Spectral Techniques in Binary and Multiple-Valued Switching Theory

A review of results in the decade 1991-2000

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Abstract

This paper presents a tutorial review of spectral methods in switching and multiple-valued logic theory and the design of digital system in the last decade. The paper continues review work in this area done by the authors in 1981 and 1991.

1 Introduction

Spectral techniques are a mathematical discipline in the area of abstract harmonic analysis devoted to the applications in engineering, primarily electrical and computer engineering. Practical applications of spectral techniques in analysis and design with switching and multiple-valued (MV) functions date back to the early history of these areas [6], [11], [23], [39], [43], [52], [63], [91], while mathematics foundation are set even earlier [104]. It is believed that spectral techniques have developed into a separate discipline during the probably highest activity in this area between 1970 and 1975, which is possibly best expressed by pointing out annual symposia on Walsh and other nonsinusoidal functions, with even two international meetings on that subject in 1973 and 1975. After that time, although with somewhat fluctuating interest, spectral techniques have been continuously developing and activity in the area has been summarized at the international Workshops in Boston, 1985 [33], Montreal, 1986, and Dortmund, 1988 [53], and presented at regular sessions at some conferences on signal processing and circuit design, among which the ISMVL are probably the most supporting for the research in this area.

That activity was discussed and a summary of development in theory and practice of spectral techniques up to the late seventies was given by Karpovsky [32], while the next decade was overviewed by Moraga [54]. The present paper is an attempt to point out and briefly review some, from our point of view, interesting and important results in spectral techniques in the last decade, without pretending to provide a full review of a rather considerable amount of achievements. The selected references should serve as an illustration of the related statements, and could hardly be considered as a complete bibliography on the subject.

2 Activity in Spectral Techniques

There is apparent a renewed and considerable interest in spectral techniques after the publication of a report about applications of Walsh functions to technology mapping [97]. This interest is due to the development of technology of digital circuits imposing requirements and strong demands regarding complexity and performances of logic networks and digital devices in general, which cannot be met by traditional approaches.

In last decade, the 4th International Workshop on Spectral Techniques was organized in 1990 within the Int. Conf. on Signal Processing, Beijing'90, followed by the 5th Workshop hosted by the Beijing University of Aeronautics and Astronautics, in 1994. The Workshop had an especial support from Volkswagen Foundation of Germany. Selected papers were later collected in [56]. We want to point out as an especially important contribution a bibliography of selected papers in this area published in Russian, prepared by Shmerko and Mikhailov [73].

In 1992, the Workshop on Logic Design and Microprocessor Architecture in Iizuka, Fukuoka, Japan, provided space for a tutorial discussion of spectral methods in logic design by Varma and Trachtenberg, further de-

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veloped in a chapter [95] of a book edited afterwards by Sasao [67], accompanied by a chapter on minimization of AND-EXOR expressions including fixed polarity Reed-Muller expressions [66].

It should be noted that renewed interest in Reed-Muller expressions is due to the publication of the paper [70] presenting a conjecture that AND-EXOR expressions require on the average a fewer number of product compared to SOP expressions. That conjecture was confirmed and experimentally verified by the same authors, and many others in a series of publications. (See a discussion and references in [68], [69]). That renewed interest resulted in organization of Int. Workshops on Application of Reed-Muller Expressions in Circuit Design in Hamburg, Germany, 1993, Chiba, Japan, 1995, Oxford, UK, 1997, and Victoria, Canada, 1999. Besides many other interesting results, an especially important achievement of the Reed-Muller Workshops was the establishment of relationships between spectral techniques and decision diagrams (DDs) as a data structure for representation of large discrete functions. DD methods for calculation of spectral transforms originating in [9] considerably improved applicability of spectral techniques, since permit to overcome problems related to the exponential complexity of FFT algorithms in terms of both space and time. Conversely, spectral interpretation of DDs [75], [78], [85], permits a unified consideration and classification of different DDs [84], and offers a way for further generalizations and optimization of DD representations [21], [27], [77]. It should be noted that publication of a book collecting a selection of papers presented at Reed-Muller Workshop in 1995, edited by Sasao and Fujita [72], is considered of crucial importance for further development of both DD representations and spectral techniques.

Besides the Walsh and Reed-Muller transforms, traditional approaches in spectral techniques are related to the arithmetic and Haar transforms. The Conference Series "Advances in Computer Systems" organized at the Technical University of Szczecin, Poland, has acted as a forum for discussing spectral techniques, and in particular arithmetic transforms, mainly due to the previous background work in this area done in East Europe and Russia. Publication of a monograph [46], a book chapter [30], a tutorial paper [22], and a historic overview [18] are setting fundamentals for further work in this area. Importance of arithmetic expressions is further raised by showing that some classes of decision diagrams represent functions in the form of arithmetic polynomials [75], [78], [83], [85].

