Problem Motivation

Goals:

- Identification of faulty components in a board or system from compressed responses.

- Estimations for minimal number of signatures.

- VLSI implementation.

Assumption:
At most \( l \) components in the board may be faulty.

Approach:
Multiple signature analysis and space compression using error correcting codes.
Diagnostic Model

1- Application of pseudorandom test patterns onto the primary inputs of \( n \) chips on a board under test.

2. Space and time compression of test responses into \( r(n,l) \) signatures where \( l \) is the number of faulty chips to be located.

3. Identification of faulty chips by decoding distorted signatures using Soft or Hard decision approach.
Straightforward Diagnostic Approach

**Overhead:**

- Number of signatures = n.

- Space complexity has \( \Theta(nm) \).

- Time complexity has \( \Theta(nT) \).
Space-Time Diagnostic Approaches

**Soft Decision Approach:**
Location of faulty chips is based on analysis of magnitudes of distortions in space-time signatures.

**Hard Decision Approach**
Location of faulty chips is based on analysis of a binary syndrome vector indicating which components in a vector of space-time signatures are distorted.
Soft Decision Diagnostic Approach

Overhead:

- Number of signatures \( r_s(n, l) = 2l \).

- Space complexity has \( \Theta(lm) \).

- Time complexity has \( \Theta(nT) + \Theta(n) \).
Diagnosability Condition (Soft Decision)

Different errors in \( y(t) = (y_0(t), \ldots, y_{n-1}(t)) \)
\((y_i(t) \in GF(2^m))\) result in different distortions
in space signatures \( z(t) = (z_0(t), \ldots, z_{r-1}(t)) \).

Example:
Let \( l = 2 \) and consider the following two different double errors \((i, j) \neq (u, v)\):

\[
e^{(1)}(t) = (0, \ldots, 0, e_i(t), 0, \ldots, 0, e_j(t), 0, \ldots, 0),
\]

\[
e^{(2)}(t) = (0, \ldots, 0, e_u(t), 0, \ldots, 0, e_v(t), 0, \ldots, 0).
\]

Let us denote columns of \( H_s \) as \( h_0, \ldots, h_{n-1} \).
Then

\[
e_i(t)h_i \oplus e_j(t)h_j \neq e_u(t)h_u \oplus e_v(t)h_v.
\]

Thus, for this case of independent errors, a \((r \times
n)\) check matrix \( H_s \) of a double error-correcting code over \( GF(2^m) \) can be chosen as a soft
decision space compression matrix.
Savings in Overhead (Soft Decision)

Example:

**Straightforward Approach:**

\( n = 100, \ m = 32. \)

\( L_0 \approx 88,000 \) equivalent gates.

**Soft Decision Approach:**

\( n = 100, \ \mathfrak{m} = 32, \ l = 5. \)

\( L_{1,s} \approx 28,000 \) equivalent gates.

Savings in hardware overhead of about 70% over straightforward approach.
Overhead Minimization Problem (Soft Decision)

Minimize $r_s(n, l)$, number of space-time signatures, such that different errors is $y(t)$ result in different distortions in $z(t) = y(t)H^{tr}_s$.

For soft decision approach optimal solution for $r_s(n, l) = 2l$ and $H_s$ is a check matrix of $(n, n - 2l)$ $l$-error-correcting Reed-Solomon code over $GF(2^m)$ if $n \leq 2^m - 1$. 
Savings in Overhead (Soft Decision)

\((1 - L_i/L_*) \times 100\)
Soft Decision Diagnostic Approach

Advantages:

Soft Decision Diagnostic Approach is efficient if a system is disconnected in testing mode.

Single faulty component results in a distortion of only one component in the signature vector.

Test responses are transferred in the diagnostic hardware via system bus or boundary scan register.

Minimum number of signatures to be stored.

Disadvantages:

Complex decoding procedure.
Hard Decision Diagnostic Approach

Overhead:

- Number of signatures $r_h(n, l) \geq \left\lceil \log_2 \sum_{i=0}^{l} \binom{n}{i} \right\rceil$.

- Space complexity has $\Theta(rm)$.

- Time complexity has $\Theta(nT) + \Theta(n^l)$. 
Diagnosability Condition (Hard Decision)

Different binary syndromes of errors in $y(t) = (y_0(t), \ldots, y_{n-1}(t))$ ($y_i(t) \in GF(2^m)$) result in different distortions in space signatures $z(t) = (z_0(t), \ldots, z_{r-1}(t))$.

For the error $e(t) = (e_0(t), \ldots, e_{n-1}(t))$ the binary syndrome $S(e(t)) = (S(e_0(t)), \ldots, S(e_{n-1}(t)))$ is defined as

$$S(e_i(t)) = \begin{cases} 1 & e_i(t) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$
Diagnosability Condition (Hard Decision)

Example:

Let $l = 2$ and consider the following two different double errors $(i, j) \neq (u, v)$:

$$e^{(1)}(t) = (0, \ldots, 0, e_i(t), 0, \ldots, 0, e_j(t), 0, \ldots, 0),$$

$$e^{(2)}(t) = (0, \ldots, 0, e_u(t), 0, \ldots, 0, e_v(t), 0, \ldots, 0).$$

Let us denote columns of $H_h$ as $h_0, \ldots, h_{n-1}$. Then

$$S(e_i(t))h_i \lor S(e_j(t))h_j \neq S(e_u(t))h_u \lor S(e_v(t))h_v.$$

Thus, for this case of independent errors, a $(r \times n)$ matrix $H_h$ of a second order binary superimposed code can be chosen as a hard decision space compression matrix.

A binary superimposed code of order $l$ consists of a set of codewords such that component-wise Boolean sum (OR) of any $l$ codewords differs from every other component-wise sum of $l$ or fewer codewords.
Overhead Minimization Problem (Hard Decision)

Minimize $r_h(n, l)$, number of space-time signatures, such that different binary syndromes of errors is $y(t)$ result in different distortions in $z(t) = y(t)H^t_r$.

General bounds:

$$\left\lfloor \log_2 \sum_{i=0}^{l} \binom{n}{i} \right\rfloor \leq r_h(n, l) \leq n$$

$$r_h(n, l) \leq 3(l + 1) \log_2 \left( (l + 1) \binom{n}{l + 1} \right)$$
Overhead Minimization Problem (Hard Decision)

Constructive bounds:

\[ r_h(n, 1) = \lfloor \log_2(n + 1) \rfloor \]

\[ r_h(n, 2) \]

<table>
<thead>
<tr>
<th>n</th>
<th>16</th>
<th>25</th>
<th>49</th>
<th>64</th>
<th>125</th>
<th>343</th>
<th>512</th>
<th>2401</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_h(n, 2) )</td>
<td>12</td>
<td>15</td>
<td>21</td>
<td>24</td>
<td>25</td>
<td>35</td>
<td>40</td>
<td>49</td>
</tr>
</tbody>
</table>
Savings in Overhead (Hard Decision)

\[(1 - L_{1,w}/L_0) \times 100\]

Example:
\[n = 100, \ m = 32, \ l = 1.\]
\[L_{1,h} \approx 10,000\] equivalent gates.
Savings in hardware overhead of about 88% over straightforward approach.
Hard Decision Diagnostic Approach

**Advantages:**

Simple decoding procedure.

Hard Decision Diagnostic Approach is efficient if a system is not disconnected in testing mode (e.g. systolic array)

**Disadvantages:**

Decoding hardware complexity.
Conclusions:

1. Space-time diagnostic approach results in considerable hardware savings in overhead for testing.

2. Up to $l$ faulty chips can be identified using Soft or Hard decision approach.