

SC 753

FAULT-TOLERANT COMPUTING

LECTURE NOTES

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Complimentary Text:

"FAULT-TOLERANT COMPUTING"

Edited by S.K. PRADHAN

Related Courses:

SC561 ERROR CONTROL CODES

SC752 TESTING OF COMPUTER
HARDWARE

SC891 RESEARCH SEMINAR.

Fault Models

Fault (failure) is a physical event

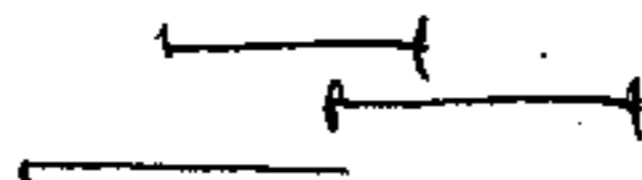
Error is a manifestation of a fault

Logic-level faults vs TIME FAULTS

Fault model is necessary for efficient
test generation AND
TEST OBSERVATION AND DFT

Statistical analysis of faults to const-
ruct fault models.

Fault model depends on implemen-
tations and environments.



Standard Fault Models

1.3

I. Stuck-at faults - a line Z is stuck-at-0 or stuck-at-1 ($Z/0, Z/1$ or $Z\equiv 0, Z\equiv 1$)

Single (SSF) and multiple (MSF) faults

SSF is the most popular model

stuck-at faults are detected by off-line and by on-line testing

Fault coverage - percentage of faults which are detected)

(Example: For SSF and μ processors 95% is a reasonable fault coverage)

Number of stuck-at faults with a multiplicity at most l

$$\sum_{i=1}^l 2^i \binom{L}{i}; \quad \binom{L}{i} = \frac{L!}{i!(L-i)!}; \quad a! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot a$$

L is a number of lines in a network

SYMMETRICAL FAULTS

Example L = 1,000, l = 2

2 · 10⁶ faults

INDEPENDENT ERRORS

Prob of a FAULT WITH MULTIPLICITY l at given l lines is p^l

Special Classes of Stuck-at Faults

∴ Bursts of length b (for discs or tapes)
FOR SOLID BURSTS

Number of bursts: $N-b+1$ N is a number of binary cells

∴ Symmetric, asymmetric and unidirectional stuck-at faults

Number of unidirectional faults with a multiplicity at most l : $\sum_{i=1}^l \binom{L}{i}$ Ex: $L=1,000; l=2 \Rightarrow 500,500$ faults

∴ Input / Output (Terminal or Pin)

stuck-at faults at interconnections between chips

Number of I/O stuck-at faults for a chip with m inputs and k outputs: $\sum_{i=1}^l 2^i \binom{m+k}{i}$; Ex: $m=k=16; l=2 \Rightarrow 2,048$ faults (symmetrical)

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(5)

Fault Models (Cont.)

1.6

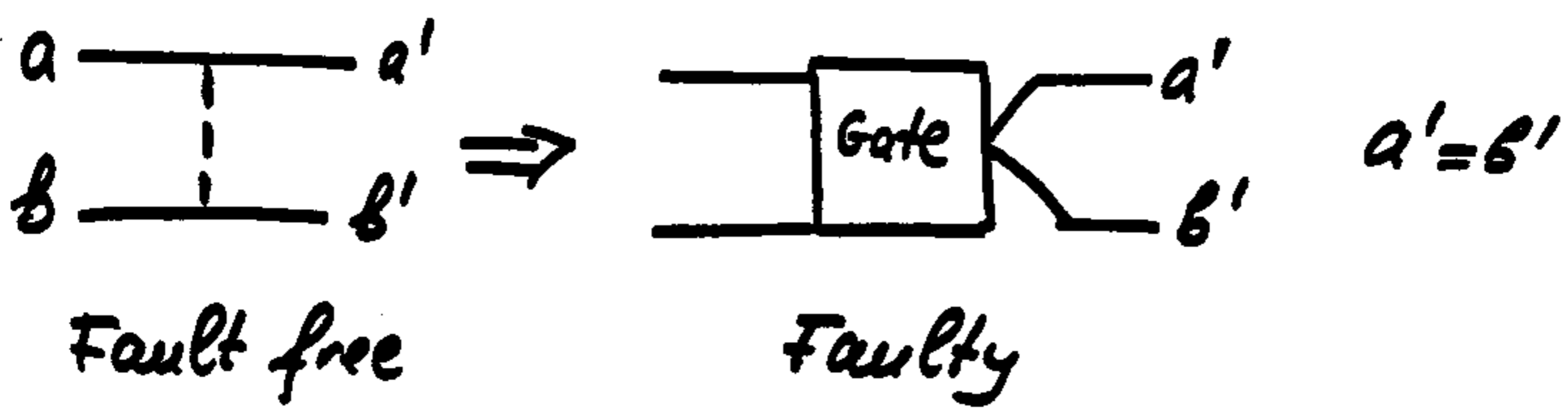
II Intermittent (transient, soft) faults

detected only by on-line tests
(ex. replication of hardware, parity checks, error-correcting codes, roll-backs of a program, etc.)

● Single intermittent faults
(α -particles) vs repeating faults

Up to 80% - 90% of faults
are intermittent (Air Force, IBM)

III Bridgings (short circuits)
 (high density of gates for VLSI)



∴ AND and OR type bridgings

AND: $(a, b)_*$

OR: $(a, b)_+$

Relation between stucks and bridgings:

$a/0 = (a, 0)_*$

$a/1 = (a, 1)_+$

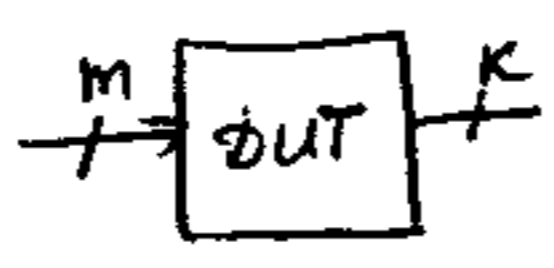
a	b	AND $a' = b'$	OR $a' = b'$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

number of bridgings:

