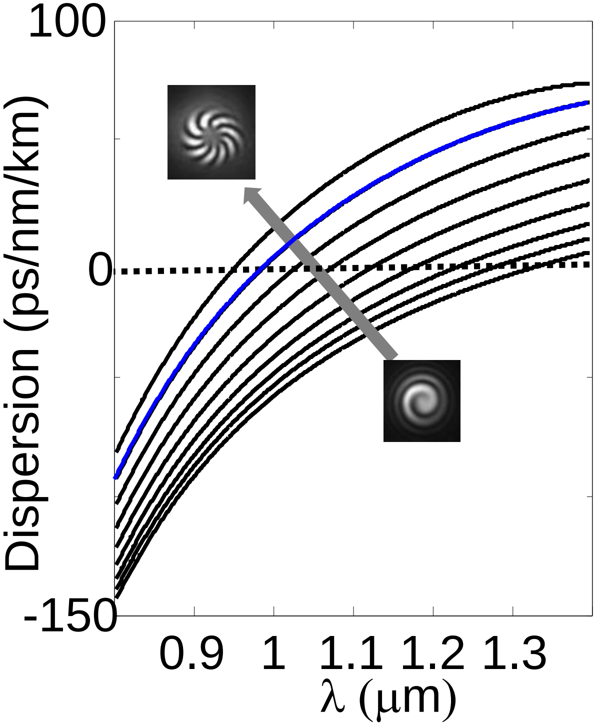
OAM-FWM Unique Properties – Comparison with LP Modes

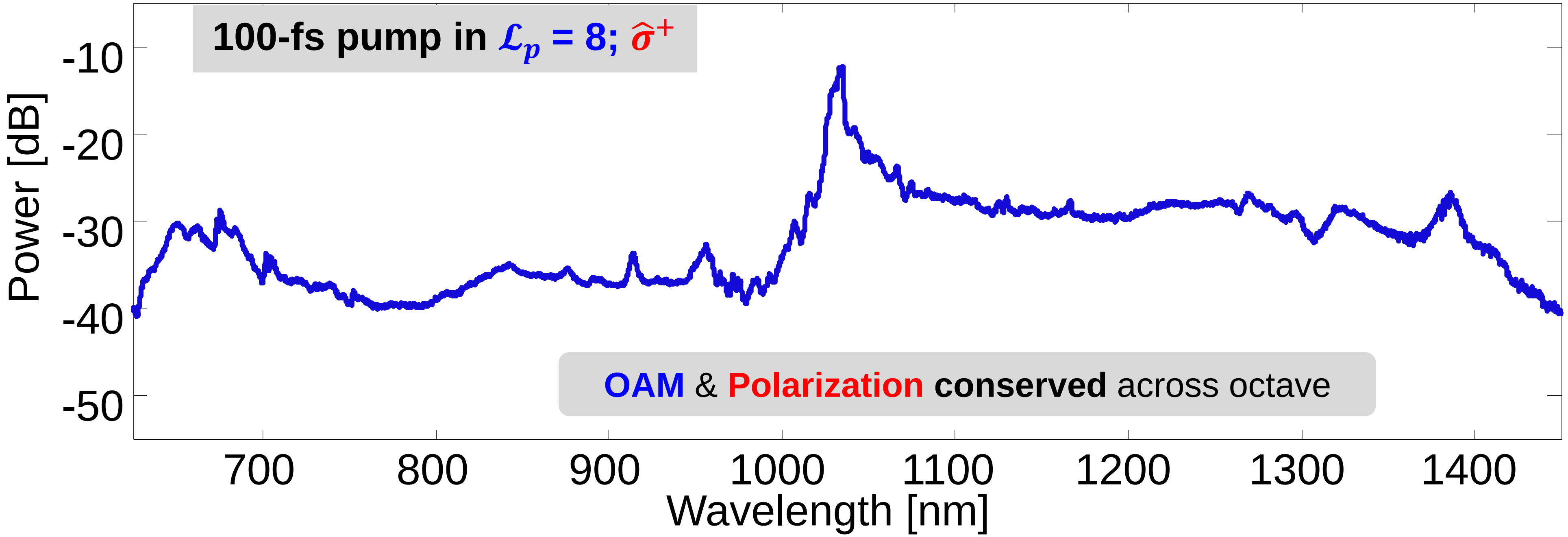


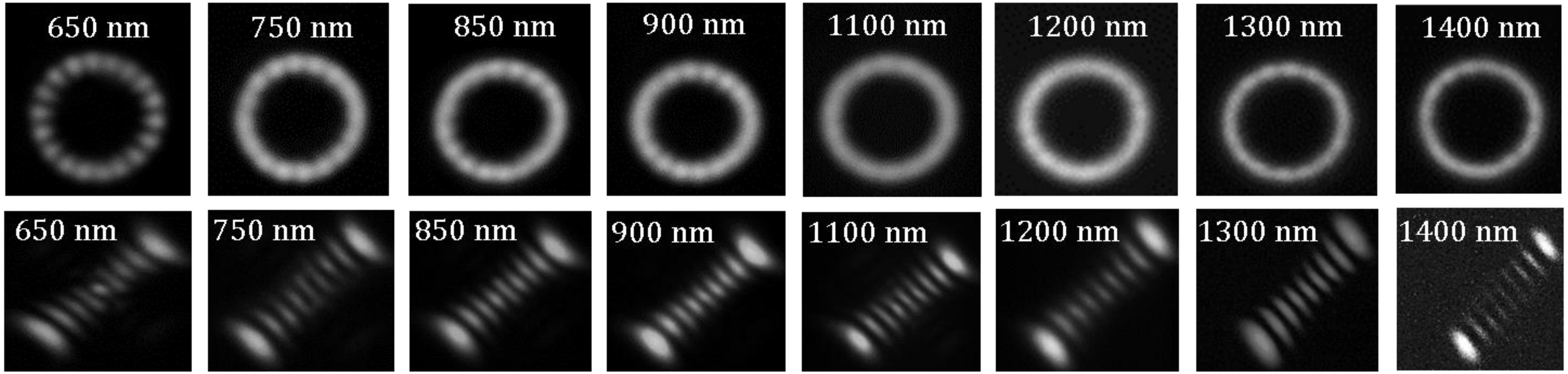




OAM Supercontinuum







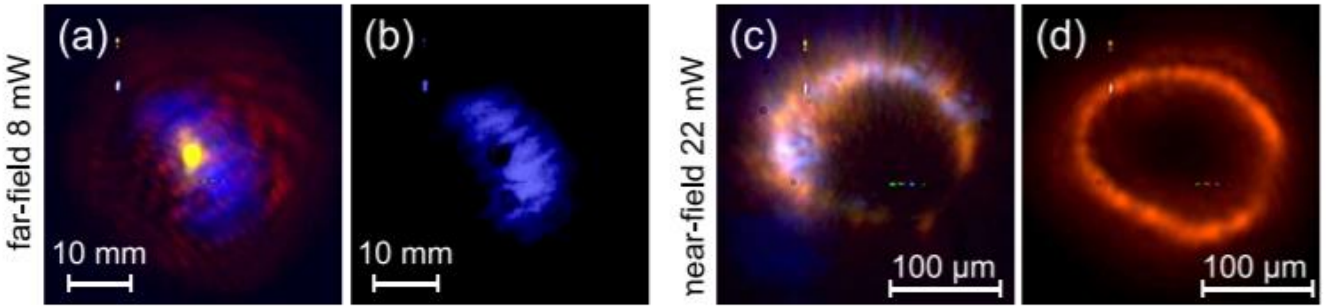
G. Prabhakar *et al.*, *Opt. Express* 27, 11547 (2019)

BOSTON UNIVERSITY

Role of group velocity in continuum generation

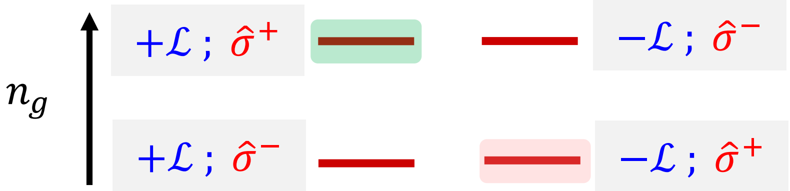
D. N. Neshev *et al.*, *Opt. Express* 18, 18368 (2010)

OAM *not* conserved in bulk supercontinuum

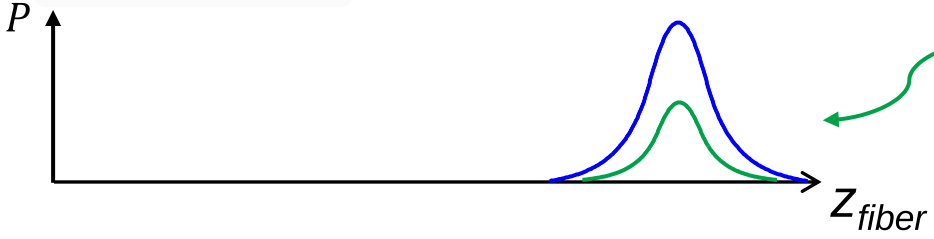


Why is fiber different?

n_g splitting due to SOI



Pump: $+\mathcal{L}$; $\hat{\sigma}^+$

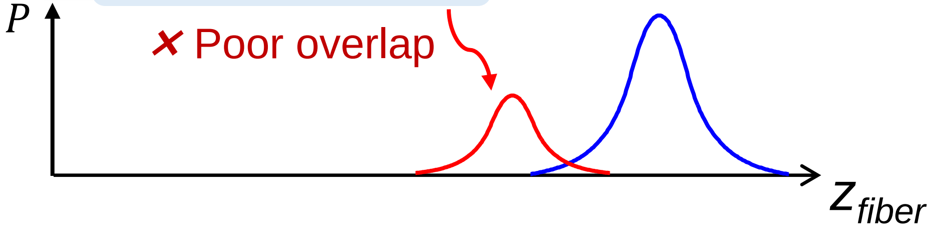


Nonlinear: $\pm\mathcal{L}$; $\hat{\sigma}^\pm$

✓ High overlap

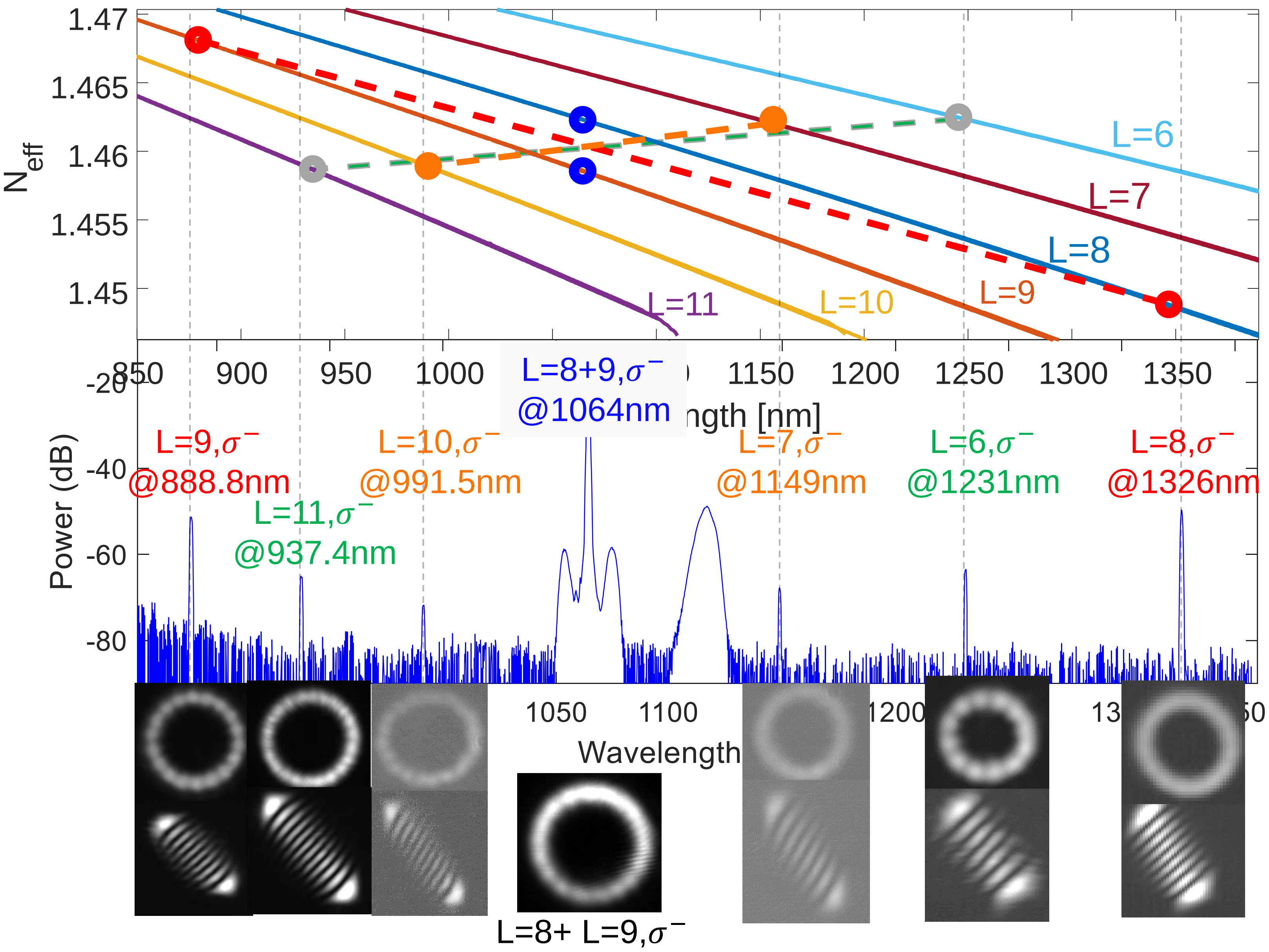
Nonlinear: $\mp\mathcal{L}$; $\hat{\sigma}^\pm$

✗ Poor overlap



All other modes have **different n_g**

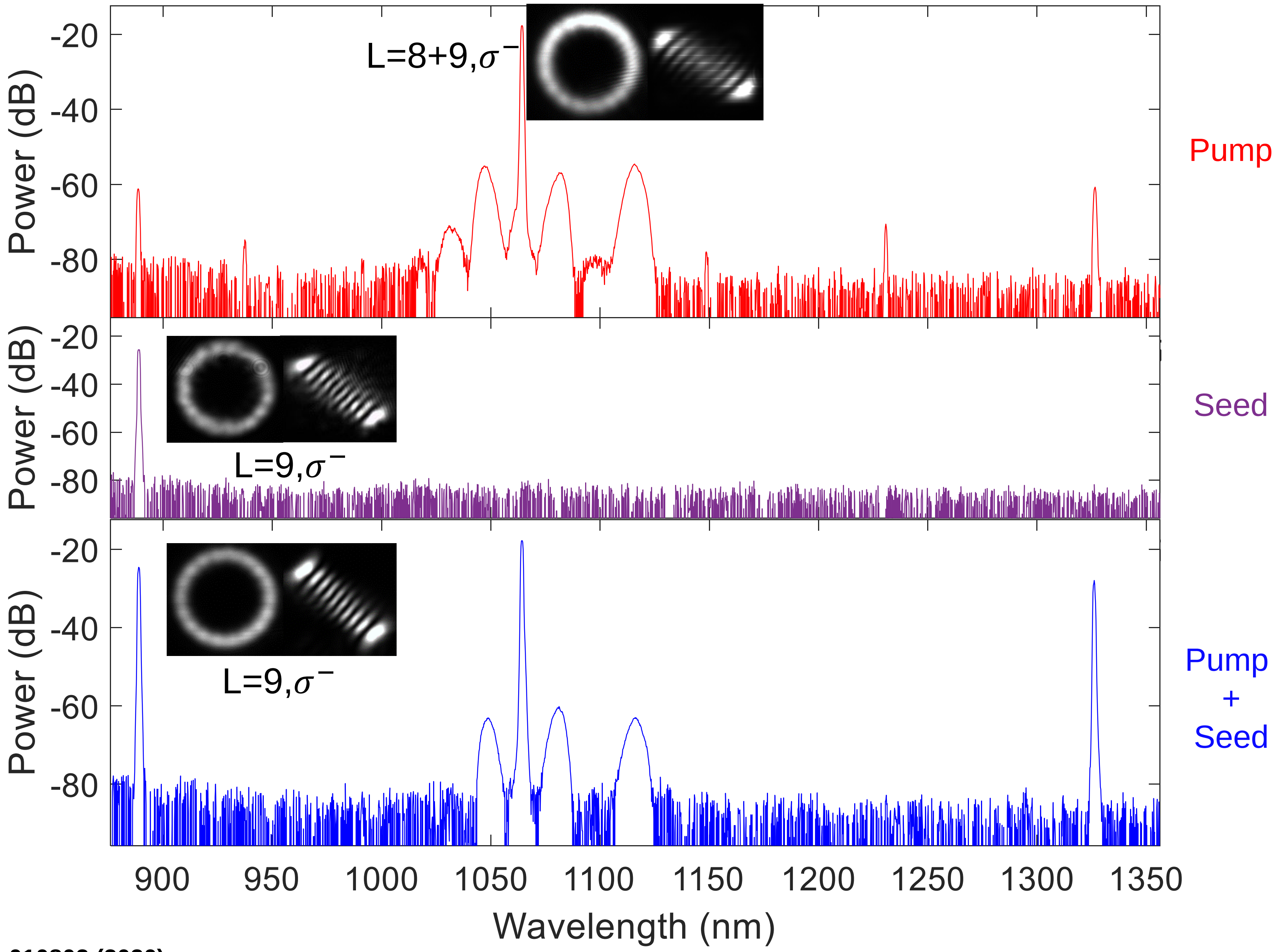
Selective OAM generation



- OAM conserved in all cases
- Diversity of photon pairs
- Selective yet efficient

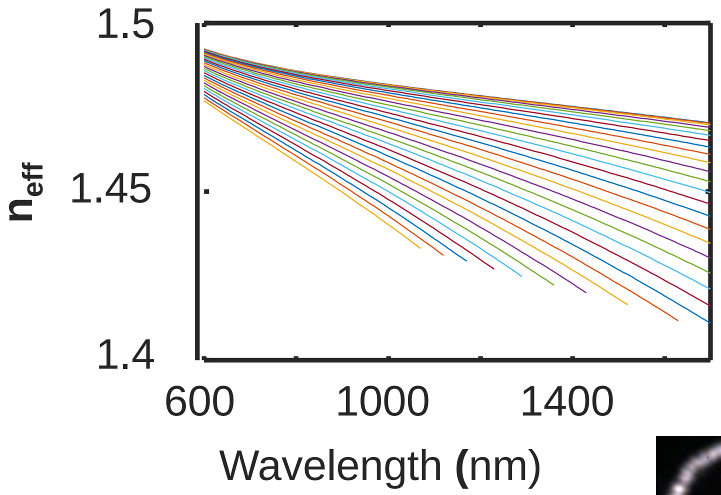
X. Liu et al, APL Photonics 5, 010802 (2020)

