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ABSTRACT

Materials processing often occurs under extreme dynamic conditions leading to a multitude of unique structural environments. These structural environments generally occur at high temperatures and/or high pressures, often under non-equilibrium conditions, which results in drastic changes in the material’s structure over time. Computational techniques, such as molecular dynamics simulations, can probe the atomic regime under these extreme conditions. However, characterizing the resulting diverse atomistic structures as a material undergoes extreme changes in its structure has proved challenging due to the inherently non-linear relationship between structures as large-scale changes occur. Here, we introduce SODAS++, a universal graph neural network framework, that can accurately and intuitively quantify the atomistic structural evolution corresponding to the transition between any two arbitrary phases. We showcase SODAS++ for both solid–solid and solid–liquid transitions for systems of increasing geometric and chemical complexity, such as colloidal systems, elemental Al, rutile and amorphous TiO₂, and the non-stoichiometric ternary alloy Ag₉₆Au₅Cu₁₉. We show that SODAS++ can accurately quantify all transitions in a physically interpretable manner, showcasing the power of unsupervised graph neural network encodings for capturing the complex and non-linear pathway a material’s structure takes as it evolves.

Materials under extreme thermal, mechanical, and chemical conditions often experience drastic changes in their structure.1–5 These structural changes begin at the atomic scale and propagate upward in length scale to ultimately affect larger-scale features, such as the material’s microstructure.6–8 Due to conditions, such as high temperature and pressure, these changes in the atomic structure are often non-trivial and highly correlated, with complex atomic diffusion events acting as a driving force for changes in local atomic geometries.9,10 As these structural changes often lead to significant differences in material properties, a thorough understanding of how atomic-level configurations evolve over time is necessary. Therefore, if one aims to quantify the changes in materials properties under such extreme conditions, algorithmic tools that can accurately map out both subtle and significant differences in atomic geometries is vital.

Over the last few decades, numerous developments on this front have occurred, leading to a plethora of methods with excellent atomic environment characterization capabilities.11–26 However, having the ability to provide a unique geometric descriptor for a particular atomic structure does not inherently mean that an interpretable pathway between two random structures exists.27 Such a pathway is critical to understanding the spatiotemporal behavior of materials under extreme conditions because materials often undergo a cascade of subtle changes that culminate in significant differences in their atomic structures over time.27–29 Quantifying this process from start-to-finish, on both collective and local atomic scales, is critical to understanding the question of why materials behave the way they do when placed in extreme external conditions, laying the foundation for predictive modeling of such systems and their optimization.
Recent work has shown that answering this question is non-trivial for the majority of atomistic descriptors due to numerous challenges, such as the uniqueness of descriptors, their computational cost, and their generality to any arbitrary material system. In particular, many methods employ a "bottom-up" approach when characterizing structures as a whole, aiming to accurately quantify local atomic sites and then project the collection of atomic features into a global descriptor. This bottom-up approach often results in a loss of information due to the inherently disconnected nature of treating atomic environments as independent entities.

Here, we build upon our previous work, the Structural Orderness Degree for Atomic Systems (SODAS), and present SODAS++, which overcomes the limitations of the original SODAS algorithm by addressing two major components: (1) extension to multicomponent systems and (2) universal applicability to any atomic structural transformation, not just those that depend on temperature. We achieve (1) by interfacing SODAS with ALIGNN, a line-graph approach that incorporates bonds and bond angles into the graph representation. (2) is accomplished by mapping the ALIGNN latent space to an intuitive lower-dimensional manifold using a dimensionality reduction technique called UMAP. Importantly, SODAS++ does not require a training stage, instead relying purely on the unsupervised GNN encoding. This allows for SODAS++ to be generalized to any arbitrary material system.

The SODAS formalism relies on a scalar order parameter, $c$, to determine how far along a given structural transition a particular atomic system is. In the original SODAS implementation, $c$ was explicitly linked to temperature, limiting its applicability to temperature-dependent structural transitions. SODAS++ alleviates this dependency, instead relying on the GNN latent space encoding to produce an accurate phase space. By combining (1) and (2), SODAS++ can quantify any arbitrary structural transformation, under dynamic external conditions, in a universal and efficient manner. Importantly, SODAS++ naturally encodes collective correlations among local environments as it takes a "top-down" approach by building global graph of the physical system. We showcase the accuracy and generality of SODAS++ by characterizing a multitude of structural transformations, such as the density-dependent phase changes in colloidal particles, the compression and melting of FCC aluminum, the amorphization of rutile TiO$_2$, and finally the melting of the non-stoichiometric ternary alloy Ag$_{26}$Au$_5$Cu$_{19}$.

