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OPTIMAL VARYING DYADIC STRUCTURE MODELS OF TIME INVARIANT SYSTEMS

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Summary: We consider the problem of the best approximation to a given linear system by a varying structure dyadic system (VDS) over a finite group. Consider a given linear invariant system with zero initial state, defined on a finite discrete time interval $[0,2^n-1]$:

$$z(t) = (w*u)(t) = \sum_{z=0}^{2^n-1} w(t-z)u(z) \quad , \quad (0 \le t \le 2^n-1). \qquad (1)$$

The symbol * stands for convolution of the impulse function w and the input u and w, u, z: $[0,2^n-1] \to C$, where C is the field of complex numbers. Let now m = 2n and consider the dyadic group of binary 2n-vectors, $t \in G$, $t=(t_0, t_1, \ldots, t_{2n-1})$, $t_i \in \{0,1\}$, $0 \le i \le 2n-1$ with the group operation $\oplus$-componentwise addition mod 2. In this case the group characters are Walsh functions

$$\chi_\omega(t) = (-1)^{\bigoplus_{i=0}^{2n-1} \omega_i t_i} \quad , 0 \le \omega \ , \ t \le 2^{2n}-1 \text{ , see e.g. [1,2].}$$

Let v:G→C and we will consider the following VDS

$$y(t) = \sum_{\zeta \in G} v(t \oplus \zeta)\, \chi_{\vec{t}}(\zeta) u(\zeta) \quad , \qquad (2)$$

where $\vec{t} \triangleq (t_n, t_{n+1}, \ldots, t_{2n-1}, t_0, t_1, \ldots, t_{n-1})$.

Treating $u:[0,2^n-1] \to C$ in (1) as a function defined on G, i.e., $u: G \to \mathcal{C}$, we state the following problem of the best approximation of system (1) by a VDS (2) over the group G: Amongst all systems (2) with the input u, it is required to find an optimal VDS with an impulse function v_{opt}:G→C such that

$$\|w - v_{opt}\| = \min_{v: G \to C} \{\|w - v\|\} \ , \ \|f\| \triangleq \Big(\sum_{z \in G} f(z) \overline{f(z)}\Big)^{1/2} , \quad f: G \to C. \quad (3)$$

In [1], the problem (3) was posed and solved, where the approximation of (1) in norm (3) was done by convolution systems (CLS) over the group G.

$$y(t) = (h \circledast u)(t) = \sum_{z \in G} h(t \oplus z)\, u(z) \quad , \quad t \in G \quad , \tag{4}$$

where $h: G \to C$ is the impulse function.

In [2,3] the problem of approximation of a time invariant system (1) by general (not necessarily Abelian group models (2) or (4)) has been solved. By considering special varying dyadic models, we may achieve some computational advantages as well as increased accuracy of approximation [4,5].

As disadvantages of such dyadic models, we mention the increased complexity of their implementation [3]. Also, there is not always a one-to-one correspondence between the original system (1) and its model (2).

In the present work, we introduce a special family of bases in the space $\{f: G \to C\}$ so that the VDS (2) has a convolution type property. Then we describe the analytical solution of (3) and investigate the problem of one-to-one correspondence between causal and symmetric systems (1) and their VDS models (2). This problem was solved in [1,2] for CLS models (4). We also establish bounds on the complexity of computing the output y of (2) using fast algorithms for computation of convolution. The usefulness of properties of group models (2), (4) is explained by their relation to Frobenius group matrices, which were studied in [3,5].

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