Competition Policy as Strategic Trade

Marc Rysman*

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Abstract

This paper analyzes how countries use competition policy as a tool for strategic trade. Specifically, the paper studies countries that choose how many domestic exporters exist in addition to choosing subsidy levels. High subsidy levels or a large number of exporters are equally effective ways to commit to high quantity. In a sequential game, the value of choosing a large number of firms is undercut by choosing high subsidies afterwards. In equilibrium, countries choose to have only one firm. However, when subsidies are not allowed, countries want to have more firms than their competitor, leading towards welfare-maximizing perfect competition.

1 Introduction

Growth in the share of goods traded in the international market has made domestic competition policy into an international issue. Competition policy is entering the realm of international trade negotiations and, even in the United States, can no longer be seen simply as a matter of intra-national industrial policy. This paper explores what basic economic theory can say about competition policy as a tool for strategic trade. The paper looks at competition policy, and competition policy in conjunction with subsidy policy, in order to predict government strategies and analyze how the resulting equilibria diverge from the global welfare-maximizing solution.

*Department of Economics, Boston University. 270 Bay State Rd., Boston MA 02415. mrysman@bu.edu.I thank Yeon-Koo Che, Ray Deneckere, Jonathan Eaton, Phil Haile, Larry Samuelson, Barbara Spencer and Robert Staiger for support and encouragement.

Conventional wisdom about international competition policy is that countries want to have very large firms in order to exercise market power and compete effectively against foreign firms. Ken George (1990) writes "behind this [British pro-merger policy] was the view that British companies were frequently too small to compete effectively against overseas competitors together with the conviction that market forces could not be relied upon to rectify such structural weaknesses." (Pg. 106) Frederic Jenny (1990) reports of the "feeling among [French] public officials that large firms are better able to withstand international competition than smaller firms and that therefore, in general, one should not interfere with the attempts of French firms to increase their size through mergers." (Pg 185). However, F.M. Scherer reports on a trade-driven pro-merger policy in Britain that "studies of what happened following such mergers have raised doubts about whether the large-scale, government-brokered mergers of the 1960s and 1970s in fact enhanced industrial strength." (Scherer, 1994, pg 64). In a collective approach, the international community together seems to have determined that vigorous enforcement of antitrust laws is important.[1] The European Community reached an agreement in 1990 outlining an antitrust policy based on the relatively stringent West German model (Scherer 1994). An OECD report (1992) says that in general, "all countries were very active in enforcement. Efforts were not spared to counter practices constituting horizontal and vertical restraints on competition." (pg. 7)

This paper analyzes the strategic use of antitrust policy by adapting a Brander-Spencer (1985) model of export subsidization. As is well known, the results of the Brander-Spencer model turn crucially on the number of firms in each country. This paper recognizes that the number of firms in a country is not exogenous. A government can use antitrust measures (or the lack thereof) to affect the number of firms in its country and thereby grab a larger share of the international market. I model competition policy by allowing governments to choose exactly the number of domestic firms. Clearly, no modern government has the power to select

[1] I use the terms "competition policy" and "merger policy" interchangeably.

precisely the number of firms in each export market. However, the model makes strong, easily interpretable predictions that are useful for analyzing observed government behavior.

This paper shows that when countries are allowed to set their number of firms and then set subsidies, each country chooses to have one firm that is heavily subsidized. However, when countries are not allowed to use subsidies, as is the case under the rules of the World Trade Organization, the countries push each other to choose an unlimited number of firms, implying welfare-maximizing perfect competition.

The intuition is straightforward. Two countries are playing a quantity-setting game (in a third consuming country) through their firms. Each country can benefit from being able to commit to produce a high quantity. Having more home firms is one way to credibly commit to producing a higher national quantity. This paper shows that in the game without subsidies, it is beneficial to have a slightly higher number of firms than the competing country. But when both countries play this strategy, there is no equilibrium. The result is suggestive of perfect competition.

The result turns around when the countries play an export subsidy game. Subsidies are another way to elicit higher national quantity. This paper shows that in the case of unilateral intervention, subsidy policy and competition policy are interchangeable. There is an optimal quantity to reach (the Stackelberg quantity) that both policies can achieve independently, so there is no gain to using one policy when the other is set optimally.

In the sequential game with bilateral intervention, where countries choose their number of firms and then choose subsidies, having a large number of firms is no longer an effective way to commit to a high quantity. A country that chooses to have a large number of firms could have achieved the same goal by picking a high subsidy. In fact, picking a large number of firms actually reduces national welfare because observing a high number of firms commits the other country to pick a high subsidy. In equilibrium, each country chooses to have one firm in the first period and subsidize in the second period. Unlike the game without subsidies, equilibrium price is greater than marginal cost so some surplus remains unexploited. This result suggests that by making subsidies illegal, the WTO pushes the world governments towards more competitive antitrust policies and improves world welfare.

