Extended Hamming Codes : $d(V)=4$

Detection : single, double and triple errors

Correction : single errors

Parameters of Extended Hamming Codes

n	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
k	1	1	2	3	4	4	5	6	7	8	9	10	11	11	12

n	19	20	21	22	23	24	25	26	27	28	29	30	31	32
k	13	14	15	16	17	18	19	20	21	22	23	24	25	26

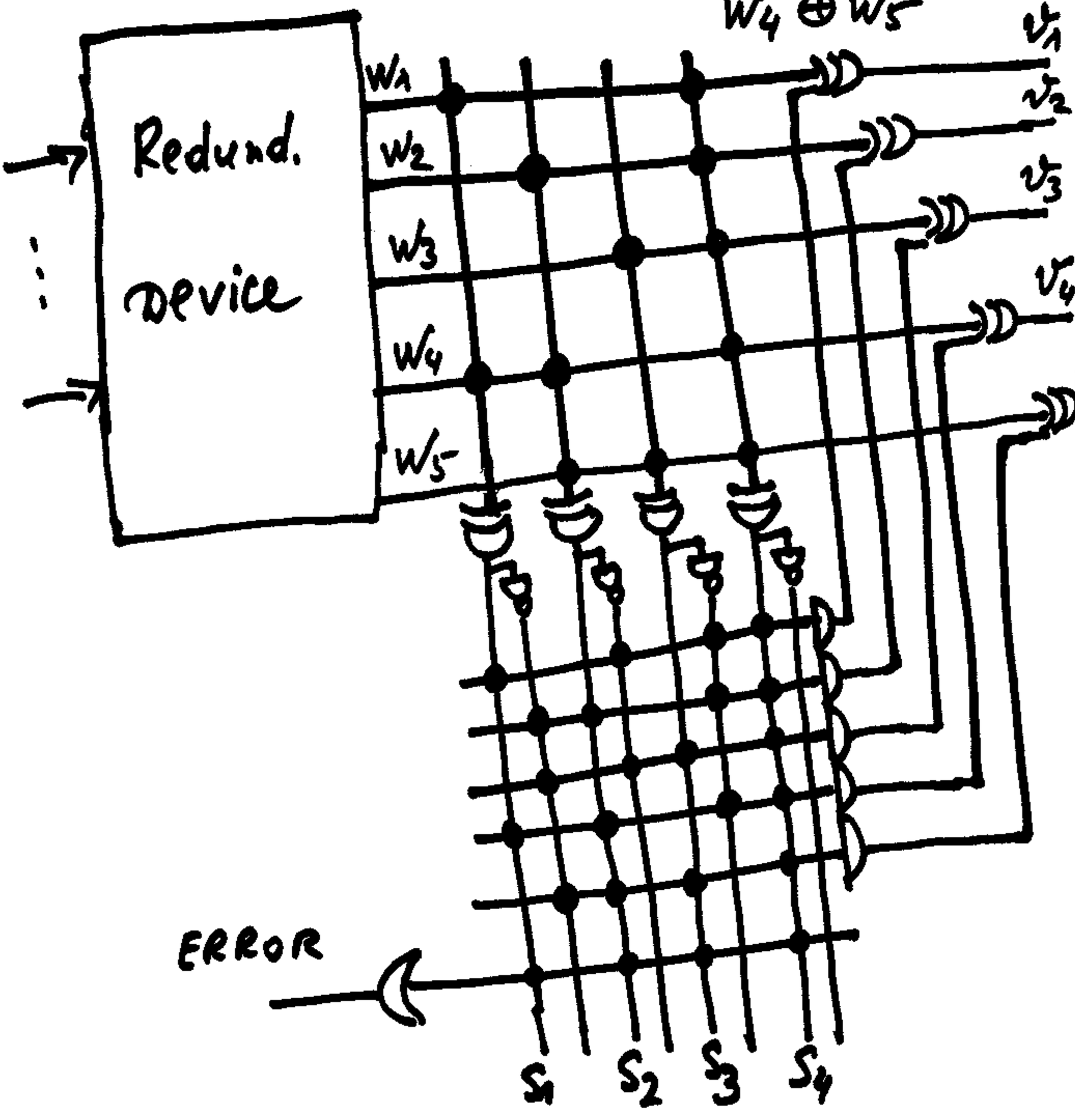
Construction: Add row of all 1's to the check matrix of the Hamming code

Extended Hamming codes consist of codewords of corresponding Hamming codes with even weight.

EXAMPLE $n=5 \Rightarrow r = \lceil \log_2(5+1) \rceil + 1 = 4$

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \Rightarrow$$

$$S_1 = W_1 \oplus W_4$$
$$S_2 = W_2 \oplus W_4 \oplus W_5$$
$$S_3 = W_3 \oplus W_5$$
$$S_4 = W_1 \oplus W_2 \oplus W_3 \oplus W_4 \oplus W_5$$



for Hamming and extended Hamming codes we have for a number of 2-input XOR gates to compute syndrome:

$$L \le \sum_{ij} h_{ij} - r$$

$H = (h_{ij})$ $(r \times n)$ - check matrix

Tradeoff between r and L

Use codes with small numbers of ones in every column of H
(codes with low density of parity checks)