For a recent renewed interest and development in Haar transforms [29], [31], credit should be given to the organization of a special session of spectral techniques and DDs, accompanied by another session devoted exclusively to Haar transforms within the Int. Conference on Informatics, Communications and Signal Processing (1st ICICS), 1997, in Singapore.

In two Workshops on Spectral Techniques and Filter Banks, in Tampere, Finland, 1998, and Brandenburg, Germany, 1999, spectral techniques for switching theory and logic design were considered within the spectral methods for signal processing. That work motivated organization of the TICSP Workshop on Spectral Transforms and Logic Design for Future Digital Systems, SPECLOG-2000, hosted by the Tampere Int. Center for Signal Processing, Tampere, Finland.

Besides organization of several Workshops and editing related Proceedings, the publication of a few monographs devoted completely or in part to spectral methods in switching and MV logic should be considered as a considerable support for future work in the area [1], [16], [41], [46], [53], [56], [72], [78], [82], [88], [89], [90], [101], [102], [103].

2.1 Research Interest

Topics of research interest in spectral techniques in the last decade can be briefly summarized as follows

1. Definition of new transforms, [13], [14], [15], [58], [59], [61],
2. Consideration of relationships among the various transforms and related operators, [17], [55], [76], [98], [100],
3. Classical and DD methods for calculation of spectral transforms and related operators, [9], [10], [28], [51], [80], [81], [86],
4. Spectral method for synthesis including testing, [3], [4], [8], [24], [26], [34], [35], [36], [40], [44], [45], [57], [65], [71], [92], [93],
5. Spectral interpretation of DDs and related topics, [7], [12], [19], [78], [83], [85].
6. Definition of new decision diagrams and their application in circuit synthesis, [21], [25], [27], [42], [49], [50], [87], [88], [96],
7. Circuit synthesis for realization of spectral transforms, [2], [47], [48], [60], [62], [64], [98], [99],
8. Historic remarks and corrections in interpretation and applications of spectral methods [18], [79].

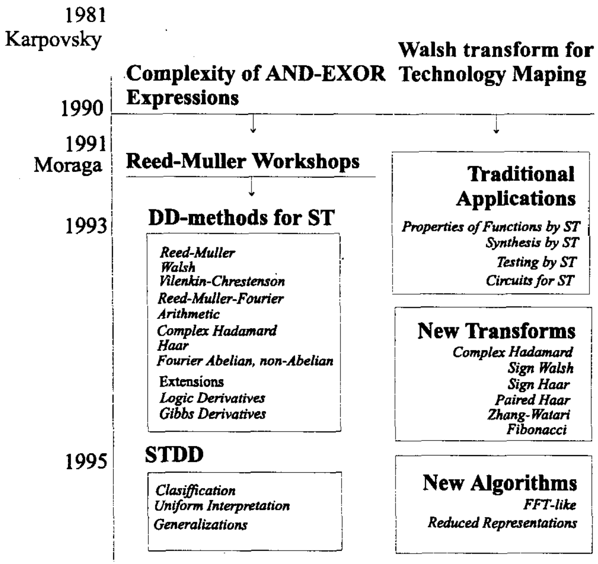

Figure 1. Research subjects.

Fig. 1 shows the timeline and summarizes research activity in the area.

Due to pages limitation is not possible to give here a more complete presentation. Alone the list of references is already 8 pages long. Readers interested in a full version of this work including a discussion of spectral interpretation of DDs, calculation of spectral transforms through DDs, and related applications of spectral techniques on finite not necessarily Abelian groups, may obtain an electronic version by sending a mail to moraga@cs.uni-dortmund.de.

3 Closing Remarks

Spectral techniques allow derivation of alternative methods for solving complex tasks efficiently in terms of space and time. Transferring a problem from the original into the spectral domain may provide several advantages. In particular, some numerical calculation tasks difficult to perform in the original domain, may be simple in the spectral domain. The convolution product, that is often used in description and mathematical modeling of linear shift-invariant systems, is a supporting example. Some properties of a signal or a system, which are shadowed or difficult to observe in the original domain, become easy observable in the spectral domain. Examples are determination of cut-off frequencies and sampling rates in signal processing, and detection of decomposability and symmetry properties in logic design, see for example, [1], [31], [56], [57]. Fast calculation algorithms for spectral coefficients further improve performances of related methods. Decision diagrams (DDs) extends applicability of these algorithms to functions defined in a large number of points, [9].

The complexity of problems in switching and MV theory and logic design give rise for further application of spectral techniques, since they may provide for simple and elegant analytic solutions where the traditional approaches reduce to the brute force search methods.

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