A detailed description of the SODAS++ workflow is described in the following paragraphs, and a visual depiction can be seen in Fig. 1. We begin by leveraging the ALIGNN formulation, where two graphs are used to encode one atomic structure: an original atomic graph $G$ and its corresponding line graph $L(G)$. The nodes and edges in $G$ represent atoms and bonds, respectively. The nodes and edges in $L(G)$, on the other hand, represent bonds and bond angles, respectively. Note that the edges in $G$ and the nodes in $L(G)$ are identical entities and share the same embedding during GNN operation. In this work, we extended the ALIGNN encoding to also explicitly represent dihedral angles. We used Atomic Simulation Environment and PyTorch Geometric to construct the graph representations and to calculate all bond angles.

In the initial encoding, the atom type, the bond distance, and the angular values are converted from scalars to feature vectors for subsequent neural network operations. The atom type $z$ is transformed by a learnable embedding matrix. The bond distance $d$ is expanded into the Radial Bessel basis proposed by Klicpera et al. The angles $a$ are expanded into the "standard" Gaussian basis implemented by Schnet. The interaction operations are also known as graph convolution, aggregation, or message-passing. Following the ALIGNN paper, we...
also adopted the edge-gated graph convolution\(^{33,44}\) for the interaction operations. The node features \(\tilde{h}_i^{l+1}\) of node \(i\) at the \((l+1)\)th layer are updated as

\[
\tilde{h}_i^{l+1} = \tilde{h}_i^{l} + \text{SiLU} \left( \text{LayerNorm} \left( \tilde{W}_e^{l} \tilde{h}_i^{l} + \sum_{j \in \mathcal{N}(i)} \tilde{e}_{ij}^{l} \odot \tilde{W}_d^{l} \tilde{h}_j^{l} \right) \right),
\]

where SiLU is the sigmoid linear unit activation function;\(^{46}\) LayerNorm is the layer normalization operation;\(^{47}\) \(\tilde{W}_e\) and \(\tilde{W}_d\) are weight matrices; the index \(j\) denotes the neighbor node of node \(i\); \(\tilde{e}_{ij}\) is the edge gate vector for the edge from node \(i\) to node \(j\); and \(\odot\) denotes element-wise multiplication. The edge gate \(\tilde{e}_{ij}^{l}\) at the \(l\)th layer is defined as

\[
\tilde{e}_{ij}^{l} = \frac{\sigma(\tilde{e}_{ij}^{l})}{\sum_{f \in \mathcal{E}(i)} \sigma(\tilde{e}_{if}^{l}) + \epsilon},
\]

where \(\sigma\) is the sigmoid function, \(\tilde{e}_{ij}^{l}\) is the original edge feature, and \(\epsilon\) is a small constant for numerical stability. The edge features \(\tilde{e}_{ij}^{l}\) are updated by

\[
\tilde{e}_{ij}^{l+1} = \tilde{e}_{ij}^{l} + \text{SiLU} \left( \text{LayerNorm} \left( \tilde{W}_e^{l+1} \tilde{e}_{ij}^{l} \right) \right),
\]

where \(\tilde{W}_e\) is a weight matrix and \(\tilde{e}_{ij}\) is the concatenated vector from the node features \(\tilde{h}_i\), \(\tilde{h}_j\), and the edge features \(\tilde{e}_{ij}\). 

