SC753

TESTING OF COMPUTER MEMORIES (OFF-LINE)

Consider $(W \times N)$ MEMORY

N-words w-bits/per word.

(Ex. $N = 10^6$, $w = 4$)

<u>General Fault Model</u>: $PS(N,K)$

1. For any K-tuple of words data in one word may be distorted for a specific combination of data in ~~the~~ remaining K-1 words (<u>STATIC</u> PS FAULTS)

$PS(N,K)$ =
PATTERN SENSITIVE FAULTS =

K-couplings

2. Errors in transition from one content of a cell to another for a specific combination of data in remaining $K-1$ cells (<u>dynamic</u> PS FAULTS)

FROM THE PRACTICAL POINT OF VIES CASES $K=1,2,3$ are important

<u>Background:</u>

$$B=\left(B^{(0)},\dots,B^{(N-1)}\right), \quad B^{(i)}\in Z_2^{w}$$

BACKGROUND MATRIX:

$$B = \begin{bmatrix} B_0^{(0)} & \dots & B_0^{(N-1)} \\ B_1^{(0)} & \dots & B_1^{(N-1)} \\ \dots & \dots & \dots \\ B_{T-1}^{(0)} & \dots & B_{T-1}^{(N-1)} \end{bmatrix}, \quad B_i^{(j)} \in Z_2^W$$

T - is a number of backgrounds

For detection of any number of PS(N,K) faults we need that in any K-tuple of columns of B all $q^K = 2^{WK}$ $(q = 2^W)$ K-digit q-ary vectors appear at least once as rows

If B satisfies this condition we call $\{B_0, \dots, B_{T-1}\}$ - K-pseudoexhaustive test

<u>Problem:</u> Given w, N, and k construct an optimal k-pseudo-exhaustive test with min T

(Problem is still open. Solutions will be presented for optimal and near optimal tests for $k=1,2,3$)

<u>Example</u> $w=1$ $(q=2)$, $k=2$

$N=10$

$T=6$

$$B=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

All 4 combinations 00, 01, 10, 11 appear in any 2 columns

Let H be a check-matrix for $(N, N-K, K+1)$ q-ary Reed Solomon (RS) code $(N \leq q-1)$ $\quad q = 2^w$

Then

$$H = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{N-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \alpha^{K-1} & \alpha^{2(K-1)} & \dots & \alpha^{(K-1)(N-1)} \end{bmatrix},$$

where α primitive in $\mathbb{Z}_2^w = \mathbb{Z}_q$.

Since this code detects K errors, any K columns of H are linearly independent over $\mathbb{Z}_q$. Thus, the linear span (a set of all linear combinations with coefficients from $\mathbb{Z}_q$) can be taken as B.

Thus for $N \le q-1 = 2^w - 1$

the linear span of rows

of the check matrix of $(n, n-k, k+1)$

RS code is <u>optimal</u> k-pseudo-

exhaustive test with

$$T = T_k = q^k = 2^{wk}.$$

<u>Example</u> $w=2$ $(q=4)$, $k=2$, $N=3$

$\mathbb{Z}_2^2$:

$\mathbb{Z}_2^2$	
0 0	0
0 1	α
1 0	1
1 1	α^2
1 α	

$\alpha^2 + \alpha + 1 = 0$

α - primitive in $\mathbb{Z}_2^2$, $\alpha^3 = 1$

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

BACKgrounds are generated as

$[v_0, v_1]\, H$ where $v_0, v_1 \in \mathbb{Z}_2^2$

$B =$

$B^{(0)}$	$B^{(1)}$	$B^{(2)}$	v_0	v_1
0	0	0	0	0
1	α	α^2	0	1
α	α^2	1	0	α
α^2	1	α	0	α^2
1	1	1	1	0
0	α^2	α	1	1
α^2	α	0	1	α
α	0	α^2	1	α^2
α	α	α	α	0
α^2	0	1	α	1
0	1	α^2	α	α
1	α^2	0	α	α^2
α^2	α^2	α^2	α^2	0
α	1	0	α^2	1
1	0	α	α^2	α
0	α	1	α^2	α^2

In any $k=2$ columns of $B=[B^{(0)}\ B^{(1)}\ B^{(2)}]$ all 9 combinations $00, 01, 0\alpha, 0\alpha^2, \ldots, \alpha^2\alpha^2$ appear at least once. $\square$

For this test dynamic faults are <u>not</u> detected even for $k=1$

For example in $B^{(0)}$ there is no transition $0\to\alpha$.

