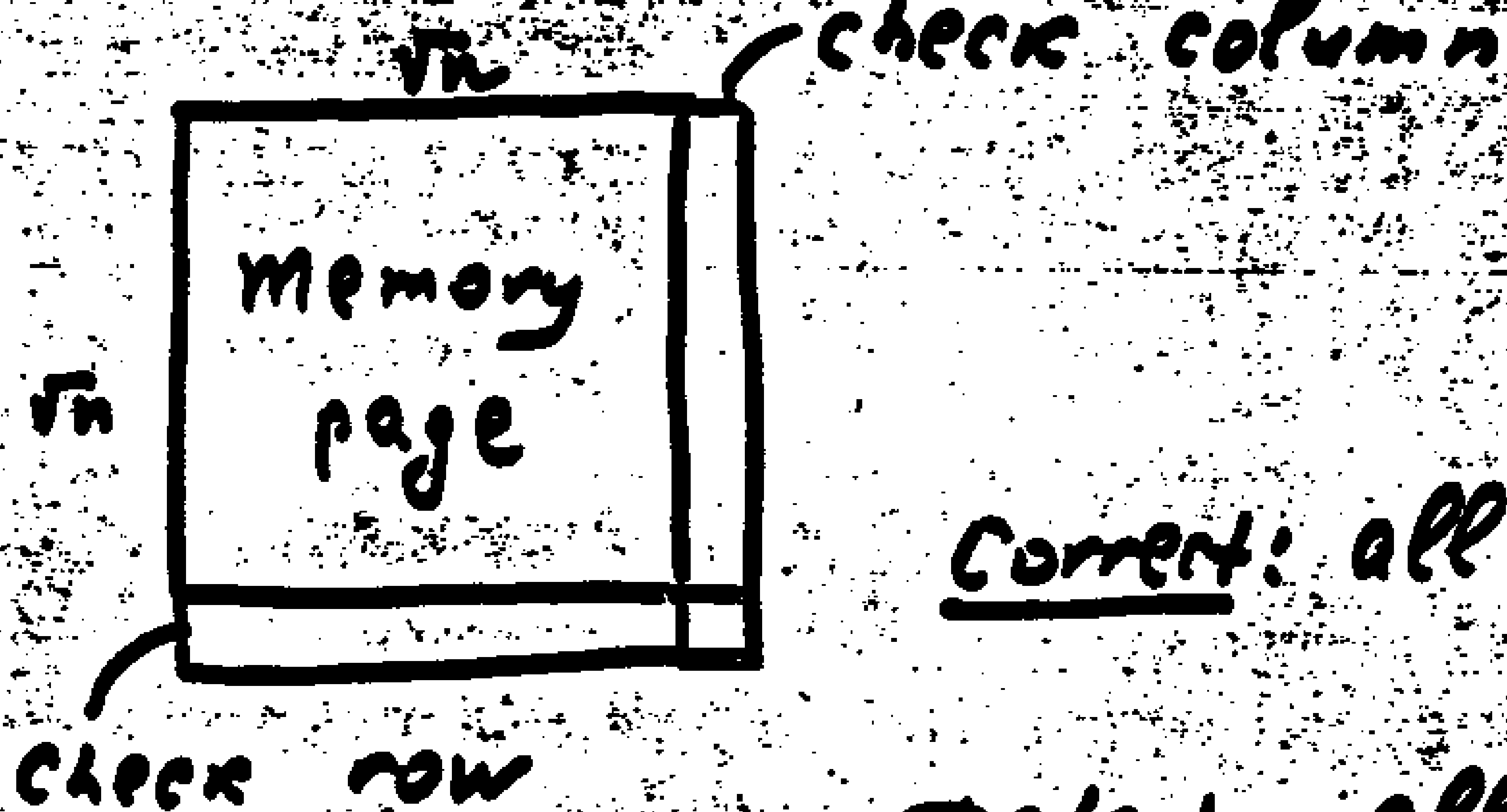
1

Two-dimensional Parity Checking



Correct: all single errors

Detect: all single double and triple errors

Example

check column

$$
\left[\begin{array}{cccc|c}
0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 \\
\hline
1 & 1 & 1 & 0 & 1
\end{array}\right]
$$