$2 \binom{2}{2} = 2(2-2)$

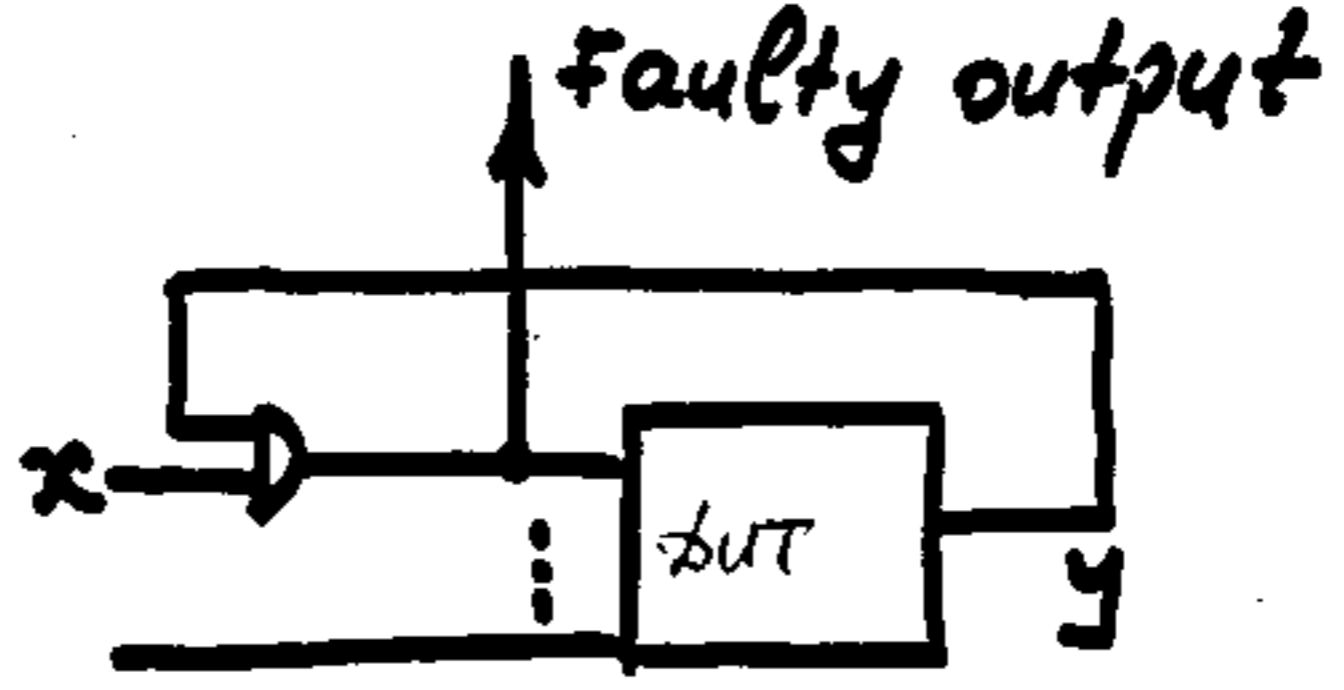
Feedback (FB) and NonFeedback (NFB) 1.8

Bridgings



⇒ m.k FB bridgings between inputs and outputs

FB (x,y)*

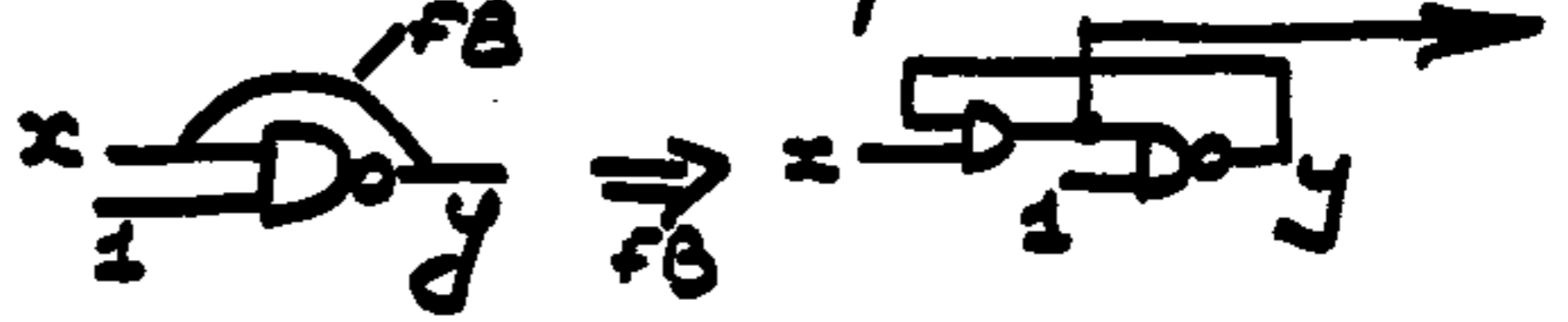


Combinational

FB → Sequential

Oscillation:

(x,y)*

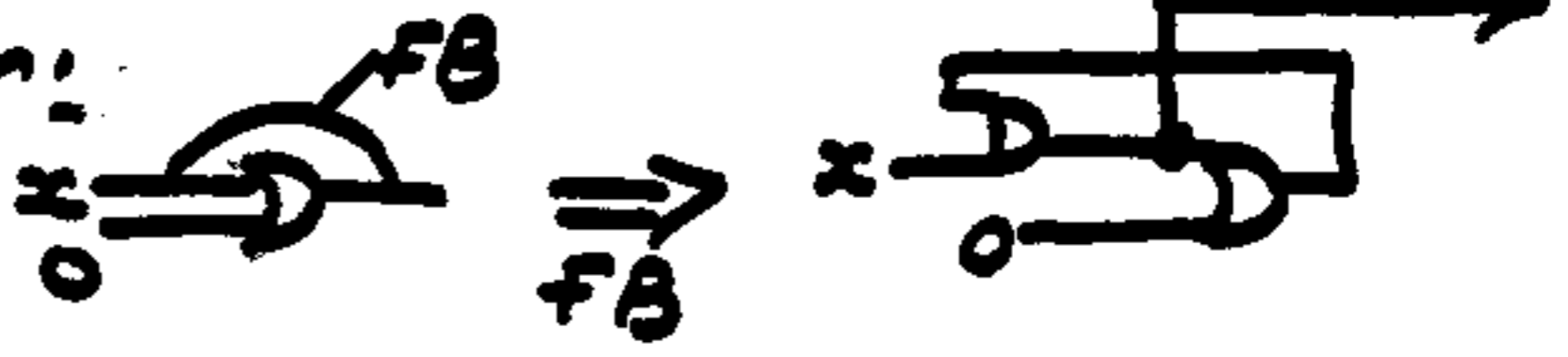


FOR Q=1
x=1:

difficult to observe

Asynchronous Behavior:

(x,y)*



(For any x output stay 0 after x=0 applied)

IV PLA Faults

CROSSPOINT FAULTS

1.10

1. A product term can grow (cover more minterms)



NOTATION: AG.

2. A product term can shrink (cover fewer minterms)



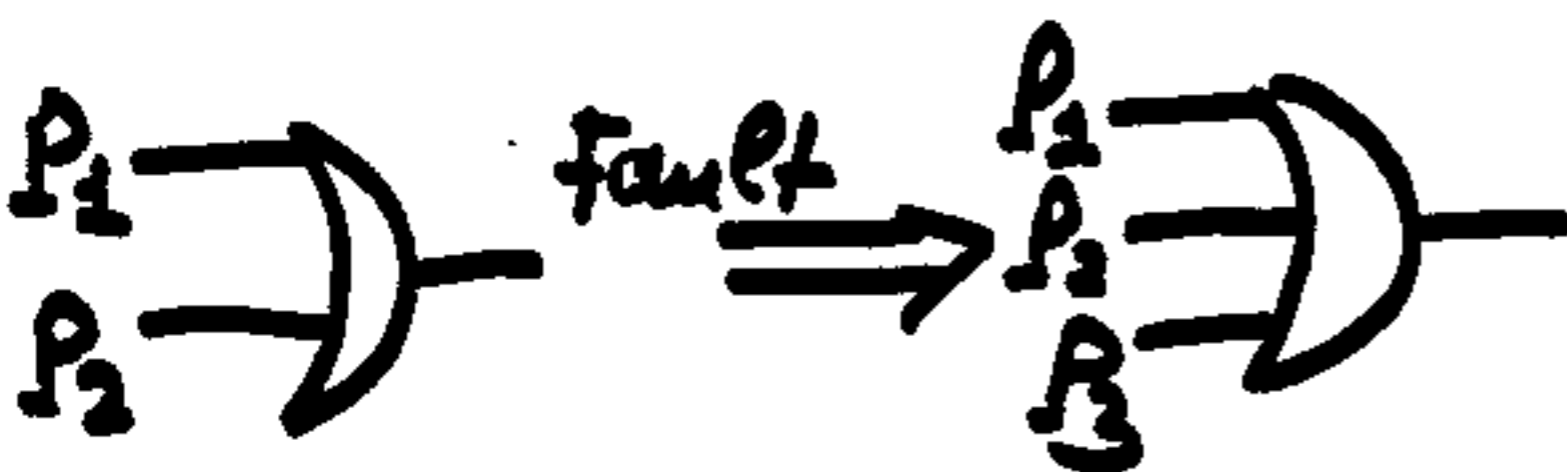
NOTATION: AS.

