# Detection / Correction of Unidirectional Errors

- FOR DETECTION OF <u>ALL</u> UNIDIRECT. ERRORS THE BEST is $\lfloor n/2 \rfloor$ out of $n$ codes (<u>non</u>systematic) with $\binom{n}{\lfloor n/2 \rfloor}$ codewords
- systematic codes for detection of <u>all</u> unidirect. errors Berger codes with $r = \lceil \log_2 (k+1) \rceil$

# Correction of Unidirectional Error with Multiplicity $\ell$

- FOR $\ell = 1 \Rightarrow$ HAMMING CODES
- FOR UNIDIRECT. errors when $0 \to 1$ distortions only

$$y \mapsto y \underset{\text{OR}}{\vee} e$$

$$\Big(\text{FOR } 1 \to 0 \text{ distortions } y \mapsto y \underset{\text{AND}}{\cdot} \bar{e}\Big)$$

- ~~$H = [h_1\ h_2 \dots h_n]$ — check matrix~~

FOR $0 \to 1$ errors the necessary and sufficient condition is that for any two codewords $v_1, v_2$

$$v_1 \vee e_1 \neq v_2 \vee e_2 \quad , \|e_1\|, \|e_2\| \leq \ell$$

FOR $1 \to 0$ errors

$$v_1 \bar{e}_1 \neq v_2 \bar{e}_2 \quad \text{or} \quad \bar{v}_1 \vee e_1 \neq \bar{v}_2 \vee e_2$$

- Thus the problem is to construct a max. code $C$ for a given $n$ and $\ell$ such that

$$\forall v_1, v_2 \in C \quad \forall e_1, e_2 : \|e_1\|, \|e_2\| \leq \ell$$

$$v_1 \vee e_1 \neq v_1 \vee e_2 \text{ and } \overline{v_1} \vee e_1 \neq \overline{v_2} \vee e_2$$

- This problem is still open even for $\ell = 2$.

# Codes detecting all symmetrical errors with multiplicity up to $t$ and unidirectional errors with any multiplicity

$t$-ED/AUED codes

$t$-ED/AUED code $V$ has
$d(V) = t+1$

$2t$-ED/AUED = $t$-error correcting and AUED =
= $t$-EC/AUED

This codes are useful since faults affecting a large number of outputs lines often result in unidirectional errors

## CONSTRUCTION FOR t-ED/AUED CODES

1. FOR A GIVEN K (NUMBER OF INFO BITS) CONSTRUCT A BEST $(K+r_H, K, d=t+1)$ code with distance $t+1$.

2. FOR ANY CODEWORD $V$ add $r_u$ bits $r_u = \left\lceil \log_2\left(\left\lfloor \frac{1}{d}(K+r_H)\right\rfloor + 1\right)\right\rceil$ which are NEGATION OF a binary representation for $\left\lfloor \frac{\|V\|}{d} \right\rfloor$ ($\|V\|$ is HAMMING WEIGHT

- this construction results in $t$-ED/AUED codes which are systematic $\cancel{(K + r_u + \lceil \log_2 \ldots (K + r_H) \rceil)}$ codes. These codes are nonlinear

- FOR $t=0$ $(d=1)$ THIS CONSTRUCTION PRODUCE THE BERGER CODE

- FOR THIS CONSTRUCTION

$r_H$ bits provide for a given distance

$d = t+1$

$r_u = \left\lceil \log_2 \left(1 + \left\lfloor \frac{1}{t+1} (K + r_H) \right\rfloor \right) \right\rceil$ bits

provide for detection of all unidirectional ~~errors~~ errors

# EXAMPLE 2ED/AUED = 1EC/AUED for $k=4$.

FOR THE (7,4) HAMMING code with $r_H=3$

$$G=\begin{bmatrix}1000\,110\\0100\,101\\0010\,011\\0001\,111\end{bmatrix}$$

$d=3$

~~$r_U = \lceil \log_2 \ldots \rceil = 2$~~

$r_U = \lceil \log_2 \lfloor (1 + \frac{7}{3}) \rfloor \rceil$

V:

| k | $r_H$ | $r_U$ |
|---|---|---|
| 0 0 0 0 | 0 0 0 | 1 1 |
| 1 0 0 0 | 1 1 0 | 1 0 |
| 0 1 0 0 | 1 0 1 | 1 0 |
| 1 1 0 0 | 0 1 1 | 1 0 |
| 0 0 1 0 | 0 1 1 | 1 0 |
| 1 0 1 0 | 1 0 1 | 1 0 |
| 0 1 1 0 | 1 1 0 | 1 0 |
| 1 1 1 0 | 0 0 0 | 1 0 |
| 0 0 0 1 | 1 1 1 | 1 0 |
| 1 0 0 1 | 0 0 1 | 1 0 |
| 0 1 0 1 | 0 1 0 | 1 0 |
| 1 1 0 1 | 1 0 0 | 1 0 |
| 0 0 1 1 | 1 0 0 | 1 0 |
| 1 0 1 1 | 0 1 0 | 1 0 |
| 0 1 1 1 | 0 0 1 | 1 0 |
| 1 1 1 1 | 1 1 1 | 0 [illegible] |

(9,4) 1-EC/AUED code

Blaum, Bruck, "Unordered Error Correcting Codes and Their Applications", FTCS-22, 1992