We will modify now the order for generation of $v=(v_0, v_1)$ for detection of all dynamic faults for $k=1$.

FOR $K=2$

Let $x^2+c_1x+c_2$ is primitive over Z_2^w $(c_1, c_2 \in Z_2^w)$

and $\beta^2+c_1\beta+c_2=0$.

Then $\beta^i \neq \beta^j$ $(i,j=0,1,\dots,q^2-2)$
$q=2^w$

We generate vectors $v=(v_0,v_1)$ $(v \in Z_q^2)$ as $0, \beta^0, \beta^1, \dots, \beta^{q^2-2}$

then all transitions within one word will be generated (all dynamic faults with $K=1$ will be detected)

<u>Example</u> (contd). $w=2$ $(q=4)$, $K=2$

$\beta^2+\beta+\alpha=0$, $\alpha\in\mathbb{Z}_2^2$, $N=3$

$B=$

$B^{(0)}$	$B^{(1)}$	$B^{(2)}$	β^i	v_0 (1)	v_1 (β)
0	0	0	–	0	0
1	1	1	β^0	1	0
1	α	α^2	β^1	0	1
α^2	0	1	β^2	α	1
1	α^2	0	β^3	α	α^2
0	α^2	α	β^4	1	1
α	α	α	β^5	α	0
α	α^2	1	β^6	0	α
1	0	α	β^7	α^2	α
α	1	0	β^8	α^2	1
0	1	α^2	β^9	α	α
α^2	α^2	α^2	β^{10}	α^2	0
α^2	1	α	β^{11}	0	α^2
α	0	α^2	β^{12}	1	α^2
α^2	α	0	β^{13}	1	α
0	α	1	β^{14}	α^2	α^2

$$\beta^2 = \beta + \alpha \quad , \quad \beta^3 = \beta^2 + \alpha\beta = \beta + \alpha + \alpha\beta = \alpha + \alpha^2\beta ,$$

$$\beta^4 = \beta^2 + \alpha^2 = \beta + \alpha + \alpha^2 = 1 + \beta ,$$

$$\beta^5 = \beta + \beta^2 = \beta + \beta + \alpha = \alpha ,$$

$$\beta^6 = \alpha\beta \quad , \quad \beta^7 = \alpha\beta^2 = \alpha(\alpha+\beta) = \alpha^2 + \alpha\beta ,$$

$$\beta^8 = \alpha^2\beta + \alpha\beta^2 = \alpha^2\beta + \alpha(\alpha+\beta) = \alpha^2 + \beta$$

$$\beta^9 = \alpha^2\beta + \beta^2 = \alpha^2\beta + \alpha + \beta = \alpha + \alpha\beta$$

$$\beta^{10} = \alpha\beta + \alpha\beta^2 = \alpha\beta + \alpha(\alpha+\beta) = \alpha^2$$

$$\beta^{11} = \alpha^2\beta \qquad \beta^{12} = \alpha^2\beta^2 = \alpha^2(\alpha+\beta) = 1 + \alpha^2\beta$$

$$\beta^{13} = \beta + \alpha^2\beta^2 = \beta + \alpha^2(\alpha+\beta) = 1 + \alpha\beta$$

$$\beta^{14} = \beta + \alpha\beta^2 = \beta + \alpha(\alpha+\beta) = \alpha^2 + \alpha^2\beta$$

$$\beta^{15} = \alpha^2\beta + \alpha^2\beta^2 = \alpha^2\beta + \alpha^2(\alpha+\beta) = 1.$$

FOR THIS TEST all TRANSITIONS

$a \to b \qquad (a, b \in Z_2^2)$

except for $(a,b)=(0,0)$

within one word will be generated and all dynamic PS faults with $K=1$ are detected.