3. A product term can disappear from a function



NOTATION: OS

4. A product term from one function can appear also in another function

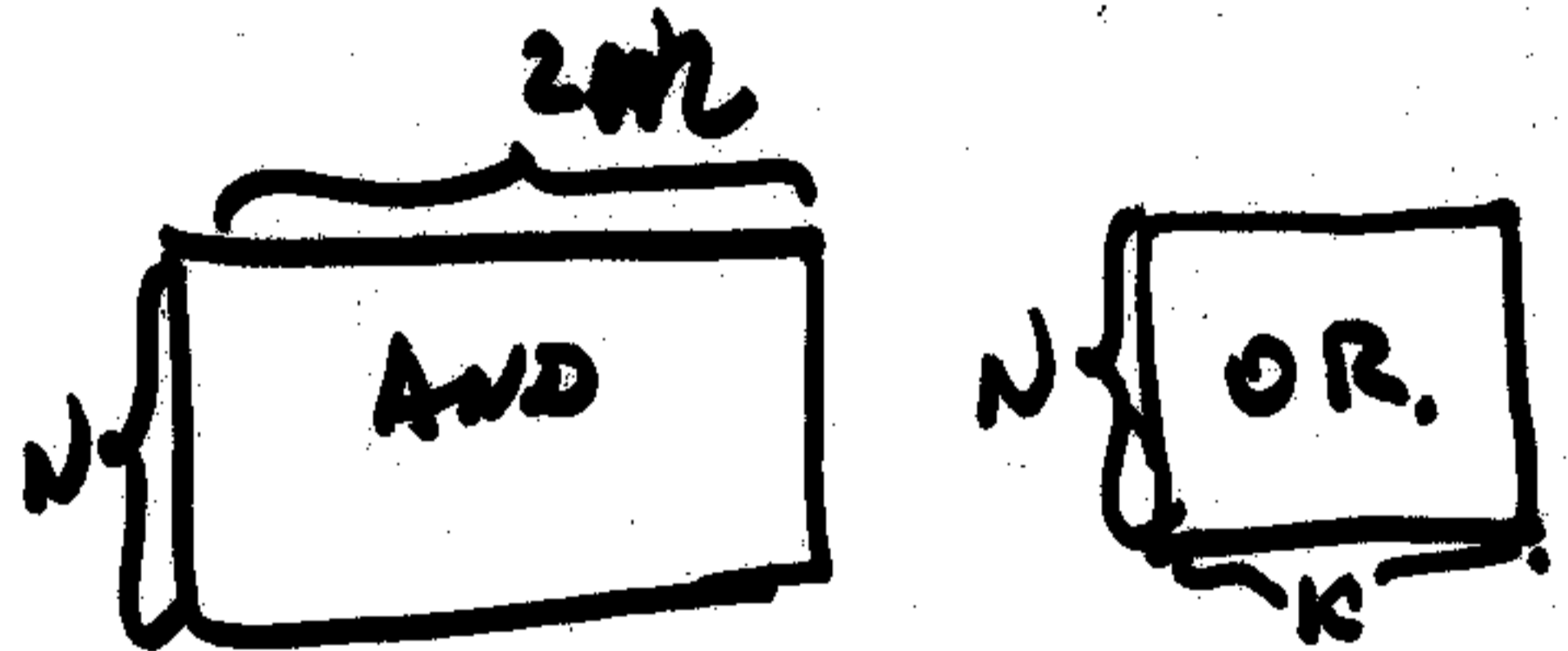


NOTATION: OG

NUMBER OF CROSSPOINT

FAULTS

m - INPUTS
 N - PRODUCT TERMS
 K - outputs



$$2m \cdot N + NK = N(2m + K)$$

AND

OR

RELATIVELY SMALL.

V memory faults

5.1. Decoding errors

open decoder \Rightarrow no addressing
multiple writes, two addresses at a time
replacement of an address by another

5.2 INTERMITTENT FAULTS IN BINARY CELLS

5.2.1 Stuck-at faults in binary cells

5.3. Bridgings between cells

5.4 Bursts on a surface of a disc or tape

For semiconductor memories:

5.5 Hold time for refreshing data \Rightarrow
sleeping sickness - graceful degradation
of performance

5.6. Write recovery - not producing data

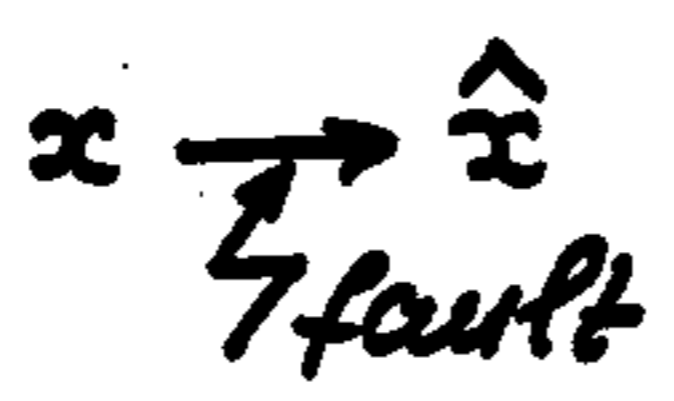
at a given access time if each READ

is preceded by WRITE

TIME
FAULTS

IV Arithmetical Faults (ALU)

Typical for adders, subtractors, counters, multipliers, etc.



Multiplicity l : $|x - \hat{x}| = \pm 2^{i_1} \pm \dots \pm 2^{i_l}$

Example: $x = 0111, \hat{x} = 1000 \Rightarrow l = 1$

(For stuck-at fault model $l = 4$)

Number of faults: $\sum_{i=1}^l 2^i \binom{m}{i}$ - symmetrical

m - number of bits $\sum_{i=1}^l \binom{m}{i}$ - unidirectional

Symmetric, asymmetric and unidirectional arithmetical faults

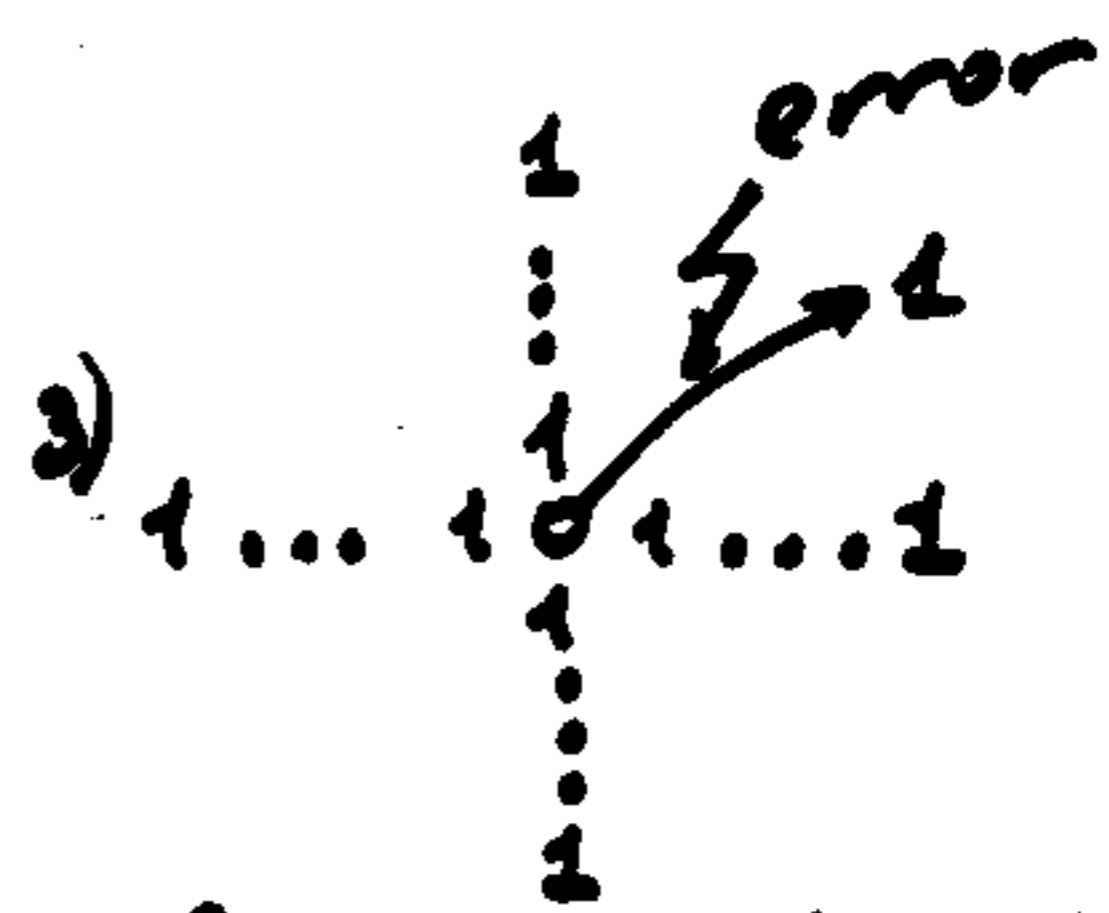
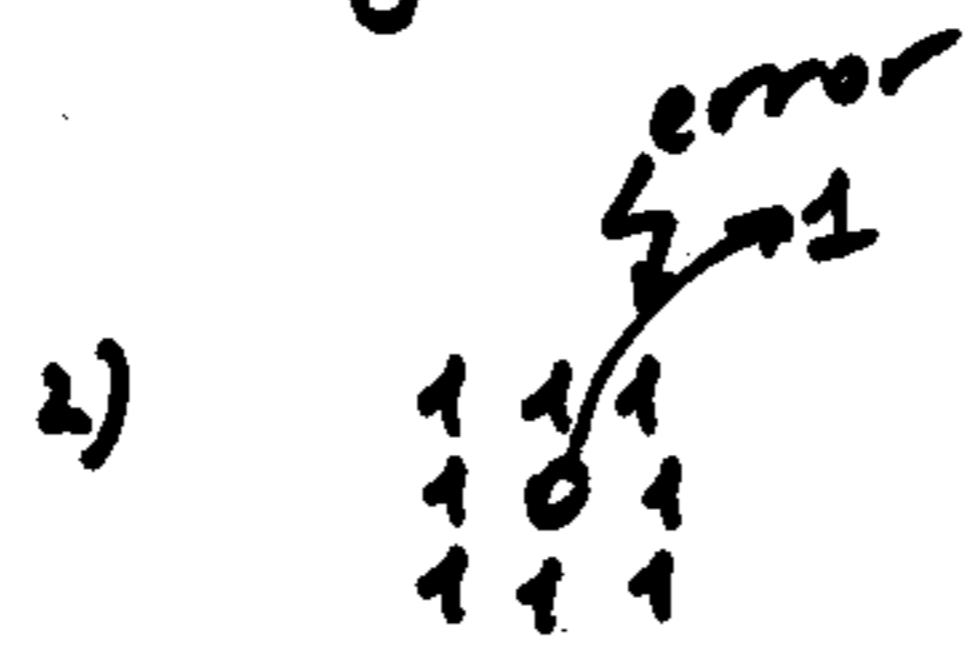
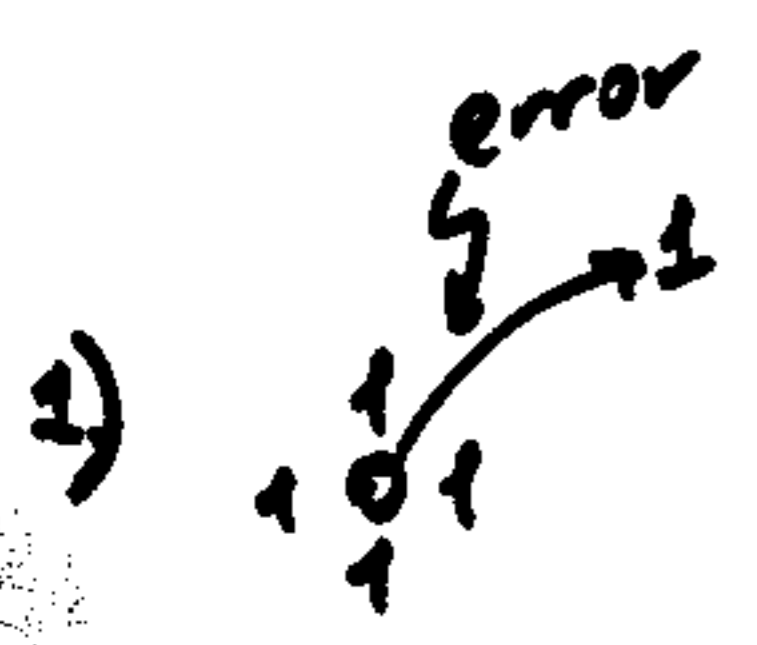
Arithmetical codes (modulo A checks)