Policy analysts and legal researchers have long recognized a possible relationship between trade policy and antitrust policy. There is some work by economists as well, but efforts to subject competition policy and trade policy to formal economic analysis are recent and rare. Horn and Levinsohn (2001) and Richardson (1999) use very similar models to this one but they analyze a range of more complicated situations, notably home consumption and per-firm fixed fees. Horn and Levinsohn also look at mergers that reduce marginal cost. Richardson focuses exclusively on tariffs as opposed to subsidies, and studies the case of custom unions as well. Both papers explore the case of heterogeneous countries, which becomes important when there is home and foreign consumption. Horn and Levinsohn look at countries that are of different sizes, while Richardson discusses countries that differ in the value they put on consumer and producer surplus. Horn and Levinsohn, and Richardson analyze richer environments than this paper. But they are forced to look mostly at linear demand functions and their gain in realism comes at the expense of some of the simple points that this paper makes. [2]

Section 2 presents the results of a model where countries select the number of firms in the first period and firms set quantity in the second period. Section 3 extends the model to three periods, where countries select the number of firms in the first period, countries select subsidy levels in the second period, and firms set quantity in the third. Most proofs appear in the appendix.

[2] There is also a related literature in industrial organization. Similar mathematics can be used to analyze a game in which firms pick their number of franchisees or divisions, and then choose two-part tariffs (which can be thought of as franchise contracts). Rysman (2001) develops such a model and makes explicit the link with the trade literature. Rysman (2001) uses a different style of proof and shows the equivalent to Lemma 6 only for linear demand curves. A number of papers analyze the case without contracting, such as Polasky (1992) and Baye, Crocker and Ju (1996).

2 A Model of Competition Policy

In this model, two countries export all production of an identical good to a third consuming country. The consuming country has an inverse demand function $P(Q)$, where Q is total world production of the good. I denote home production as X and foreign production as Y, so $Q = X + Y$. Firms are assumed to have identical constant marginal cost c. An individual home firm produces quantity x and a foreign firm produces y.[3] There are n_h home firms and n_f foreign firms, so $X = n_h x$ and $Y = n_f y$. The demand curve is characterized by the following assumptions:

Assumption 1 *(i)* $P : \Re_+ \to \Re_+$ *is twice continuously differentiable;*

(ii)There exists Q' *such that* $P(Q') = 0$ *for* $Q \geq Q'$ *and* $P' < 0$ *for* $Q < Q'$*;*

(iii)$P'(Q) + qP''(Q) < 0, \forall Q \in [0, Q'], q \in [0, Q];$

(iv)$P(0) > c.$

The first three assumptions come from Gaudet and Salant (1991), which explores minimum requirements for a Cournot equilibrium. The fourth assumption guarantees that there is an interior solution. The third assumption is the substantive assumption. It implies that the marginal revenue of a firm is downward sloping in the quantity chosen by other firms. This condition implies that a firm's best response functions slopes down and is important for signing many of the results in this paper. This condition also implies that $P''Q/P' > -1$. For purposes of tractability, the final result of this paper restricts this term to be constant (which includes linear demand curves and affine transformations of constant elasticity demand functions) but that restriction is not necessary for the results in this section.

This section analyzes a two-period game, where governments choose the number of domestic firms in the first period and then those firms set quantity in the second period. Home

[3]In the next section, I will allow countries to subsidize at different levels, which will imply $x \neq y$. Without subsidies, all firms face the same marginal cost so $x = y$, but it will be useful to distinguish between home and foreign firm production now, in preparation for the next section.

firm i pick x_i to maximize $(P(\sum_{j=1}^{n_h} x_j + Y) - c)x_i$. Gaudet and Salant (1991) show that equilibrium is unique, which I take to be symmetric so $x_i = x$. The home country solves:

$$\max_{n_h} n_h(P(n_h x(n_h, n_f) + n_f y(n_h, n_f)) - c)x(n_h, n_f).$$

Foreign countries and firms have symmetric objective functions. I assume countries cannot choose less than one firm. In general, I solve problems using calculus and only impose integer constraints when they are binding. We start at the end of game in order to derive the Subgame Perfect Equilibrium.

Lemma 1 *The direct effect on a firm's quantity from a change in the number of firms is:*

$$\frac{dx}{dn_h} < 0 \;\; \frac{dy}{dn_h} < 0 \;\; \frac{dQ}{dn_h} > 0.$$

Increasing the number of firms increases the quantity produced by the home country, which is beneficial to the home country, but also increases world quantity, which pushes down the price and hurts the home country. There is an optimal quantity that the country seeks to achieve by choosing its number of firms.

