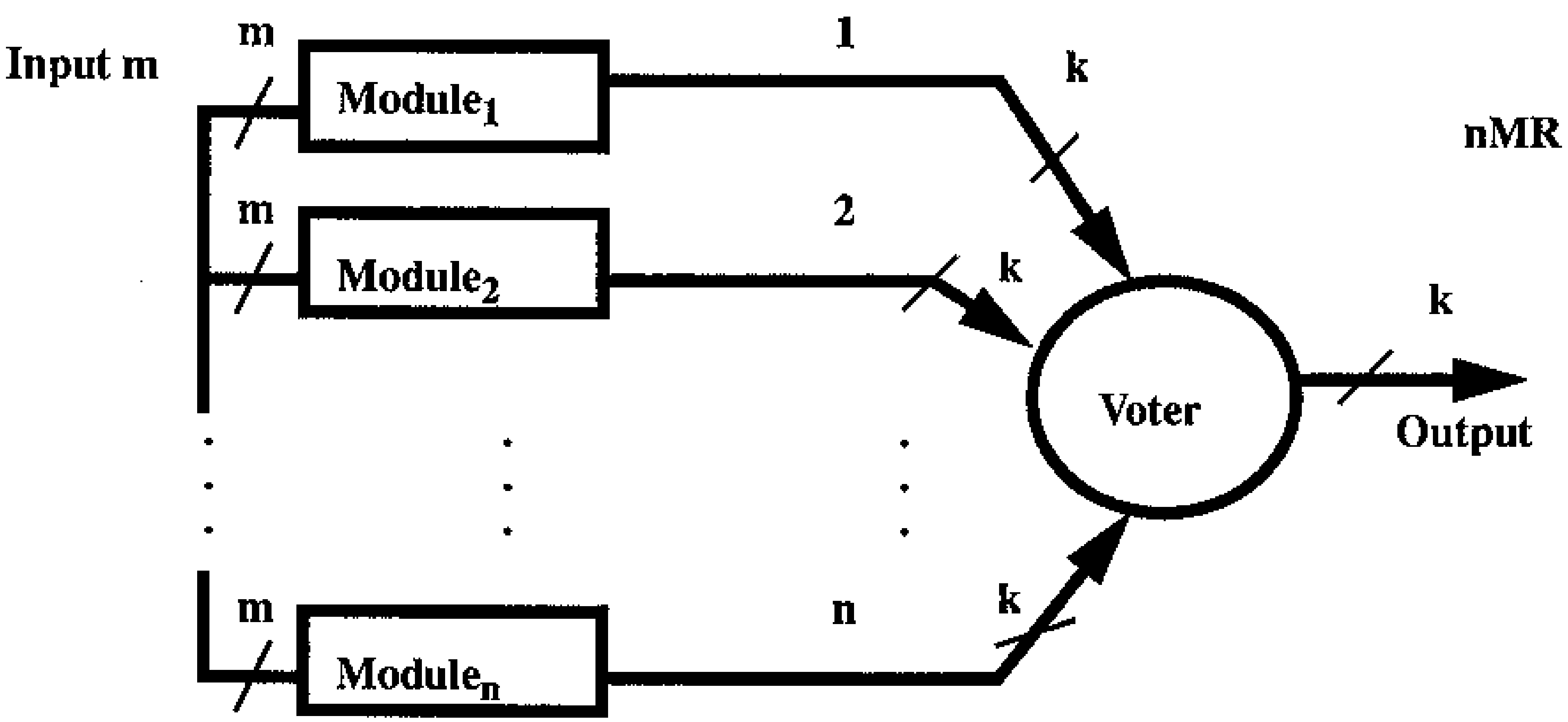
Self - Error Detection / Correction by Replication. (On - line Testing).

* For *detection* of intermittent and stuck - at faults and for *correction* of *independent* errors.



For nMR

$n = 2s + 1$

Detects 2s faults

Corrects s faults

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SC753

Lecture 2

Widely used : $n = 2$ (duplication)
$n = 3$ (triplication)
$n = 4$, $n = 5$ (military space applications)

If p - probability of a fault - free copy (Voter is *fault - free*),

Reliability :

$$R = p^3 + 3p^2(1-p) = 3p^2 - 2p^3 \qquad \text{for } n = 3$$

no faults   *one fault*

Example : $p = .1$

$R = .028$

FOR SYMMETRICAL ERRORS

For nMR ($n = 2s + 1$), $R = p^n + \binom{n}{1} p^{n-1}(1-p) + \ldots + \binom{n}{s} p^{s+1}(1-p)^s$

ESTIMATIONS FOR BINOMIAL SUMS

$$A=\sum_{i=0}^{s}\binom{2s+1}{i}p^{i}(1-p)^{2s+1-i}$$

Let $j=2s+1-i$. Then

$$A=\sum_{j=s+1}^{2s+1}\binom{2s+1}{j}p^{2s+1-j}(1-p)^{j}$$

Denote $\mu=\frac{s}{2s+1}$, $\lambda=\frac{s+1}{2s+1}$ ($\lambda,\mu\simeq 0.5$ FOR LARGE s)