We applied the same edge-gated convolution scheme [Eqs. (1)–(3)] to operate on both the atomic graph \(G\) and the line graphs \(L(G)\). In the case of \(G\), the edge-gated convolution updates nodes, which represent atoms, and edges, which represent bonds, while exchanging information between the two. In the case of \(L(G)\), the convolution updates nodes, which represent atoms, and edges, which represent angles. Note that by iteratively applying the convolution operation on both the original graph and the line graph, the angular information stored in \(L(G)\) can propagate to \(G\). Due to the nature of the edge-gated convolution, all the feature/embedding vectors for atoms, bonds, and angles during the interaction layers have the same length, or the same number of channels \(D\). Finally, the final output layers pool (by summation) the node features of \(G\) and transform the pooled embedding into an output vector, which preserves the dimensionality of the original input layer. All model parameters can be found in Table I. We emphasize here that the GNN encoding requires no training process and is purely unsupervised. The GNN is simply learning the information present in the ALIGNN representation and projecting it onto a higher dimensional space.

The resulting GNN latent space is used to determine a pathway between the initial and final configurations of a given trajectory. This is accomplished in a four-step process: (1) normalization of the original latent space, (2) projection of the normalized latent space onto a manifold using UMAP,\(^{47}\) (3) conversion of the manifold space into a planar graph, and (4) recursive graph search to determine an optimal pathway. The following paragraphs provide more detail into each step.

We apply a normalization algorithm \(\mathcal{N}\) to the initial GNN latent space to ensure that all dimensions are weighted equally. Here, we define a phase space order parameter, \(\gamma\), as the point where two endpoints lie within the phase space. The purpose of \(\gamma\) is to project the inherently non-linear transition pathway onto an order parameter that varies linearly within the space. Conceptually, \(\gamma\) is the location along a given transition, a proxy for the configurational entropy at any arbitrary point along the trajectory. Using our graph-pathing scheme, we can break the path into iterative segments, where \(\gamma\) can be calculated as the distance along the total path. Therefore, we define \(\gamma\) as

\[
\gamma_k = \frac{\sum_{i=1}^{k} d_i}{\sum_{j=1}^{N} d_j}.
\]

\(\mathcal{N}\) is chosen due to its algorithm being rooted in topology, making the resulting space ideal for the construction of a planar graph. The planar graph is generated from the UMAP manifold space via a \(k\)-nearest neighbor (\(k\)NN) approach. For all systems studied in this work, \(k\) was set to 15. The nearest neighbors were identified using a Minkowski distance metric\(^{48}\) in the UMAP space.

From the \(k\)NN graph, a recursive graph search algorithm is employed to discover a pathway between the initial and final configurations in the original trajectory. Upon reaching any given node, we rank its neighboring nodes and chose the closest neighbor that has a possible pathway to the final node. No node can be visited twice during the pathway generation. If at any point a pathway cannot be created due to lack of suitable neighbors, the algorithm goes back one node in the path and moves to the next suitable neighbor. This iterative process results in a realistic pathway generation by mimicking the true oscillations in phase space during a molecular dynamics trajectory and is a graph-analog for damped potential energy surface exploration.\(^{49}\)

Finally, we define a phase space order parameter, \(\gamma\), as the point where two endpoints lie within the phase space. The purpose of \(\gamma\) is to project the inherently non-linear transition pathway onto an order parameter that varies linearly within the space. Conceptually, \(\gamma\) is the location along a given transition, a proxy for the configurational entropy at any arbitrary point along the trajectory. Using our graph-pathing scheme, we can break the path into iterative segments, where \(\gamma\) can be calculated as the distance along the total path. Therefore, we define \(\gamma\) as

\[
\gamma_k = \frac{\sum_{i=1}^{k} d_i}{\sum_{j=1}^{N} d_j}.
\]
To validate SODAS++, we performed MD simulations on several systems ranging from fictitious toy problems to the melting of oxides and ternary alloys. All MD simulations were performed using LAMMPS. Interatomic potentials for Al, TiO₂ (Ref. 51), and Ag₂₆Au₅Cu₉ (Ref. 52) were used. NVT simulations were performed for both melting and compression trajectories. For melting, temperatures were initialized to 100 K and culminated at 3000 K, though only snapshots within 2 ps of melting were taken from the high temperature regime. Compression simulations were performed at 300 K.