The following propositions present the results of the game:

Lemma 2 *The best response in the first period is for the home country is to set n_h in such that:*

$$n_h = n_f + 1 + \frac{P''}{P'} n_f y. \blacksquare \tag{1}$$

First consider the linear case, where $P'' = 0$. Then, Equation 1 implies that the home country sets $n_h = n_f + 1$. The home country wants to have more firms than the foreign country (We will see in Theorem 2 that this result extends to nonlinear demand functions). Therefore, the country's choices over the number of firms are strategic complements. Remember that countries choose the number of firms to affect quantity, and quantity is a strategic substitute in the traditional Cournot game. Why the difference? When countries increase the number of firms, the *firms* reduce quantity. So increasing the number of firms in the home country does increase the total quantity in the home country and decrease the quantity in the foreign country. In fact, each country can use the number of firms to try to achieve the optimal quantity. This notion is formalized in the next theorem.

Theorem 1 *The quantity that the country achieves by choosing the number of firms optimally is equal to the quantity it would choose if it could choose quantity directly in the first period - i.e. the Stackelberg quantity in a game with n_f followers.*

This result is similar that found in a number of papers, such as Brander and Spencer (1985) and Fershtman and Judd (1987). Those papers find that principals use pre-production strategies to influence agents to choose what the principals would have chosen directly if they could.

In this case, when both countries use the number of firms to try to achieve the Stackelberg quantity, there is no equilibrium. In the linear case, the home country sets $n_h = n_f + 1$, and by symmetry, the foreign country sets $n_f = n_h + 1$. Clearly, there will be no equilibrium to this game. The result extends to the case of general demand functions.

Theorem 2 *In the case with general demand functions, both countries would like to have more firms than the other. There is no equilibrium to this game.* [4]

Proof: Assumption 1(iii) implies $(P''/P')Y > -1$, which implies that when Firm 1 sets n_h optimally (according to Equation 1), we have $n_h > n_f$. By symmetry, when Firm 2 sets n_f optimally, we have $n_f > n_h$. Therefore, there does not exist an $< n_h, n_f >$ combination that satisfies the best response functions of both firms simultaneously. ■

Formally, there is no Subgame Perfect Equilibrium to this game and therefore we cannot make a prediction about what will happen. However, the results are suggestive of an economy

[4] Some may wish to impose integer constraints here. If the foreign country chooses 3 firms, a demand curve might be concave enough for the home country to want to choose some number below 3.5. Arguably then, there is an equilibrium at $n_h = n_f = 3$. I prefer to focus on the point that each country would like to have slightly more firms than the other, resulting in no equilibrium.

that goes to perfect competition. If we imagine an "iterated application of best response functions", then the result will be perfect competition. More formally, $n_h = n_f = \infty$ are the only strategies that survive iterated strict dominance or rationalizability. [5] Another approach is to note that Baye, Crocker and Ju (1996) show that an equilibrium number of firms exists in the game with per-firm fixed costs, and that number tends to infinity as fixed costs tend to zero. [6] This result will hold for any type of increasing returns to scale. Similarly, any amount of product differentiation across countries will lead to an equilibrium in finite n_h and n_f. But as the product differentiation disappears, n_h and n_f go towards infinity. [7] So whether the reader would like to see the game in terms of iterated best response functions, iterated strict dominance or as the limiting case as frictions such as product differentiation or increasing returns to scale go to zero, it is reasonable to interpret the result of this game as being perfect competition.

When a country chooses to have a large number of firms, the firms reduce quantity, and keep price from going too low. In that sense, the firms regulate themselves, which makes raising the number of firms a particularly useful way of increasing quantity. But it is a way that confronts the countries with a severe Prisoner's Dilemma. In this game, the three countries are competing for the surplus under the demand curve. Surplus for the two exporting countries is jointly maximized when both exporting countries choose to have one firm each. But each country will try to have slightly more firms than the other, to no end. This Prisoner's Dilemma is very beneficial to the world because it leads to perfect competition, which maximizes world surplus.

[5] To see this result, note that country profits go up in its number of firms until the best response function is satisfied. So in the linear case, the home country prefers $n_h = 2$ to $n_h = 1$, regardless of what the foreign country plays. Once $n_h = 1$ is eliminated, $n_f = 3$ strictly dominates $n_f = 2$ or $n_f = 1$ for the foreign country. And so on. The proof is the same under general demand functions.

[6] They do so under the under linear demand curves for the industrial organization interpretation - that of a firm choosing how many divisions to have. But the models are mathematically identical to the game in this section under linear demand curves.

