1 SC753

TESTING AND DIAGNOSIS OF COMPUTER / COMMUNICATION NETWORKS AND MULTIPROCESSORS

NODE FAULTS
LINK FAULTS
ROUTER FAULTS

} SINGLE AND MULTIPLE

COMBINING SELF-TEST (BIST) FOR NODES AND system test

when a node tests its neighbours

NODES FOR SELF-TEST ⇒ <u>monitors</u>
(testers)

FIRST MONITORS TEST THEMSELVES

THEN MONITORS TEST THEIR
NEIBBOURS

<u>MONITOR PLACEMENT PROBLEM</u>

PLACE A <u>MINIMAL</u> NUMBER OF
MONITORS SUCH THAT BALLS
OF RADIUS one cover all nodes
(LINKS, routers)

MINIMIZATION OF A NUMBER of monitors M result in a decrease in the traffic required for testing and/or diagnosis and in the space required for storing test generation SOFTWARE.

Testing and Diagnosis FOR Node faults

let M - min # number of monitors

D - diameter of the system

N - number of nodes

d_i - number of node neighbouring node i.

Lower bounds

T1

1) $M \geq D/3$

2) $M \geq K$ where

K is a min number such that

$$\sum_{i=1}^{K} (d_i + 1) \geq N$$

$(d_1 \geq d_2 \geq \ldots \geq d_N)$

3) $d_1 = d_2 = \ldots = d_N$

$$M \geq \frac{N}{d+1}$$

FOR SINGLE NODE FAULTS MONITORS are codewords of a code with covering radius 1.

Ex. For d-dimensional cube

if $d = 2^i - 1$ $\quad (N = 2^d)$

$$M = \frac{2^d}{d+1} = 2^{2^i - i - 1}$$

monitors are codewords of the Hamming code of length d.

Example $d = 7$ $(i = 3)$

Monitors are nodes with coordinates which are linear combinations mod 2 of the generating matrix G_3 of (7, 16, 3) HAMMING CODE

$$G_3 = \begin{bmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&1&1&0 \end{bmatrix}.$$

Number of Monitors for Hypercubes (upper bounds)

d	M	
3	2	EXACT VALUES
4	4	
5	7	
6	12	
7	16	
8	32	
9	62	UPPER BOUNDS
10	120	

$\frac{M}{N} \to 0$ as $d \to \infty$.

T2 1) FOR p-ary cubes ($N = p^d$)

(p - prime)