The utility of SODAS++ is also demonstrated in quantifying entropy-driven disorder-order phase transition in 3D hard-sphere systems, which are excellent models for many colloidal systems dominated by strong repulsive interactions. The transition is induced by increasing density or packing fraction \( \phi \) (via compression), and the equilibrium phase diagram possesses three branches: disordered liquid (0 < \( \phi < 0.494 \)), co-existence (0.494 < \( \phi < 0.545 \)), and crystalline solid (0.545 < \( \phi < 0.74 \)). Here, we use event-driven molecular dynamics to capture the disorder-order transition, starting from a metastable liquid state with \( \phi = 0.54 \). Once the density increases beyond the freezing point (\( \phi_F = 0.545 \)), the liquid becomes unstable and undergoes a rapid collective structural re-arrangement to the crystalline state (with defects), which is then continuously compressed to the maximal density.

It can be seen in Figs. 2(a) and 3 that the disorder-to-order transition of the colloidal system is clearly captured by SODAS++. The metastable liquid states are clustered and separated from the crystalline states (connected via a continuous path); and a sharp, discontinuous-like transition between the two phases can be seen in both Figs. 2(a) and 3(a). Particularly, from Fig. 2(a), one can see two distinct subgraphs connected via a single edge. The two subgraphs represent the disordered and ordered states with the single graph edge representing the transition from order-to-disorder. One can also see from Fig. 2(a) that the subgraph representing the ordered states can be broken into two further subgraphs by examining the average edge connections to each node. This further breakdown represents the transition of the ordered region from one phase to a second ordered phase. This transition can also be seen in Fig. 3(a) with a second discrete jump around (0, 10) in (L1, L2) coordinates. The coloring of nodes in Figs. 3(b) and 2(a), which again is determined via Fig. 5, follows the trend observed in Fig. 3(a), indicating that the unsupervised determination of \( c \) is accurate.

Figure 4 depicts the SODAS++ characterization of (a, b) the compression of Al at 100 K and (c, d) the melting of Al via superheating. From Fig. 4(a), the continuous compression of Al is captured well via the latent space projection, with a continuous pathway from low-to-high density observed in Fig. 4(b). One important note regarding Fig. 4(b) is that the blue and orange points seem to overlap in the 2D view; however, in 3D, they do not overlap. From Fig. 4(c), one can see that SODAS++ also captures the continuous heating of Al from perfect crystal to liquid phase extremely well. Similar to the compression case, an intuitive pathway is found between the initial and final configurations in Fig. 4(d). It is important to note that the compression and melting scenarios present wildly different transition types, with one depending on a continuous disordering of the structure and another the slow change in interatomic distances but without breaking the symmetry of the crystal. SODAS++ quantitatively captures both of these structural transitions in a way that is interpretable.

Figure 5 focuses on order-to-disorder transitions but for more complex structural systems, with (a)–(d) capturing the amorphization of TiO₂ and (e)–(i) predicting the melting of a randomly distributed...
FIG. 3. SODAS++ UMAP latent space projection for a colloidal system’s density-based structural transition. (a) The latent space projection color-coded based on the density of the structure. Inserted images show representative points along the trajectory. (b) The latent space color-coded based on the predicted $\gamma$ values. A path through the graph shows the result of the graph pathfinding algorithm, color coded based on the edge’s $\gamma$.

FIG. 4. SODAS++ UMAP latent space projection for [(a) and (b)] compression and [(c) and (d)] melting of Al. (a) The latent space projection color-coded based on the change in volume relative to the perfect ground-state crystal volume. (c) The latent space projection color-coded based on the temperature of the system. Inserted images show representative points along the trajectory. (b) and (d) The latent space color-coded based on the predicted $\gamma$ values. A path through the graph shows the result of the graph pathfinding algorithm, color coded based on the edge’s $\gamma$. 
non-stoichiometric ternary alloy. For the amorphization of TiO$_2$ in Figs. 5(a)–5(d), we again see that SODAS++ provides an intuitive characterization a function of temperature. The amorphization of TiO$_2$ proceeds in two stages: (1) disordering the crystal into an amorphous state and (2) transition of the amorphous system to a liquid. We can see from Fig. 5(d) that $\gamma$ predicts a continuous transition pathway from the crystal to the liquid. From the radial distribution functions (RDF) shown in Figs. 5(a)–5(c), one can see the two-stage transition process as a function of $\gamma$. At small $\gamma$ values, crystalline rutile is still observed. At approximately the halfway point ($\gamma = 0.5$), we can see the first two peaks of Ti–Ti combine into a single peak, along with a broadening of the peaks after 4 Å. This long-range broadening is also present in the O–O distribution after 3 Å. These RDF changes correspond to the transition from crystalline rutile to amorphous TiO$_2$. Finally, we observe further broadening of the peaks, especially the first peak, which corresponds to the transition from an amorphous state to a liquid. Again, we emphasize the ability of SODAS++ to not only uniquely characterize these three atomic systems but also connect them in a way that is physically justified and intuitive.