Example $n = 15$

$$H_1 = \begin{pmatrix} 0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&1&1&1&0&0&0&1&1&1&1&1&1 \\ 0&1&1&0&0&1&1&0&1&0&0&1&1&0&1 \\ 1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{pmatrix}$$ (15,11) code $r=4$, $L=28$

$$H_2 = \begin{pmatrix} 0&0&0&1&0&0&0&1&0&0&1&0&1&1&1 \\ 0&0&1&0&0&0&1&0&0&1&0&1&0&1&1 \\ 0&0&1&0&0&1&0&0&1&0&1&1&0&0&1 \\ 0&1&0&0&0&1&0&0&1&1&0&0&1&0&0 \\ 1&0&0&0&1&1&1&0&0&0&0&0&0&0&0 \end{pmatrix}$$ (15,10) code $r=5$, $L=20$

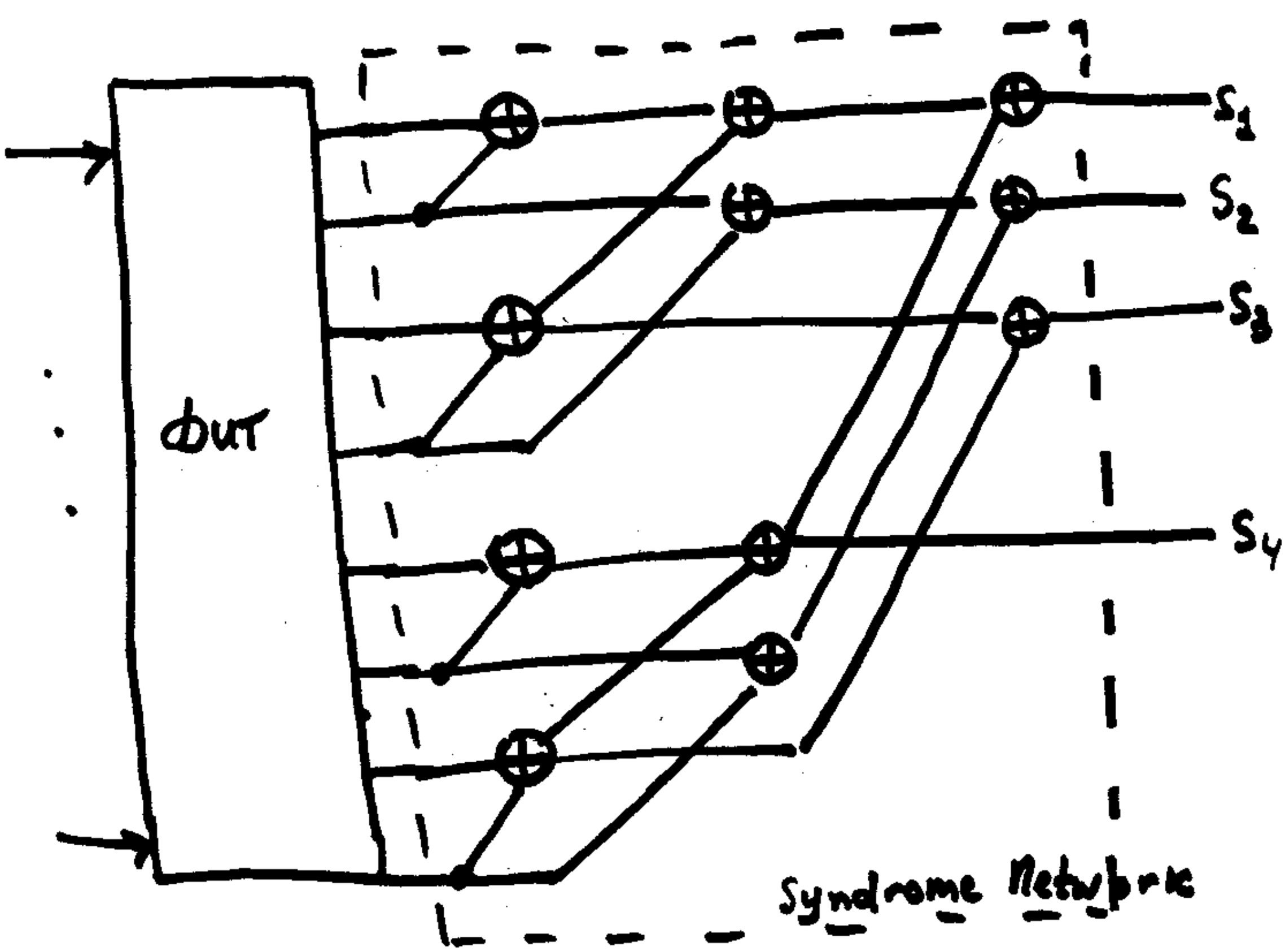
(no joint minimization for syndrome computing)

Joint minimization for syndome computing

Errors in syndrome calculator are not corrected

Example $n=8$ (8,4) extended Hamming code, distance $=4$

$$H=\begin{pmatrix} 1&1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1 \\ 0&0&0&0&1&1&1&1 \end{pmatrix}$$ ($L=16$, without joint minimization)



Existing Computer Systems with Error - Correcting Codes.

Hamming codes for main memories :

IBM 360/370

IBM STRETCH

PDP 10 and PDP 11

DEC System 20

VAX 11/750 , VAX 11/780

UNIVAC 1100/60

XEROX ALTO

BELL ESS - 1

Application of Error - Correcting Codes in Computer Systems (C + d).

Boeing Aerospace Computer	**(35,28) SEC**
DEC MF20 (memory)	**(44,36) SEC/DED**
MECRA (registers)	**Hamming codes and replications**
BELL ESS - 3a (microstore)	**4 out of 8 codes**