if $d = (p^i - 1)/2$ then

$$M = \frac{N}{2d+1} = p^{d-i}$$

2) FOR HEXAGONAL MESHES

$$M = \lceil N/4 \rceil$$

3) FOR TRIANGULAR MESHES

$$M = \lceil N/7 \rceil$$

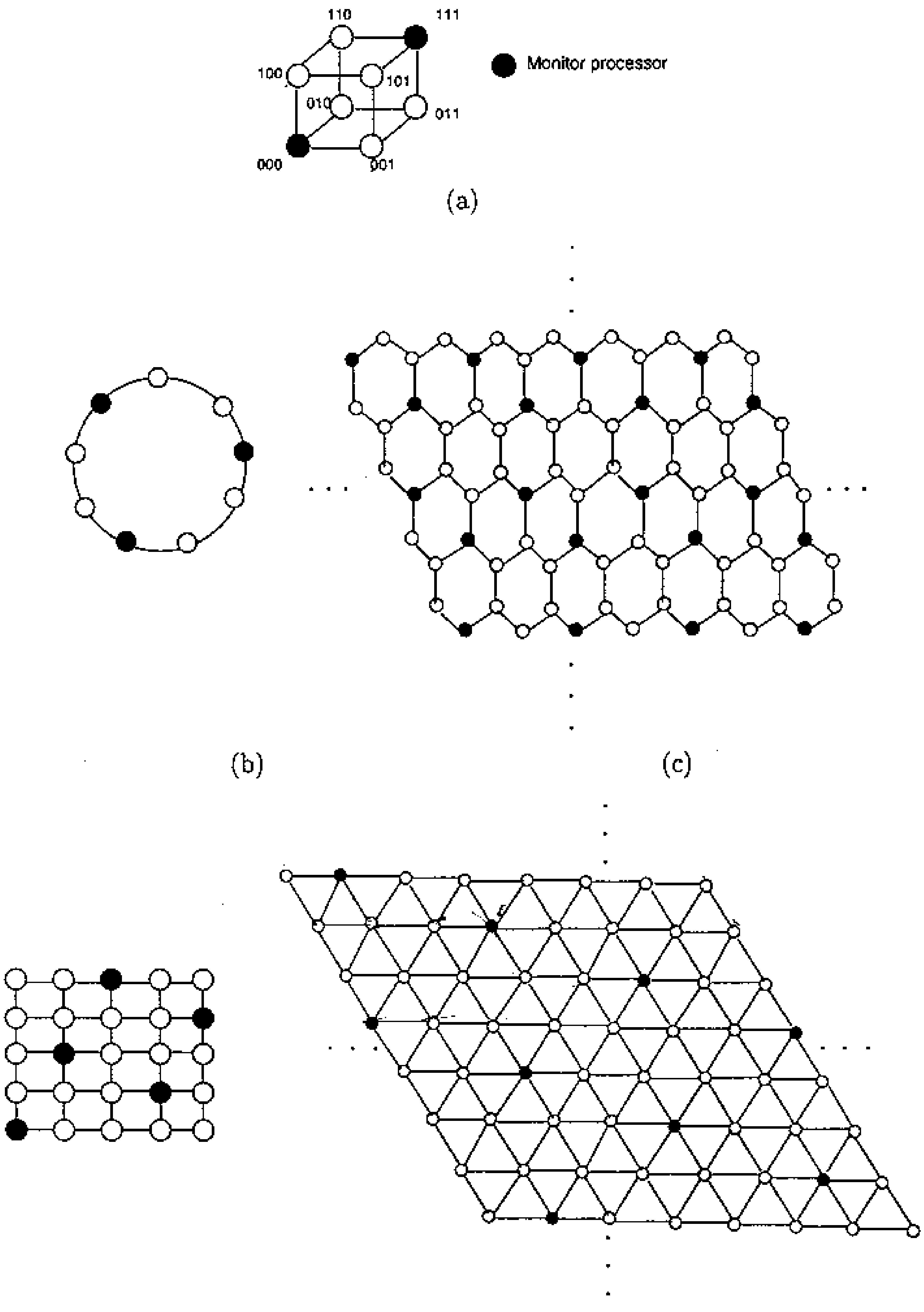


Figure 3 Perfect processor cover for (a) 3-dimensional cube, (b) ring, (c) hexagonal mesh, (d) 2-dimensional mesh with 25 processors, (e) triangular mesh.

Multiple Node FAULTS

Multiple fault is not diagnosable iff it includes at least one monitor and a neighboring nonmonitor nodes

Fraction of faults involving $\ell \geq 2$ nodes that are diagnosable

$$C(\ell) \geq 1 - M \cdot d \binom{N-2}{\ell-2} \binom{N}{\ell}^{-1}$$

Example: for $d=15$ hypercube

$N = 2^{15}$ $\quad C(\ell) \geq 95\%$

for all $\ell \leq 30$

LOCATION OF LINK FAULTS

FAULT ISOLATION

Monitor Placement Problem:

PLACE A MINIMAL NUMBER OF MONTORS SUCH THAT EVERY LINK IS CONTAINED IN AT LEAST ONE BALL OF Radius one.

T1 $$M \geq \lceil N/2 \rceil .$$

CHECKERBOARD PLACEMENT

$$M = \lceil N/2 \rceil .$$

T2. FOR CHECKERBOARD PLACEMENT ALL NODE AND LINK FAULTS WITH ANY MULTIPLICITY CAN BE LOCATED

Karpovsky, Chakrabarty, Levitin

"FAULT ISOLATION AND DIAGNOSIS IN MULTIPROCESSORS...", FAULT TOLERANT PARALLEL AND DISTRIBUTED SYSTEMS, KLUWER ACADEMIC PUBLISHERS 1998