Example $l = 1 \Rightarrow A = 3$ TO DETECT SINGLE SYMMETRICAL ARITHM. ERRORS.

5.7. Pattern-sensitive faults (crosstalks) -

impossible to write 0 surrounded by 1's
(1) (0's)

Electrical neighbourhoods



these faults are most difficult to test

Testing time (test complexity, number of READ and WRITE operations) is proportional to the size of a neighbourhood

VII Functional Faults (processors)

7.1. Replacement of one instruction I_i by another I_j , I_i/I_j

7.2. Replacement of an instruction I_i by no instruction, I_i/ϕ

7.3. Replacement of an instruction I_i by two instructions I_j and I_k , I_i/I_j+I_k

these faults are detected by functional testing (function verification)

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DEVICE ORIENTED FAULT MODELS

⑨

1.10

PLA Faults

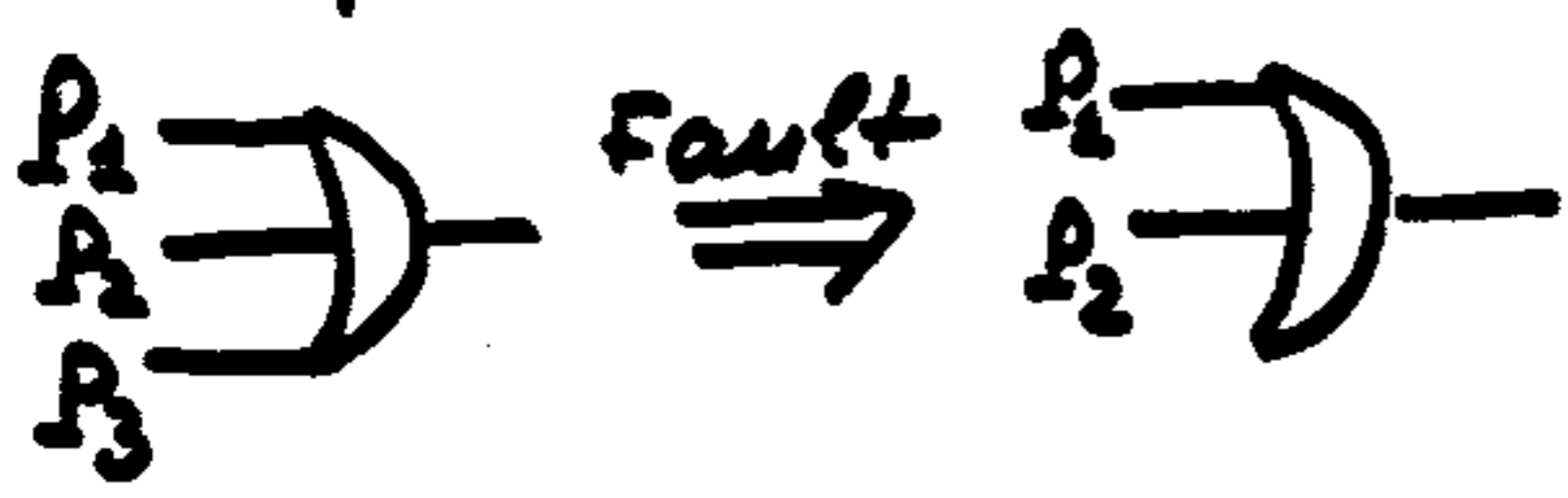
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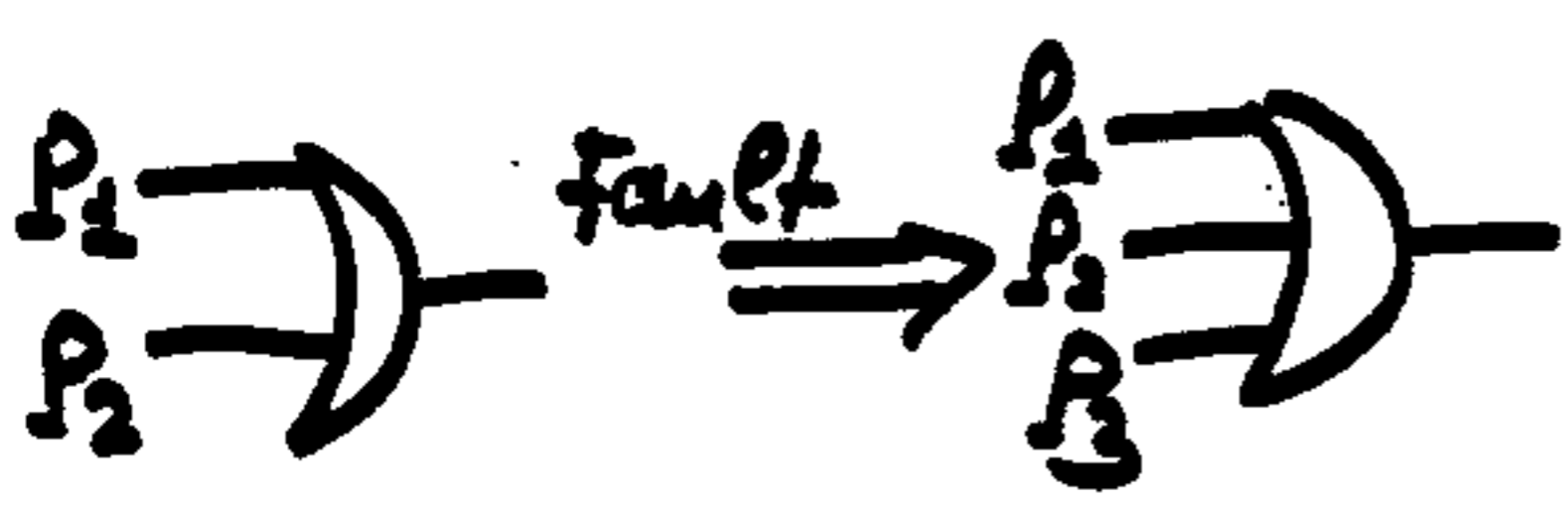
2. A product term can shrink (cover fewer minterms)



3. A product term can disappear from a function



4. A product term from one function can appear also in another function



VI memory faults

$l=1$ UNIDIRECTIONAL

5.1. Decoding errors

open decoder \Rightarrow no addressing
multiple writes, two addresses at a time
replacement of an address by another

$l=2$ SYMMETRICAL
Stuck-at faults in binary cells

5.2.

5.3.