Parametric Amplification

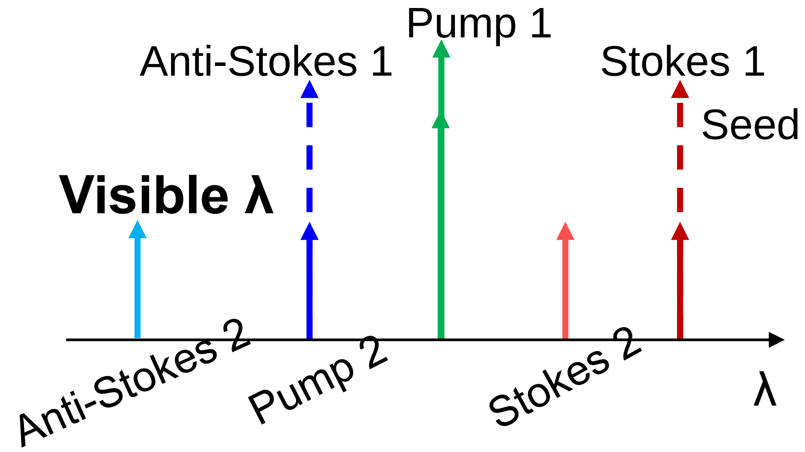


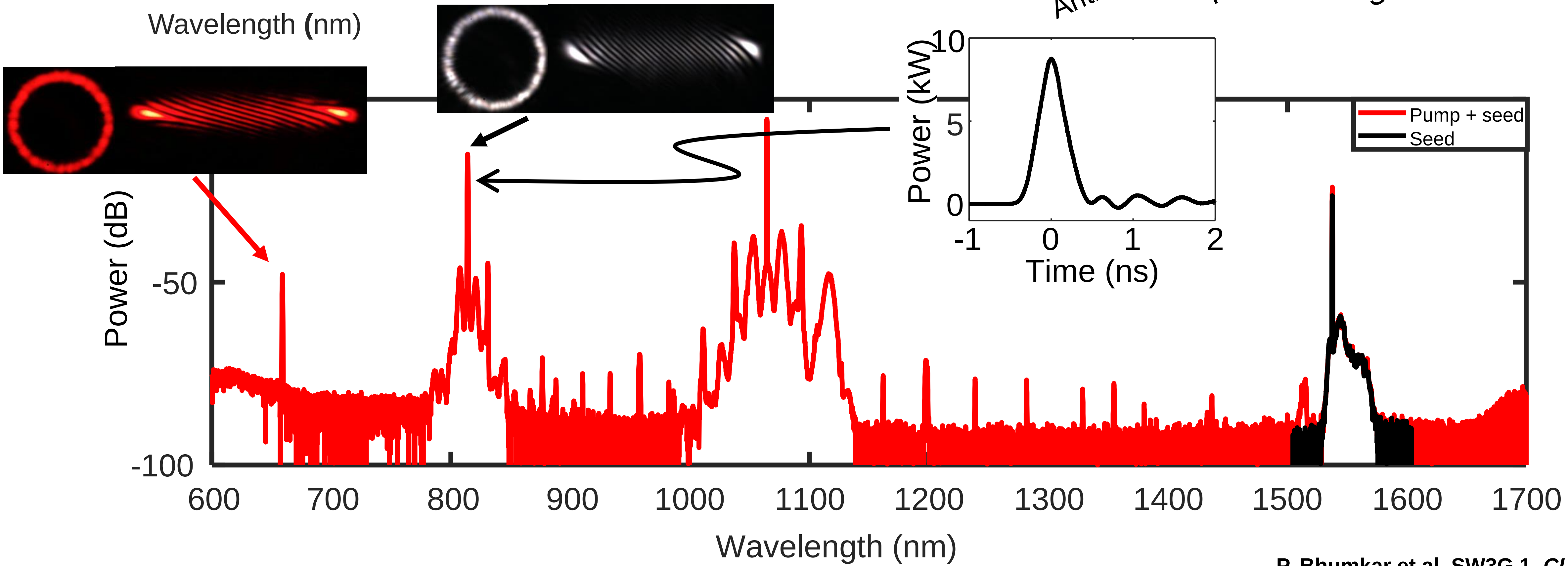
X. Liu et al, APL Photonics 5, 010802 (2020)

Cascaded Four-wave Mixing… all fiber high-power visible sources



Many modes ⇒ Many pathways





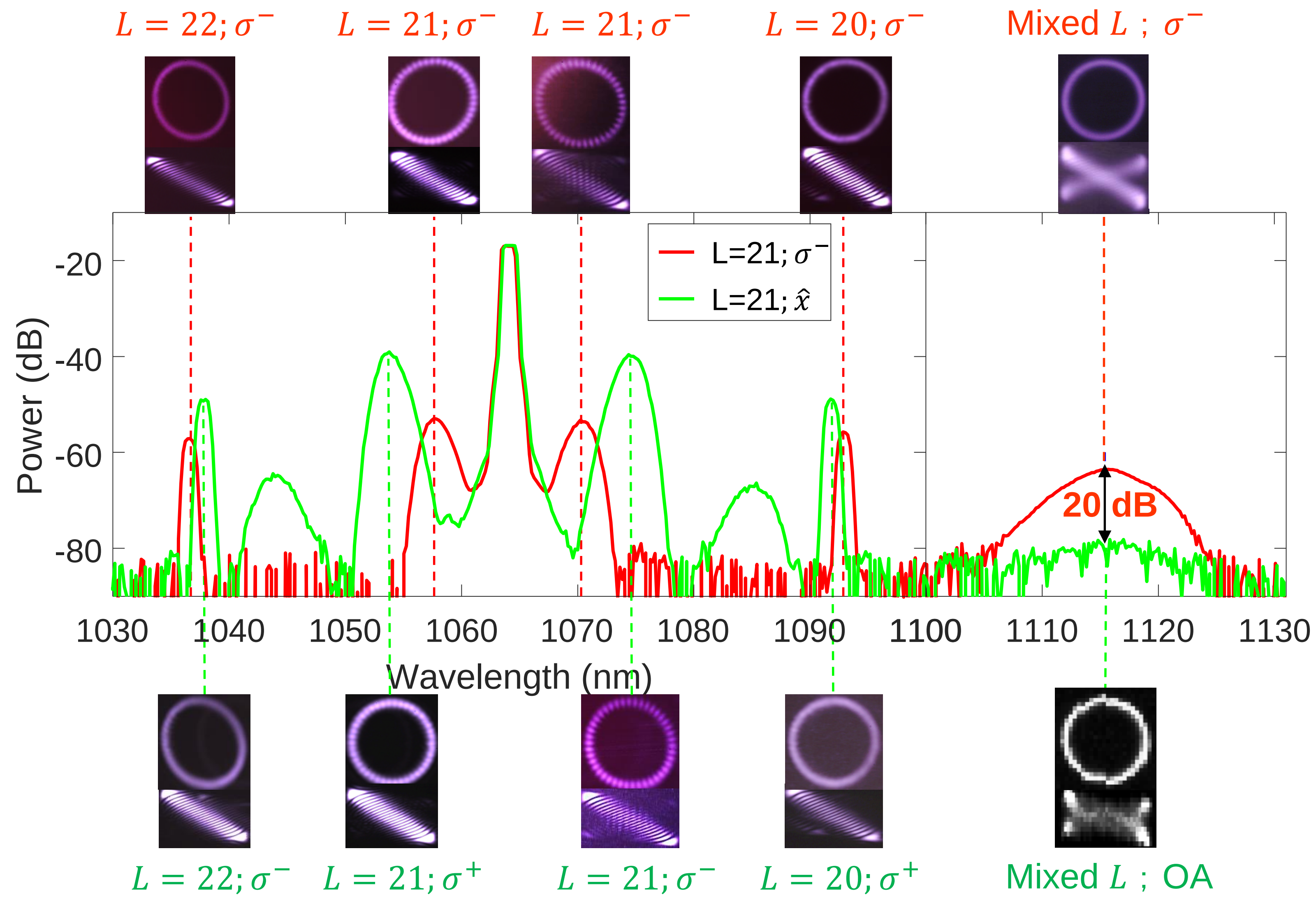
P. Bhumkar et al, SW3G.1, *CLEO* (2023)

Outline

- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

Raman dependence on OAM?

P_{peak}: 8 kW



- Un-depleted pump
- Strong FWM strength
- **Raman in all modes**
- **20 dB Raman suppression!**

X. Liu et al, Opt. Express 30, 26967 (2022)

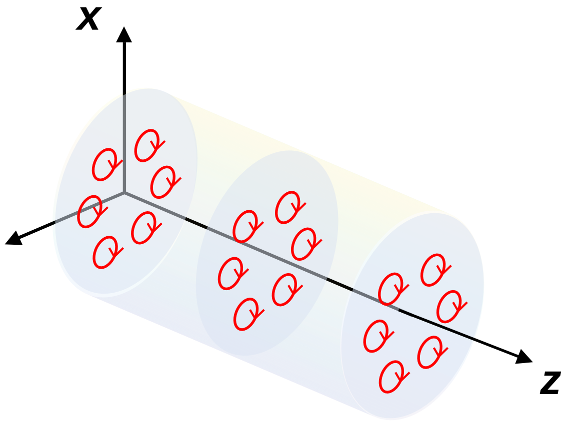
Raman Spectra for Different Pump Modes

Pump:

$L_P = 15; \hat{\sigma}^-$

Stokes:

All the L_s's

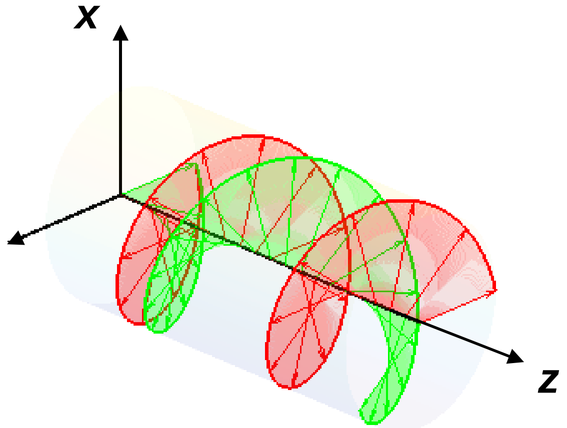


Pump:

$L_P = 15; \hat{x}$ **(OA)**

Stokes:

$L_s \neq L_p - 1$

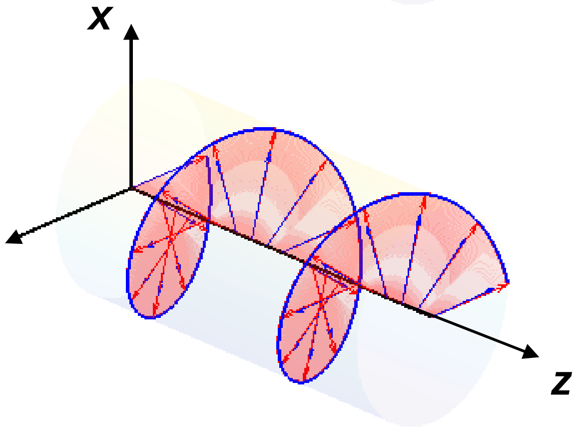


Pump:

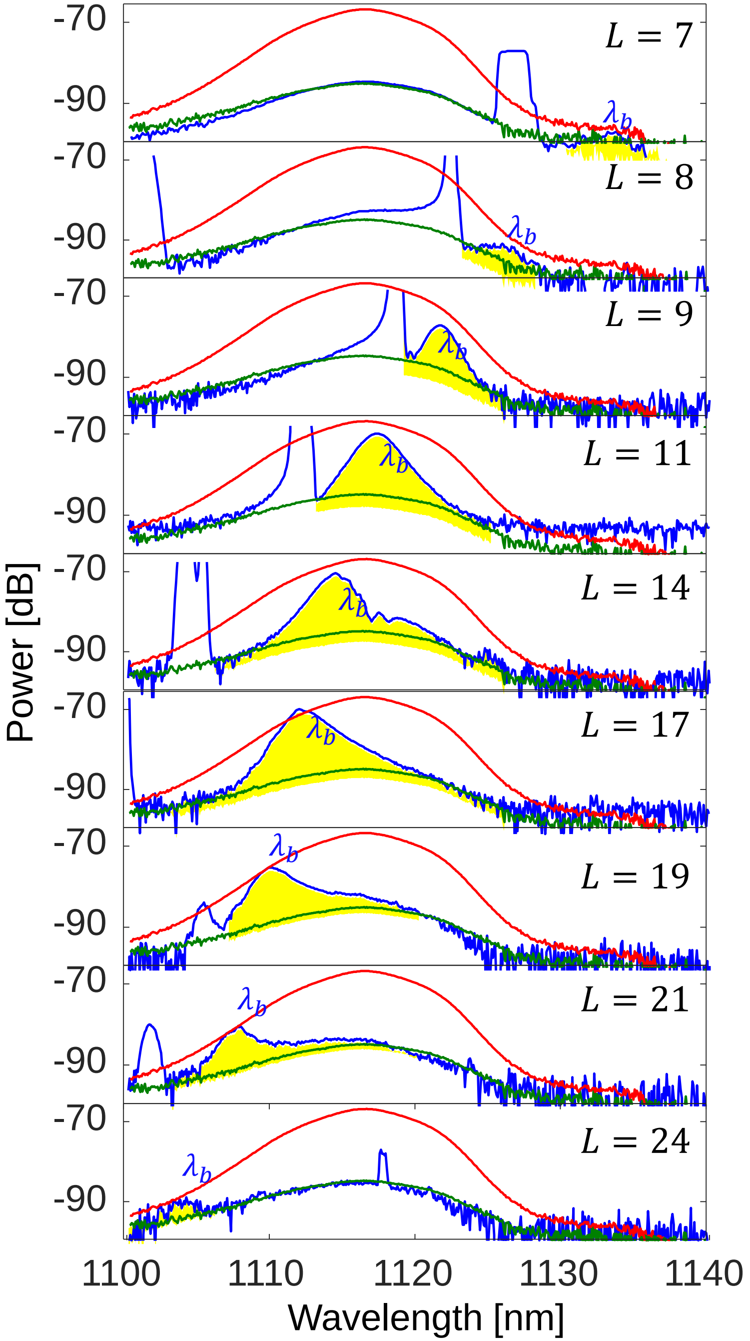
$L_P = 15; \hat{x}$ **(OA)**

Stokes:

$L_s = L_p - 1$



Phase-Matched Raman!

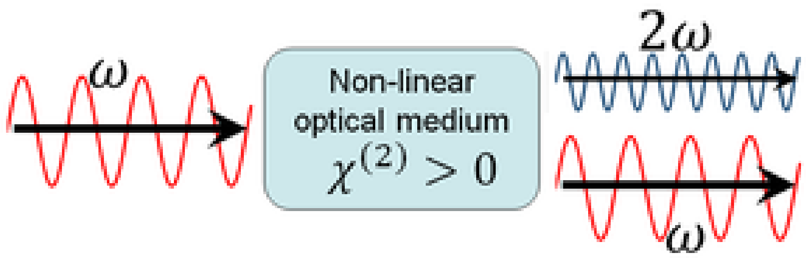


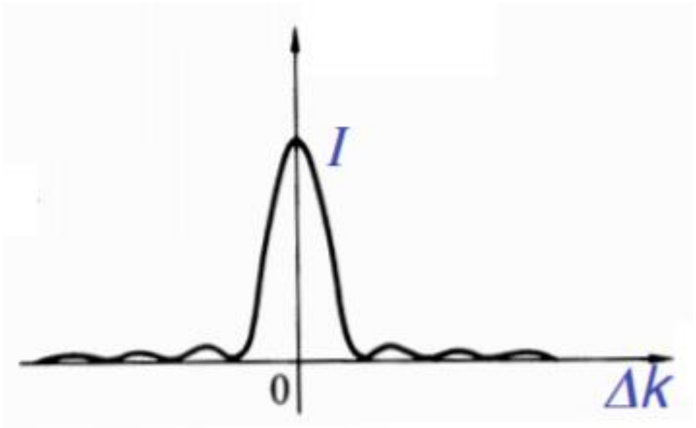
X. Liu et al, Opt. Express 30, 26967 (2022)

Phase-matching Behavior – Raman Scattering

$$\Delta k = k_p - k_s = \pi \left[\frac{\Delta n_{eff}^{(p)}(\lambda_p)}{\lambda_p} - \frac{\Delta n_{eff}^{(s)}(\lambda_s)}{\lambda_s} \right]$$

Second Harmonic Generation (SHG)