pp 285-301

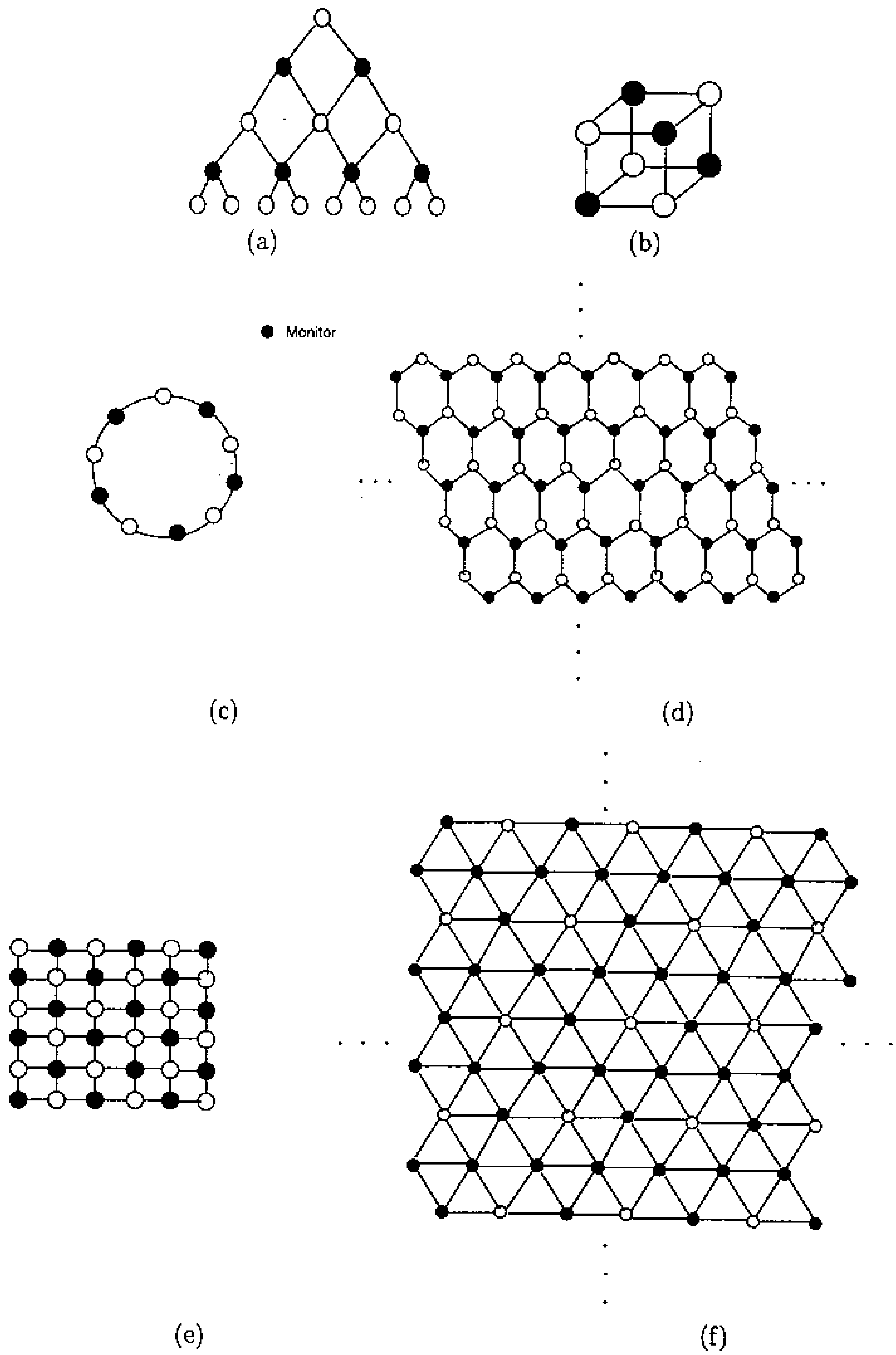


Figure 1 Monitor placement for link testing in a (a) binary tree, (b) binary 3-dimensional cube, (c) ring, (d) hexagonal mesh, (e) 2-dimensional rectangular mesh, and (f) triangular mesh.

