

SC 561

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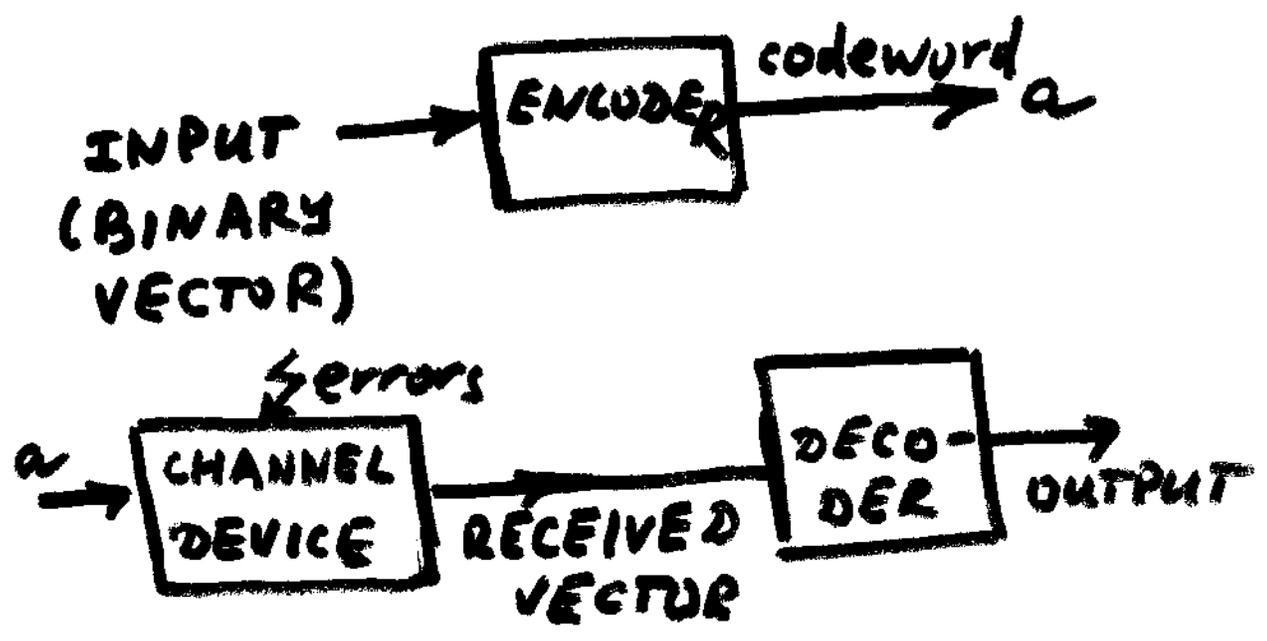
THEORY OF ERROR