Bridging between cells

5.4

Bursts on a surface of a disc or tape

For semiconductor memories:

5.5 Hold time for refreshing data \Rightarrow
sleeping sickness - graceful degradation
of performance

(for dynamic memories)

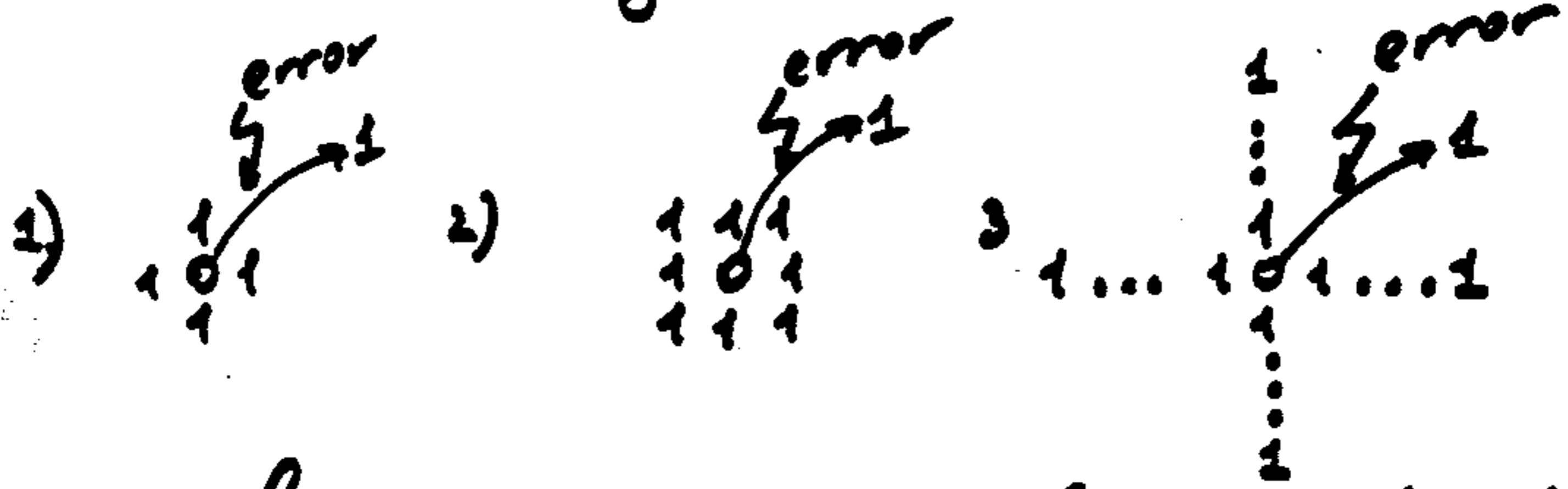
5.6. Write recovery - not producing data

at a given access time if each READ
is preceded by WRITE

5.7. Pattern-sensitive faults (crosstalks) -

impossible to write 0 surrounded by 1's
STATIC

Electrical neighbourhoods



these faults are most difficult to test

Testing time (test complexity, number of READ and WRITE operations) is proportional to the size of a neighbourhood

DYNAMIC PATTERN SENSITIVEFAULTSCOUPLINGTWO-COUPLINGS

1. $\langle \uparrow, \downarrow \rangle \Rightarrow 0 \rightarrow 1$ TRANSITION
IN CELL i results in setting
CELL j into 0 from 1
2. $\langle \uparrow, \uparrow \rangle \Rightarrow 0 \rightarrow 1$ in cell i
results in setting cell j
into 1. from 0
3. $\langle \downarrow, \downarrow \rangle \Rightarrow 1 \rightarrow 0$ in i
sets j into 0
from 1

CHANGES IN MEMORY PATTERNS
RESULTS IN FAULTS

4. $\langle \downarrow, \uparrow \rangle \Rightarrow 1 \rightarrow 0$ in i sets
 j into 1 from 0

5. $\langle \uparrow, \downarrow \rangle \Rightarrow 0 \rightarrow 1$ transition
in cell i results in negation
of a content of cell j

6. $\langle \downarrow, \downarrow \rangle \Rightarrow 1 \rightarrow 0$ in i
results in negation of contents
of j

K-couplings \Rightarrow couplings between two cells with a neighbourhood of K cells when remaining $K-2$ cells are in a certain state

Ex ($\downarrow 000\downarrow$) ($K=5$)
 (\downarrow, \downarrow) coupling between cells 1 and 5 when cells 2, 3, 4 are 000.

VII Functional Faults (μ processors)

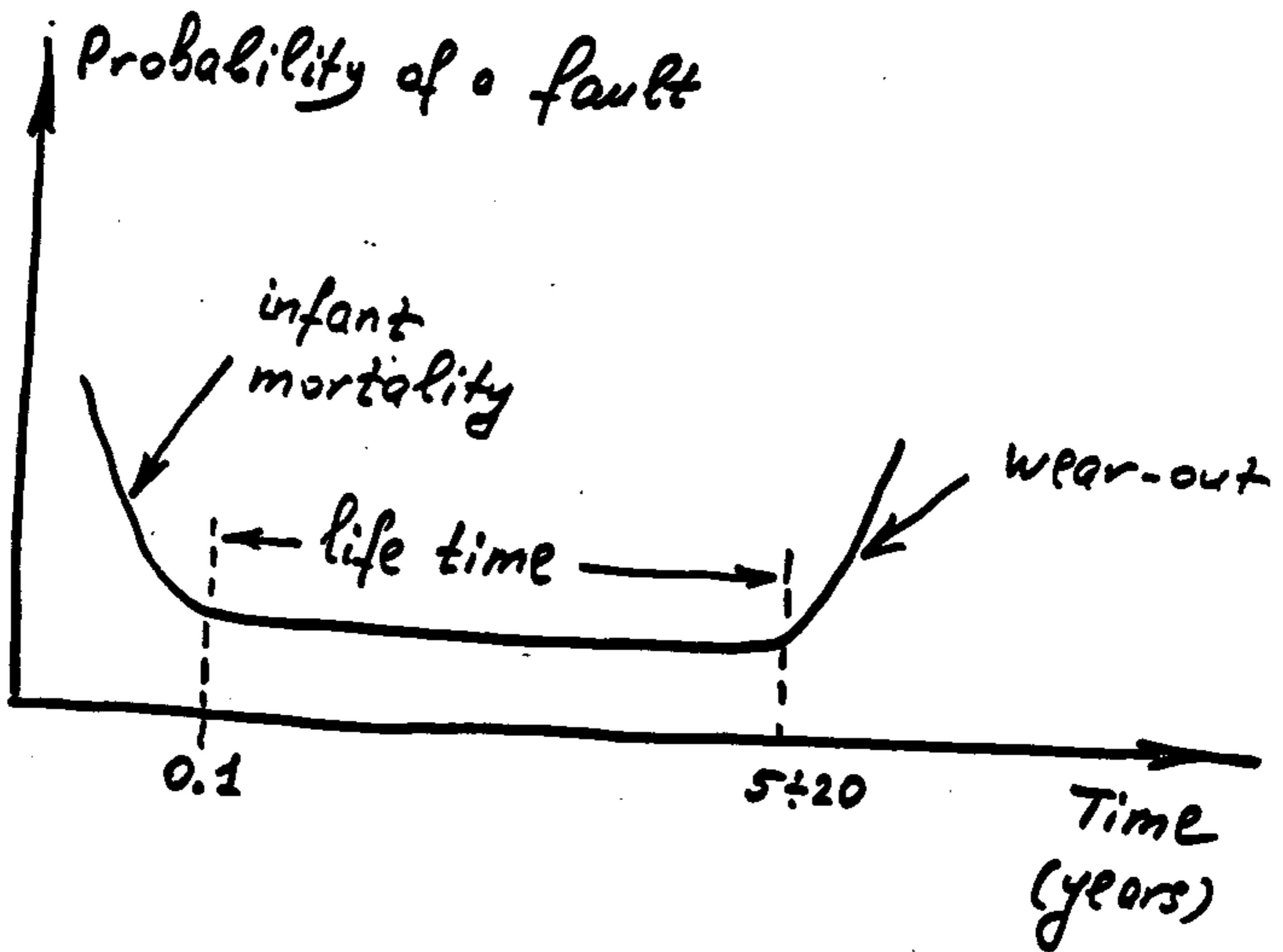
7.1. Replacement of one instruction I_i by another I_j , I_i/I_j

7.2. Replacement of an instruction I_i by no instruction, I_i/\emptyset

7.3. Replacement of an instruction I_i by two instructions I_j and I_k , I_i/I_j+I_k

These faults are detected by functional testing (function verification)

Fault model may ²¹ depend on time (14) 1.



VIII Network faults

Faults in distributed systems

8.1. Node faults

Multiplicity of a node fault = number of processors in the system which are faulty at the same time.

8.2. Link faults

Multiplicity of a link fault = number of links between nodes which are faulty at the same time.