If $\lambda>1-p$, Then

$$\binom{2s+1}{s+1}(1-p)^{s+1}p^{s}<A<\lambda p\,(\lambda-1+p)^{-1}\binom{2s+1}{s+1}(1-p)^{s+1}p^{s}$$

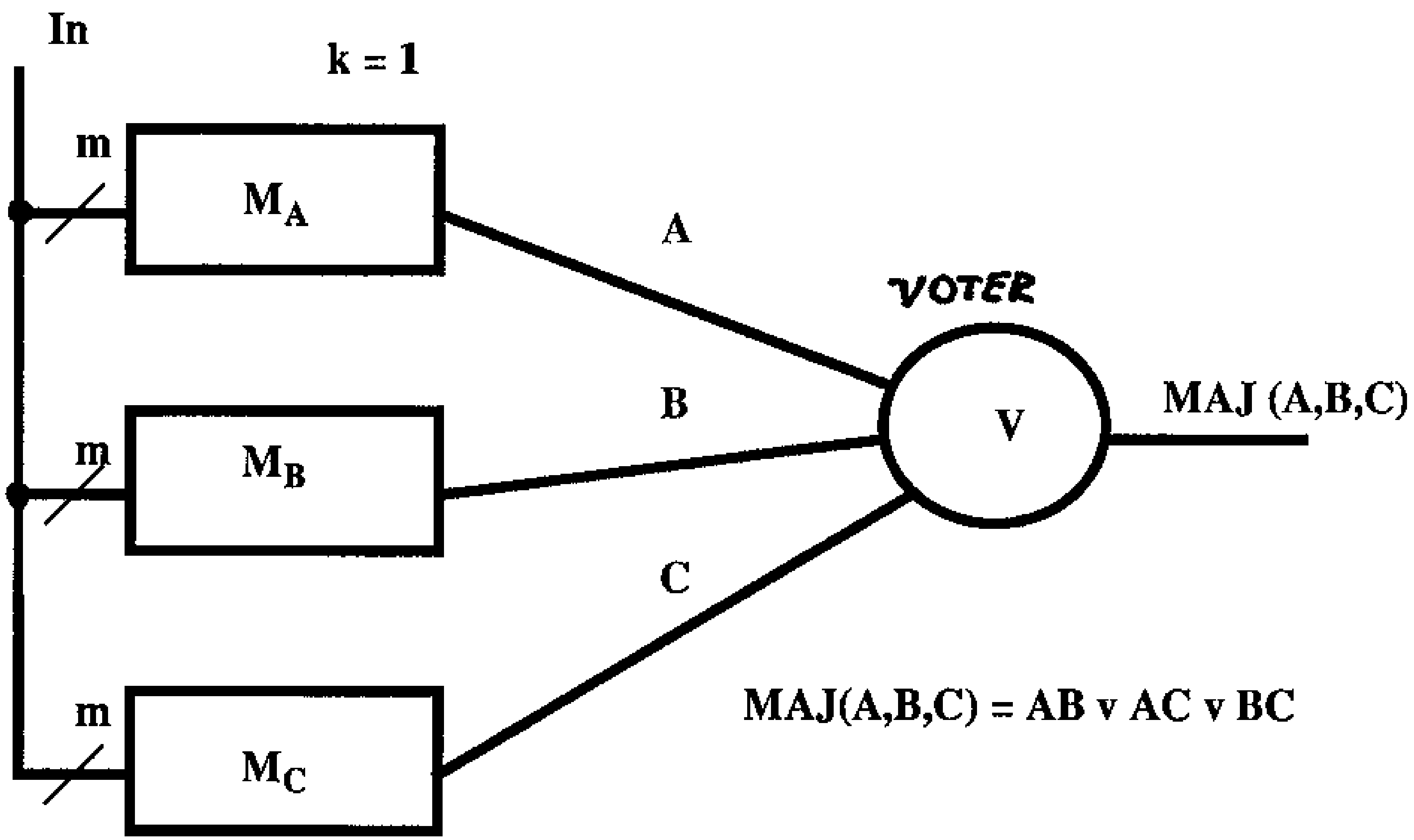
where

$$\frac{\lambda^{-(s+1)}\mu^{-s}\exp\left(-\frac{1}{12(s+1)}\right)}{\sqrt{2\pi(2s+1)\lambda\mu}}<\binom{2s+1}{s+1}<\frac{1}{\sqrt{2\pi(2s+1)\lambda\mu}}\lambda^{-(s+1)}\mu^{-s},$$