Proof: FOR THE COLUMN j

$j = 0, 1, \ldots, N-1$ we have

$$B^{(j)} = v_0 + v_1 \alpha^j$$

If $v_0 + v_1 \beta = \beta^i$, then

$$\beta^{i+1} = v_0 \beta + v_1 \beta^2$$

Let $\varphi(x) = x^2 + c_1 x + c_0 \quad c_i \in Z_2^w$

primitive and

$$\varphi(\beta) = \beta^2 + c_1 \beta + c_0 = 0$$

$$\beta^2 = c_1 \beta + c_0$$

~~[illegible]~~ Then

$$\beta^{i+1} = v_0\beta + v_1(c_1\beta + c_0) =$$
$$= v_1 c_0 + \beta(v_0 + v_1 c_1)$$

Then to provide for the transition $a \to b$ $(a, b \in \mathbb{Z}_2^N)$ in column j (or word j) we have two equations with 2 unknowns v_0, v_1

$$\begin{cases} v_0 + v_1\alpha^j = a \\ v_1 c_0 + (v_0 + v_1 c_1)\alpha^j = b \end{cases}$$

Solving this system we have

$$v_0 = a + \alpha^j(a\alpha^j + b)(\alpha^{2j} + c_1\alpha^j + c_0)^{-1}$$

$$v_1 = (a\alpha^j + b)(\alpha^{2j} + c_1\alpha^j + c_0)^{-1}$$

We note that

$$\alpha^{2j} + c_1 \alpha^j + c_0 \neq 0$$

since otherwise

$\varphi(x) = x^2 + c_1 x + c_0$ is divisible by $(x+\alpha^j)$ and $\varphi(x)$ is <u>not</u> primitive

<u>Example</u> (Cont'd) $w=2, q=4$

$N=3$ Let $j=1$, $a=\alpha^2$, $b=0$

Then we have the following system

$$\begin{cases} v_0 + v_1 \alpha = \alpha^2 \\ v_1 \cdot \alpha + (v_0 + v_1)\alpha = 0 \end{cases}$$

$\varphi(x) = x^2 + x + \alpha$

This system has a unique solution:

$v_0 = 0 \qquad v_1 = \alpha$

(SEE TABLE ON PAGE 10)

Solution for the problem of construction of K-pseudoexhaustive tests for $N < q = 2^w$

1. Select a primitive polynomial of degree K over Z_2^w

$$\varphi(x) = x^K + c_{K-1} x^{K-1} + \dots + c_1 x + c_0$$

$$c_i \in Z_2^w = Z_q$$

2. Represent all $(v_0, v_1, \dots, v_{K-1}) \in Z_q^K$, $v_i \in Z_q$ as $0, 1, \beta, \beta^2, \dots, \beta^{q^K-2}$ where $\varphi(\beta) = 0$.

3. Backgrounds (rows of test matrix B) are obtained by multiplication of

$0, 1, \beta, \beta^2, \dots, \beta^{q^K-2}$ by $\qquad N \le q-1$

$$H = \underbrace{\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{N-1} \\ 1 & \alpha^2 & \alpha^4 & \dots & \alpha^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \alpha^{K-1} & \alpha^{2(K-1)} & \dots & \alpha^{(K-1)(N-1)} \end{bmatrix}}_{N}$$

- This test is <u>optimal</u> with a min number of backgrounds

$$T = T_K = q^K$$

- This test detects all <u>static</u> <u>PS(N,K)</u>
- This test detects all <u>dynamic</u> PS(N,K-1)

EXTENSION OF K-PSEUDO EXHAUSTIVE TESTS FOR $N \geq q$ ($K=1,2,3$)