Figures 5(e)–5(i) show the melting of Ag$_{26}$Au$_5$Cu$_{19}$ via continuous heating simulations. Again, SODAS++ does an excellent job of capturing the continuous order-to-disorder transition as the thermostat temperature is increasingly ramped up. Here, an interesting solid–solid transition occurs prior to the melting of the system. At around 1000 K, the phase space breaks into two distinct regions, as evidenced by Fig. 5(i). Interestingly, this transition can be explained by observing the RDF as a function of temperature, as shown in Figs. 5(e)–5(h). At lower temperatures, Cu–Au distances ($h$) are the second most common peak at interatomic distances smaller than 2.5 Å. However, at approximately 1000 K Ag–Cu distances become equivalent in likelihood at those distances, with Au–Au distances becoming more likely as temperature increases as well. This indicates that the atomic structure changes from a Cu–Au dominant geometry to one in which Ag plays a larger role as the system becomes more disordered. Figure 5(i) captures this transition well by predicting a "bridge" region between the two distinct clusters, one representing low-temperature motifs and the other higher-temperature ones. The $\gamma$ characterization in Fig. 5(i) quantifies this transition extremely well, further signifying that the SODAS++ latent space can quantify the structural changes in a material in a unique and interpretable manner.

In this work, we showcased an improvement over the SODAS methodology, SODAS++, that alleviates the pitfalls of SODAS by generalizing the framework to any class of structural transition and any class of material. Using SODAS++, we quantified structural transitions of colloidal systems of increasing density, the low-temperature compression and melting of bulk Al, the amorphization of TiO$_2$, and...
the melting of Ag25Au5Cu19. These systems represent a wide spectrum of materials classes and changes in atomic structure. In all cases, SODAS++ quantified these structural transitions in a manner that was not only accurate but was, more importantly, interpretable. This interpretability is a critical piece of machine learning methodologies for physical systems, as it is not sufficient to simply apply black-box models to systems without understanding the underlying mechanisms at play.

SODAS++ also does not require training a model on reference data, as the characterizations shown in this work are created via the GNN encoding, which simply operates on the information contained within the initial graph. In this way, SODAS++ is an unsupervised technique that can encode any structural transition in a physically interpretable manner. This process makes SODAS++ generalizable to any material system that can be appropriately encoded as a graph. Based on this, we speculate that SODAS++ could be used as a foundation for generative modeling of atomistic trajectories, in which a path can be generated between known endpoints, alleviating the need for complex and expensive simulations. We do note the inherently non-linear pathways found between endpoints in this work; however, implying that further work is needed to interpolate between points within the latent space using sparse datasets. We hope that methods, such as SODAS++, can help to alleviate the current bottlenecks with characterizing atomistic structures and provide a pathway to accurate, unique, and interpretable atomistic descriptors.

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AUTHOR DECLARATIONS

Conflict of Interest
The authors have no conflicts to disclose.

Author Contributions

Bamidele Aroboto: Formal analysis (equal); Methodology (equal); Validation (equal); Writing – original draft (supporting).
Shaohua Chen: Data curation (equal); Formal analysis (equal).
Tim Hsu: Conceptualization (equal); Writing – review & editing (equal).
Brandon wood: Conceptualization (equal); Writing – review & editing (equal).
Yang Jiao: Conceptualization (equal); Project administration (equal); Supervision (equal); Writing – original draft (equal).
James Chapman: Conceptualization (equal); Project administration (equal); Software (lead); Supervision (equal); Writing – original draft (equal).

DATA AVAILABILITY

All data required to reproduce this work will be given upon request by contacting the corresponding author. The source code for this work, along with working examples, can be found at https://github.com/Materials-Informatics-Laboratory/SODAS.

REFERENCES