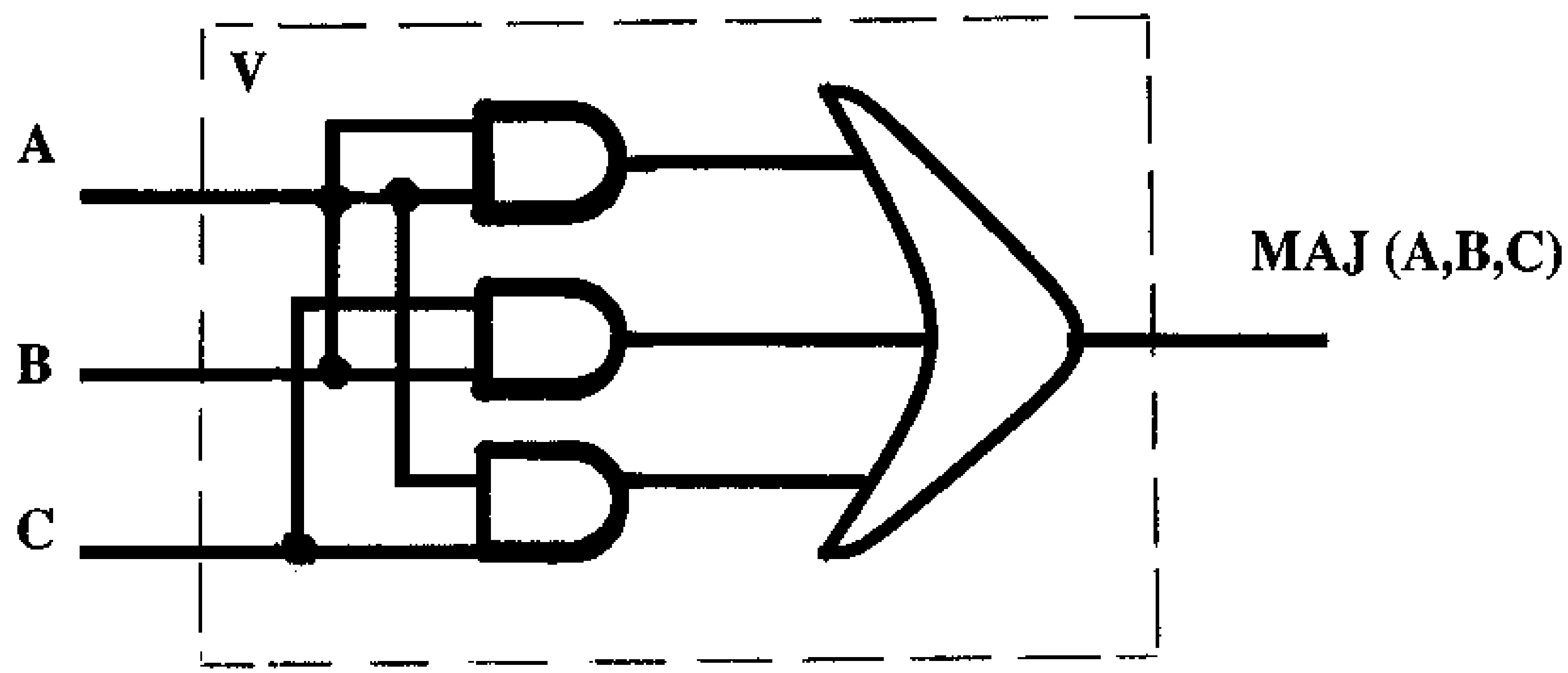
where $\pi=3.14\ldots$.

For $s\to\infty$ ($\lambda=0.5$, $\mu=0.5$): $\binom{2s+1}{s+1}\simeq\frac{2^{2s+2}}{\sqrt{2\pi(2s+1)}}$

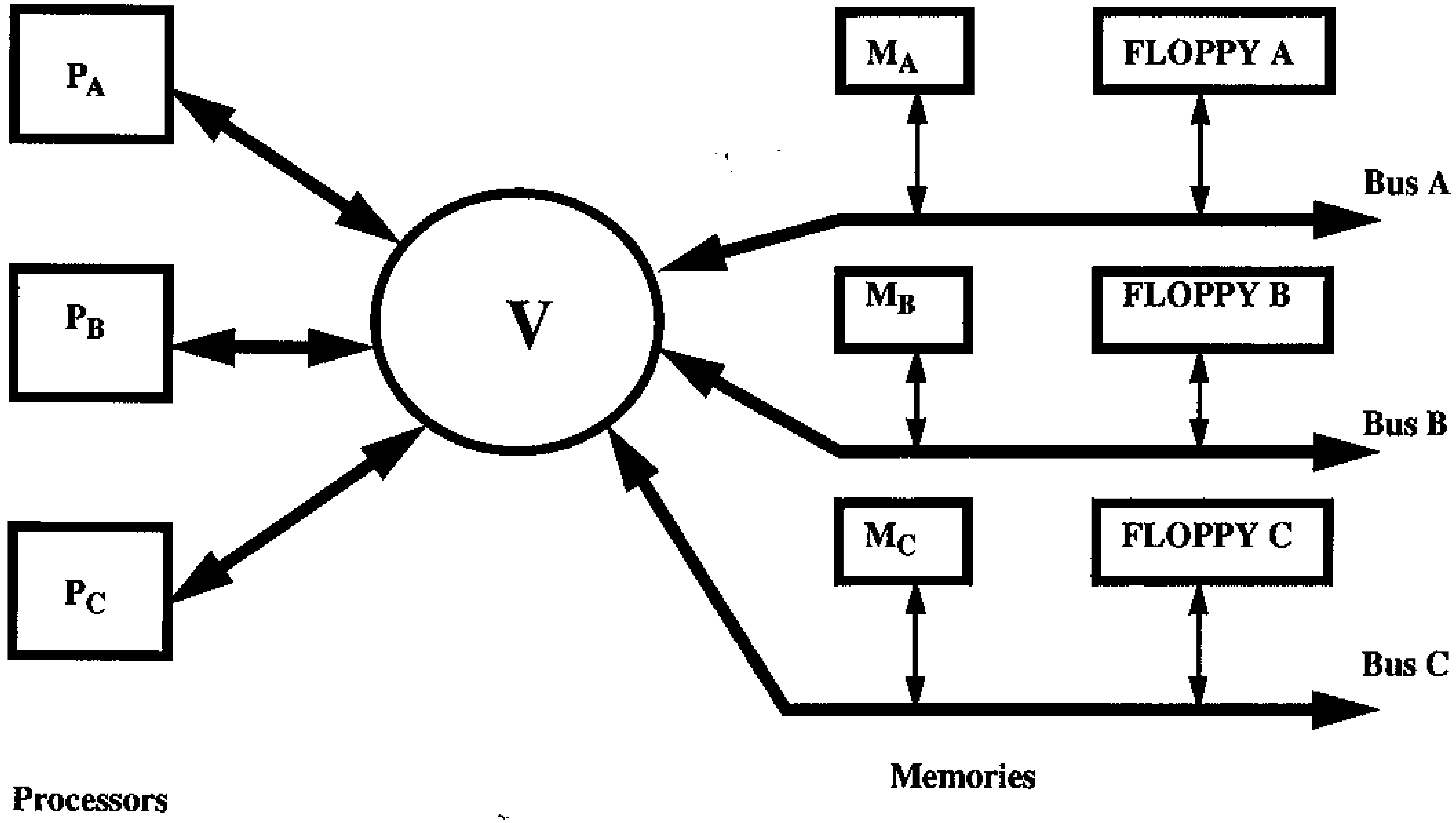
Implementation of Majority Voters.