check row

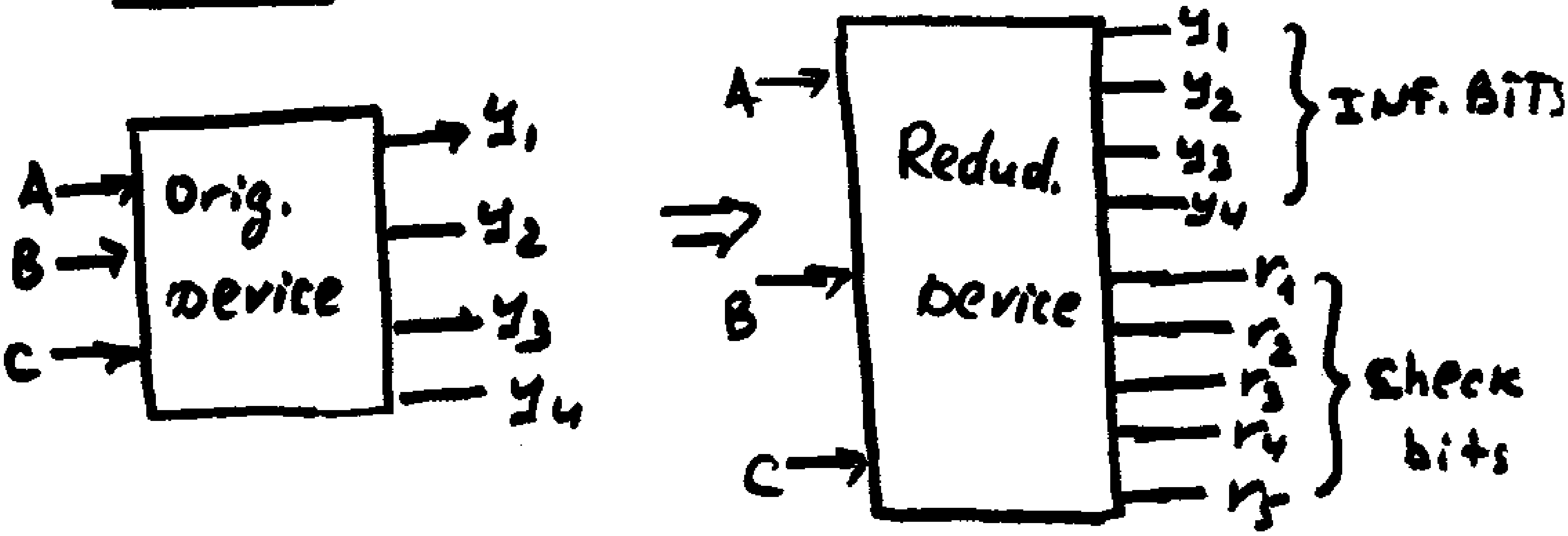
$d(V)=4$

$n = K + 2\sqrt{K} + 1$

$K = 64 \Rightarrow n = 81$

DETECTION OF SINGLE, DOUBLE AND TRIPLE ERRORS BY TWO-DIMENSIONAL PARITY CHECK CODES.