[7] A proof under linear demand curves is available from the author.

3 Competition Policy with Subsidies

In practice, governments have many tools for implementing strategic trade policy. The use of export subsidies is probably the most important and most closely studied of these tools. How sensitive are the results in Section 2 to the introduction of subsidies? It turns out that the ability to subsidize turns around equilibrium strategies completely. If a country tries to appropriate quantity by choosing a large number of firms, the competing country can just set a very high subsidy. Under a class of demand functions, countries choose to have one domestic firm and heavily subsidize, at great cost to world welfare.

This section analyzes a three-period game. Each country chooses its number of firms in the first period, its per-unit subsidy level in the second period, and its firms set quantity in the third period. Formally, in the first period, the home country solves:

$$\max_{n_h} n_h[\pi_h(n_h, n_f, s_h(n_h, n_f), s_f(n_h, n_f)) - s_h(n_h, n_f)x(n_h, k_f, s_h(n_h, n_f), s_f(n_h, n_f))]$$

where π_h is the profitability of a home firm, defined as:

$$\pi_h = [P(Q(n_h, n_f, s_h(n_h, n_f), s_f(n_h, n_f)) - c + s_h(n_h, n_f)]x(n_h, n_f, s_h(n_h, n_f), s_f(n_h, n_f)).$$

In the second period, the home country solves:

$$\max_{s_h} n_h[\pi_h(n_h, n_f, s_h, s_f) - s_h x(n_h, n_f, s_h, s_f)]$$

where

$$\pi_h = [P(n_h x(n_h, n_f, s_h, s_f) + n_f y(n_h, n_f, s_h, s_f)) - c + s_h]x(n_h, n_f, s_h, s_f).$$

I have countries choose the number of firms first because competition policy is a national policy that is presumably difficult to change - often requiring legislation, while subsidy policy can be quickly changed and altered for individual industries. However, the timing substantially affects the results, so I consider an alternate set-up later. Lemma 1 still characterizes how firms react to a change in the number of competing firms. [8] For this section, we must also characterize how firms will react to a change in subsidy.

Lemma 3 *The change in a firm's quantity from a change in the home subsidy level s_h is:*

$$\frac{dx}{ds_h} > 0 \quad \frac{dy}{ds_h} < 0.$$

Not surprisingly, a home subsidy increases home quantity and reduces foreign quantity. With Lemma 1 and Lemma 3 in place, we can consider the second-period solution.

Lemma 4 *In the second period, the home country sets the subsidy such that*

$$s_h = P'x\left(\frac{n_h}{n_f + 1 + \frac{P''}{P'}n_f y} - 1\right). \tag{2}$$

A low n_h relative to n_f implies a positive subsidy. A high n_h implies a tax. Intuitively, a subsidy allows a country to grab market share from the foreign country, but increases total quantity produced and drives down price. If a country has many firms relative to the number of foreign ones, the Cournot production externality hurts home revenues more than the increase in market share helps.

Unfortunately, showing existence and uniqueness of an equilibrium when both countries choose subsidies is very difficult. Brander and Spencer (1985) assume that

$$\frac{\partial^2\Pi_h}{\partial s_h^2}\frac{\partial^2\Pi_f}{\partial s_f^2} - \frac{\partial^2\Pi_h}{\partial s_h \partial s_f}\frac{\partial^2\Pi_f}{\partial s_h \partial s_f} > 0$$

where Π_h and Π_f denote national welfare for the home and foreign country. Brander and Spencer appeal to global univalence theorems by Gale and Nikaido (Nikaido, 1968, pg. 371) to get existence and uniqueness. However, this condition requires knowledge of the third derivative of the inverse demand function, and even then is very difficult to verify for the general case. I resort to assuming that equilibrium exists and is unique, and mention that a unique equilibrium can be verified under linear and constant elasticity demand functions.

[8] Now, dx/dn_h refers to $x(n_h, n_f, s_h, s_f)$, as opposed to $x(n_h + n_f)$.

The subsidy game is similar to the firm-number game in that both sides try to mimic a Stackelberg leader in both games. Why can equilibrium exist in the subsidy game but not in the firm-number game? In the firm-number game, each country picks its number of firms to mimic a Stackelberg leader who plays at cost c against followers with cost c. The no-equilibrium result relied on the fact that when a Stackelberg leader has the same marginal cost as its followers, the leader will produce a higher quantity than all of the followers combined, regardless of how many followers there are. In contrast, the country in the subsidy game picks s_i to mimic a Stackelberg leader with marginal cost c and with followers' marginal cost of $c - s_j$. There exists a s_j high enough such that the leader would pick the same quantity as the total follower quantity. In equilibrium, both firms play the Stackelberg quantity *simultaneously* in the sense that they both reach the quantity chosen by a leader with marginal cost c playing against the opponents' number of firms with marginal cost equal to the opponents' *subsidized marginal cost* (as opposed to actual marginal cost).