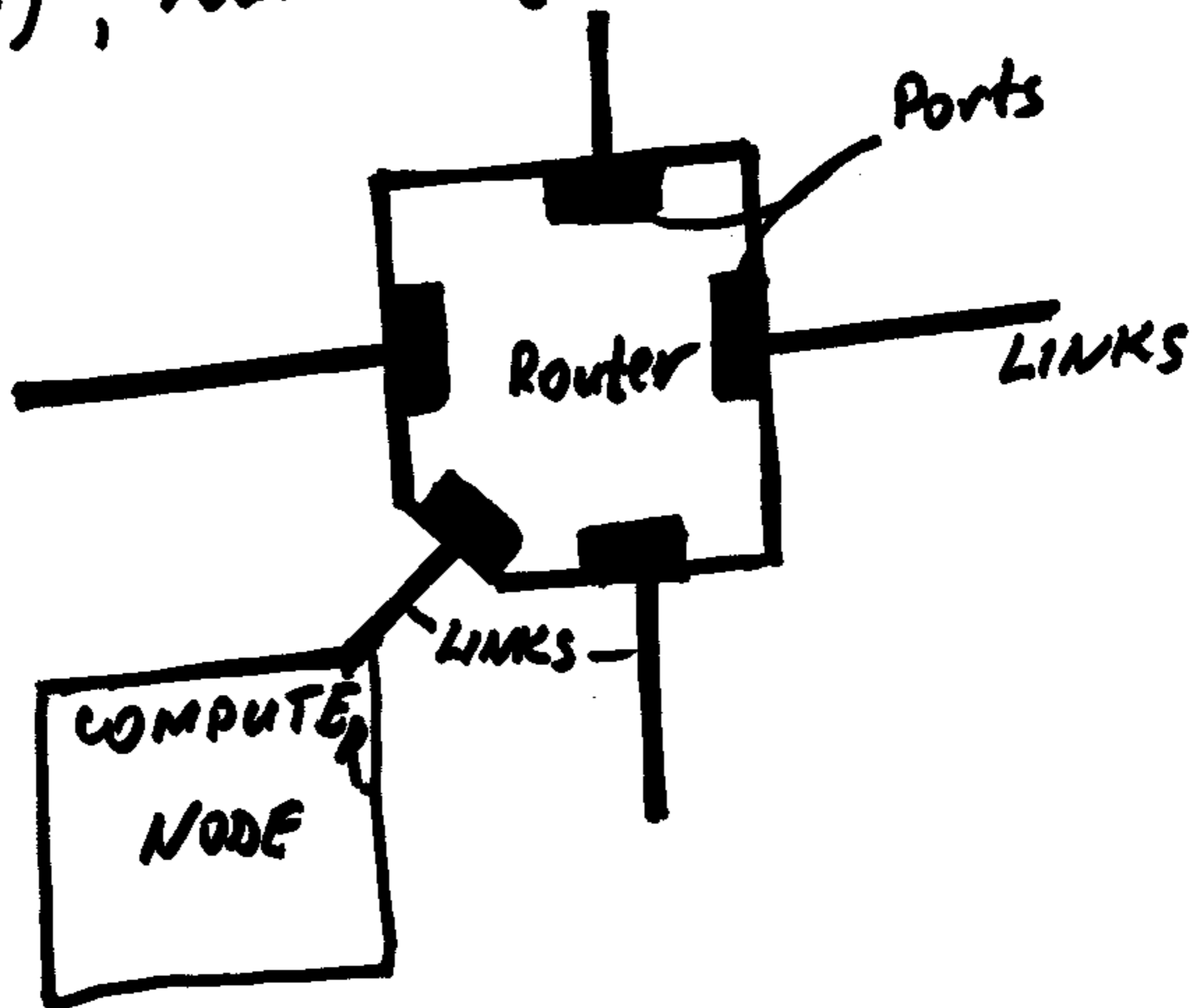
∴ Optimal test strategy for a given topology of links.

∴ Optimal topology of links for a given class of faults.

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ROUTER FAULTS IN
COMPUTER AND COMMUNICATION
NETWORKS AND MULTIPROCESSORS

Networks consist of computing nodes (with processor, RAM, ROM, bus), routers (switches) and links.



• TOTAL ROUTER FAULTS

(NO CONNECTION BETWEEN INPUT AND OUTPUT PORTS)

• PARTIAL ROUTER FAULTS

(NO CONNECTION FOR ONE PAIR OF INPUT-OUTPUT PORTS; e.g.

message cannot be sent from a horizontal to a vertical link)

GENERAL ERROR MODELS

AT THE CHIP LEVEL

∴ TEMPORAL AND SPATIAL
COMPONENTS OF ERROR

MODELS

∴ FOR COMBINATIONAL DEVICES

AND COMBINATIONAL FAULTS

ALL ERRORS ASSUMED TO BE

TIME-INDEPENDENT

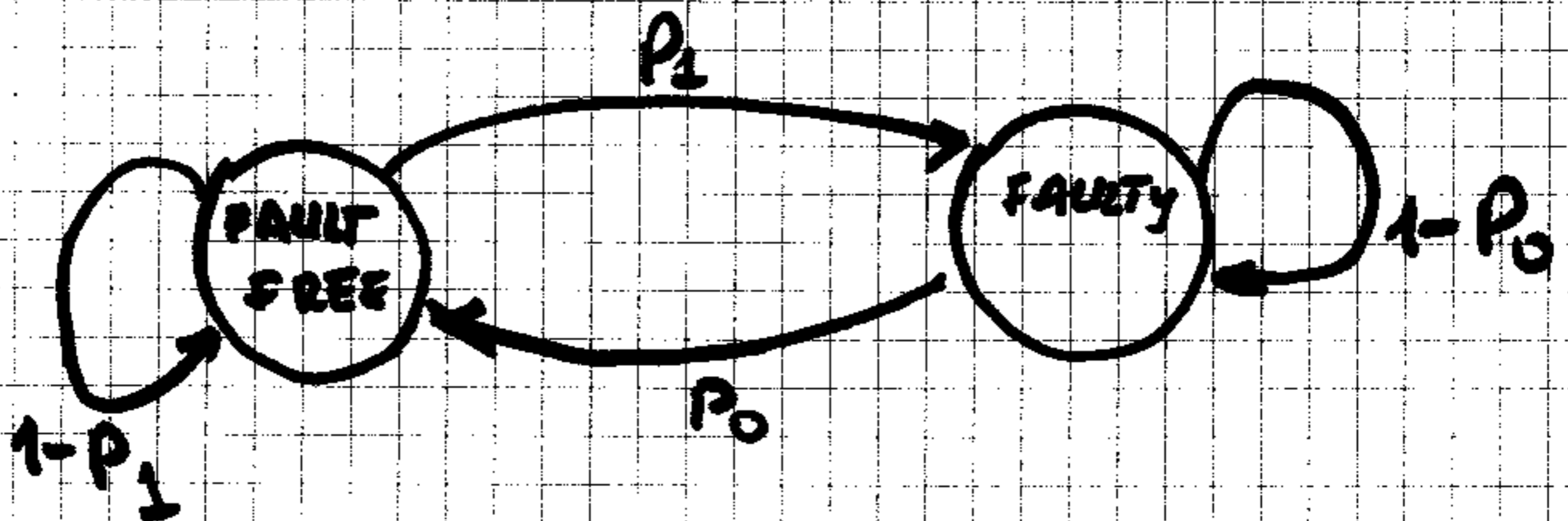
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TIME CORRELATED

ERROR MODELS

(MARKOV CHAIN MODELS)

(P_0, P_1 - model)



P_1 = Prob that the device is fault-free at clock t and faulty at clock $t+1$

P_0 = Prob that the device is faulty at clock t and fault-free at clock $t+1$.