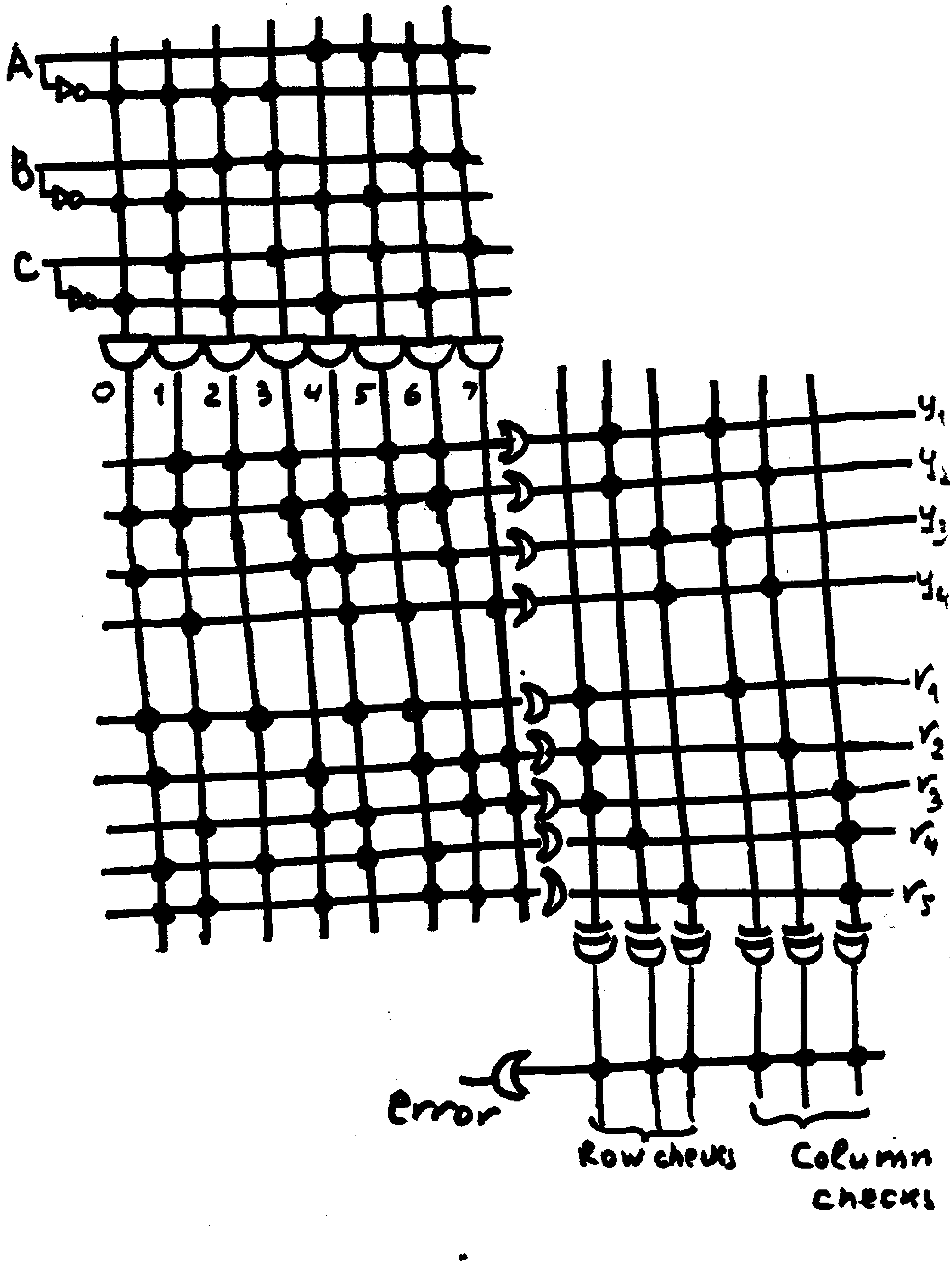
Example. $K=4$ $\quad r = 2\sqrt{K}+1 = 5$



$$\begin{array}{cc} y_1 & y_2 \\ y_3 & y_4 \end{array} \quad \Rightarrow \quad \begin{array}{cc|c} r_1 & r_2 & r_3 \\ \hline y_1 & y_2 & r_4 \\ y_3 & y_4 & r_5 \end{array}$$

Row checks	Column checks
$r_1 \oplus r_2 \oplus r_3 = 0, \quad r_3 = r_1 \oplus r_2$	$r_1 \oplus y_1 \oplus y_3 = 0, \quad r_1 = y_1 \oplus y_3$
$y_1 \oplus y_2 \oplus r_4 = 0, \quad r_4 = y_1 \oplus y_2$	$r_2 \oplus y_2 \oplus y_4 = 0, \quad r_2 = y_2 \oplus y_4$
$y_3 \oplus y_4 \oplus r_5 = 0, \quad r_5 = y_3 \oplus y_4$	$r_3 \oplus r_4 \oplus r_5 = 0, \quad r_3 = r_4 \oplus r_5$