MAJ(A,B,C) = AB v AC v BC



Example : *C.vmp (Computer voted multiprocessor)*



Basic Structure of C.vmp with TMR.

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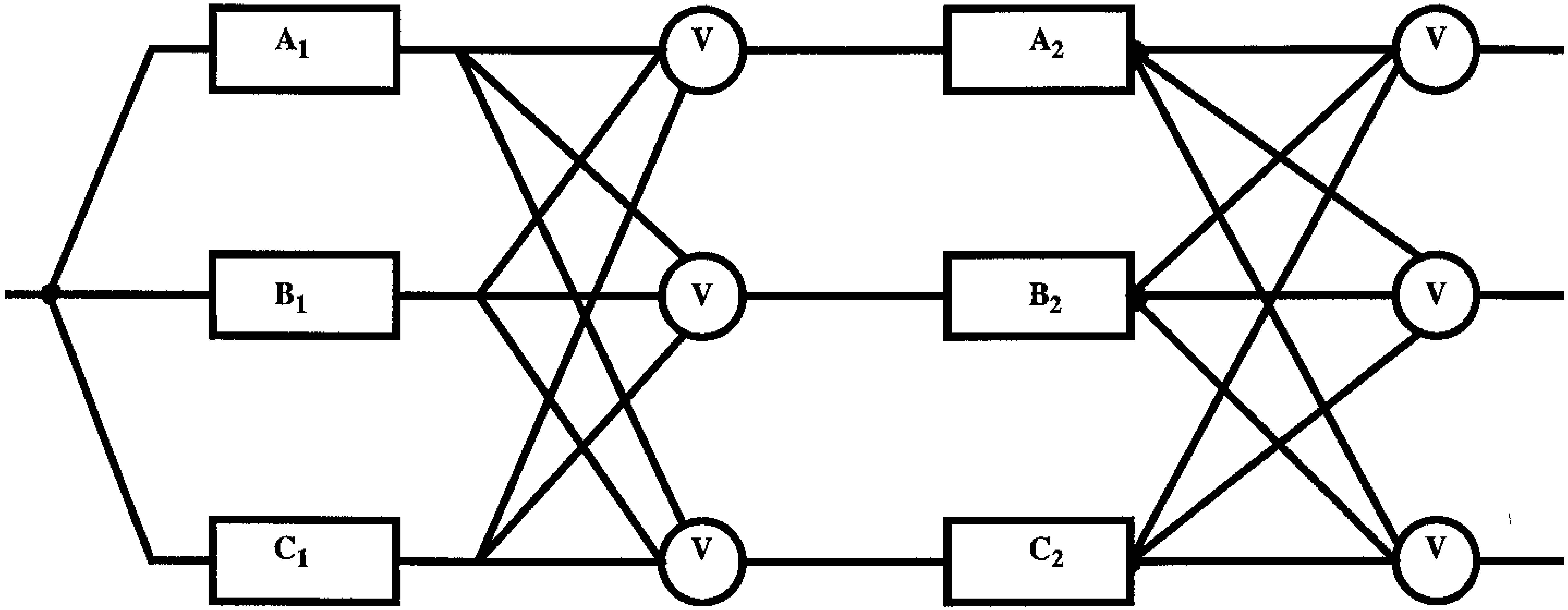
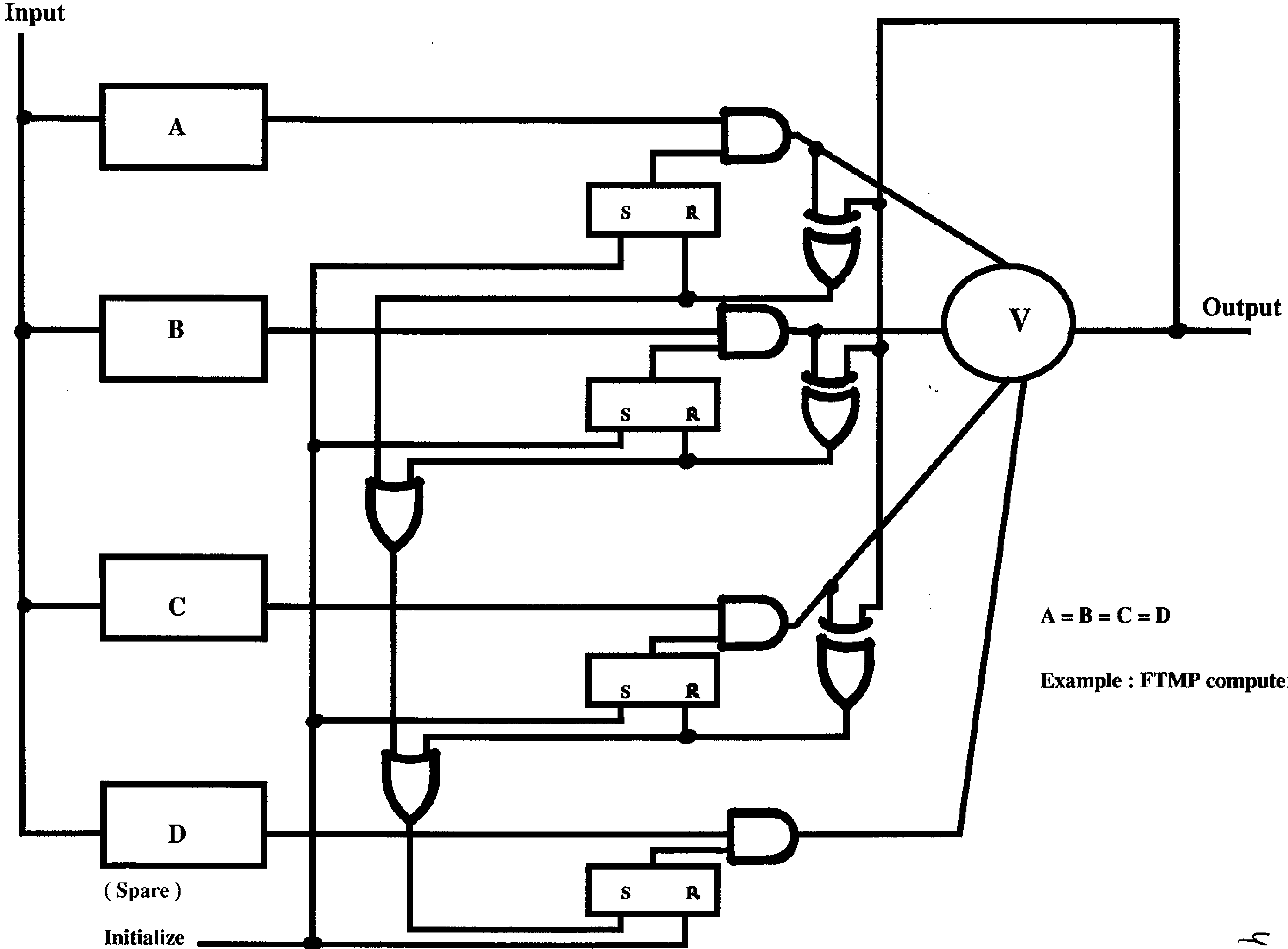