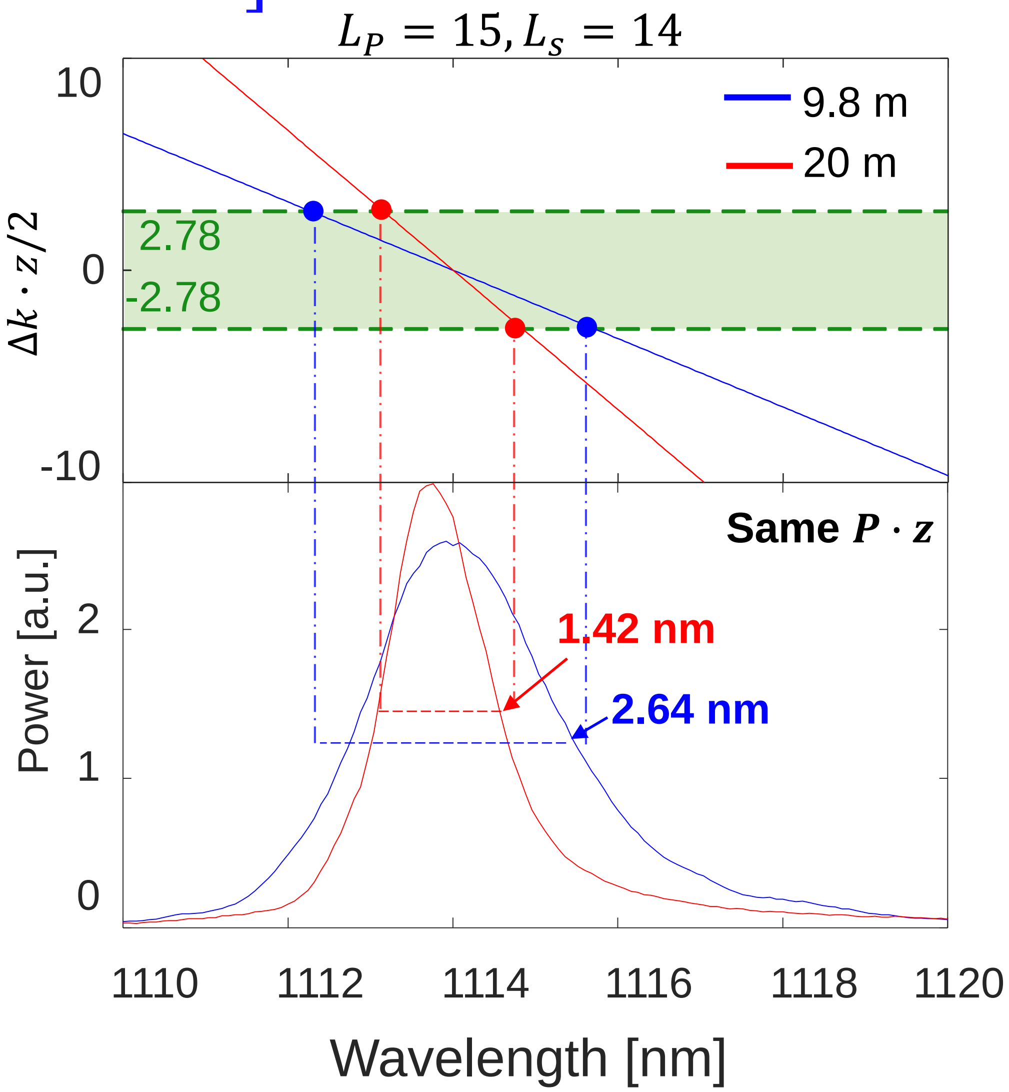


$$I \propto sinc^2(\Delta k \cdot z/2)$$

R.W. Boyd, "Nonlinear optics," Academic press (2020)

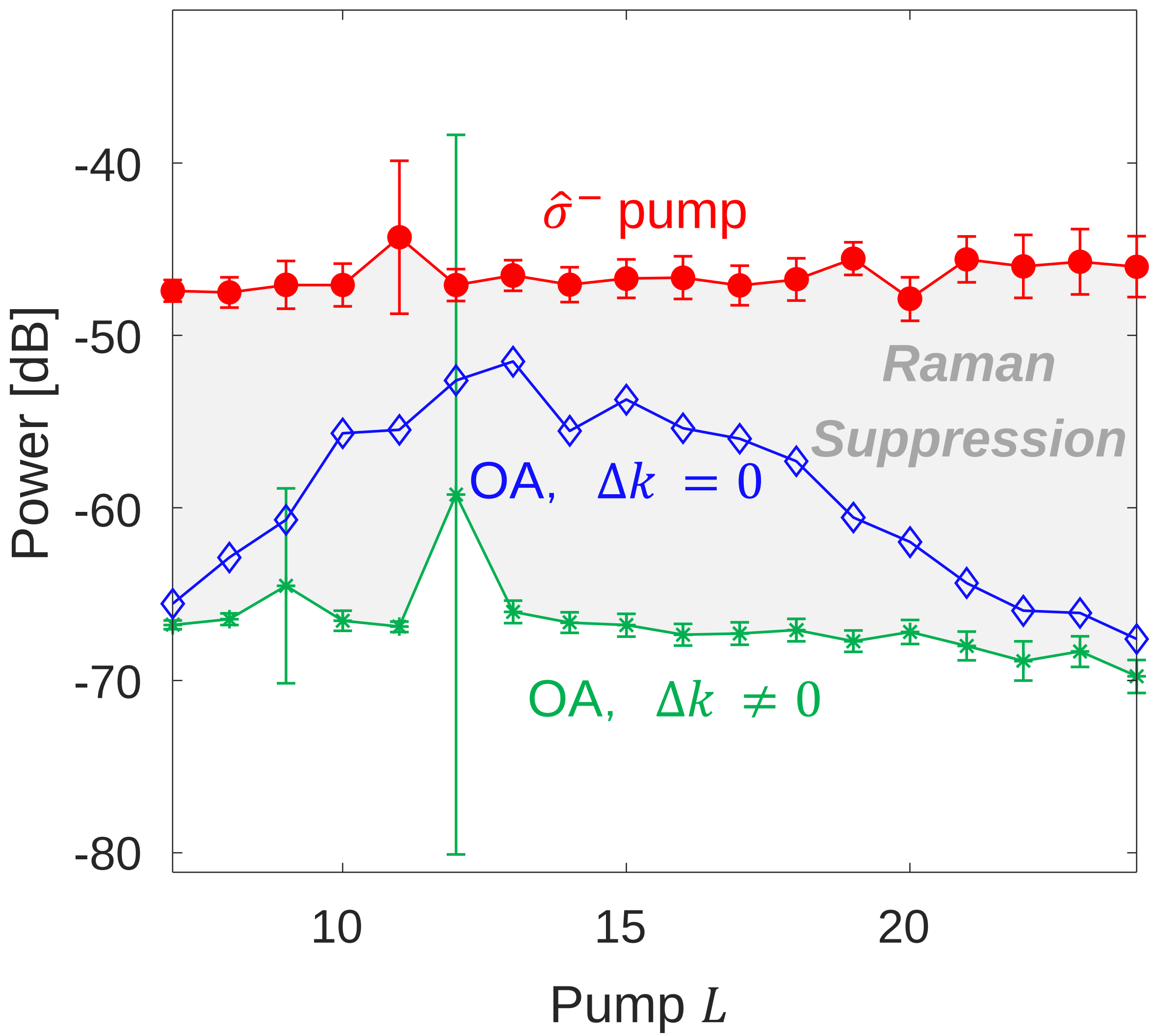
Chirality mediated Raman bandwidth

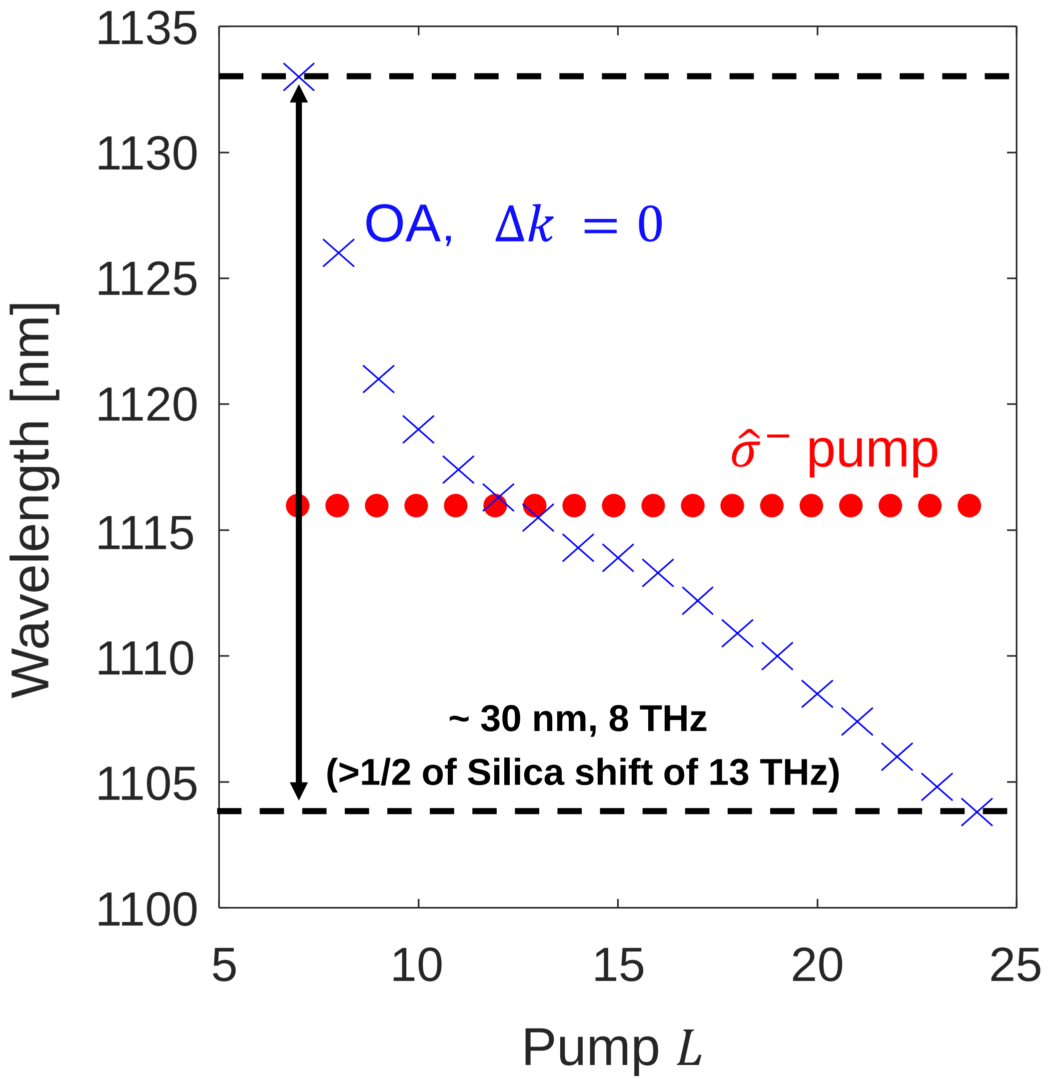
$\Delta\lambda \propto 1/z$



X. Liu et al, Opt. Express 30, 26967 (2022)

OA-mediated Raman Characteristics





- 20 dB Raman suppression
  - Fiber laser – increase SRS threshold
  - Reducing noise in entanglement source

- Wide Raman wavelength tuning
  - Phase plays a role in Raman scattering
    - Can dispersion engineer Raman gain shape

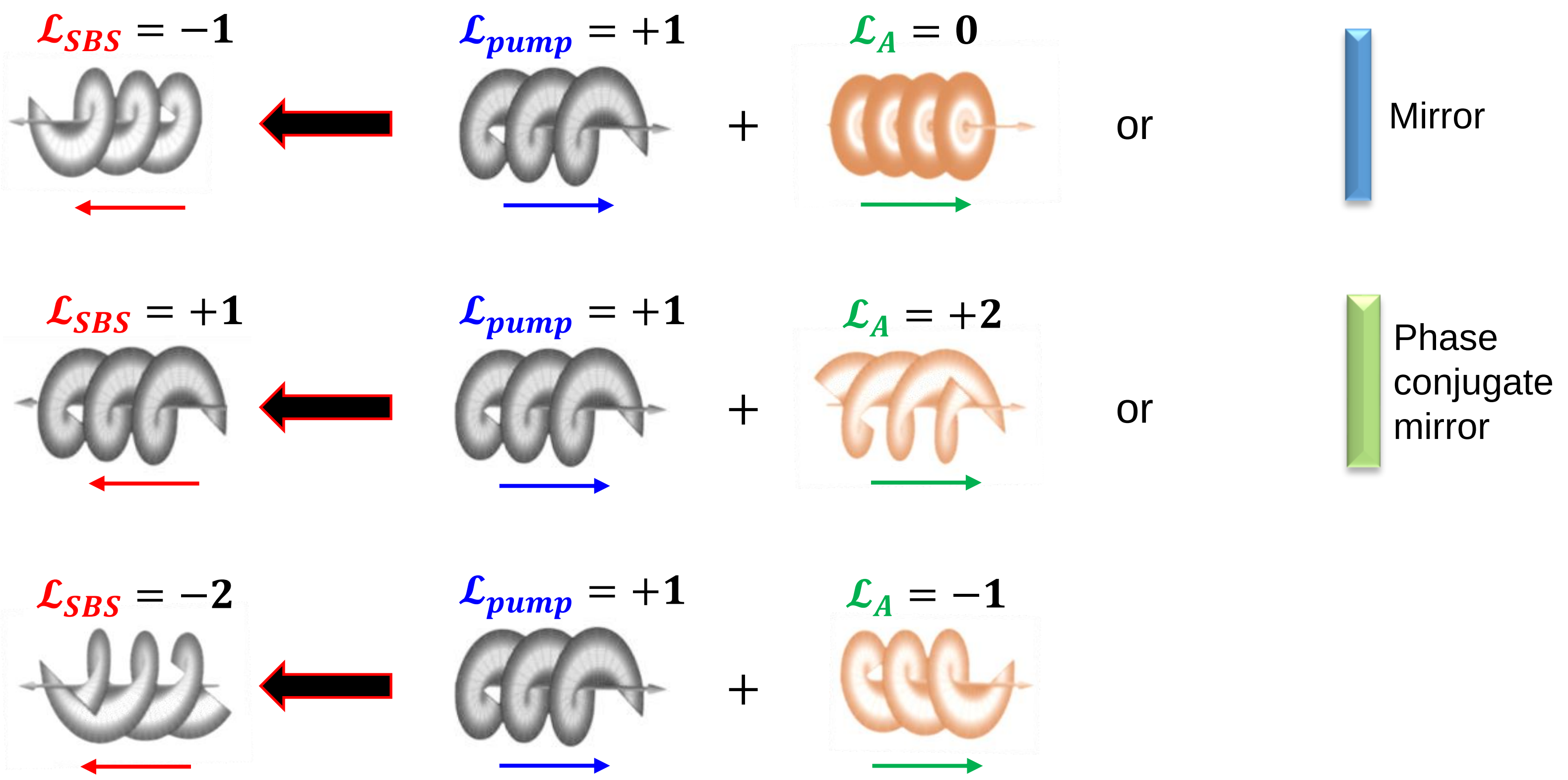
X. Liu, et al., *Opt. Exp.* **30**, 26967 (2022).

Stimulated Brillouin Scattering – dependence on OAM

General SBS gain: $G_{SBS} \propto {}^{1}/_{A_{eff}^{ao}} \propto \iint \vec{E}^{*}_{pump} \cdot \rho \cdot \vec{E}_{SBS}\, dA$

OAM-SBS gain: $G_{SBS} \propto {}^{1}/_{A_{eff}^{ao}} \propto \iint F^{*}_{\mathcal{L}_{pump}}(r,\phi) \cdot \rho_{\mathcal{L}_A}(r,\phi) \cdot F_{\mathcal{L}_{SBS}}(r,\phi)\ e^{i[\mathcal{L}_A - (\mathcal{L}_{pump} + \mathcal{L}_{SBS})]\phi}\, dA$

OAM conservation: $\mathcal{L}_A = \mathcal{L}_{pump} + \mathcal{L}_{SBS}$ (for $G_{SBS} \neq 0$); spin conserved by optical fields



Reflected Process

Conjugate Process

Which OAM modes are ***preferred*** for SBS?

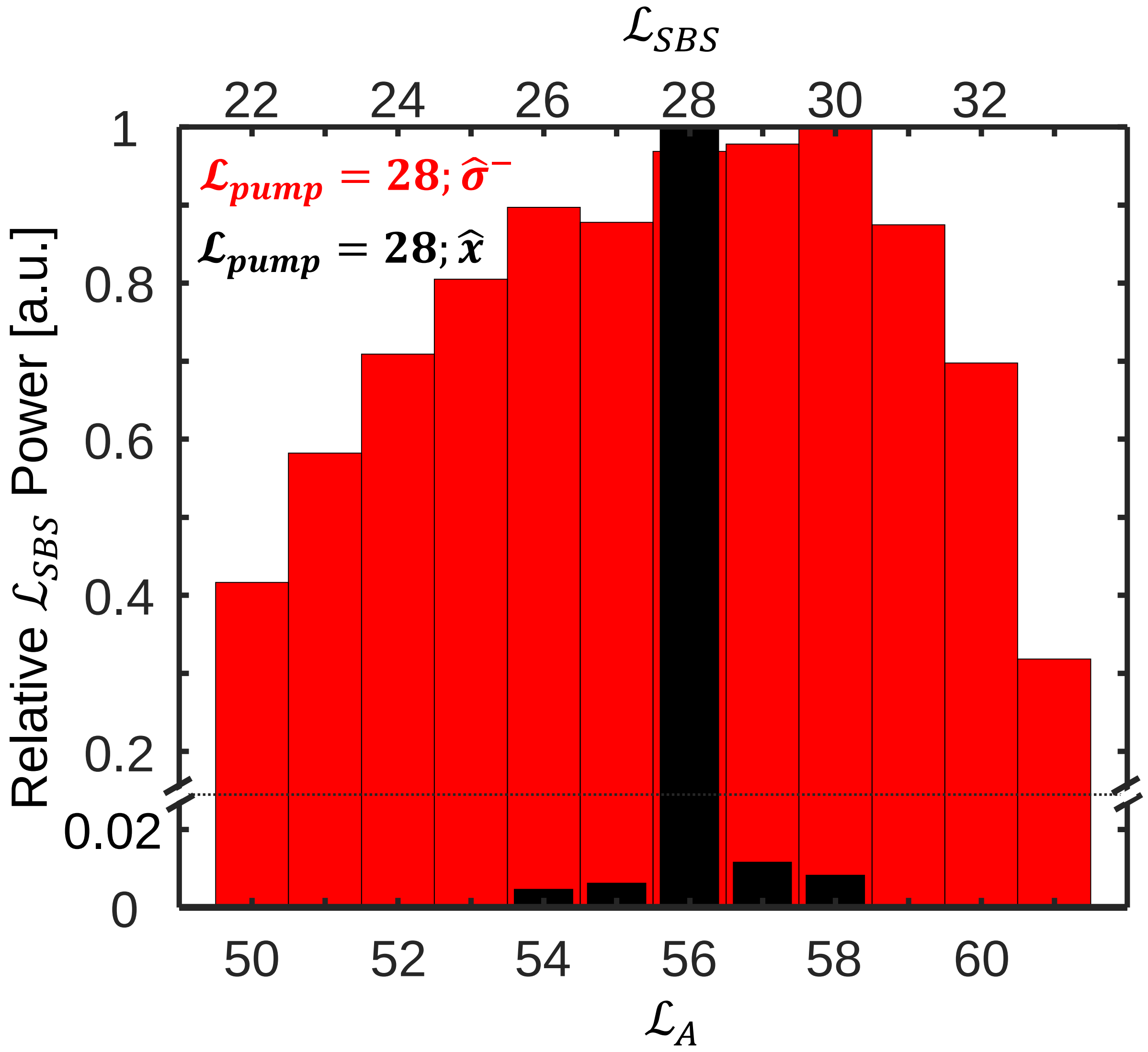
and all other combinations…

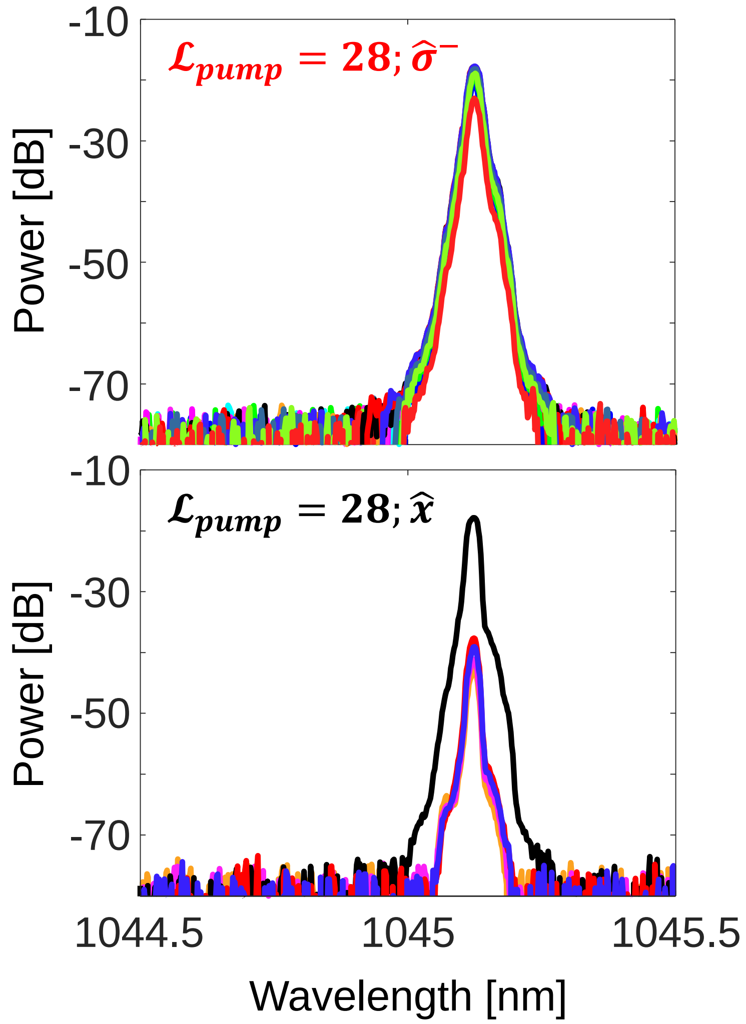
A. Y. Okulov, *J. Phys. B* 41,101001 (2008)
G. Prabhakar *et al.*, *CLEO* FTh1M.4 (2018)