Example (Ctd) Implementation



ARRAY MULTIPLIER WITH SELF-ERROR-DETECTION

by parity prediction

EXAMPLE 4-bit MULTIPLIER

$$p = a * b$$

$$a = (a_3, a_2, a_1, a_0)$$

$$b = (b_3, b_2, b_1, b_0)$$

$$p = (p_7, p_6, \ldots, p_0)$$

$$a_c = a_3 \oplus a_2 \oplus a_1 \oplus a_0$$

$$b_c = b_3 \oplus b_2 \oplus b_1 \oplus b_0$$

$$p_c = p_7 \oplus \ldots \oplus p_0$$

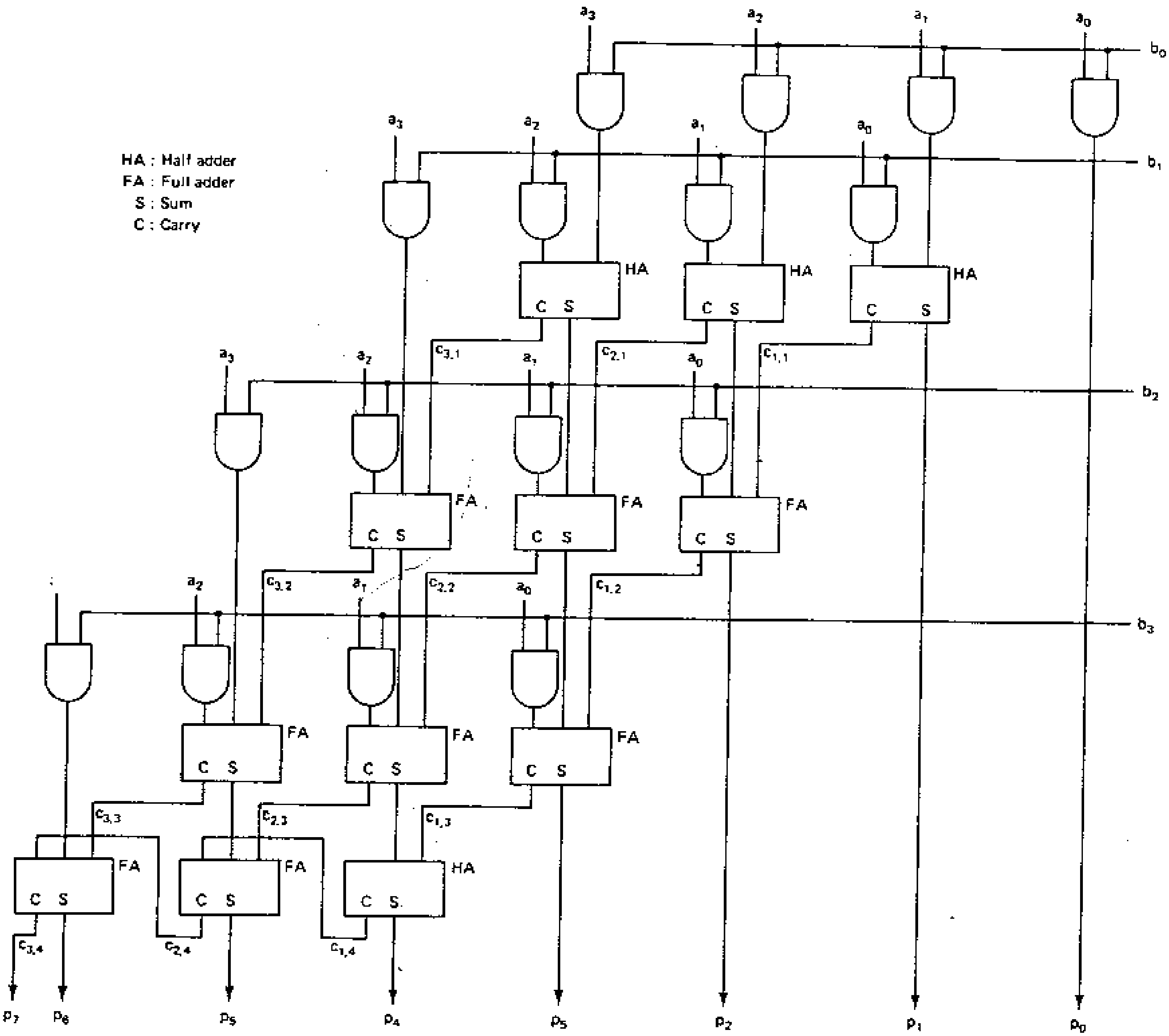
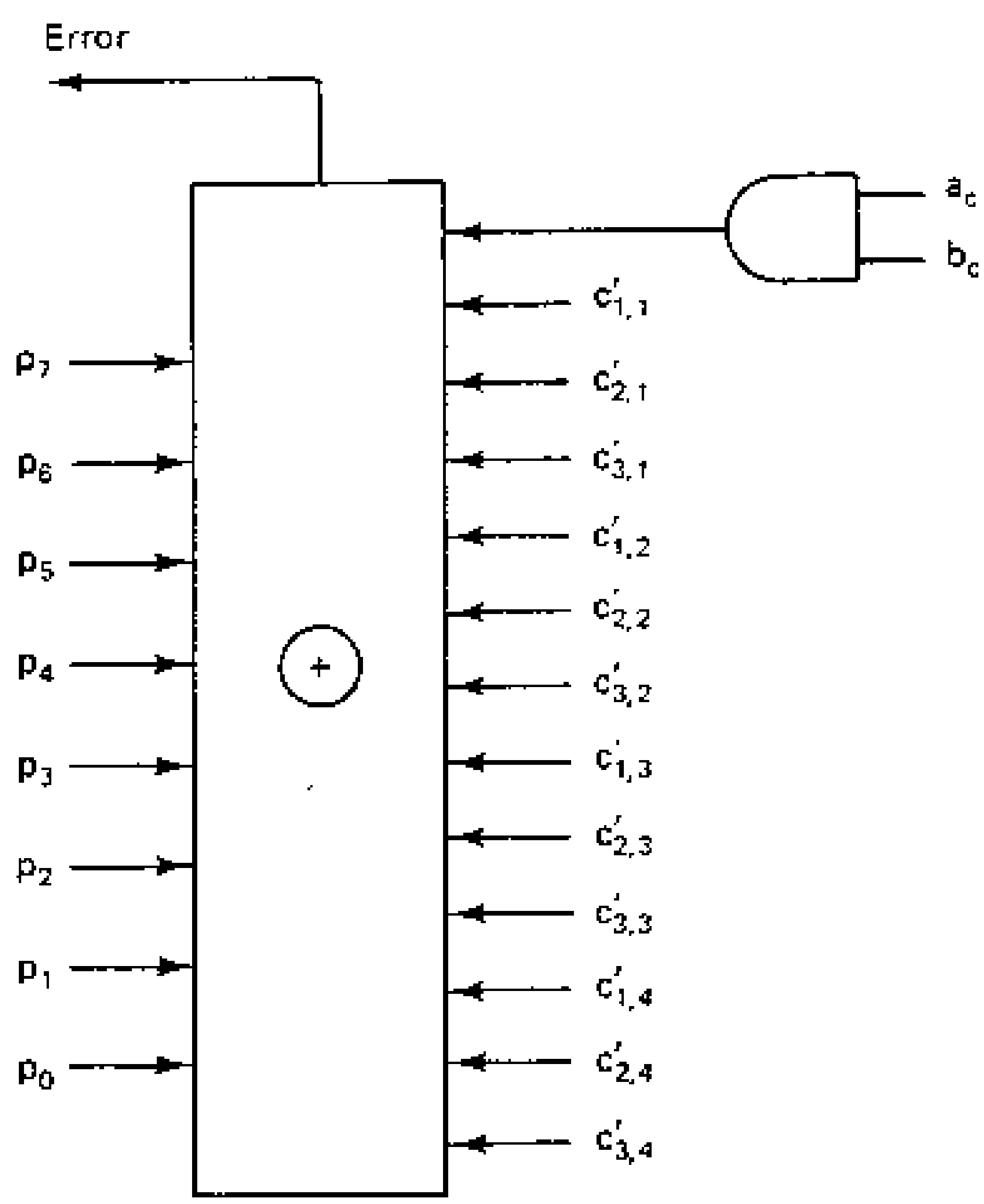