An important feature of the result of Lemma 4 is that if the home country can set n_h such that Equation 1 is satisfied, the subsidy is zero. A country cannot do better than the Stackelberg quantity. The implication is that if there was unilateral intervention (i.e. the foreign country has a fixed number of firms and does not subsidize in the second period), then subsidization cannot do better than antitrust policy. In fact, antitrust policy cannot improve upon optimal subsidization either in this case. Denoting home national welfare as Π_h:

Lemma 5 *In the game where both countries choose their number of firms in the first period and subsidies in the second period,*

$$\text{sign}\left[\frac{d\Pi_h}{dn}\right] = -\text{sign}\left[\frac{ds_f}{dn}\right].$$

That is, the choice of the number of firms depends only on how it affects foreign subsidies. First, consider the case of unilateral intervention, where $s_f = 0$ by definition, so of course $ds_f/dn_h = 0$. In that case, it does not matter what the home country chooses in the first period, assuming it chooses the optimal subsidy in the second period. Competition policy does not improve upon optimal subsidy policy. Any national quantity that can be achieved through competition policy can be achieved through subsidization. Optimal second-period subsidization implies that profits of a country are invariant to the choice of the number of firms in the first period.

In the case of unilateral intervention, the home country is indifferent between reaching its objective through competition policy and reaching it through subsidization. If we were to add a cost for raising funds for subsidies, or any home consumption, then presumably antitrust policy would be a more useful tool. On the other hand, the presence of per-firm fixed costs would favor subsidization.

The interchangeability of subsidies and the number of firms is the key intuition for the paper. The two countries are playing a quantity setting game through their firms. Having more firms is one way to increase national output. Subsidization is another. The solution to this game depends on the sequencing. The result in Lemma 5 follows from the fact that choosing s_h in the second period can accomplish anything that choosing n_h in the first period can (in terms of home production), so the only importance that n_h can have is in how it affects the foreign subsidy. And all else being equal, the home country wants the foreign subsidy to be lower. So, if ds_f/dn_h is negative, the country would like to set n_h as high as possible. If ds_f/dn_h, is positive the country will set n_h as low as possible, equal to one.

Intuitively, ds_f/dn_h should be positive. As the number of home firms increases, the effect of the Cournot production externality by foreign firms on each other becomes less important, and a subsidy becomes more valuable. However, note that the definition of s_h (and by symmetry, s_f) in Lemma 4 is really a fixed point equation, because s_h enters into x and y, and through them into P' and P''. Determining the sign of ds_h/dn_f requires knowing something about the third derivative of the demand function, and is very difficult to address algebraically. For tractability reasons, I assume that $P''Q/P'$ is constant. This class of demand curves includes linear demand curves, CES demand curves and demand curves of the form $P = A - Q^{-b}$. [9] Note that the restriction is sufficient but not necessary for the result.

Lemma 6 *Under a demand curve in which $P''Q/P'$ is constant in a symmetric equilibrium, $ds_h/dn_f > 0$.*

Lemma 5 and Lemma 6 imply the central result of this section:

Theorem 3 *In the game where both countries choose their number of firms in the first period and subsidies in the second period with $P''Q/P'$ constant, the Subgame Perfect Equilibrium symmetric strategies are for each country to choose to have one firm in the first period and subsidize in the second period.*

The result from Section 2 is completely turned around. Instead of going towards perfect competition, the industry goes to a subsidized duopoly. One way to see how the game changes with and without subsidies is to look at country-level quantities. In the game in Section 2, the change in home quantity from a first period increase in the number of firms was:

$$\frac{dX}{dn_h} = x + n_h \frac{dx}{dn_h}.$$

In this game, that change becomes:

$$\frac{dX}{dn_h} = x + n_h \frac{dx}{dn_h} + n_h \frac{dx}{ds_h}\frac{ds_h}{dn_h} + n_h \frac{dx}{ds_f}\frac{ds_f}{dn_h}.$$

Raising the number of firms commits the home country to decrease the subsidy in the second period and leads the foreign country to increase its subsidy. Both subsidy effects reduce home production and undercut the benefits of increasing the number of firms. If a country tries to capture quantity by creating a large number of firms, the competing country can just increase its subsidy. In case, the third term cancels out the first one term. All that is left is the third

[9] Gonzales-Maestre (2000) and Ziss (2001) also use this condition for tractability reasons.

term. The power to subsidize in the second period saps the power of adjusting the number of firms in the first period.