Illustration Source: Wikimedia commons

OAM SBS Polarization Dependence

- Circularly polarized pump – highly multimoded SBS
- Linearly polarized pump – ***complete*** spatial phase conjugation





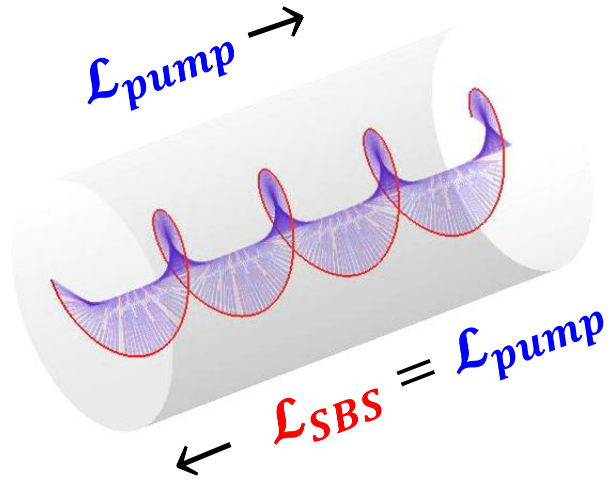
A.P. Greenberg et al, Opt. Express 30, 29708 (2022)

Optical Rotation Phase Matching / Overlap

Rotational Phase Matching Condition:

$$\frac{\partial A_{stokes}}{\partial z} \propto \exp\left(i\left[\Delta\beta(\mathcal{L}_{pump}) - \Delta\beta(\mathcal{L}_{SBS})\right]z\right)$$

Spin-Orbit Interaction $\Rightarrow \Delta\beta = \beta_{\sigma^+}(\mathcal{L}) - \beta_{\sigma^-}(\mathcal{L})$



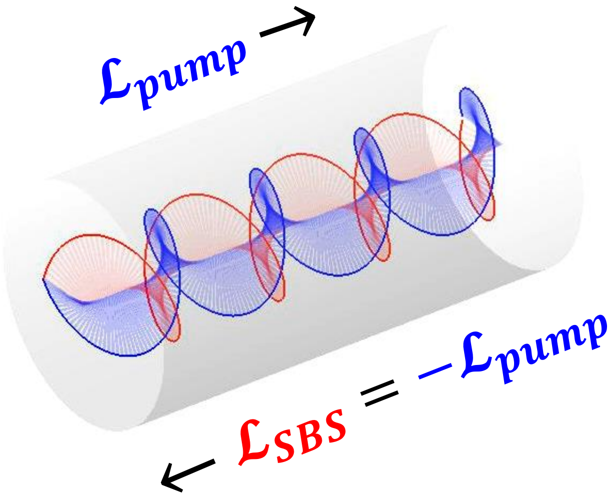
Polarization Overlap

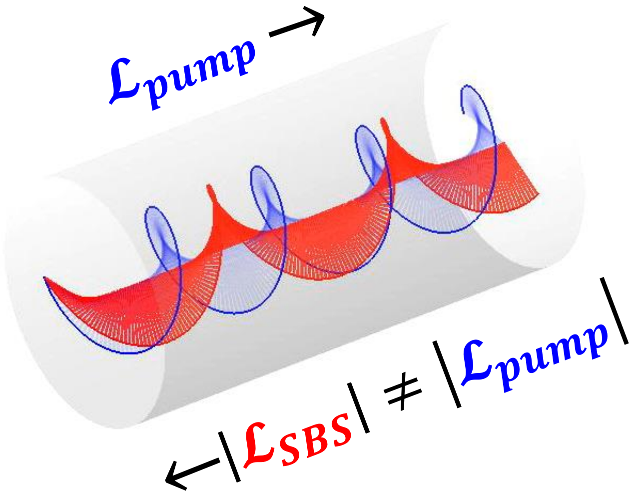
- $\mathcal{L}_{SBS} = \mathcal{L}_{pump} \rightarrow$ retraces rotation

  high nonlinear gain

- $\mathcal{L}_{SBS} \neq \mathcal{L}_{pump} \rightarrow$ rotational walk-off

  low nonlinear gain





Control over SBS gain via polarization and $\mathcal{L}$

R. Stolen, *J. Quantum Electron.* 15, 1157 (1979)

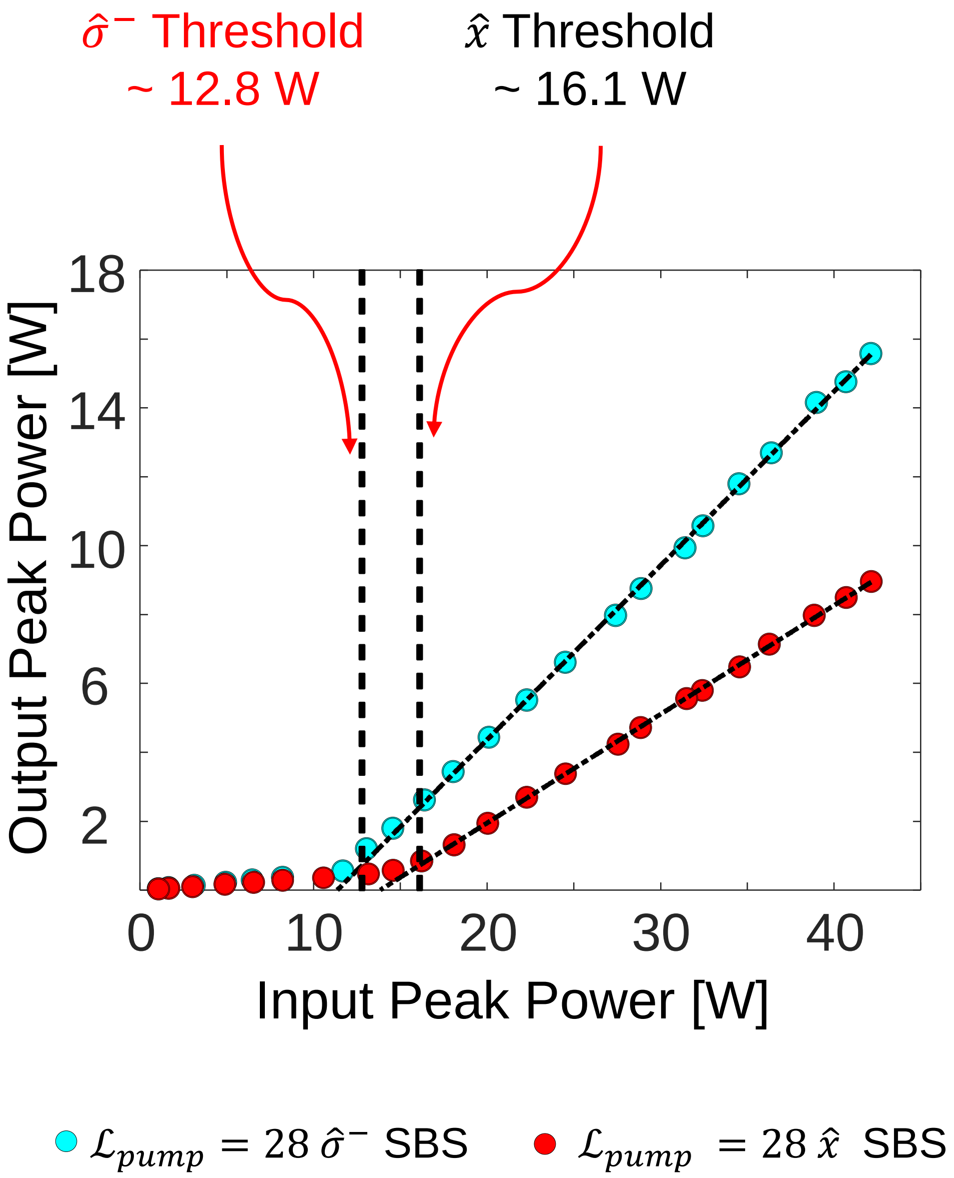
Angular Momentum Modulated SBS Power Thresholds

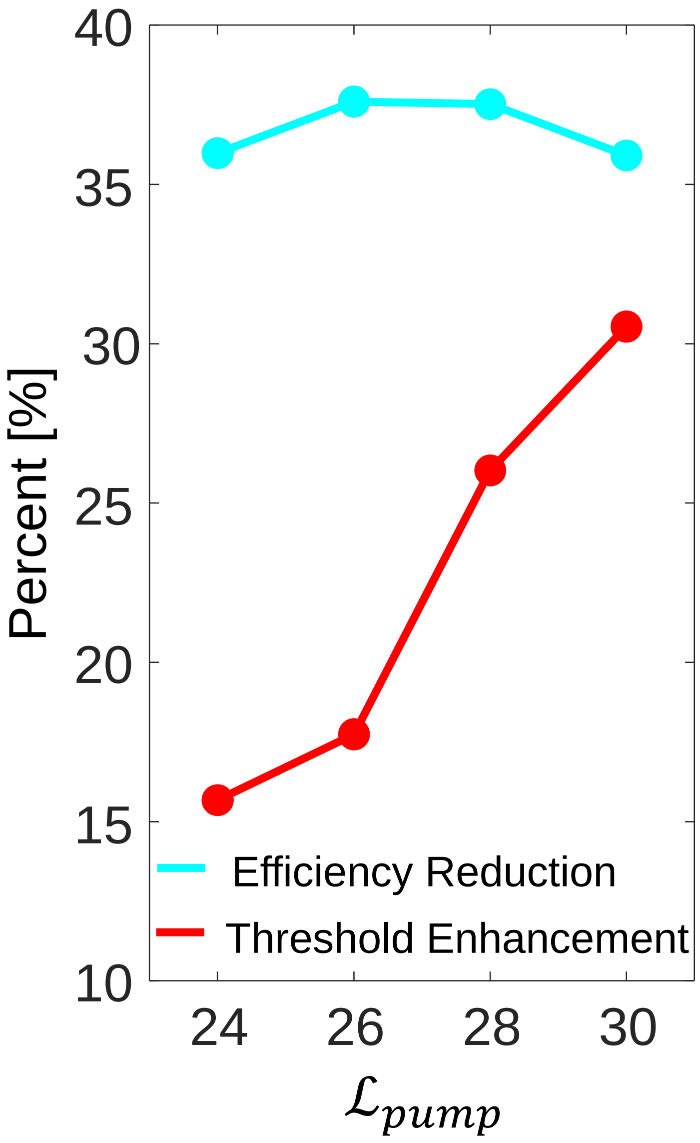
- Circular vs. linear polarization nonlinear gain
- Different gain dynamics changes SBS thresholds
- SBS Threshold:

$$P_{th} \approx 0.01 * P_{pump}$$

Up to 30% Threshold ***Enhancement***

~38% Efficiency ***Reduction***





A.P. Greenberg et al, STh5K.3, CLEO-2022

Outline

- **Background: Single-Mode Fiber Nonlinear Optics**
  - Capabilities and Limitations
- **Multimode Fiber NLO**
  - Governing principles for phase matching
  - A time-line of results from the last 5 decades
- **Segue: Optical Fiber Modes**
  - In-depth understanding of their linear properties
  - Unique, counter-intuitive behaviour for high order fiber modes
- **Unique Nonlinear Effects in Multimode Fibers**
  - Role of group velocity
  - Large modal dimensionality
  - Role of chirality
- **Applications**
  - Brief survey of current and emerging fields that exploit multimode fiber NLO

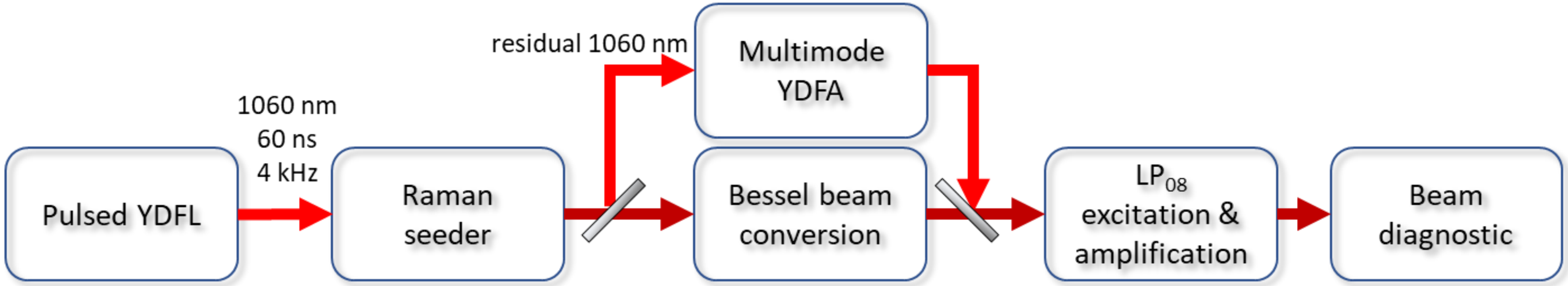
Power-scalable Raman fiber lasers

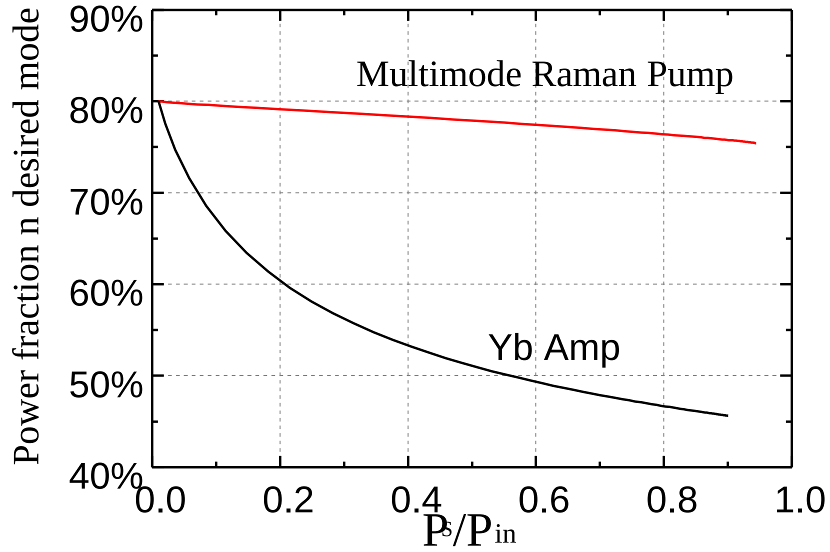
BOSTON UNIVERSITY

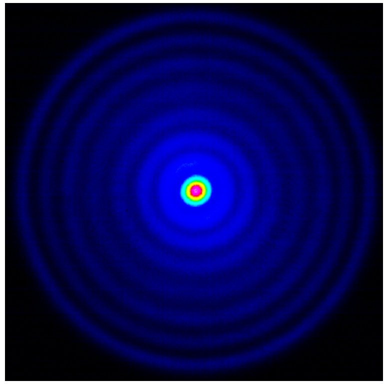
ATh4A.4.pdf OSA Laser Congress 2017 (ASSL,LAC) © OSA 2017

Multimode Raman Pumping for Power-Scaling of Large Area Higher Order Modes in Fiber Amplifiers

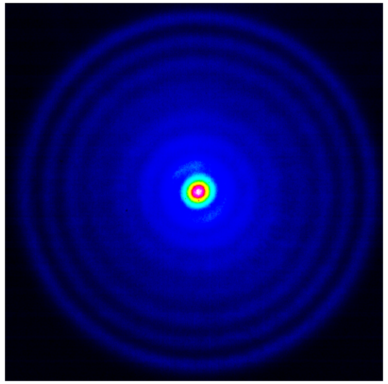
Sheng Zhu[1], Shankar Pidishety[1, 2], Yutong Feng[1], Jeff Demas[3], Siddharth Ramachandran[3], Balaji Srinivasan[2] and Johan Nilsson[1*]