Figure 5.2.6 Multiplier using array of full-half adders.

[redacted]

[redacted]



(b)

Figure 5.2.7 (cont.)

[redacted][4]

$$P_0 = a_0 b_0$$

$$P_1 = a_0 b_1 \oplus a_1 b_0$$

$$P_2 = a_0 b_2 \oplus a_1 b_1 \oplus a_2 b_0 \oplus c_{1,1}$$

$$P_3 = a_0 b_3 \oplus a_1 b_2 \oplus a_2 b_1 \oplus a_3 b_0 \oplus c_{2,1} \oplus c_{1,2}$$

$$P_4 = a_1 b_3 \oplus a_2 b_2 \oplus a_3 b_1 \oplus c_{3,1} \oplus c_{2,2} \oplus c_{1,2}$$

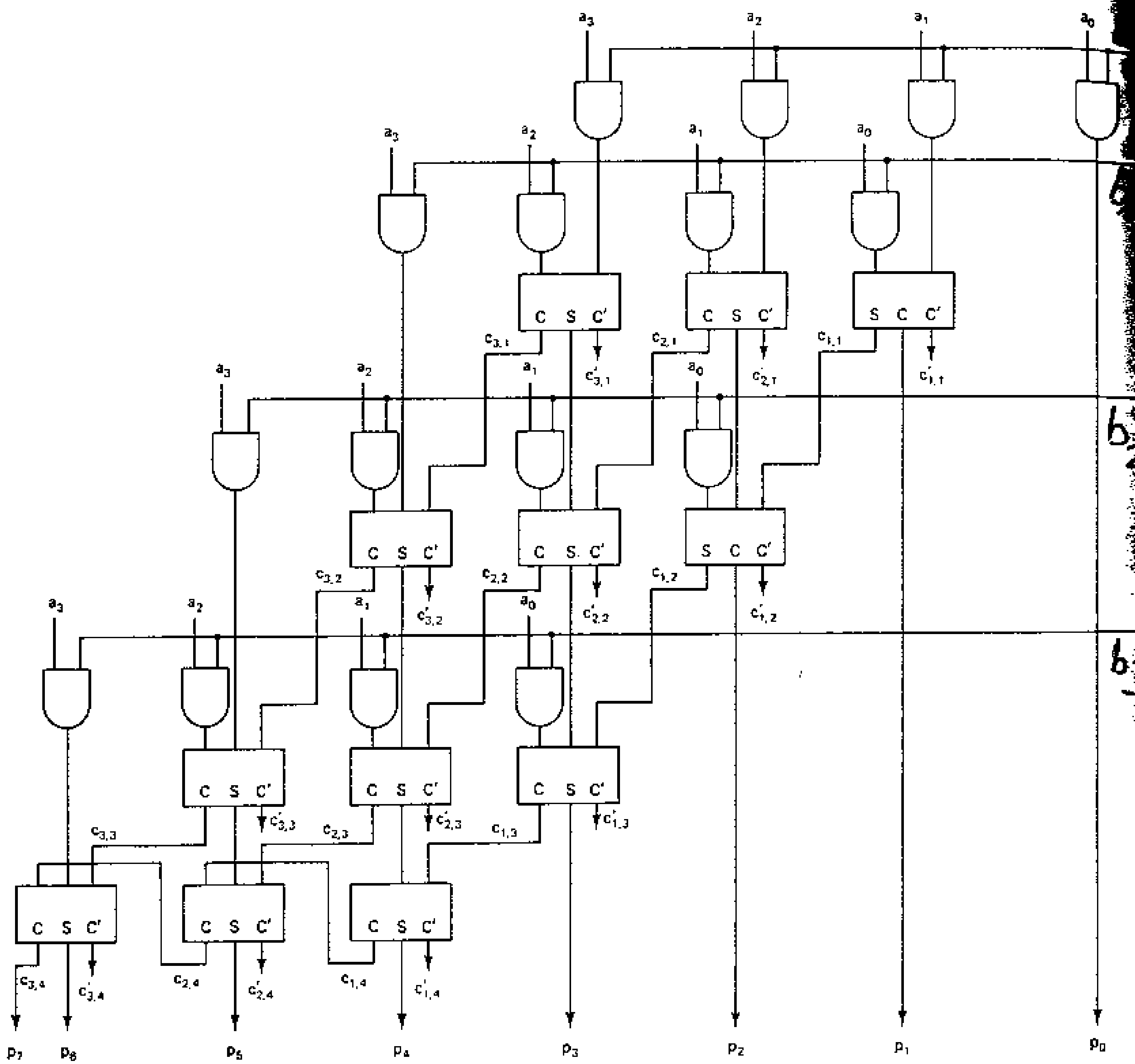
$$P_5 = a_2 b_3 \oplus a_3 b_2 \oplus c_{3,2} \oplus c_{2,3} \oplus c_{1,4}$$

$$P_6 = a_3 b_3 \oplus c_{3,3} \oplus c_{2,4}$$

$$P_7 = c_{3,4}$$

$$P_c = \bigoplus_{i=0}^{7} P_i = \left(\bigoplus_{i=0}^{3} a_i\right) \left(\bigoplus_{i=0}^{3} b_i\right) \oplus$$

$$\bigoplus_{i=1}^{3} \bigoplus_{j=1}^{4} c_{i,j} = \underbrace{a_c b_c \oplus \bigoplus_{i=1}^{3} \bigoplus_{j=1}^{4} c_{i,j}}.$$



(a)

Figure 5.2.7 Parity-checked multiplier.

[redacted] the le[redacted]

Transmission Rates

k - number of information bits
n - number of bits in an expanded word
r - number of redundant bits

$n = k + r$

Transmission rate for an (n,k)-code

$$R = \frac{k}{n} \qquad 0 < R < 1$$

For one-dimensial parity:

$$R = \frac{n-1}{n} = 1 - \frac{1}{n} \to 1 \quad \text{for large } n$$

for two dimensional parity:

$$R = \frac{k}{k + 2\sqrt{k} + 1} \to 1 \quad \text{for large } k$$