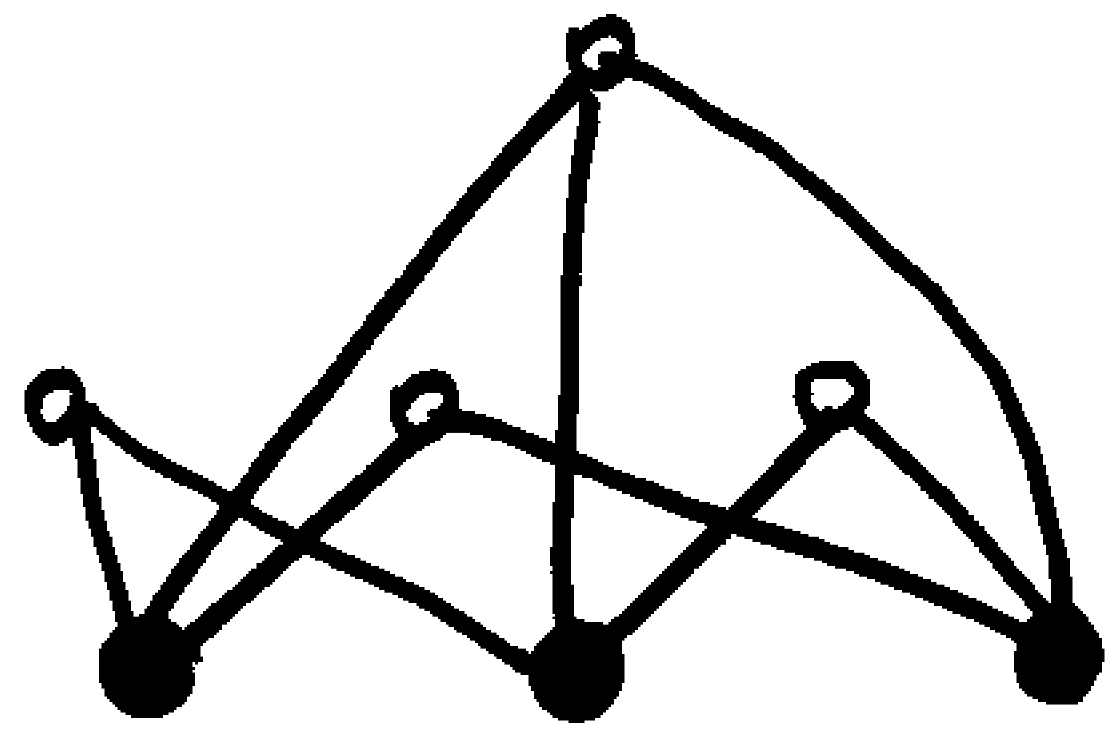
Centralized diagnosis of Node FAULTS

A monitor sends to the host one bit indicating a presence of a fault in the ball centered at this monitor..

Monitor Placement Problem:

Place a minimal number of monitors such that every node will belong to a unique set of balls centered on monitors

Example



● – monitors

Lower bounds:

1) $M \geq \lceil \log_2 (N+1) \rceil$

2) If $d_1 \geq d_2 \geq \dots \geq d_N$

$M \geq K$ where K is a min number such that

$$\sum_{i=1}^{K} h\left(\frac{d_i+1}{N+1}\right) \geq \log_2 (N+1)$$

$$h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$$

entropy

Binary cubes $\mathbb{Z}_2^d$

Let C is an optimal (minimal) code with covering radius 2 (Any point in $\mathbb{Z}_2^d$ at the distance at most 2 from a point in C) then monitors can be selected as neighbors of all points in C

Example $d=5$ $N=32$

$C=\{00000, 11111\}$

Monitors are all vectors with 1 and 4 ones $M=10$

(For $d=5$ this construction is optimal)

Let $K(d,2)$ is a size of a minimal code with covering radius 2. (Ex. $K(5,2)=2$)

Then
$$M \leq d\,K(d,2)$$

3-dim p-ary cubes ($p>4$ p-even)

Monitors are vectors with all 3 components even and all 3 components odd

$$M = p^3/4$$

Example $p=6$ $M=54$

Monitors: 000, 002, 004, 020, …, 444, 111, 113, 115, 131, …, 555

T2 If $N \to \infty$ and
$l \leq o(\sqrt{N})$ then

$$C(l) \to 1$$

Probability of locating of upto $l = \sqrt{N}$ faults converges to 1 for large N

- Karpovsky, Chakrabarty, Levitin "On A New Class of Codes Identifying Vertices in Graphs" IEEE Trans on Inf. Theory March 1998, pp 599-612