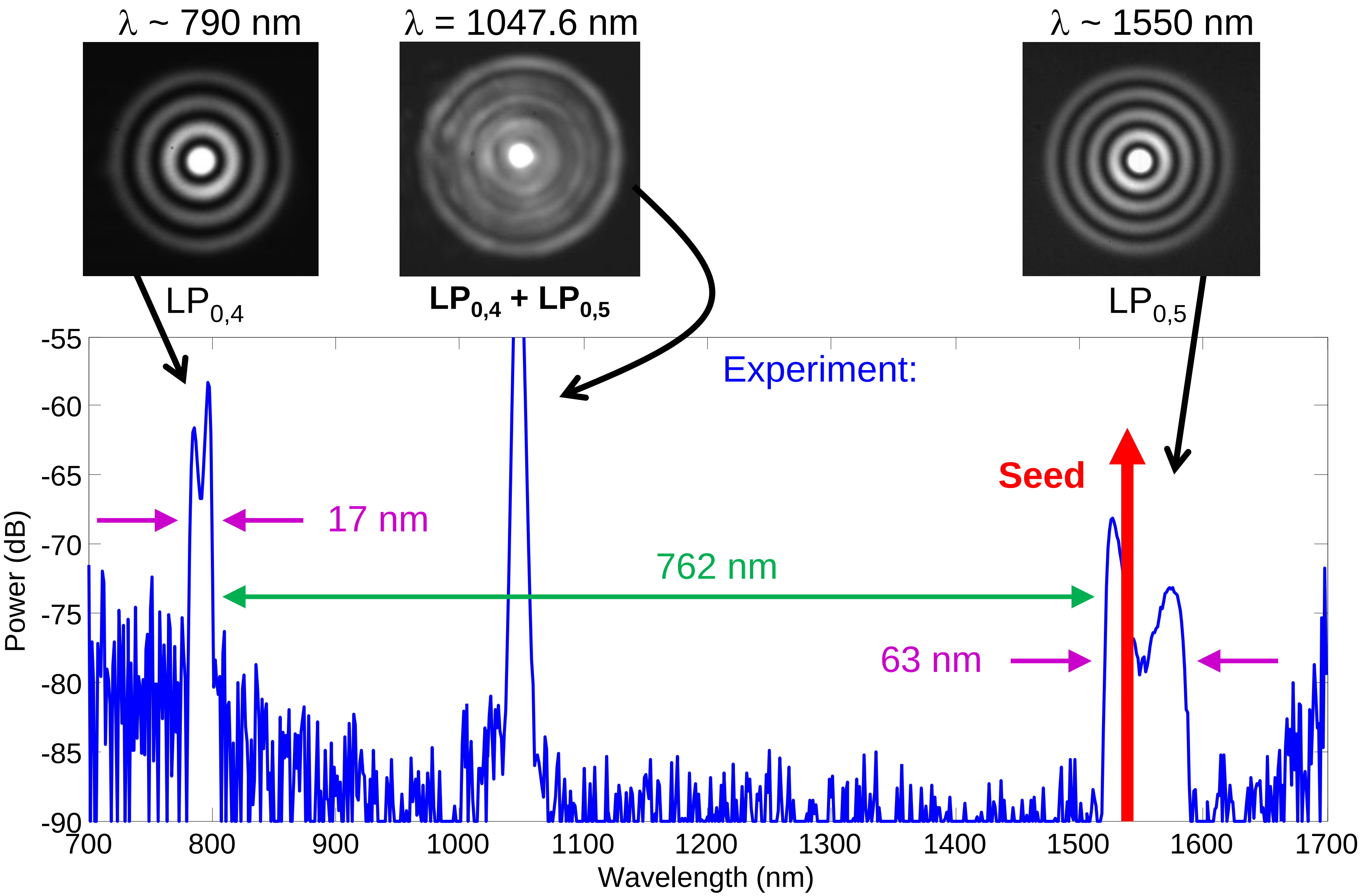


no amplification

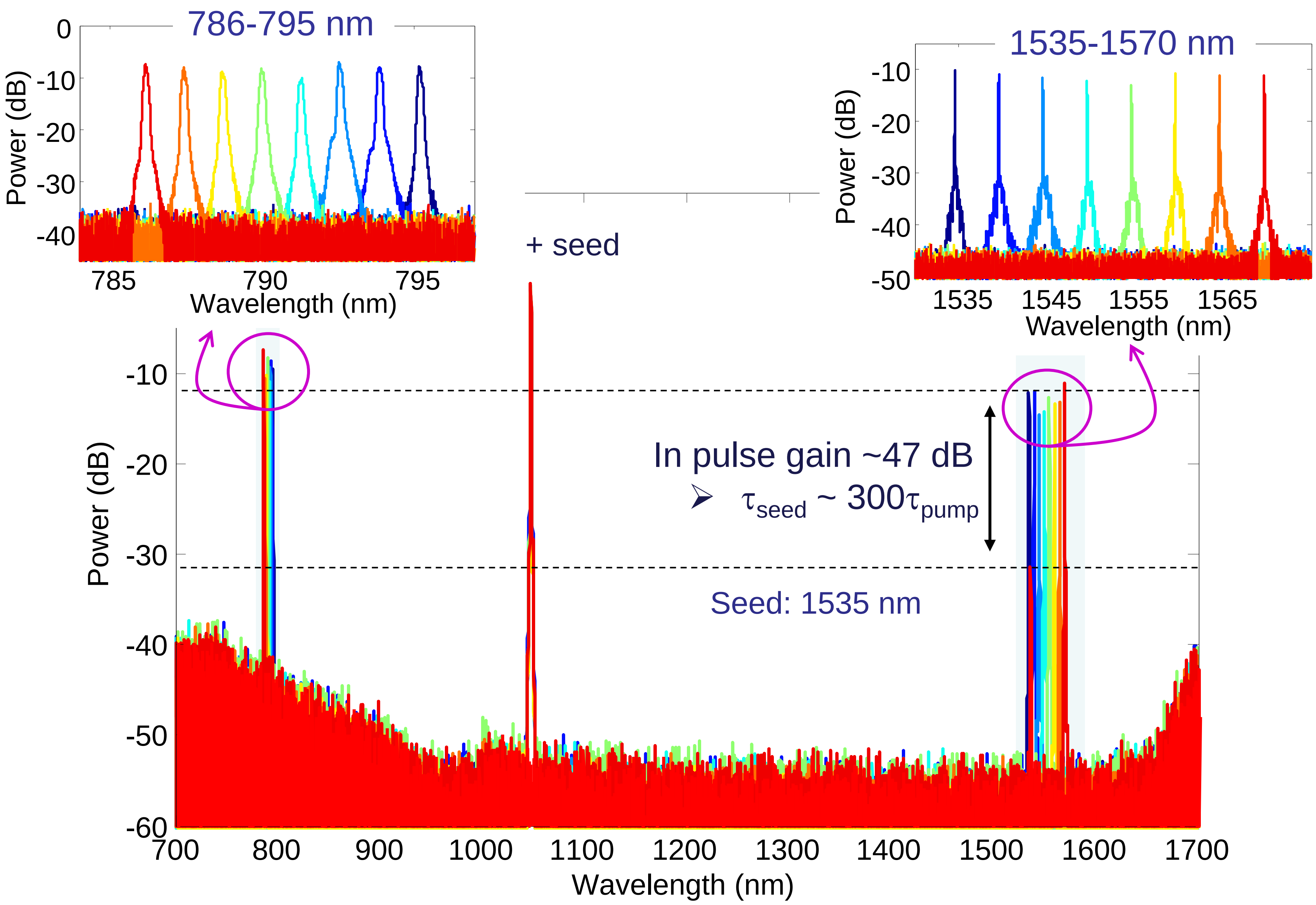


18 dB gain

Broadband *and* Wideband FWM Gain

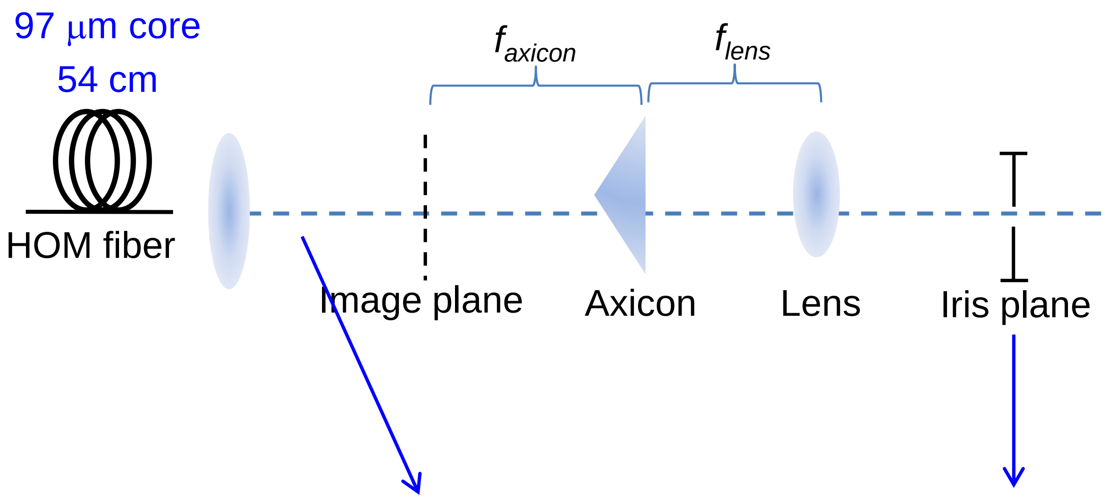


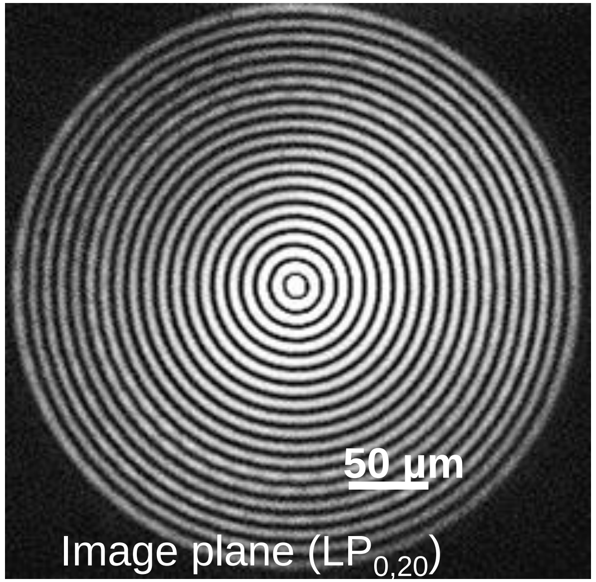
All-fiber “Ti:Sapphire” Laser

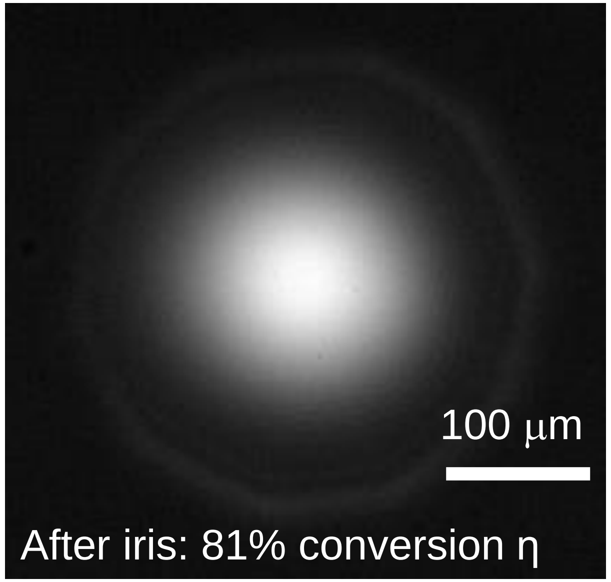


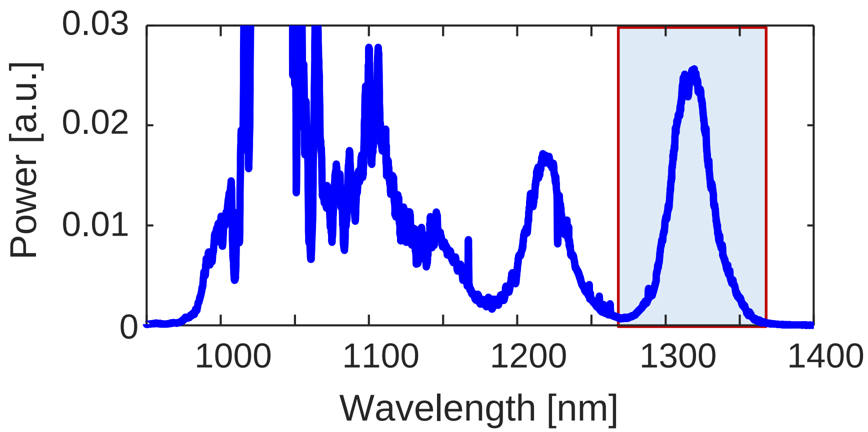
J. Demas et al, Photon. Res. 7, 1-7 (2019)

Output of Soliton Self-Mode Conversion (SSMC)









80 nJ, 74 fs

Peak power 1.1 MW

Experimentally:

- Gaussian-like beam
- Conversion efficiency:
  -0.9 dB (81 %)

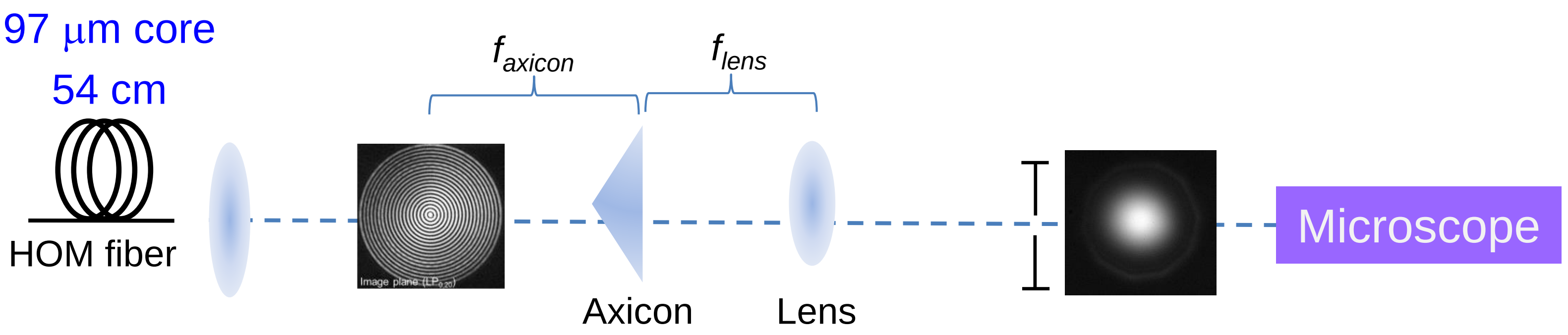
Theoretically:

- $M^2 = 1.06$ *(after spatial filtering)*
- Conversion efficiency:
  -0.65 dB (86 %)