TMR with triplicated majority voting.



For the fault - free case : $A_1 = B_1 = C_1$, $A_2 = B_2 = C_2$

TMR with Reconfiguration.



A = B = C = D

Example : FTMP computer

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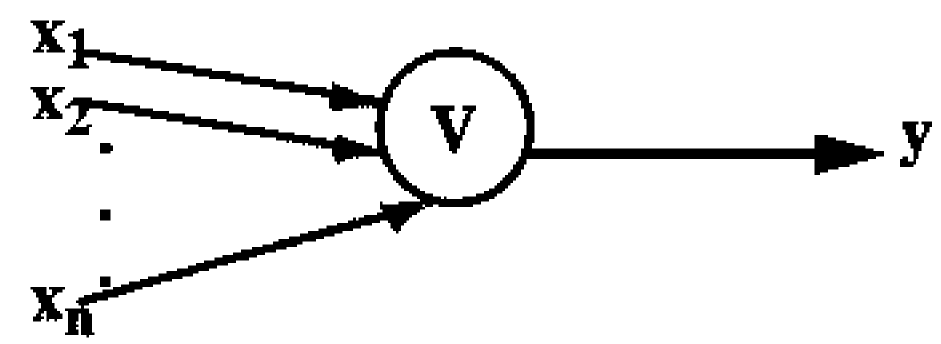
Adaptive Voting.