1. FOR $K=1$ and any N.

$$B = \begin{bmatrix} \underbrace{\begin{matrix} 0 & 0 & 0 & & 0 \\ 1 & 1 & 1 & \dots & 1 \\ \alpha & \alpha & \alpha & \dots & \alpha \\ \alpha^2 & \alpha^2 & \alpha^2 & \dots & \alpha^2 \\ \cdot & \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \alpha^{q-2} & \alpha^{q-2} & \alpha^{q-2} & \dots & \alpha^{q-2} \end{matrix}}_{N} \end{bmatrix}$$

2. FOR $K=2$

Let $B = B(0) = [B^{(0)}\, B^{(1)} \dots B^{(N-1)}]$

is optimal test for $N \leq q-1$

with $T_2 = q^2$ Backgrounds

Denote

$$B(1) = [\, B^{(1)}\, B^{(2)} \dots B^{(N-1)}\, B^{(0)} \,],$$

$$B(2) = [\, B^{(2)}\, B^{(3)} \dots B^{(0)}\, B^{(1)} \,],$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$B(N-1) = [\, B^{(N-1)}\, B^{(0)} \dots B^{(N-3)}\, B^{(N-2)} \,],$$

$$B(i) = (\text{Shift})^{i}\, B(0).$$

$$C(i) = [\, \underbrace{B^{(i)}\, B^{(i)} \dots B^{(i)}}_{N} \,].$$

We will construct K-pseudo-exhaustive test for N^2 using the K-pseudoexhaustive test for N ($N \le q-1$, $K=2$):

$$\begin{bmatrix} C(0) & C(1) & C(2) & \dots & C(N-1) \\ \underbrace{B(0) & B(0) & B(0) & \dots & B(0)}_{N^2} \end{bmatrix}$$

For this test $T = T_2 = 2q^2 = 2^{2w+1}$

<u>Example</u> (Cont'd) $w = 2$ $(q = 4)$, $K = 2$

$N = 9$

$B(0) = \left[B^{(0)}\ B^{(1)}\ B^{(2)} \right]$ (see page 10)

FOR $N = 9$ we have

$$\left[\begin{array}{ccc|ccc|ccc} B^{(0)} & B^{(0)} & B^{(0)} & B^{(1)} & B^{(1)} & B^{(1)} & B^{(2)} & B^{(2)} & B^{(2)} \\ B^{(0)} & B^{(1)} & B^{(2)} & B^{(0)} & B^{(1)} & B^{(2)} & B^{(0)} & B^{(1)} & B^{(2)} \end{array}\right]$$

$T_2 = 32.$ $\square$

Thus transition from a 2-pseudo-exhaustive test for N $(N \leq q-1)$ to a 2-pseudoexhaustive test for N^2 results in multiplication of the number of backgrounds by 2

This construction may be repeated many times

we can start with $N = q-1$ and $T_2 = q^2$

Then for test with $N = (q-1)^2$ we have $T_2 = 2 \cdot q^2$

for test with $N = (q-1)^4$ we have $T_2 = 4 \cdot q^2$

and for $N = (q-1)^{2^i}$ $T_2 = 2^i q^2$

3) $K=3$ $B(0)$ is the 3-pseudo-exhaustive test for N $(N \leq q-1)$.

3-pseudoexhastive test for N^2 can be constructed as:

~~[illegible]~~

$$\begin{bmatrix} C(0) & C(1) & \dots & C(N-1) \\ B(0) & B(0) & \dots & B(0) \\ B(0) & B(1) & \dots & B(N-1) \\ B(N-1) & B(N-2) & \dots & B(0) \end{bmatrix}$$

for $N \geq 4$. ($B(i)$ and $C(i)$ defined on page 19)

Thus for $N = q-1$ we have ($q = 2^w$)

for 3-pseudoexhaustive test $T = T_3 = q^3$

for $N = (q-1)^2$ $T_3 = 4 \cdot q^3 = 2^{3w+2}$

for $N = (q-1)^4$ $T_3 = 16 q^3 = 2^{3w+4}$

. . . .

for $N = (q-1)^{2^i}$ $T_3 = 2^{3w+2i}$