L. Rishøj et al., J. Biomedical Opt 27, 056501

L. Rishøj et al., Optica 6, 3 (2019)

SSMC-based microscopy

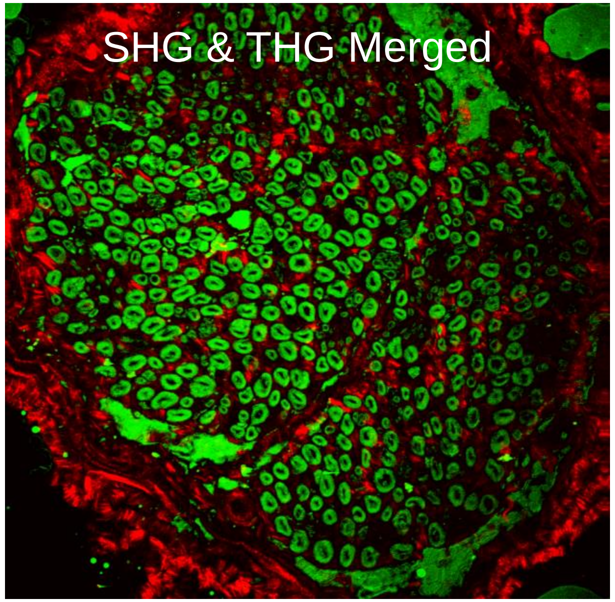


Nearly diffraction limited system

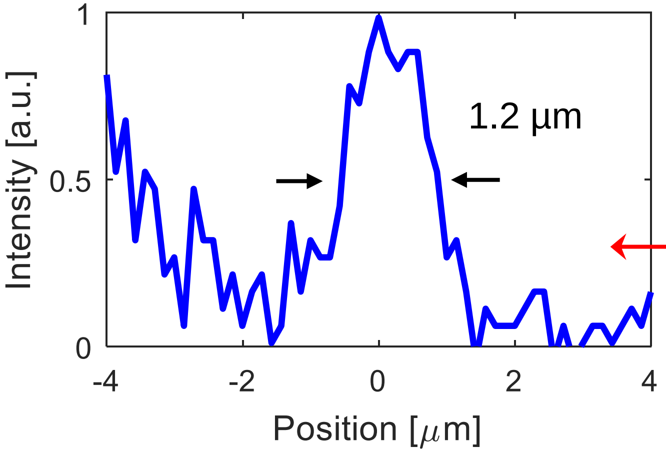
↓

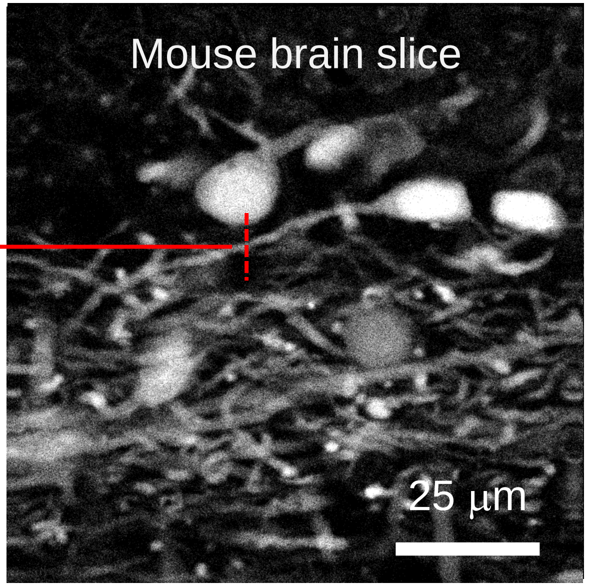
High spatial coherence

High-Harmonic Nerve Imaging



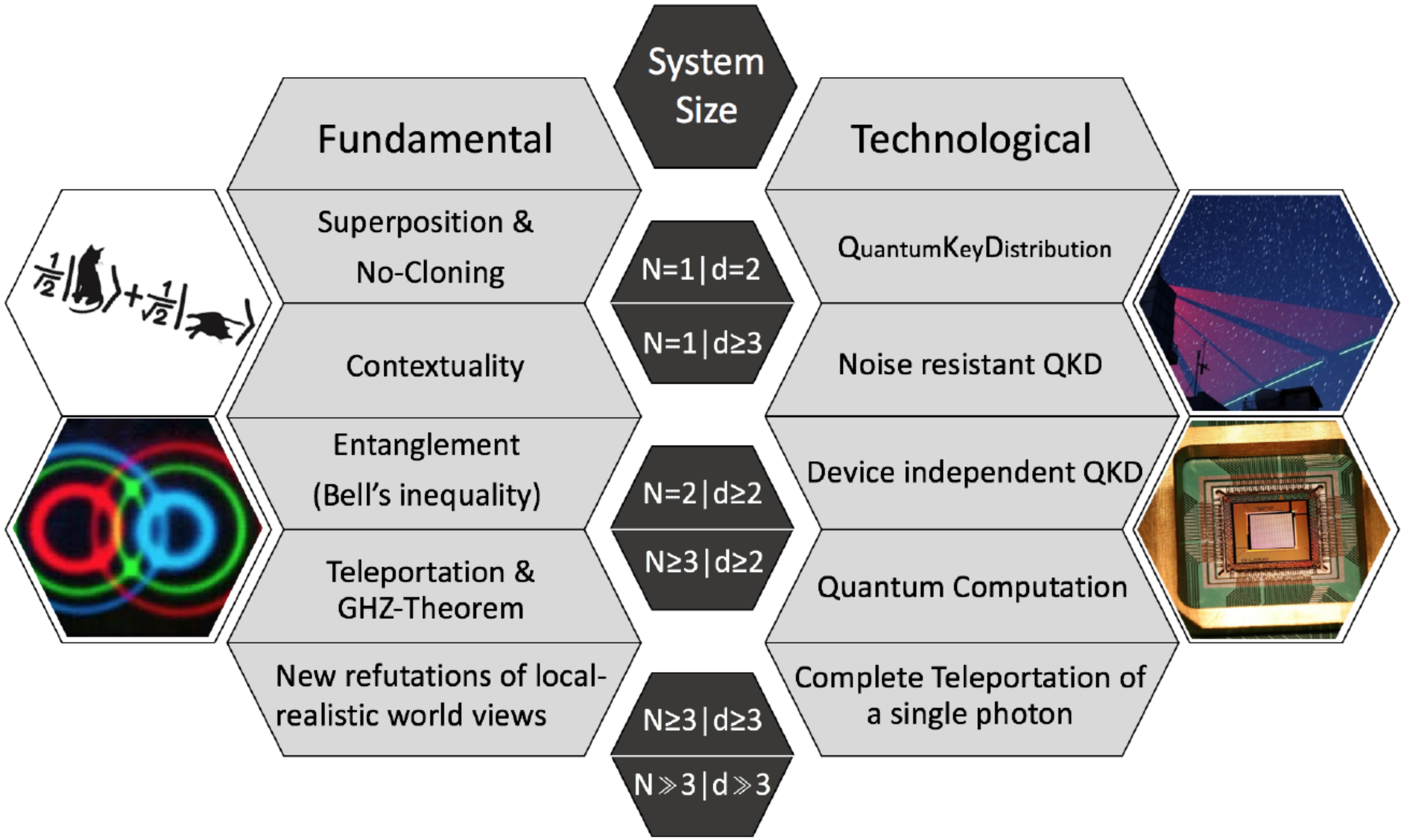
3-Photon Microscopy

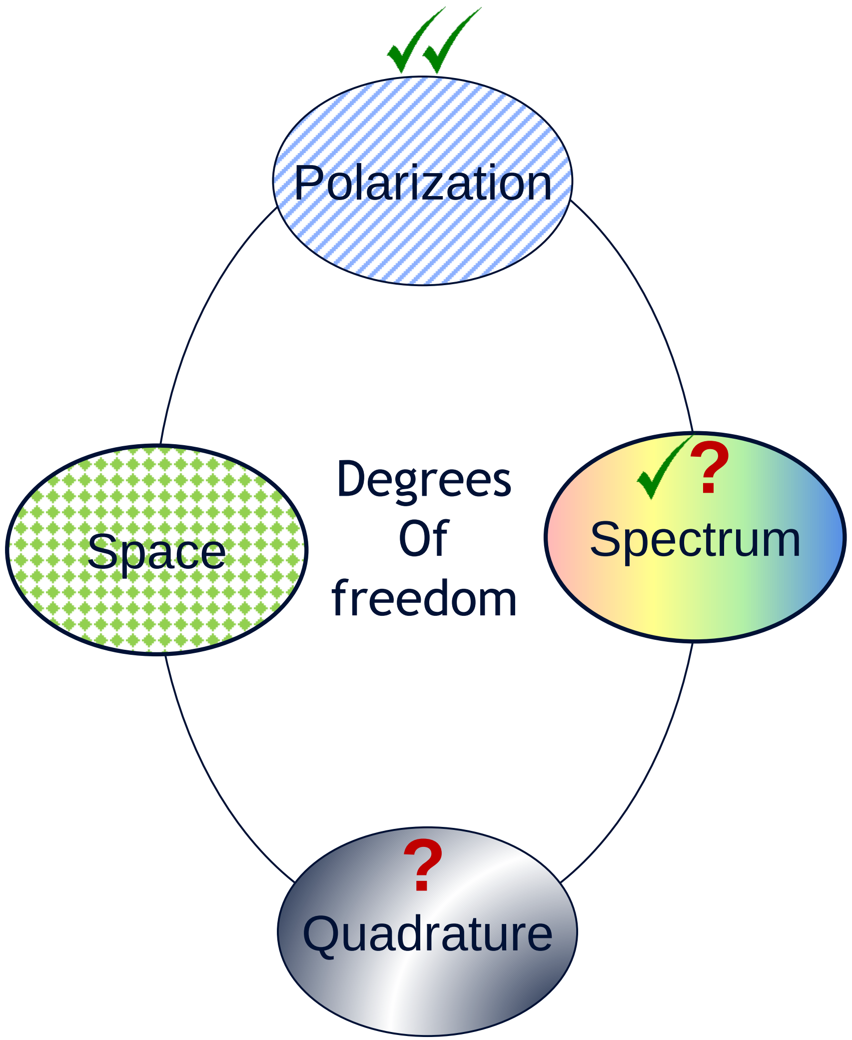




L. Rishøj et al., J. Biomedical Opt 27, 056501

Dimensionality of Entanglement





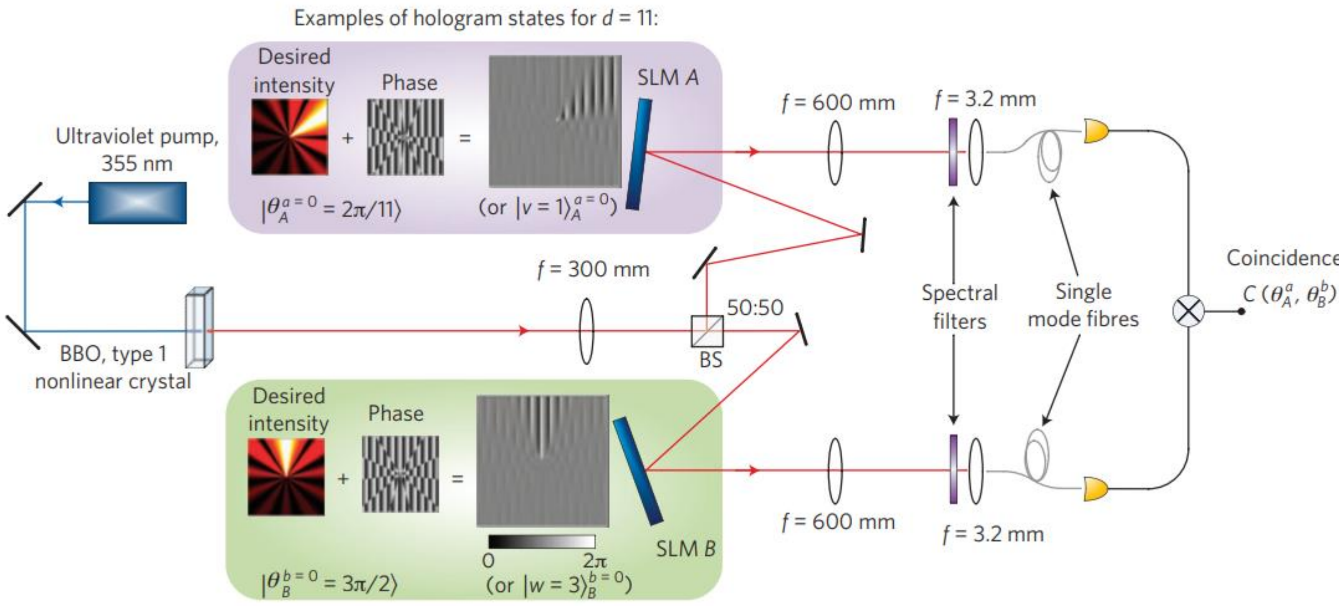
M. Erhard, M. Krenn, A. Zeilinger, "Advances in high-dimensional quantum entanglement," *Nat Rev Phys* 2, 365–381 (2020)

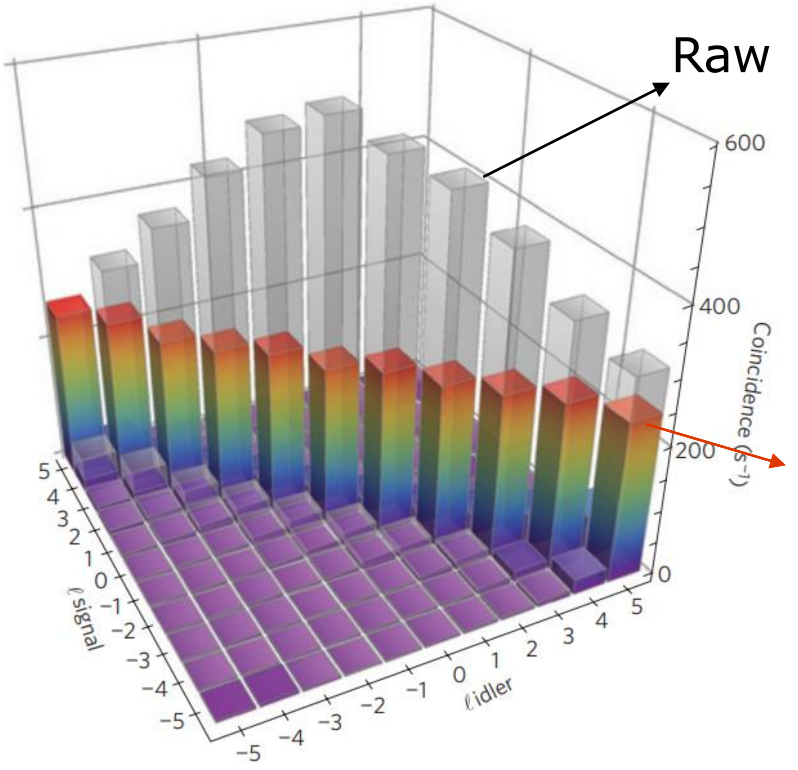
Crystal-based Entanglement in OAM

NATURE | VOL 412 | 19 JULY 2001 | 313

Entanglement of the orbital angular momentum states of photons

Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger



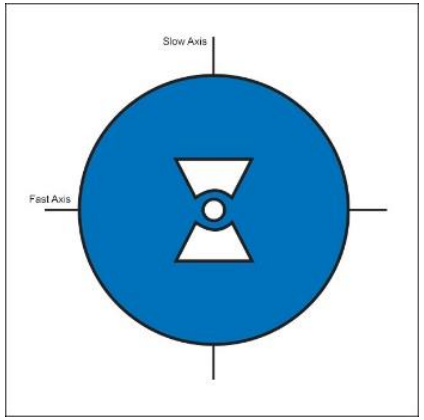


A. C. Dada et al., *Nature Physics* 7 677-68

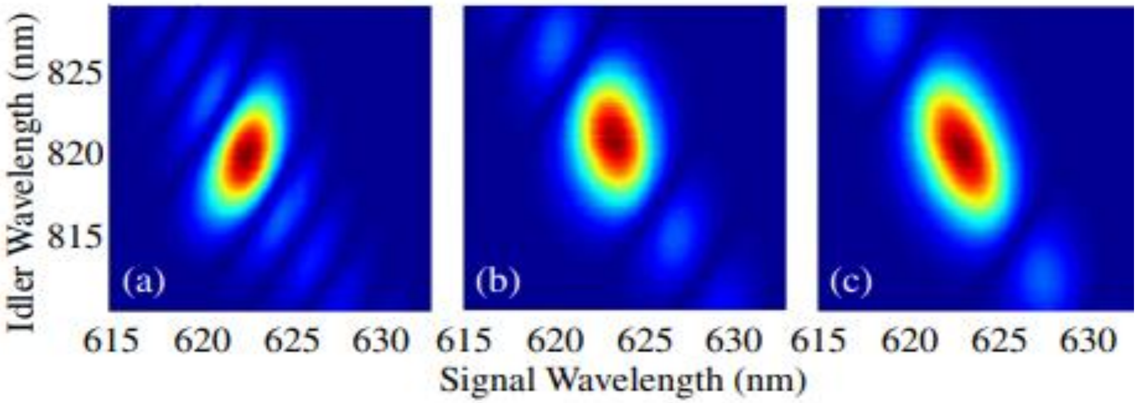
Controlling Joint Spectral Densities

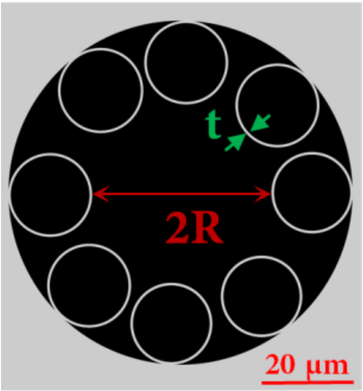
$$\phi(\omega_{as}, \omega_s) = sinc(\Delta k \mathbf{z}/2)\exp(i\Delta k \mathbf{z}/2)$$

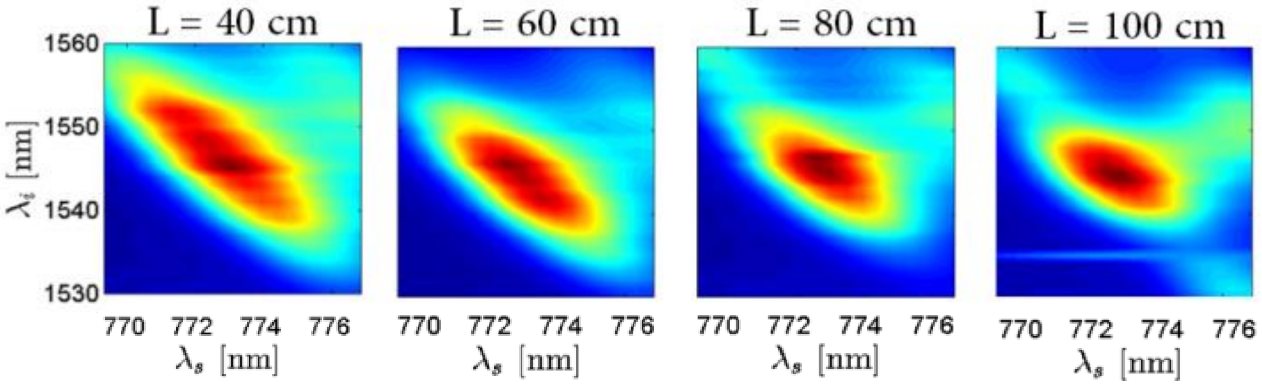
Propagation distance: *z*



2.6 cm 1.6 cm 1.1 cm

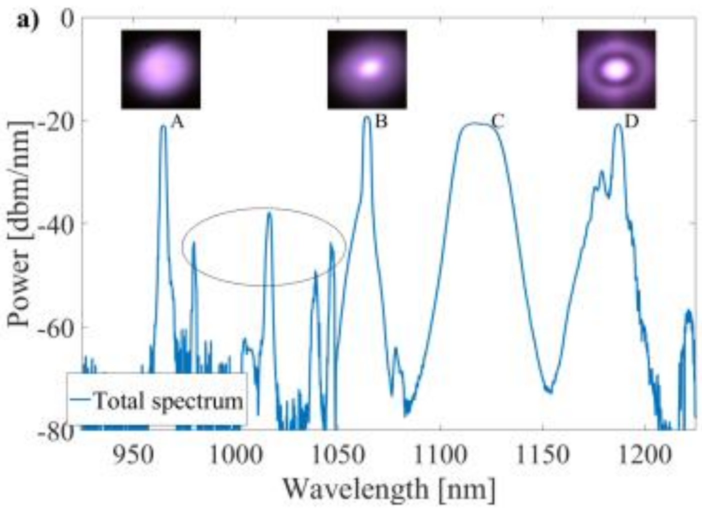


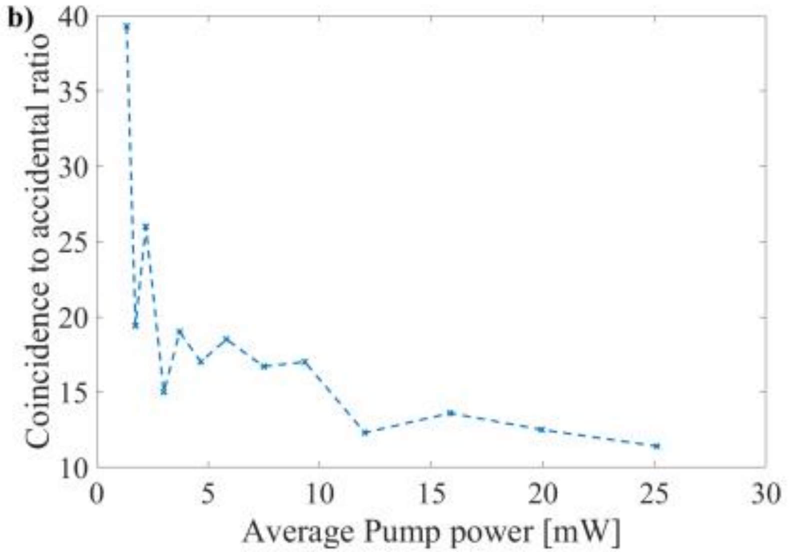




B. Fang et al., *Optica* 1, 281 (2014).
M. Cordier et al., *Opt. Express* 27, 9803 (2019).

$$\phi(\omega_{as}, \omega_s) = sinc(\Delta \boldsymbol{k} z/2)\exp(i\Delta \boldsymbol{k} z/2)$$