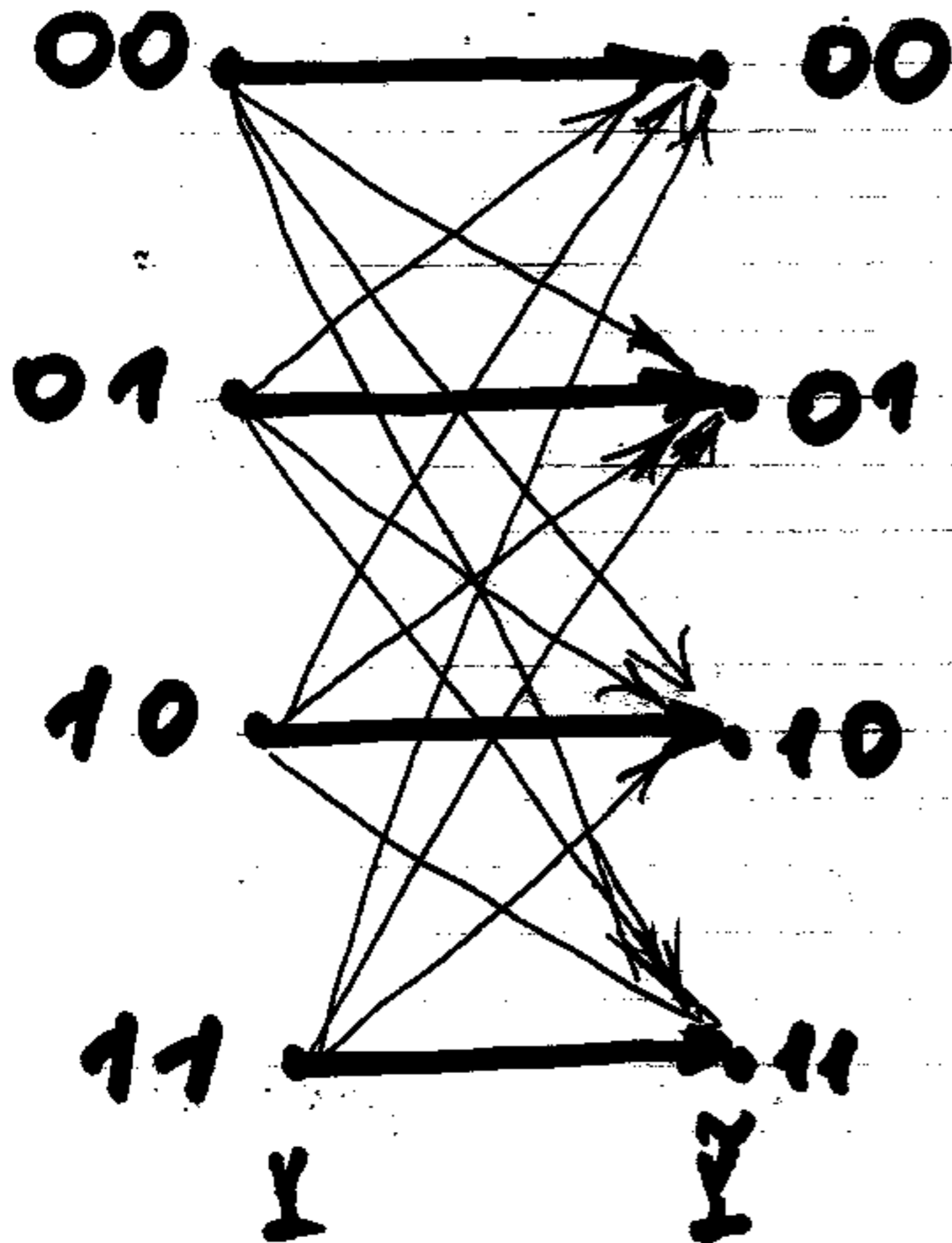
SPATIAL ERROR MODELS

1. q-ary symmetrical model

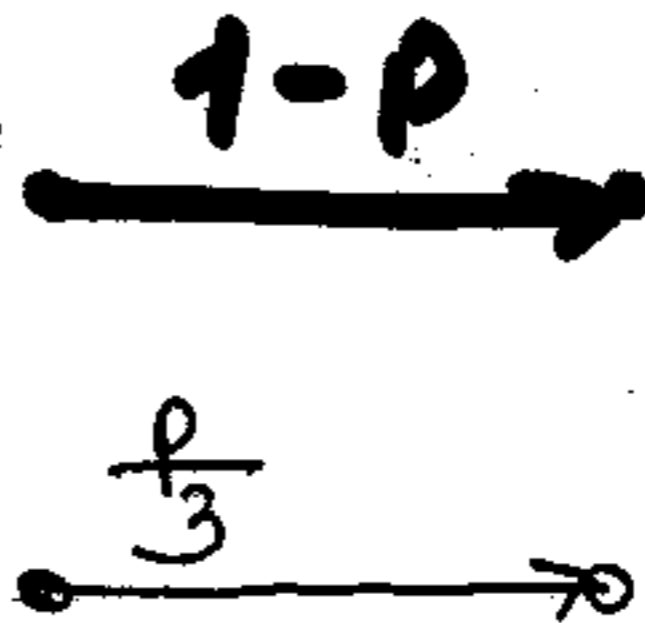
For a device with m output lines let $q = 2^m$

We assume all $q-1$ possible distortions at any given time being equiprobable

● EXAMPLE : $m=2$ $q=4$
symmetrical



4 ARY
SYMMETRICAL
CHANNEL



2. INDEPENDENT ERRORS

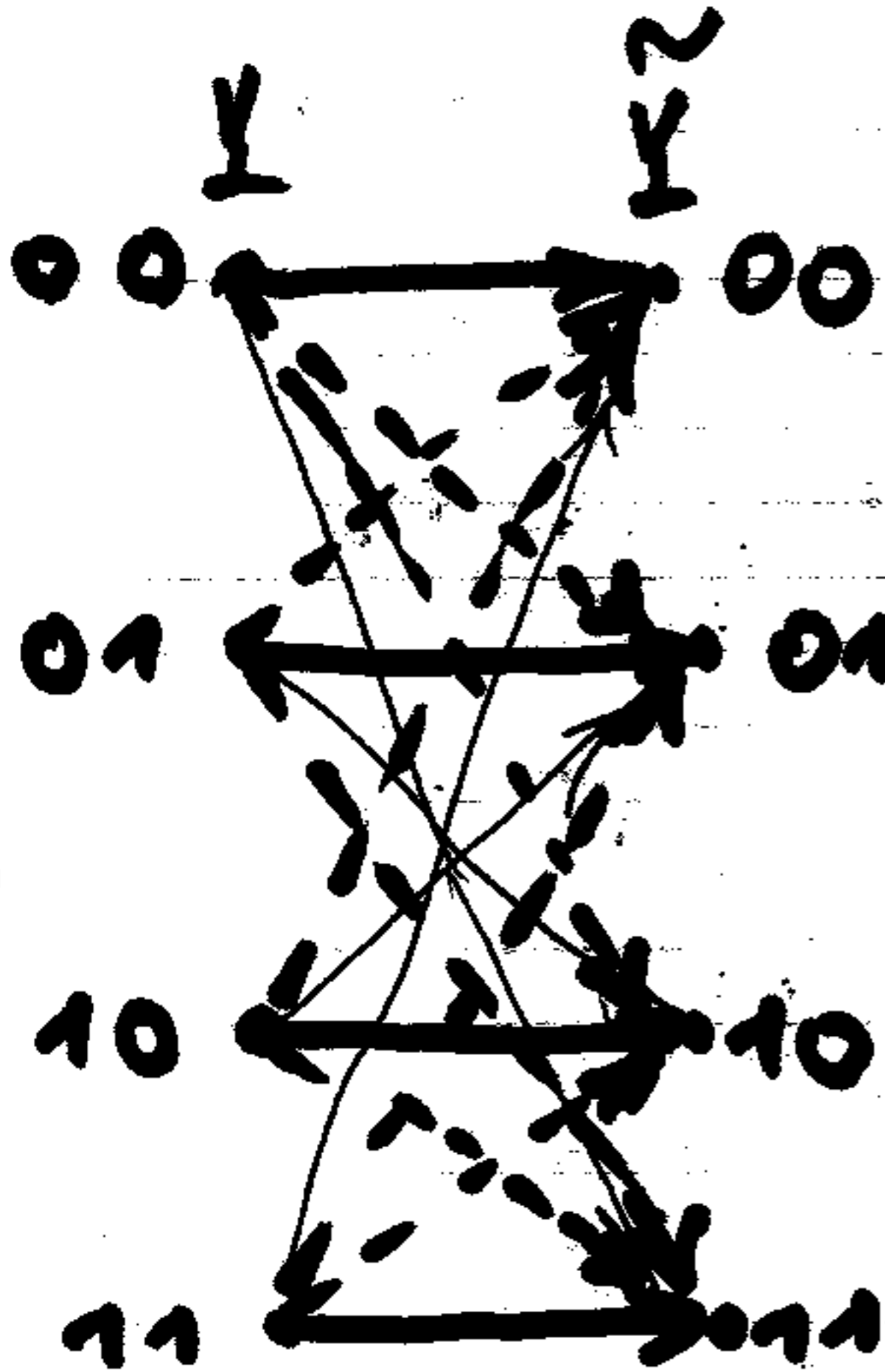
DISTORTION OF EXACTLY
 n out of m output bits
happens with probability

$$p_b^n (1-p_b)^{m-n} \text{ at any moment}$$

where p_b is a probability
of a distortion in
one bit.