$V(4)$ - number of nodes at distance at most 4 from any given node

T1 $\quad c(\ell) \geq \prod_{i=0}^{\ell-1} N^{-1}(N - i\,V(4))$

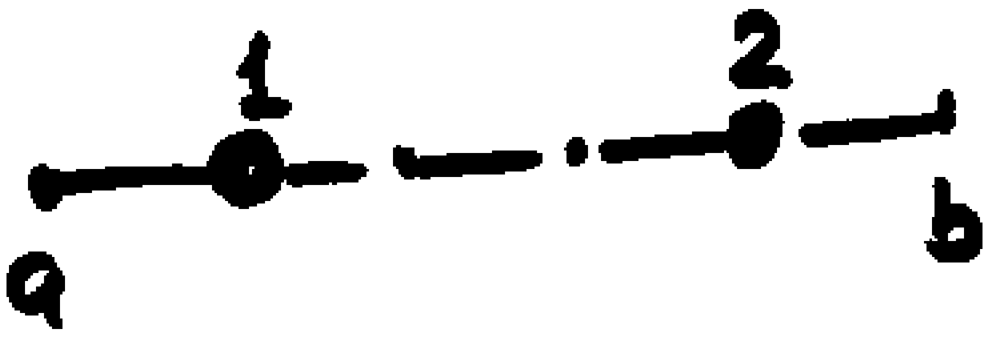
Example For 2-dim p-ary cube $(p \geq 9)$ $\quad V(4) = 40$

For Z_2^d $\quad V(4) = \sum_{i=0}^{4} \binom{d}{i}$

For $d=16$ more than 96% of double faults $(\ell = 2)$ are diagnosable

Location of Multiple Node Faults

Multiple faults are diagnosable if (but not only if) the distance between faulty nodes is at least 5.



$d(a,b)=5$

a - belongs to the ball with center 1

b - belongs to the ball with center 2

$c(\ell)$ - fraction of faults with multiplicity at most ℓ which are diagnosable

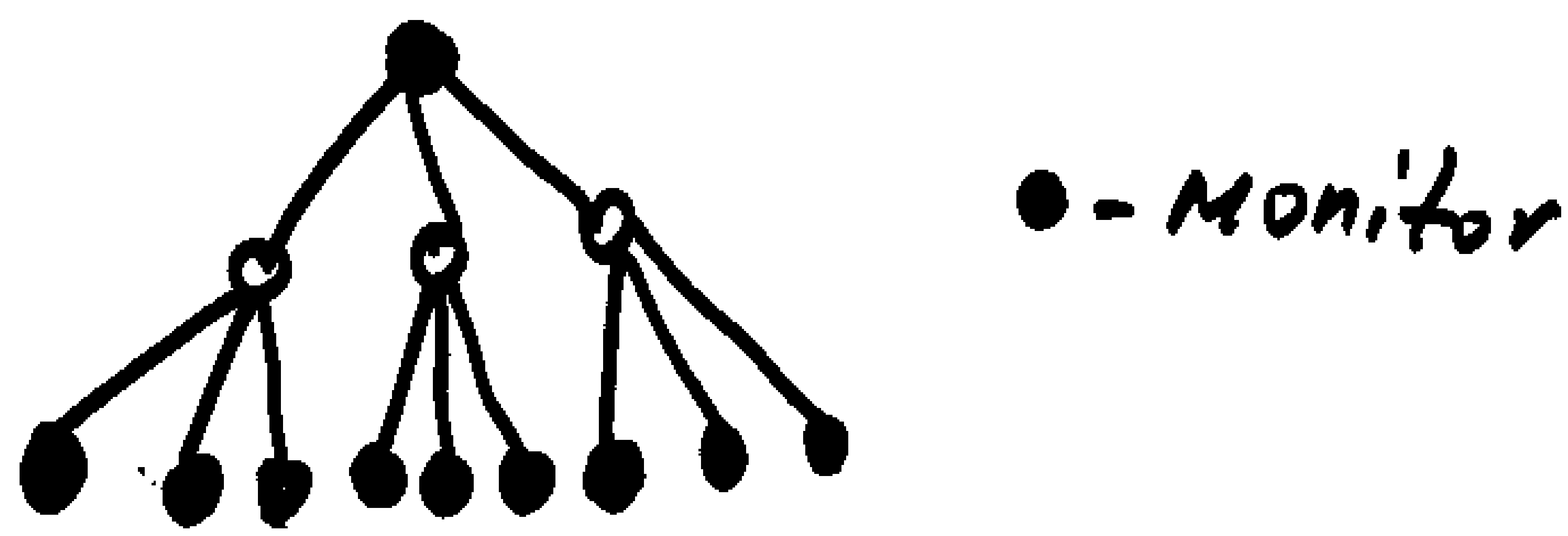
p-ary trees (balanced)