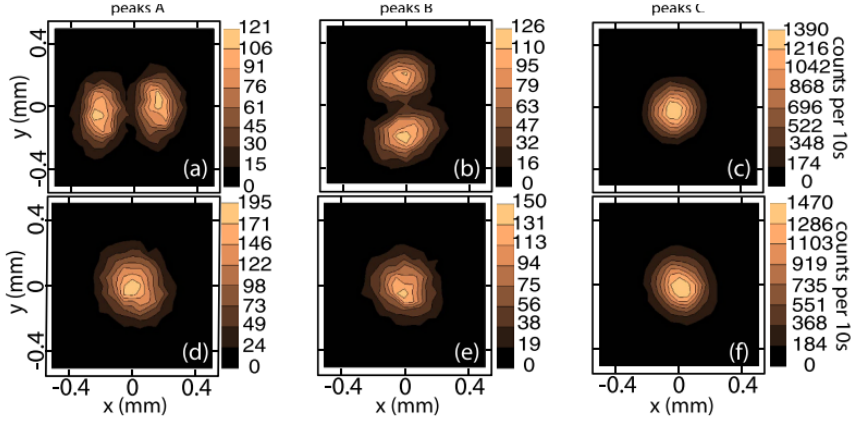


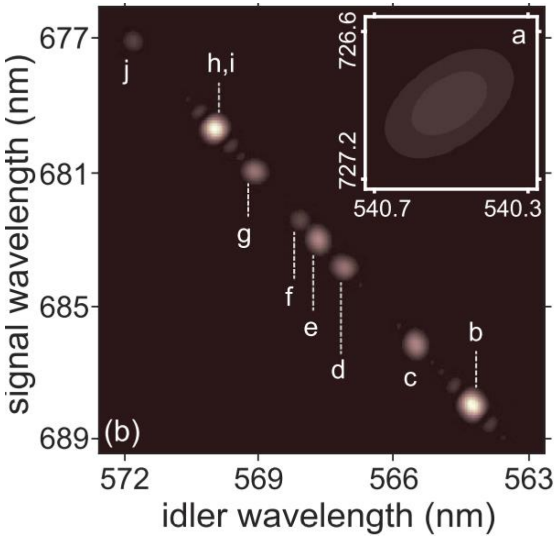


Multimode FWM

K. Rottwitt, J.G. Koefoed, E.N. Christensen, *Fibers* 6, 32 (2018).

JSDs in Few-mode fibers

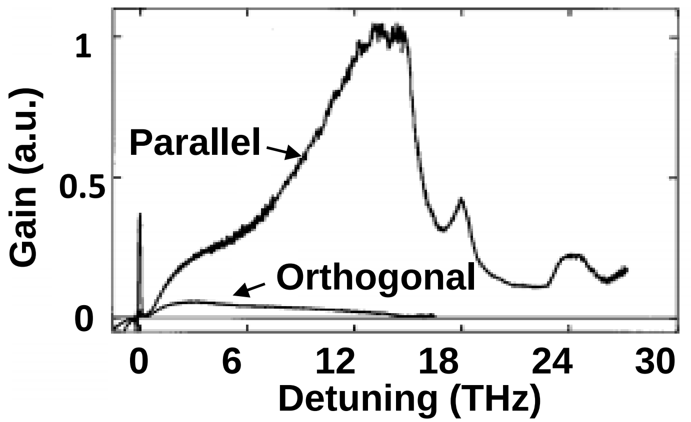




D. Cruz-Delgado et al., *Sci. Rep.* 6, 1-9 (2016).

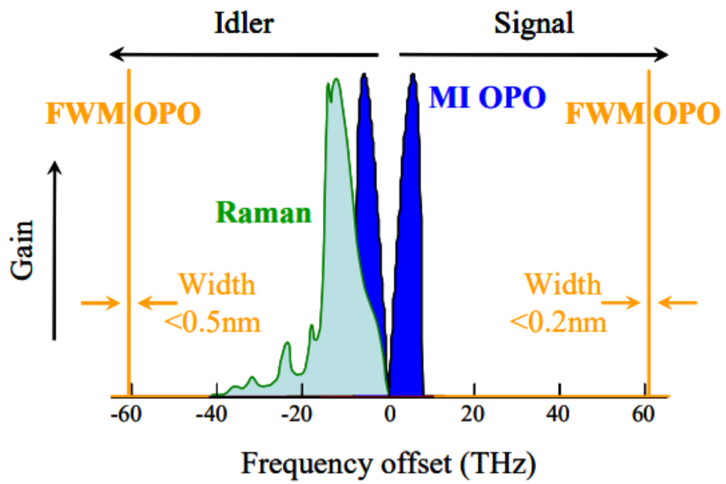
The Raman Noise Problem

Raman scattering: Noise in Silica fiber



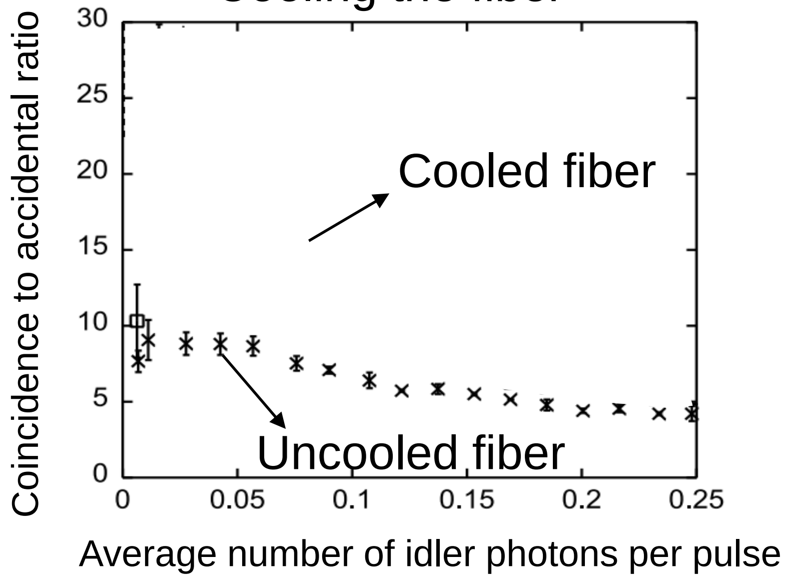
R. H. Stolen et al., *Appl. Phys. Lett.* 20, 62-64 (1972)

Large wavelength separation



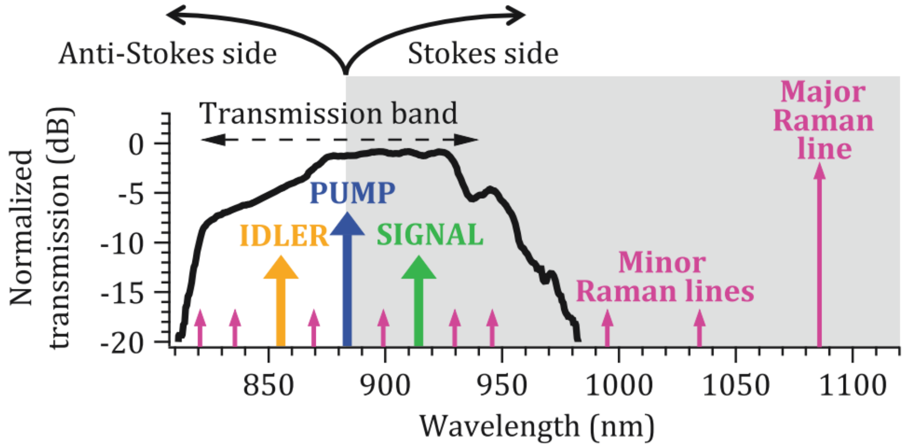
J. G. Rarity, et al. *Optics express* **13** 534-544 (2005).

Cooling the fiber



H. Takesue and K. Inoue. *Optics express* **13** 7832-7839 (2005).

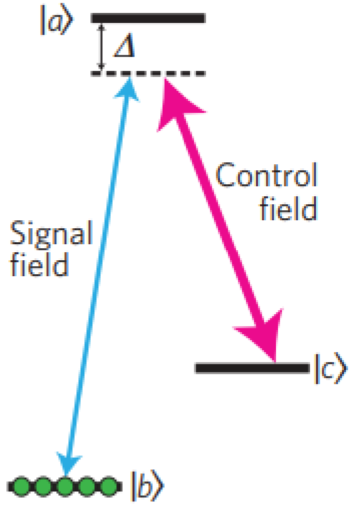
Liquid-core fibers

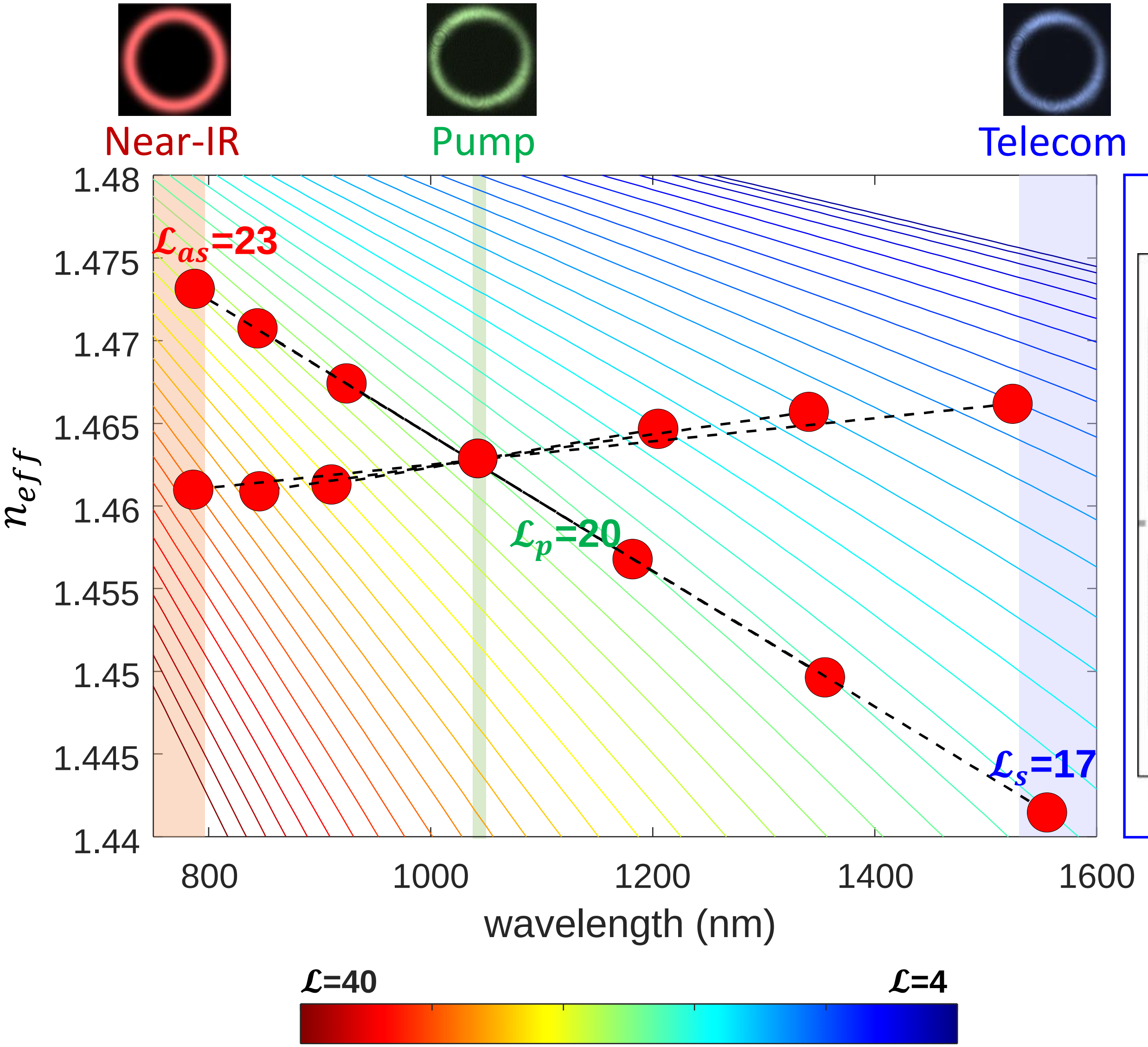


M. Barbier, et al. *new Journal of Physics*, **17**, 053031. (2015).

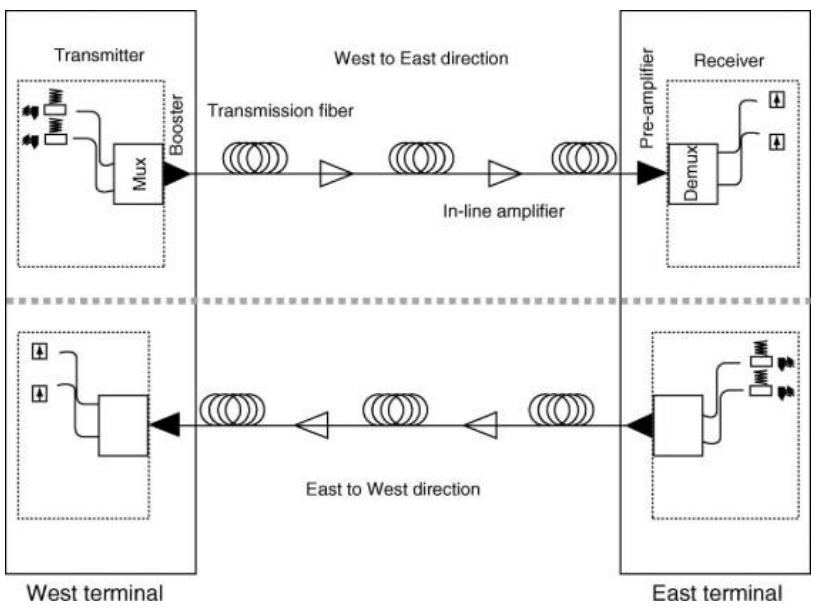
Engineering goal

Quantum memory





Long-haul transmission

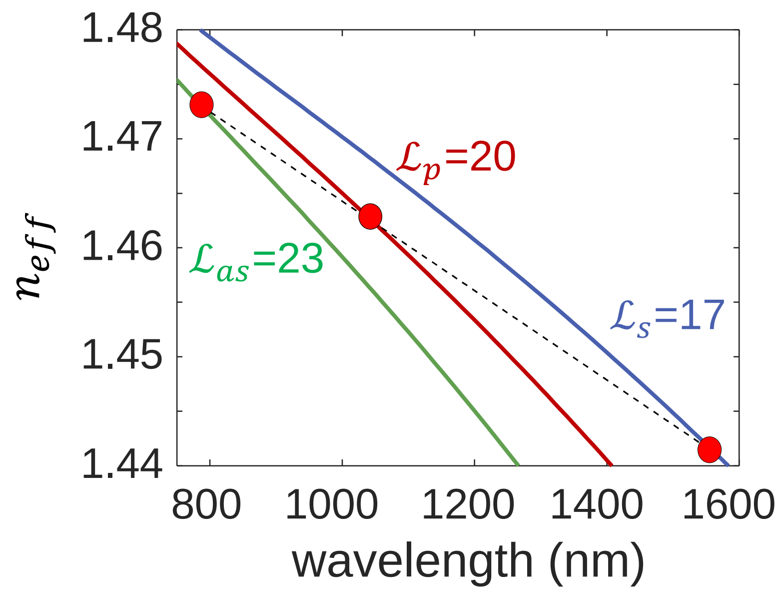


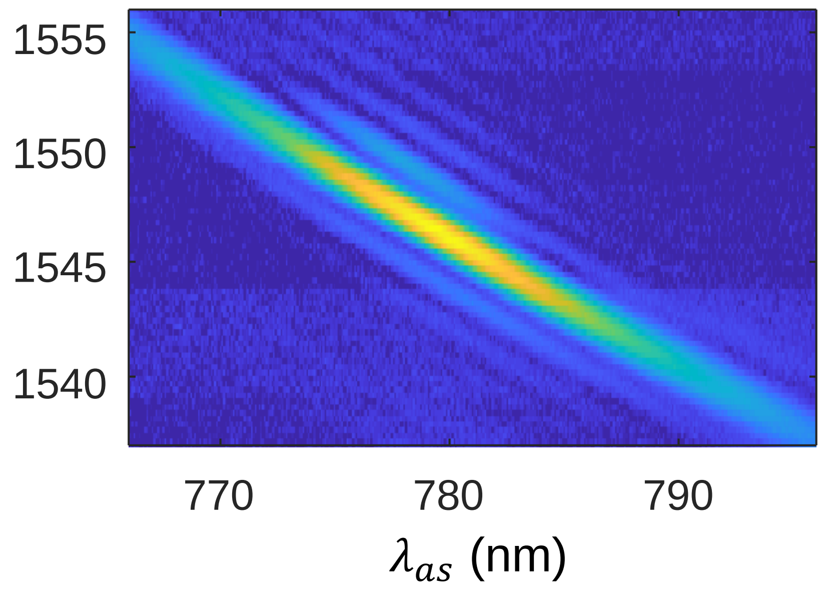
A. I. Lvovsky et al., *Nat. Photonics* 3, 706 (2009).
J. Chesnoy ed., “Undersea fiber communication systems”,*Academic press* (2015).