The results presented here give strong welfare implications to the World Trade Organization policy of eliminating production subsidies. In this game, perfect competition captures all of the surplus under the demand curve. When allowed to subsidize, competing countries choose to have a small number of very large firms and subsidize them. When subsidies are restricted, countries institute vigorous antitrust policies and go to perfect competition. World welfare is maximized, with all of the welfare going to the consuming country. Restricting subsidies as the WTO does leads naturally to optimal antitrust policy. So the theory presented here implies that there is no need for the WTO to consider competition policy explicitly. The WTO "has done its job" simply by restricting subsidies. But this implication is unlikely to hold true in practice. The presence of home consumption and non-linear costs substantially complicates the analysis. These issues are explored in depth in Horn and Levinsohn (2001).

Another issue that is overly simplified in this paper is the nature of antitrust policy. Clearly, no country can choose exactly how many firms will exist in its borders. But because there is no consumption in the producing countries in this game, firms have the same incentives as countries do in the first period. Suppose that each country begins with a certain number of firms that decide whether to merge or break themselves up in the first period. The firms will choose to merge into one if there is a subsidy game before production, but will choose to break themselves up into infinitely many firms if subsidies are not allowed. [10] The solution is exactly the same. In general, a more realistic model of merger policy is an important area for future research in this area. An important step in this direction in Horn and Persson (2000).

The timing of this game is important - the results of the game change substantially when

[10] The result that firms might break themselves up infinitely many times is closely related to the results of Reynolds, Salant and Switzer (1983). They compute computationally that merger is unprofitable unless firms capture almost all of the market. Of course, if firms cannot profit from merger, than the firms can profit from breaking themselves up.

countries choose the number of firms and the level of subsidy simultaneously. The simultaneous choice game exhibits multiple equilibria. The home country solves

$$\max_{n_h, s_h} n_h[P(n_h x(n_h, n_f, s_h, s_f) + n_f y(n_h, n_f, s_h, s_f)) - c]x(n_h, n_f, s_h, s_f).$$

The optimal subsidy is the same as in the previous case. It is defined by Lemma 4. In this case, the best response function defining how many firms to choose (the equivalent of Equation 1) is to set n_h such that:

$$(-n_h + n_f + 1)P' + n_f P'' y - \frac{n_f(P' + P'' y) + P'}{P' x} s_h = 0. \tag{3}$$

A best response for the home country in the simultaneous choice game is when n_h and s_h are set such that Equation 3 and Lemma 4 are both satisfied. Not surprisingly, if s_h is set optimally, then Equation 3 is always satisfied. Again, we see that choosing the number of firms cannot improve upon setting the optimal subsidy. From this observation, the results of the game are easy to see. Consider any choice of n'_h and n'_f. There exists an optimal s'_h and s'_f defined by Lemma 4. By the fact that s'_h and s'_f are optimal, so are n'_h and n'_f. Therefore, the choices $[n'_h, s'_h, n'_f, s'_f]$ define an equilibrium in the simultaneous choice game. Note that these strategies are weakly best responses. For the home country, there are an infinite combination of number of firms and subsidies that would lead to the same national quantity of production, and to the same national welfare. But none of the other choices would elicit the same response from the foreign country, so they would not define an equilibrium.

4 Conclusion

This paper recognizes that the number of firms in an economy should not be thought of as exogenous with regard to strategic policy. The paper endogenizes the number of firms by treating it as a strategic choice. As a unilateral trade policy tool, picking the number of domestic firms turns out to be equally as powerful as subsidies. When countries are allowed to subsidize, world surplus is frittered away by producing countries that subsidize very large firms. When subsidies are eliminated, as is the case under the WTO, the producing countries push each other towards perfect competition.

5 Appendix

Proof of **Lemma 1**: I prove this result with subsidies to make the lemma robust for Section 3. But subsidies are unimportant, so the lemma applies for Section 2 as well. Each individual firm i in the home country maximizes

$$(P + s_h - c)x_i.$$

The first order condition, assuming $x_i = x$, is:

$$P(n_h x + n_f y) + P'(n_h x + n_f y)x + s_h - c = 0.$$

Equilibrium exists as shown in Gaudet and Salant (1991). The derivative of the first-order condition with respect to n_h is always negative, so the first order condition is invertible in n_h. Therefore, we can define x and y as functions of n_h.