CORRECTING CODES

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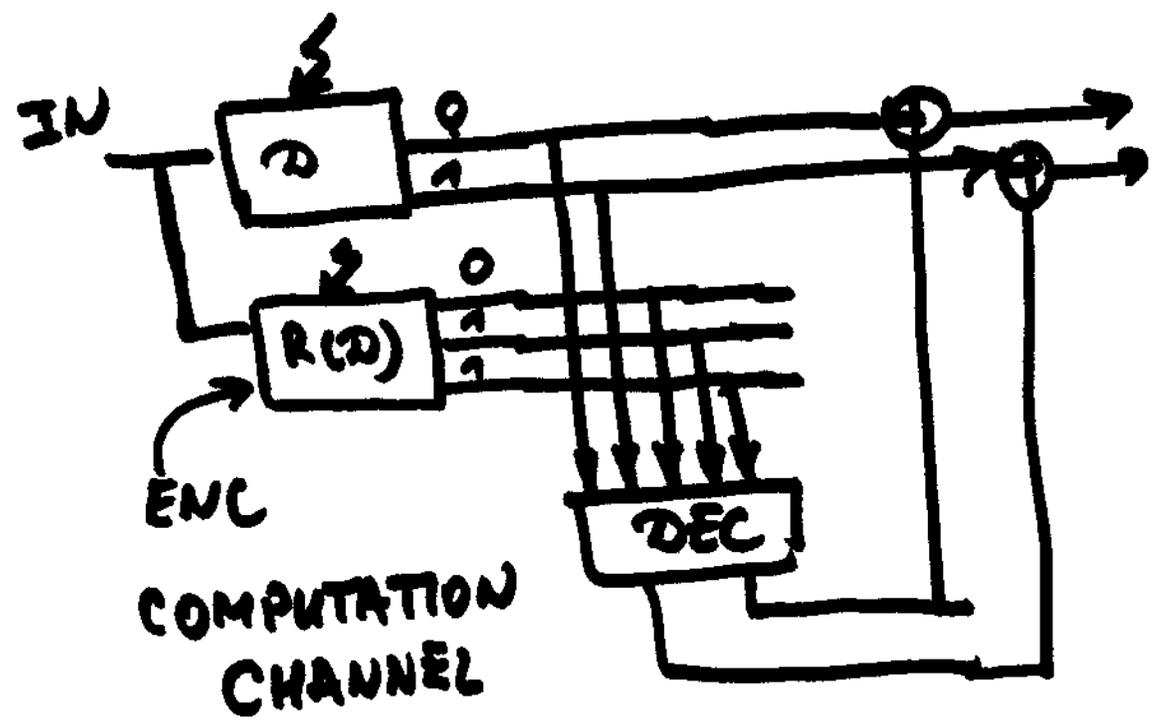
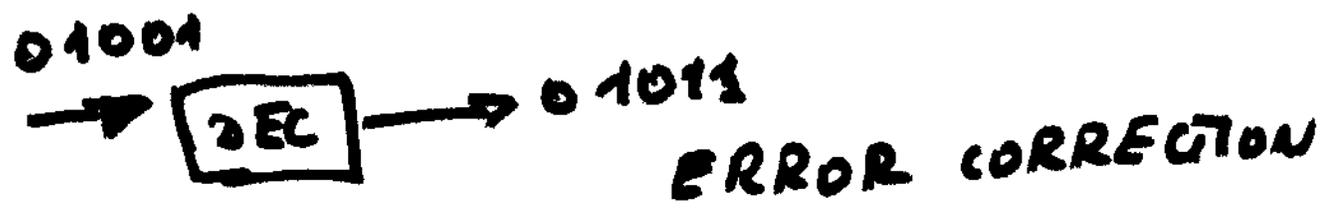
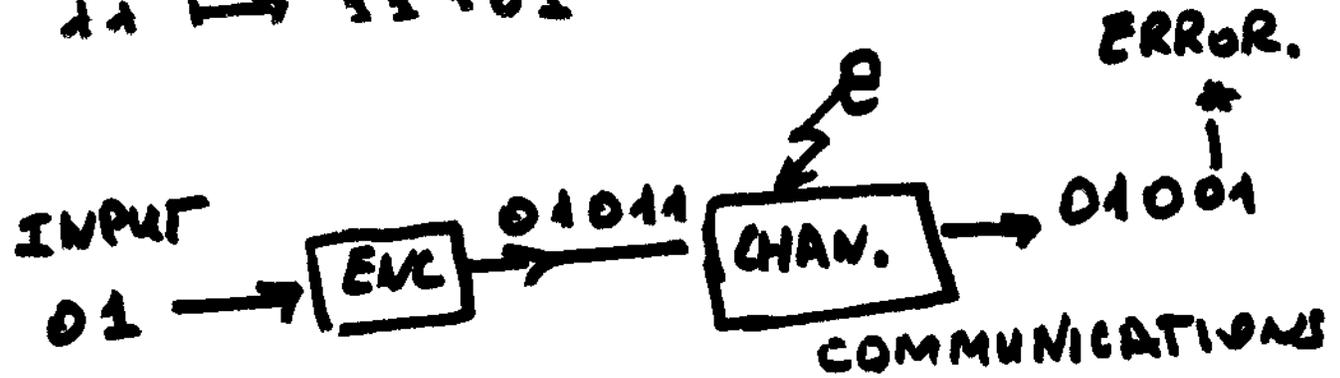
TEXT: R. HILL, "A FIRST COURSE  
IN CODING THEORY"  
CLARENDON PRESS, 1986.

# MATHEMATICAL THEORY OF DETECTION, LOCATION AND CORRECTION OF ERRORS IN COMPUTERS AND COMMUNI- CATION CHANNELS.



# EXAMPLE : ENCODER (ENC)

00	→	00000
01	→	01011
10	→	10110
11	→	11101



• ERROR CORRECTION MAY ALSO BE OBTAINED BY ERROR DETECTION AND RETRANSMISSION OR RECOMPUTING.

CHANNELS



If for error-free case  $f(x) = x$ , then CH is a communication channel

## EXAMPES OF COMMUNICATION<sup>4.</sup>

CHANNELS: COMMUNICATION  
LINKS, memories

If  $f(x) \neq x$ , then CH. is  
a computation channel

(computer components: adders,  
multipliers, etc.).

Notation:  $Z_2 = \{0, 1\}$

$Z_q = \{0, 1, 2, \dots, q-1\}$

$Z_q^n$  - set of all  $q$ -ary vectors  
of length  $n$ .

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Example  $Z_2^2 = \{00, 01, 10, 11\}$

$Z_3^2 = \{00, 01, 02, 10, 11, 12, 20, 21, 22\}$

$Z_2^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

Definition: A  $q$ -ary code  $C$  of length  $n$  is any subset of  $Z_q^n$  ( $C \subseteq Z_q^n$ )

EXAMPLE  $C = \{00000, 01011, 10110, 11101\}$   
is a binary code of length 5.

EXAMPLE. A SET OF ALL TELEPHONE NUMBERS IN USA AND CANADA is a DECIMAL (10-ARY) CODE OF LENGTH  $n=10$ .