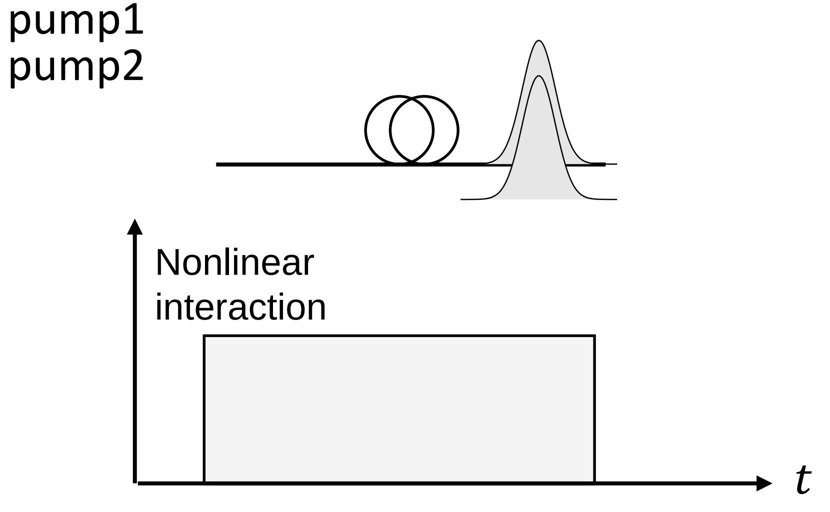
Pump modal sculpting to control bi-photon spectral sidebands

Degenerate pump:

X. Liu et al, FF2J.1, CLEO (2022)



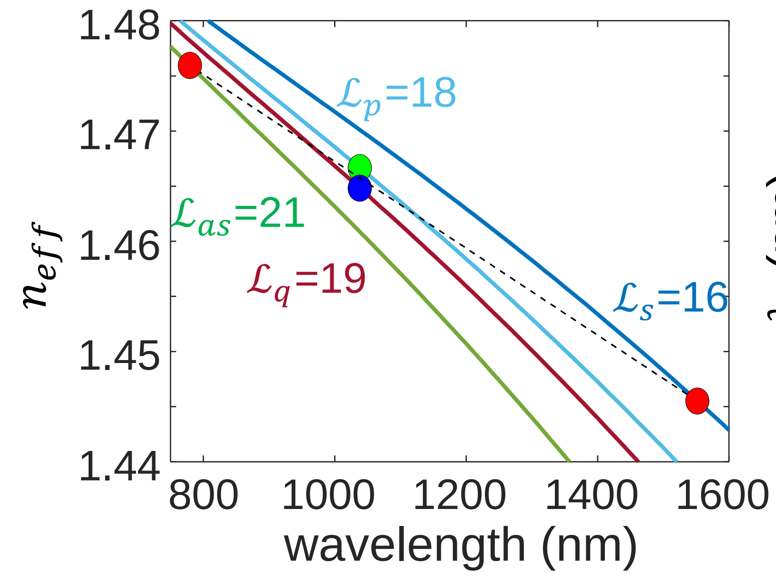


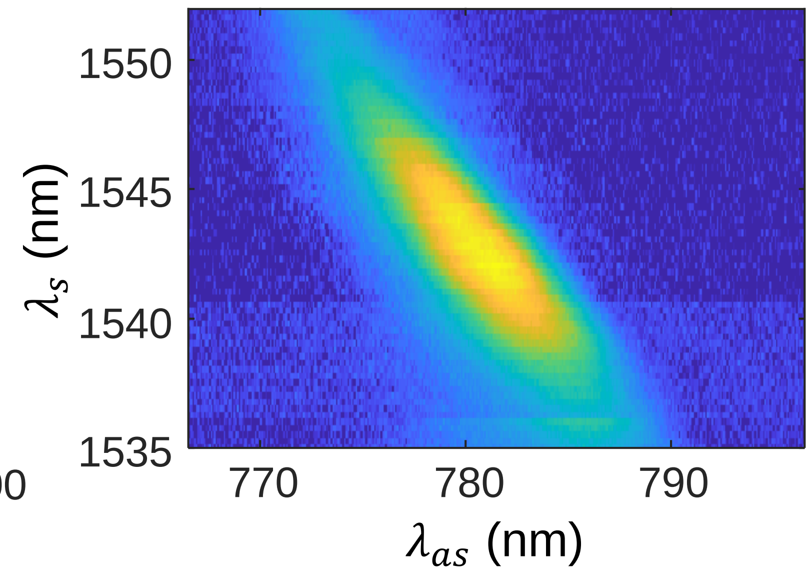


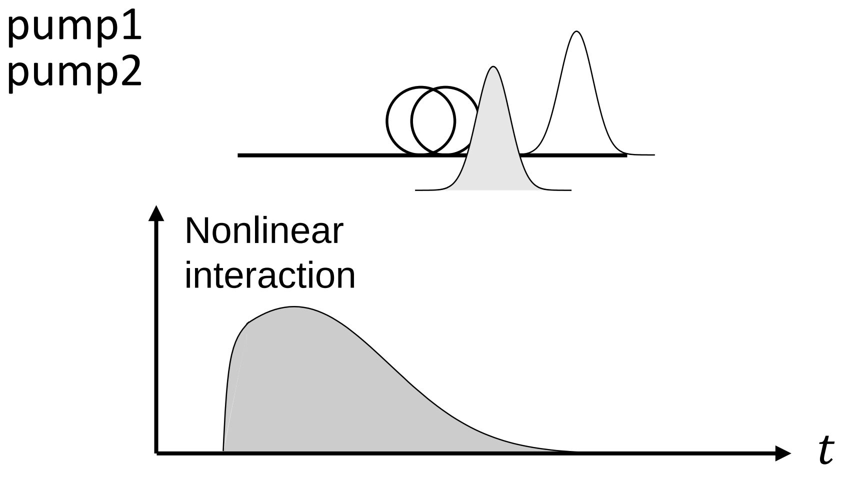
Sidebands suppression – Pump walk-off

- Pump frequency diversity
- **Pump mode diversity**

Non-degenerate pump:

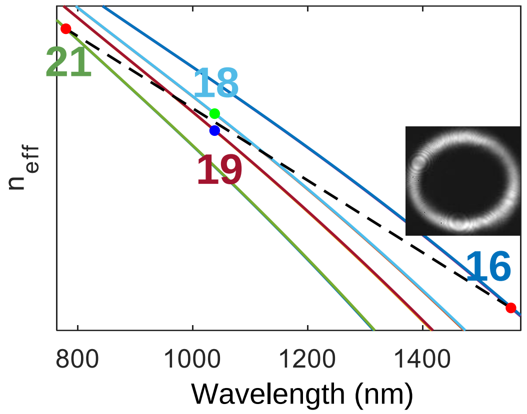


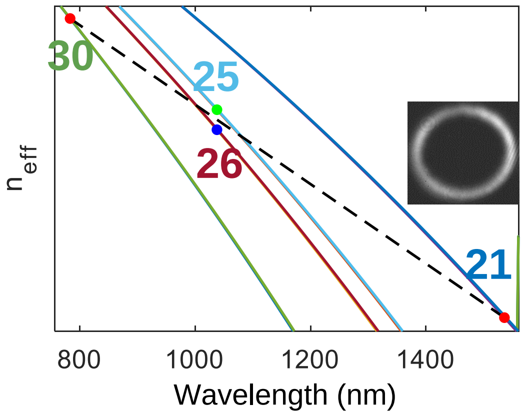


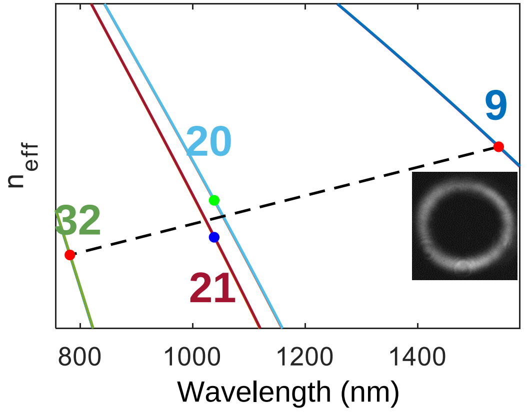


B. Fang et al., *Opt. Express* 21, 2707-2717 (2013)

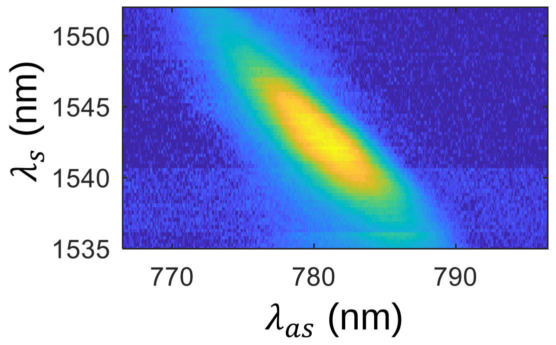
Bi-photon spectral engineering with low noise



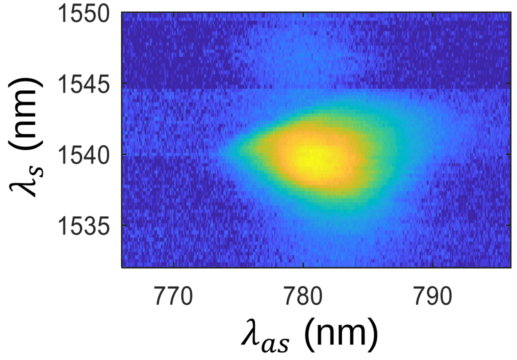




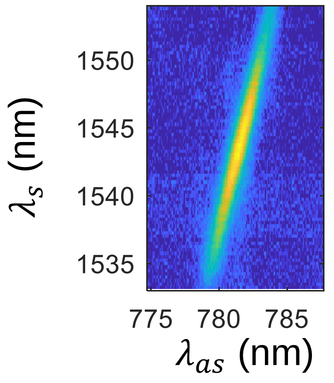
Anti-correlated

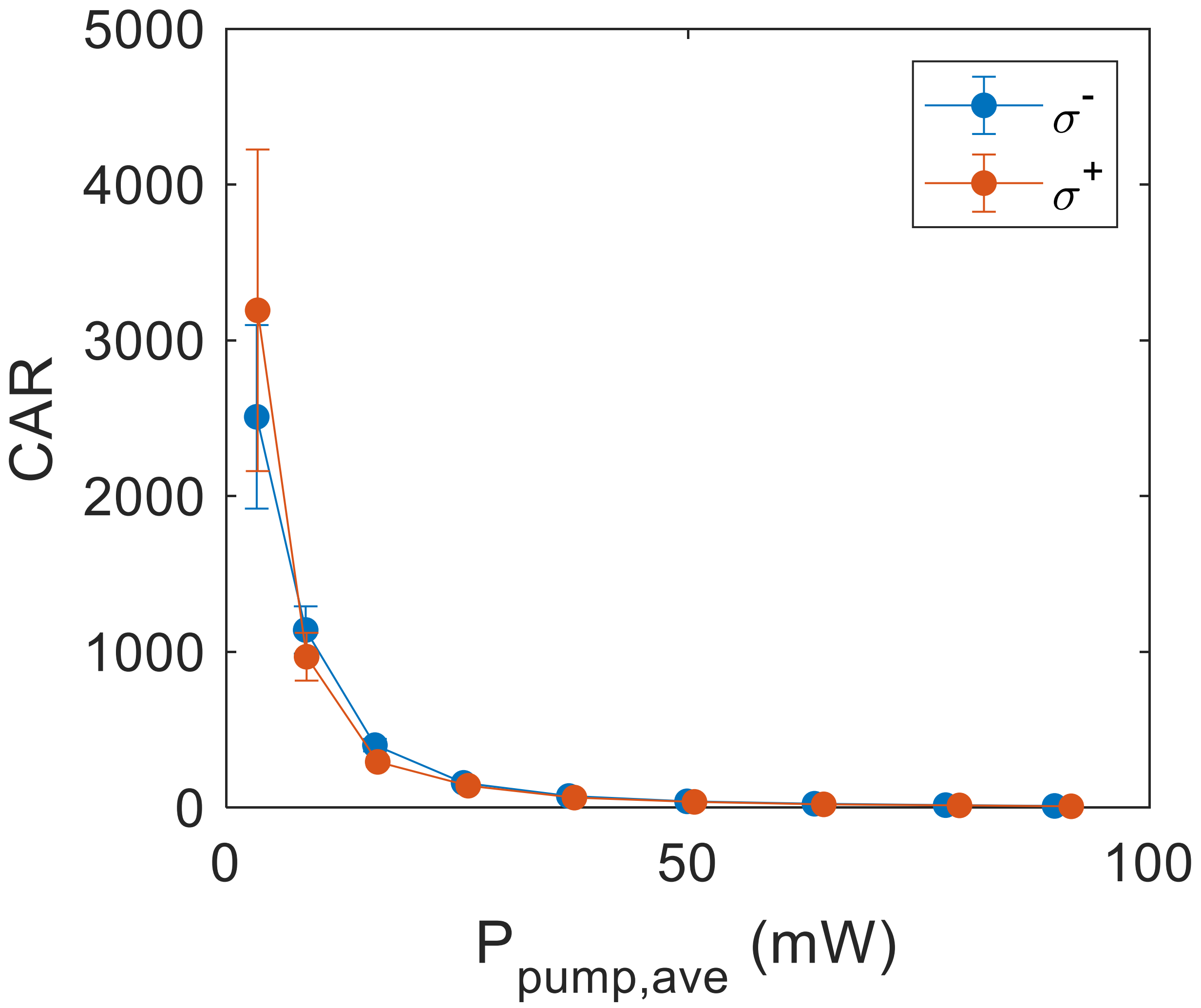


Uncorrelated



Correlated



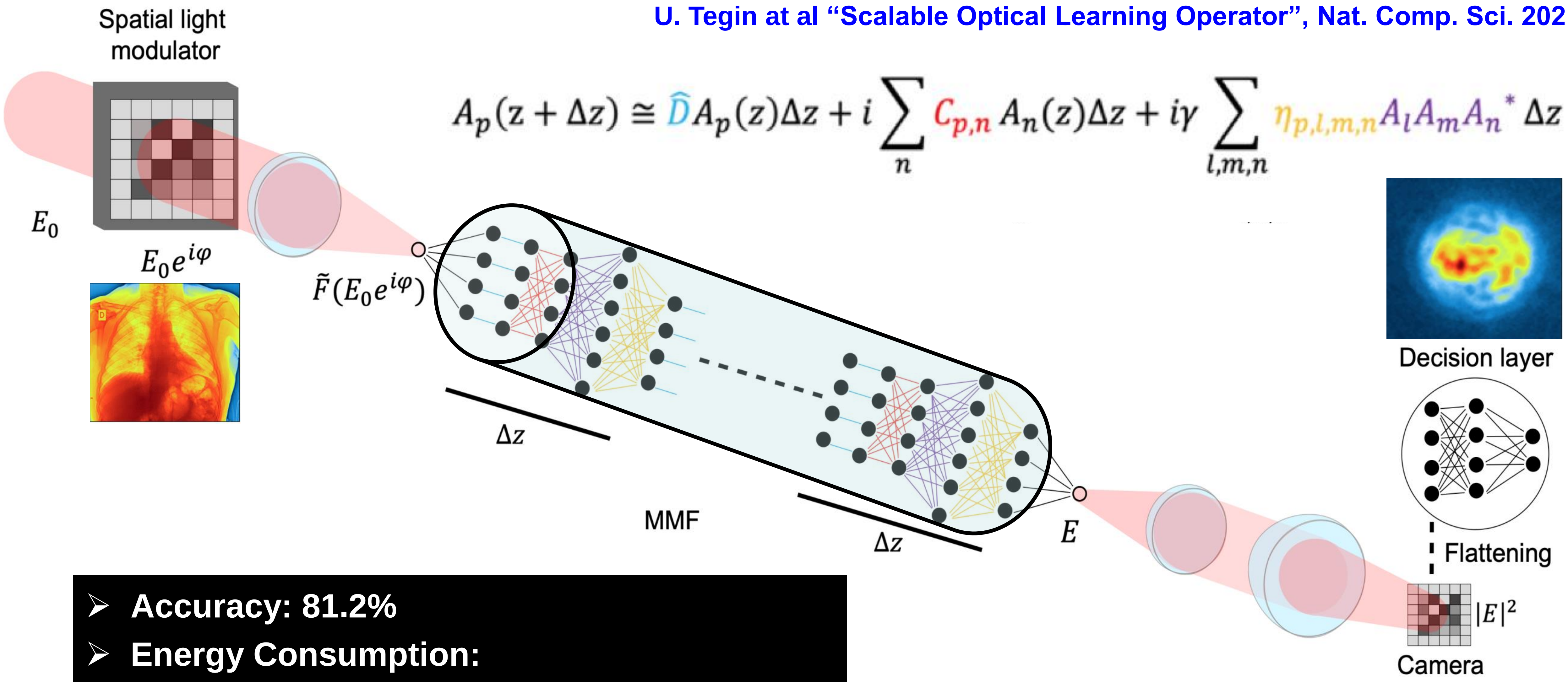


X. Liu et al, FF2J.1, *CLEO* (2022); D.I. Shahar et al, FF1L.5, CLEO (2023)

Machine Learning via multimode nonlinear Optics

U. Tegin at al "Scalable Optical Learning Operator", Nat. Comp. Sci. 2021

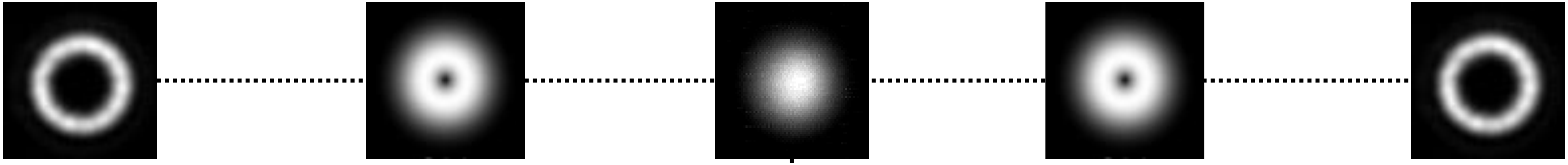
$$A_p(z+\Delta z) \cong \hat{D} A_p(z)\Delta z + i\sum_n C_{p,n} A_n(z)\Delta z + i\gamma \sum_{l,m,n} \eta_{p,l,m,n} A_l A_m A_n^* \Delta z$$

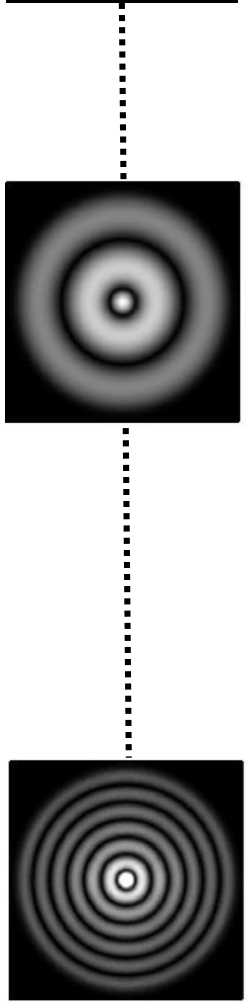


- **Accuracy: 81.2%**
- **Energy Consumption:**
  - **Equivalent NVIDA chip: 77 GFLOS/J**
  - **Optical NN; 14 TFLOPS/J (180x ↓↓ energy!)**

Nonlinear Optics with Spatial Modes of Fibers

BOSTON UNIVERSITY





Rich physics in individual modes

– Angular momentum conservation laws
– Chirality & influence of light's 3D path
– New nonlinear selection rules
– Guidance even in "forbidden" regime!

Applications

Power scalable λ conversion
(at any fiber–transparent λ: no dispersion constraint)

- CW/long pulse *and* ultrafast
- Fiber alternative to OPOs
- Endoscopic/Remote-deliverable Sources

Quantum Source Engineering
(many modes… many phase matching possibilities)

- Integrated high-dimensional sources
- User-defined joint spectral densities
- Compatible with quantum networking fiber

Emerging applications
(exploit the existence of many modes)

- All-optical machine-learning
- Emulate complex/chaotic physical phenomena
- …???...