ℓ - number of levels

Monitors are nodes at levels

$\ell, \ell-2, \ell-4, \ldots$

Example $p=3$, $\ell=3$



$M \simeq p^{\ell-1}$ for large p

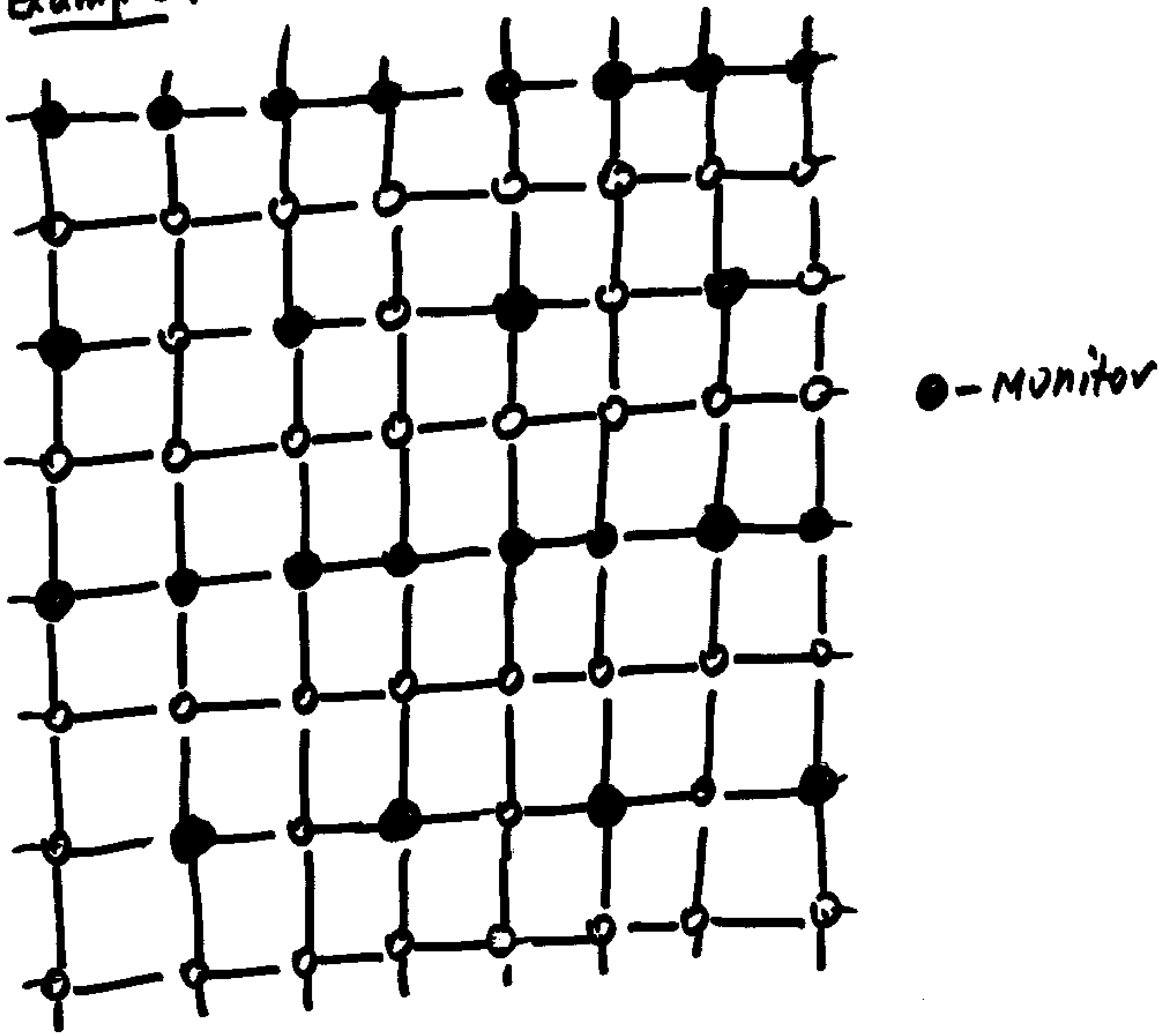
2-dim p-ary cubes (tori)

The problem is still open!

The best construction (for $p = 8s$)

Example:

$p = 8$



$$M = \frac{3}{8} p^2$$