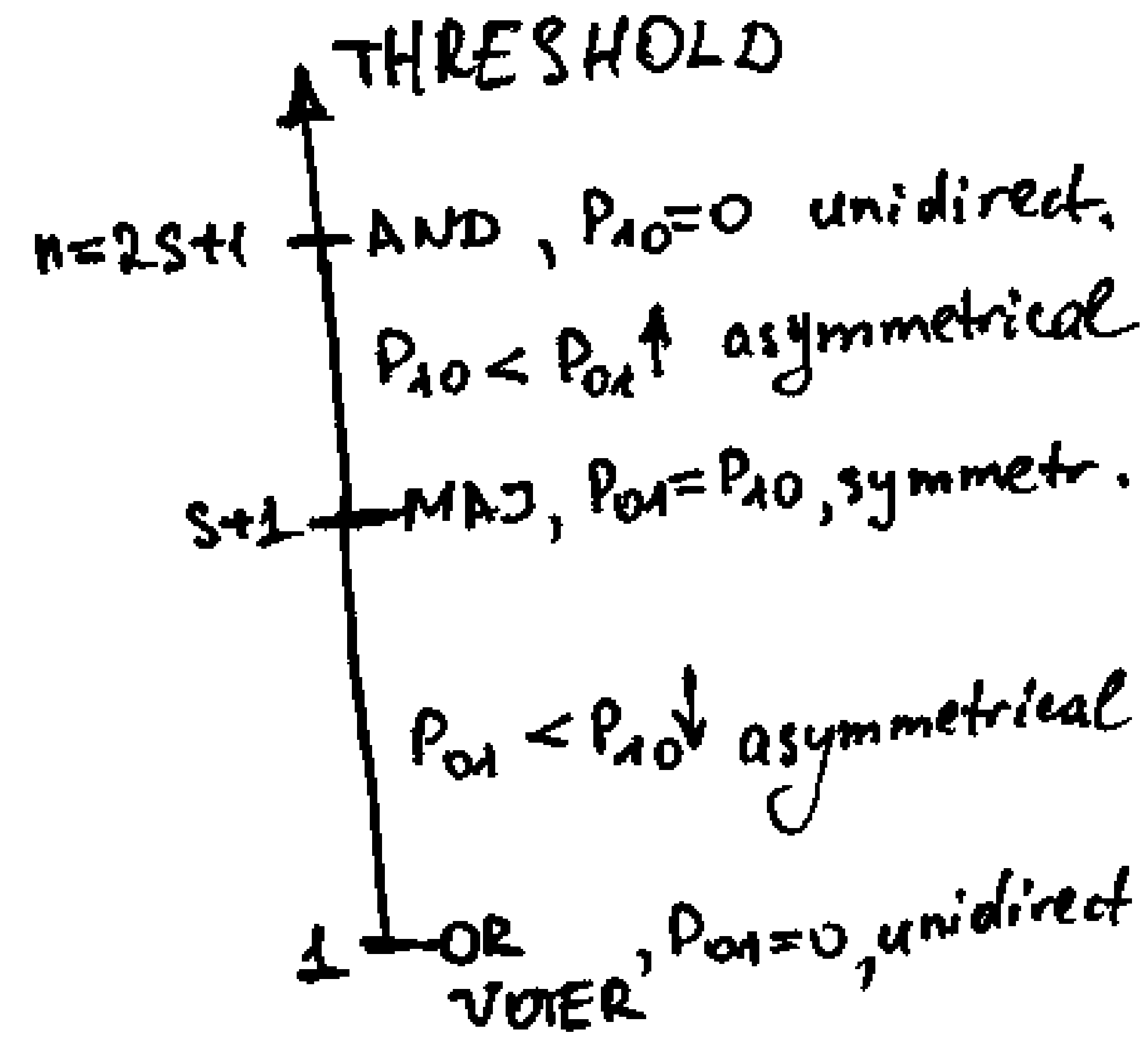
Voter = Treshold Element

$$(y = 1) \Leftrightarrow \sum_{i=1}^{n} x_i \geq T$$



T is a *threshold* , $T = T(p_{01}, p_{10})$

$n = 2s + 1$



p_{01} - probability of 0 ---> 1 errors

p_{10} - probability of 1 ---> 0 errors

1. $p_{01} = p_{10}$ (*symmetrical errors*) ---> T = s + 1 ---> majority voting.

2. *Undirectional errors* : $p_{01} = 0$ ---> T = 1, V = OR

   correct n-1 errors $\quad p_{10} = 0$ ---> T = n, V = AND

3. *Asymmetrical errors* : $p_{01} > p_{10}$ ---> T > n/2

   $p_{10} > p_{01}$ ---> T < n/2

For undirectional errors n - 1 errors corrected by nMR.

D. P. Sewiorek, R. S. Swarz, " The Theory and Practice of Reliable System Design", Digital Press, 1982, Ch.3.

Self - Purging Redundancy.