Using the Implicit Function Theorem and totally differentiating with respect to n_h, we have:

$$(P')(x + n_h \frac{dx}{dn_h} + n_f \frac{dy}{dn_h}) + (P''x)(x + n_h \frac{dx}{dn_h} + n_f \frac{dy}{dn_h}) + (P')\frac{dx}{dn_h} = 0.$$

The foreign firm first order condition is $P + P'y + s_f - c$. Again, applying the Implicit Function Theorem, we have:

$$(P')(x + n_h \frac{dx}{dn_h} + n_f \frac{dy}{dn_h}) + (P''y)(x + n_f \frac{dx}{dn_h} + n_f \frac{dy}{dn_h})) + (P')\frac{dy}{dn_h}) = 0.$$

Solving for $\frac{dx}{dn_h}$ and $\frac{dy}{dn_h}$ simultaneously gives:

$$\frac{dx}{dn_h} = -x \cdot \frac{P' + P''x}{n_h(P' + P''x) + n_f(P' + P''y) + P'} < 0$$

$$\frac{dy}{dn_h} = -x \cdot \frac{P' + P''y}{n_h(P' + P''x) + n_f(P' + P''y) + P'} < 0.$$

Assumption 1(iii) implies the signs.■

Proof of **Lemma 2**:The home country solves $\frac{d}{dn_h} n_h(P-c)x = 0$ where $P = P(n_h x + n_f y)$.

so

$$(P - c)\frac{d}{dn_h}X + XP'\frac{d}{dn_h}Q = 0. \tag{4}$$

Individual firms set $P - c + P'x = 0$. So plugging in by the Envelope Theorem implies:

$$\frac{d}{dn_h}X = n_h \frac{d}{dn}Q$$

Differentiating:

$$x + n_h \frac{dx}{dn_h} = n_h \left(x + n_h \frac{dx}{dn_h} + n_f \frac{dy}{dn_h} \right)$$

Plugging in from Lemma 1, we have:

$$x \frac{n_f(P' + P''y) + P'}{n_h(P' + P''x) + n_f(P' + P''y) + P'} = x \frac{n_h P'}{n_h(P' + P''x) + n_f(P' + P''y) + P'}$$

Canceling from both sides achieves Equation 1. ■

Proof of **Theorem 1**: Dividing Equation 4 by dX/dn_h gets:

$$P + XP' \left(1 + n_f \frac{dy/dn_h}{dX/dn_h} \right) = c.$$

Plugging in from Lemma 1 shows:

$$\frac{dy/dn_h}{dX/dn_h} = -\frac{P' + P''y}{n_f(P' + P''y) + P'}.$$

I show that a Stackelberg first order condition is equivalent. A Stackelberg leader chooses X to maximize $P(X + n_f y(X))X$. The first order condition is:

$$P + XP'(1 + n_f \cdot dy/dx)) = c.$$

A follower first order condition is $P + P'y = c$. Applying the Implicit Function Theorem and differentiating shows:

$$P'(1 + n_f \frac{dy}{dx}) + P' \frac{dy}{dx} + P''y(1 + n_f \frac{dy}{dx}) = 0.$$

Solving for dy/dx establishes the result.■

Proof of **Lemma 3**: The first order condition for an individual home firm is $P'x + P + s_h$. As in Lemma 1, x and y can be defined implicitly in terms of s_h. Totally differentiating with respect to s_h and rearranging gives:

$$\frac{dx}{ds_h}(n_h(P' + P''x) + P') + \frac{dy}{ds_h} n_f(P' + P''x) + 1 = 0$$

The first order condition of a foreign firm is $P'y + P + s_f$. Again, defining x and y in terms of s_h and differentiating with respect to s_h gives:

$$\frac{dx}{ds_h} n_h(P' + P''y) + \frac{dy}{ds_h}(n_f(P' + P''y) + P') = 0$$

Solving simultaneously for $\frac{dx}{ds_h}$ and $\frac{dy}{ds_h}$ gives:

$$\frac{dx}{ds_h} = \frac{-(n_f(P' + P''y) + P')}{P'(n_h(P' + P''x) + n_f(P' + P''y) + P')} > 0$$

$$\frac{dy}{ds_h} = \frac{n_h(P' + P''y)}{P'(n_h(P' + P''x) + n_f(P' + P''y) + P')} < 0.$$

Assumption 1 (iii) establishes the signs. ■

Proof of **Lemma 4**: Note that:

$$\frac{dPi_h}{dn_h} = (P - c)\frac{dX}{dn_h} + P'X\left(\frac{dX}{dn_h} + \frac{dY}{dn_h}\right)$$