(bit distortion rate)

EXAMPLE: $M=2$ $q=4$
 INDEPENDENT
 ERRORS



4ary
 independent
 channel

$$\frac{1 - P_b^2 - 2P_b}{2P_b}$$

$$\frac{P_b(1 - P_b)}{2P_b}$$

$$\frac{P_b^2}{2P_b}$$

3. General space model

General error model is defined by error distribution

$$P_0, P_1, \dots, P_{q-1}$$

where $P_i = \text{Prob} \{E=i\}$

$$E = y \oplus \tilde{y} - \text{FAULTY OUTPUT}$$

/

FAULT FREE OUTPUT

FOR MOST CASES :

1. INDEPENDENT MODEL is
PESSIMISTIC

2. SYMMETRICAL MODEL is

OPTIMISTIC

KARPOVSKY, GUPTA, PRAZHAN

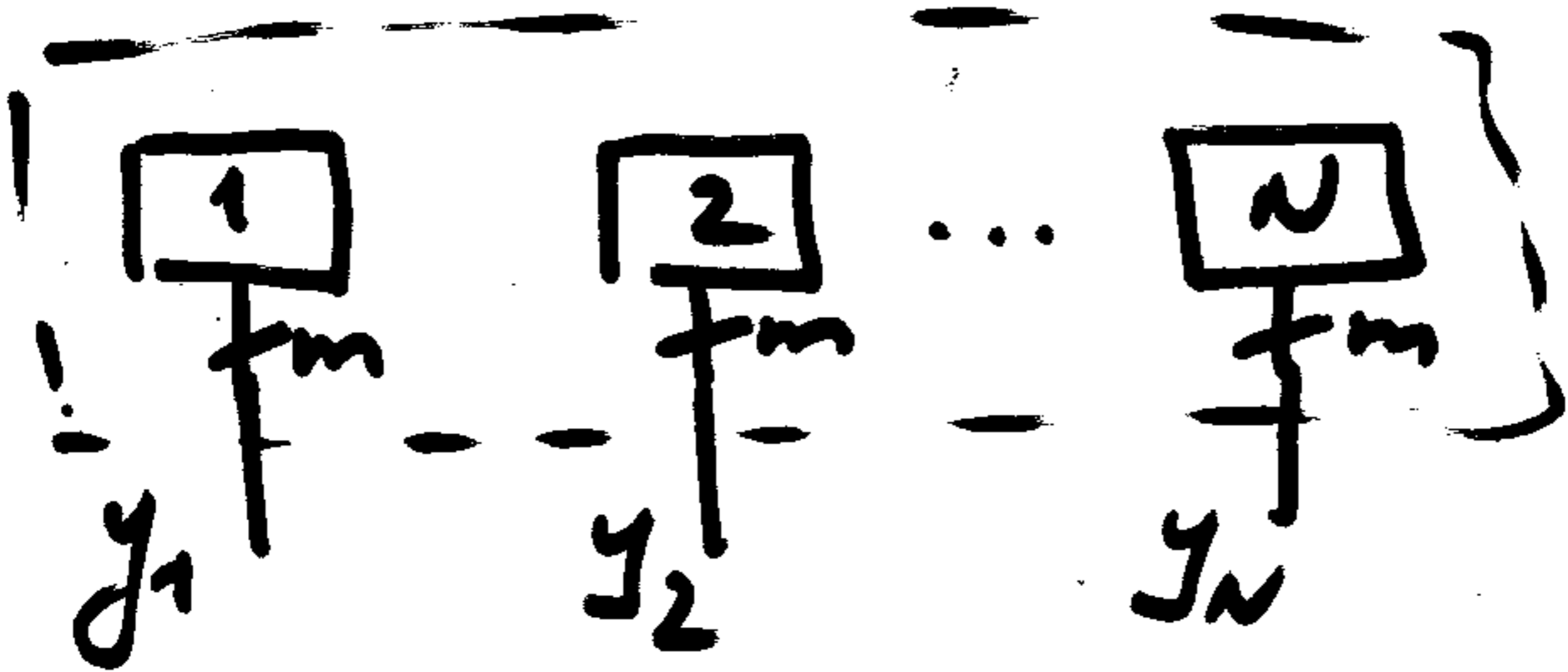
"ALIASING AND DIAGNOSTIC
PROBABILITY IN MISR..."

Proc. INT TEST CONF. 1991

PP 828 - 840.

ERROR MODELS AT THE SYSTEM LEVEL.

CONSIDER A SYSTEM WITH N COMPONENTS (CHIPS)



1. CHIP-INDEPENDENT MODELS

ERRORS IN DIFFERENT CHIPS
ARE INDEPENDENT.

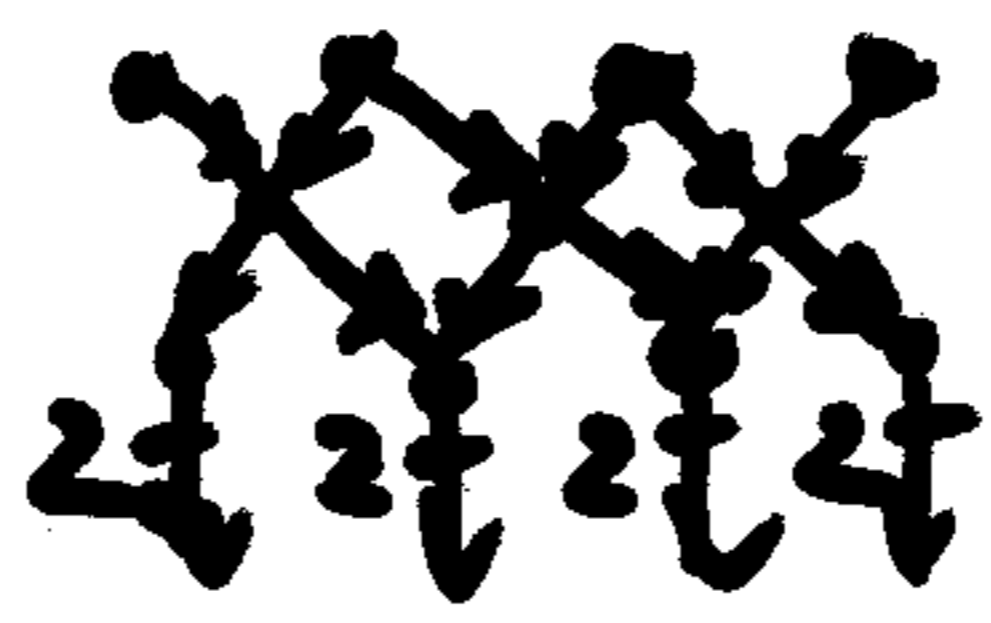
INSIDE A CHIP ERRORS
ARE DISTRIBUTED ACCORDING
TO ONE OF THE CHIP MODELS
(INDEPENDENT, SYMMETRICAL
OR GENERAL)

2. System-dependent ERRORS

(FOR ARRAY PROCESSORS,
MESHERS)

correlated errors

EXAMPLE



WE HAVE THE FOLLOWING EQUIPROBABLE ERRORS (GENERATED BY SINGLE FAULTY NODES)

$(e_1, e_2, 0, 0)$, $(e_1, e_2, e_3, 0)$, $(0, e_2, e_3, e_4)$

$(e_2, e_3, 0, 0)$, $(0, 0, e_3, e_4)$

$(e_1, 0, 0, 0)$, $(0, e_2, 0, 0)$, $(0, 0, e_3, 0)$,

$(0, 0, 0, e_4)$

$e_1, e_2, e_3, e_4 \in \{0, 1\}$

IV Arithmetical Errors (ALU)

1.9

Typical for adders, subtractors, counters, multipliers, etc.

$$x \xrightarrow{\hat{x}} \hat{x}$$

↓
fault

Multiplicity l : $|x - \hat{x}| = \pm 2^{i_1} \pm \dots \pm 2^{i_l}$

Example: $x = 0111$, $\hat{x} = 1000 \Rightarrow l = 4$

(For stuck-at fault model $l = 4$)

Number of Errors: $\sum_{i=1}^l 2^i \binom{m}{i}$ - symmetrical
 m - number of bits $\sum_{i=1}^l \binom{m}{i}$ - unidirectional

Symmetric, asymmetric and unidirectional arithmetical faults

Arithmetical codes (modulo A checks)

Example $l=1 \Rightarrow A=3$ (DETECTION)

ARITHMETICAL BURSTS

OF LENGTH b

$$|x - \hat{x}| = c_1 2^{i_1} + c_2 2^{i_1+1} + \dots + c_b 2^{i+b-1}$$

$$c_i \in \{-1, 0, +1\} \quad \text{symmetrical}$$

$$c_i \in \{0, 1\} \quad \text{- unidirectional}$$

$$c_i \in \{-1, +1\} \quad \text{- solid}$$