Combination of reconfiguration and adaptive voting

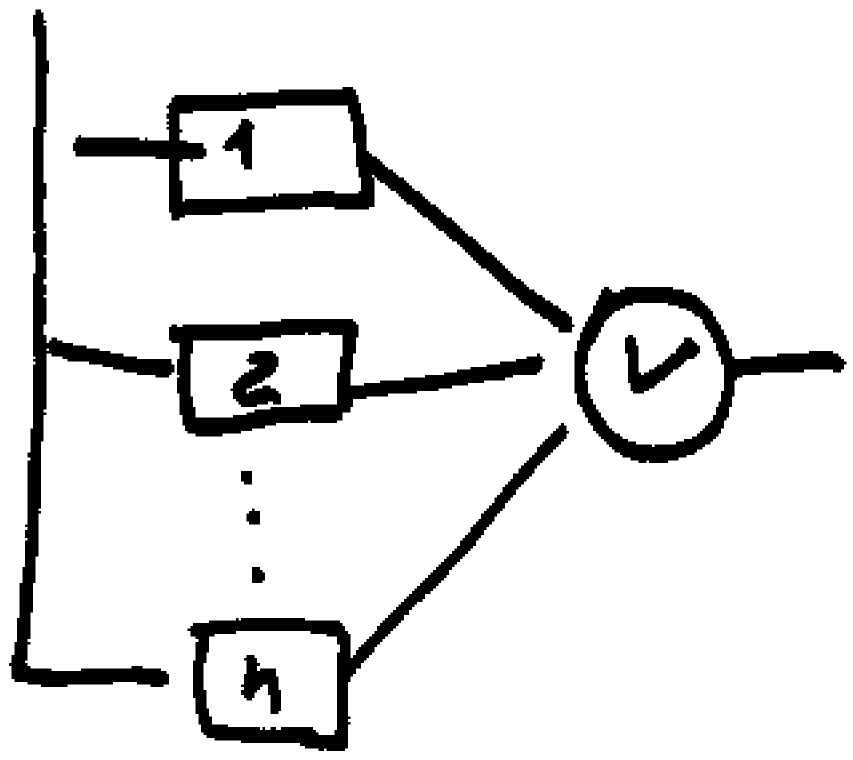
1. Locate a faulty module (comparing voter outputs and outputs of modules).

2. Switch out a faulty module.

3. Modify the threshold of the voter
   (e.g. after switching out 2 modules decrease the threshold by 1).

Graceful degradation of performance.
(J. Losq, IEEETC, C-25, June 1976, pp 569 - 578).

DEPTH OF REPLICATION



p - prob of no error in a copy

P_V - prob of no error in the voter

$$n = 2s+1$$

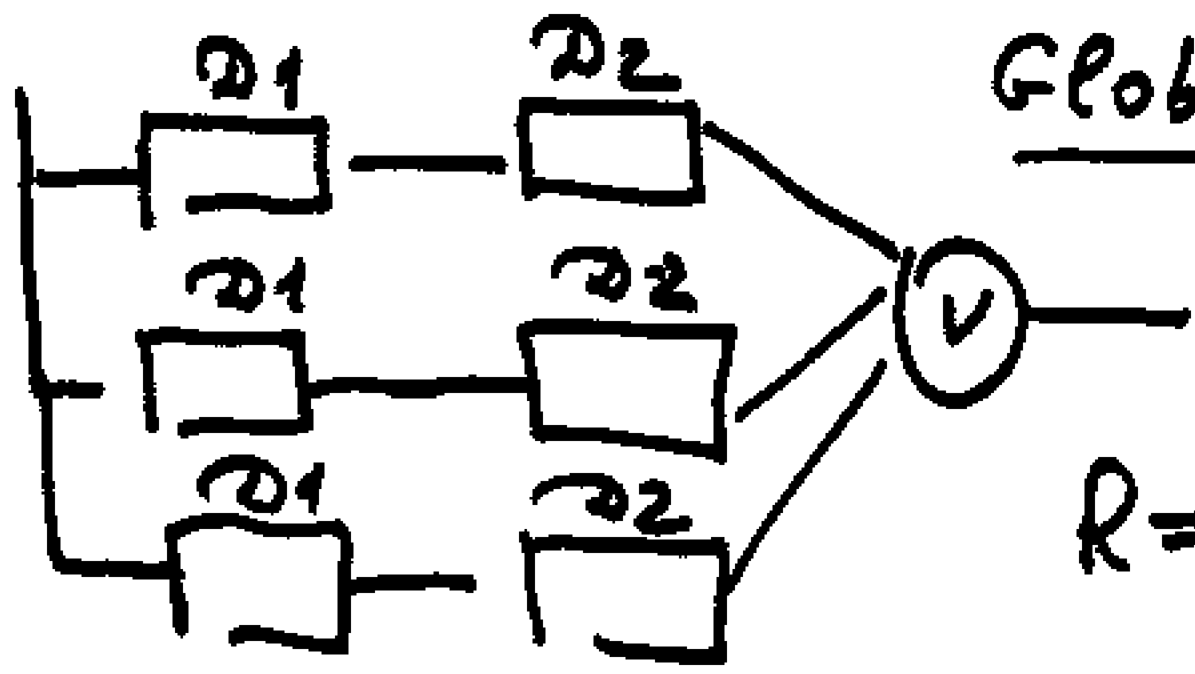
For $n=3$

$$R = P_V \sum_{i=0}^{s} \binom{2s+1}{i} p^{2s+1-i} (1-p)^i \; ; \quad R = P_V(-2p^3 + 3p^2)$$