Foreign production Y depends on home choices n_h and s_h only indirectly, through X and s_f. That is, $Y(n_h, s_h(n_h, n_f), n_f, s_f(n_h, n_f))$ can be rewritten as $Y(X(n_h, s_h(n_h, n_f), n_f, s_f(n_h, n_f)), n_f, s_f(n_h, n_f))$. Therefore:

$$\frac{dY}{dn_h} = \frac{\partial Y}{\partial X}\frac{dX}{dn_h} + \frac{\partial Y}{\partial s_f}\frac{ds_f}{dn_h}$$

so we can rewrite the first equation as:

$$\frac{d\Pi_h}{dn_h} = \left[P - c + P'X\left(1 + \frac{\partial Y}{\partial X}\right)\right]\frac{dX}{dn_h} + P'X\frac{\partial Y}{\partial s_f}\frac{ds_f}{dn_h}$$

The home country sets s_h so the term in brackets (which is the Stackelberg first order condition) equals zero in the second period, so it equals zero by the Envelope Theorem. The term $P'X(\partial Y/\partial s_h)$ is negative. So the sign $d\Pi_h/dn_h$ is the opposite of the sign of ds_h/dn_h. ■

Proof of **Lemma 6**: [11] Let $\beta_x = xP''/P'$ and $\beta_y = yP''/P'$. Note that the condition that QP''/P' is constant implies that β_x and β_y are also constant. From Equation 2, we have:

$$\frac{ds_h}{dn_f} = P'x\left(\frac{-n_h(\beta_y+1)}{(n_f(\beta_y+1)+1)^2}\right) + \left(\frac{n_h}{n_f(\beta_y+1)+1} - 1\right)\frac{dP'x}{dn_f}$$

$$\frac{dP'x}{dn_f} = P'\frac{\partial x}{\partial n_f} + xP''\frac{\partial Q}{\partial n_f} + \left(P'\frac{\partial x}{\partial s_h} + xP''\frac{\partial Q}{\partial s_h}\right)\frac{ds_h}{dn_f} + \left(P'\frac{\partial x}{\partial s_f} + xP''\frac{\partial Q}{\partial s_f}\right)\frac{ds_f}{dn_f}$$

Plugging in from Lemmas 1 and 3 and rearranging gets:

$$\begin{aligned}
&\frac{ds_h}{dn_f}\left(1 + \frac{n_h - n_f(\beta_y+1) - 1}{n_f(\beta_y+1)+1}\frac{n_f + 1 + n_h\beta_x + n_f\beta_y}{n_h(\beta_x+1) + n_f(\beta_y+1)+1}\right) \\
= \;& P'x\frac{-n_h(\beta_y+1)}{(n_f(\beta_y+1)+1)^2} + \frac{n_h - n_f(\beta_y+1) - 1}{n_f(\beta_y+1)+1}\frac{-P'y}{n_h(\beta_x+1) + n_f(\beta_y+1)+1} \\
&+ \frac{n_h - n_f(\beta_y+1) - 1}{n_f(\beta_y+1)+1}\frac{n_f}{n_h(\beta_x+1) + n_f(\beta_y+1)+1}\frac{ds_f}{dn_f}
\end{aligned}$$

Now I impose symmetry. That is, $n_h = n_f$, $x = y$ and $\beta_x = \beta_y$. Rearranging produces:

$$\frac{ds_h}{dn_f} = \left(\frac{-n^2\beta^2 - 3n^2\beta - 2n^2 + n\beta + 1}{n(3n\beta + 2n + 2)(n(\beta+1)+1)}\right)P'x - \frac{n\beta+1}{3n\beta+2n+2}\frac{ds_f}{dn_f}$$

A similar approach can be used to calculate ds_f/dn_f.

$$\frac{ds_f}{dn_f} = \frac{P'x}{n} - \frac{n\beta+1}{3n\beta+2n+2}\frac{ds_h}{dn_f} \tag{5}$$

Solving the two equations simultaneously:

$$\frac{ds_h}{dn_f} = -P'x\frac{3n\beta+2n+2}{(3n\beta+2n+2)^2 - (n\beta+1)^2}\frac{2n^2\beta^2 + 4n^2\beta + 2n^2 + n\beta + n}{n(n(\beta+1)+1)}$$

[11] A complete account of the tedious algebra required for this proof is available from the author or at http://econ.bu.edu/rysman/research/cpastLemma6Algebra.pdf.

Now it remains to sign the term. The term $-P'x$ is positive. The denominator of the second fraction is positive. The numerator of the second fraction is monotonically increasing in β and is equal to zero at $\beta = -1$. Therefore, $\beta > -1$ implies the numerator of the second fraction is positive. The numerator of the first fraction is positive when $\beta > -(2n+2)/3n$. The denominator of the first fraction is positive when $\beta > -(2n+3)/4n$. Assumption 1 (iii) implies that $\beta > -1/n$, which satisfies all three of these conditions on β. Therefore, $ds_h/dn_f > 0$ for all n. ■

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