Let the original system be

P_i - prob. of no error in D_i

1)

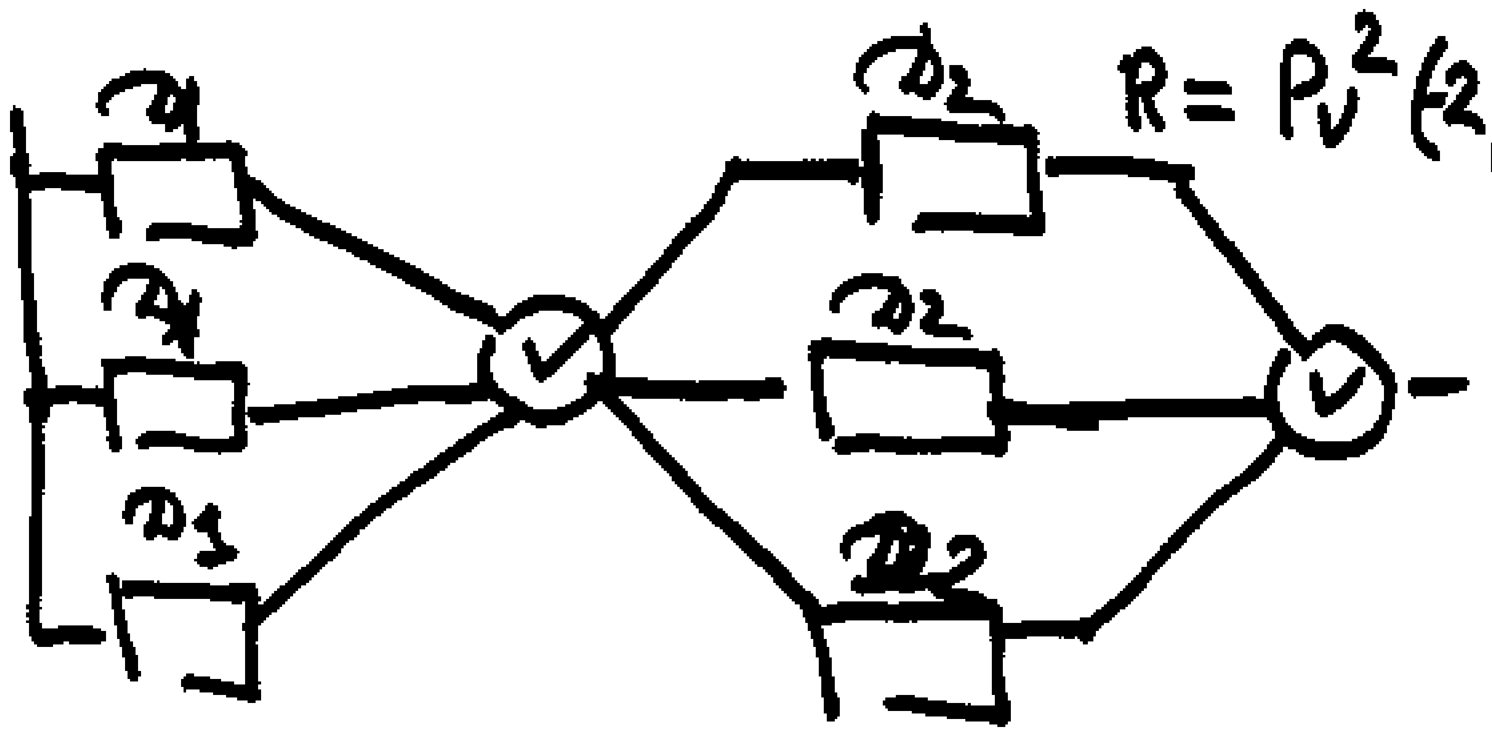


Global replication $n=3$

TMR

$$R = P_V(-2\, p_1^3 \cdot p_2^3 + 3\, p_1^2 \cdot p_2^2)$$

2) Local



$$R = P_V^2(-2p_1^3 + 3p_1^2)(-2p_2^3 + 3p_2^2)$$

Computer Systems with Replications and Voting.

C.vmp (Computer Voted processor)	TMR (voting on bus level)
JPL - STAR TARP (Test and Repair Processor)	TMR (reconfiguration, Arithmatic codes (mod 15))
FTMP (Fault Tolerant processor)	TMR (voting at outputs)
FTSC (Fault Tolerant Spaceborn Computer)	TMR for control unit
SIFT computer(Software Implemented Fault Tolerance)	Software voting
Sperry / Univac 1100 / 60	Duplication
ESS - 1 CC , ESS - 2	Duplication
Space Shuttle Computer	5MR
Sequoia multiprocessor	Every processor, memory, I/o duplicated
Saturn V	Triplication and Voting
Orbiting Astronomical Observatory (OAO)	4MR.