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Fault Detection in Combinational Networks by Reed-Muller Transforms [1]

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ABSTRACT: A new approach for fault detection in combinational networks based on Reed-Muller (RM) transforms is presented.

An upper bound on the number of RM spectral coefficients required to be verified for detection of multiple stuck-at-faults (s-a-fs) and single bridging faults at the input lines of an n input network is shown to be n. For almost all combinational networks the time complexity (time required to test a network) for detection of multiple terminal faults is shown to be $\lceil 1.25n \rceil$, and the storage required for storing the test is $\lceil 0.75n \rceil$, where $\lceil m \rceil$ denotes the smallest integer greater or equal to m.

If any terminal or internal, single or multiple fault distorts at most A spectral coefficients, then the minimum number of test patterns required to detect the fault is shown to be upper bounded by $\sum_{i=0}^{\lceil log_2(A+1) \rceil -1} \begin{pmatrix} n \\ i \end{pmatrix}$. We present standard tests, based on this result, with a simple test generation procedure and upper bounds on minimal numbers of test patterns.

Index Terms: Reed-Muller Transform, Reed-Muller spectrum, Walsh-Hadamard spectrum, signature, stuck-at-fault, bridging fault, fault detection, Reed-Muller codes, generalized Reed-Muller Canonical expansion.

1. INTRODUCTION

In recent years there has been a renewed interest in applications of spectral techniques for fault detection in logical networks. Many authors [1-4] have focussed their attention on

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Walsh-Hadamard (WH) spectral techniques for fault detection in combinational networks. It is shown [1-4] that by verifying few WH-spectral coefficients for a Boolean function it is possible to detect the faults in a network implementing this function. A minimal set of spectral coefficients required for fault detection constitute a spectral signature.

Every WH spectral coefficient depends on the global behaviour (i.e., on all 2^n values) of a given function and its computation requires 2^n additions and subtractions for a function of n variables. Hence fault detection by verification of WH spectral coefficients is impractical, if n is large.

In Section 2, we introduce Reed-Muller (RM) transforms and their properties. Every Boolean function f has an unique Reed-Muller Canonical (RMC) representation [11]. The RMC form of a Boolean function is determined by the coefficients (see (1)) in the RMC expansion. These coefficients can be obtained by transforming the functional values $(f(0), f(1), ..., f(2^n - 1))$ by a suitable RM matrix. Each coefficient in the RMC expansion is called an RM spectral coefficient and the complete set is called the RM spectrum. Unlike WH matrices, RM matrices are sparse and computation of RM spectral coefficient requires less time. Fault detection in RMC networks is considered in [5, 6].

In Section 3, fault detection is performed by verification of RM spectral coefficients. We identify the RM spectral coefficients (or simply RM coefficients) which should be computed for detection of input stuck-at and bridging faults. The upper bound on the number of spectral coefficients to be verified for detection of all multiple terminal stuck-at-faults (s-a-fs) and all single input bridging faults is shown to be n.

Unlike the WH spectral coefficients, computation of RM coefficients is done in GF(2) and requires only one bit for storing an RM coefficient, hence providing a good data compression (storing of one WH coefficient requires an n-bit storage). We also note that the parity checking methods introduced in [7, 8] are a subset of a broad class of RM spectral techniques introduced in this paper.

In Section 4, we estimate the time and hardware complexities for computing RM coefficients. An upper bound on the number of test patterns for detection of multiple input s-a-fs in combinational networks is shown [9] to be $2n+4$. A standard universal test with $2n-2$ test patterns to detect all multiple terminal s-a-fs in almost all combinational networks is presented in [10]. We show in Section 4.2 that as $n \longrightarrow \infty$ at most $\lceil 0.75n \rceil$ spectral coefficients (or $\lceil 1.25n \rceil$ test patterns) are sufficient for detection of all terminal multiple s-a-fs by verification of RM coefficients and that the time complexity for the corresponding test is $\lceil 1.25n \rceil$ for almost all combinational networks. (These are not standard but device oriented tests.) Time complexities for tests detecting all single and multiple terminal s-a-fs in standard components of computer systems, which include both combinational and sequential circuits, are also presented.

Results presented in Section 4 indicate that both from the point of view of hardware overhead and testing time, techniques based on verification of RM coefficients are more efficient for terminal faults than the techniques described in [1-4].

The problem of detection of stuck-at-faults at internal lines is considered in Section 5. In this section RM transforms are used as analytical tools for deriving relations between the maximum number of distorted RM coefficients as a result of a fault and the minimum number of test patterns. The number of test patterns required for testing a general combinational network is shown to be upper bounded by $\sum_{i=0}^{r} \binom{n}{i}$, if any fault in the network causes distortion of at most $2^{r+1}-1$ RM coefficients.

In Section 6 RM transforms are used for construction of small universal tests for detection of stuck-at and contact faults in PLAs.

Throughout this paper we do not distinguish between stuck-at faults at the different branches of fanouts.

2. REED-MULLER TRANSFORMS AND THEIR PROPERTIES

In this section we formally define RM transforms and provide some of their properties which are akin to classical transforms like Walsh-Hadamard (WH) and Fourier transforms. These properties will be used for fault detection in combinational networks in later sections.

Corresponding to every binary vector ('polarity'), $k = (k_0, k_1, ..., k_{n-1})$, $k_i \in \{0,1\}$, there is an unique representation of a Boolean function in the RMC form:

$$f(x_0, ..., x_{n-1}) = \hat{f}_k(0) \oplus \hat{f}_k(1) x_{n-1}^{k_{n-1}} \oplus \hat{f}_k(2) x_{n-2}^{k_{n-2}} \oplus \ ... \ \oplus \hat{f}_k(2^n-1) x_0^{k_0} ... x_{n-1}^{k_{n-1}} \qquad (1)$$

where $x^0 = x, x^1 = \bar{x}$; $\hat{f}_k(W) \in \{0,1\}$ and the variables associated with $\hat{f}_k(W)$ in (1) are all those variables which correspond to the 1's in $W = (w_0, w_1, .., w_{n-1})$.

Example 1 : *Function* $f(x_0, x_1) = x_0 \vee \bar{x}_1$ *has the following four RM expansions.*

1. $f = 1 \oplus x_1 \oplus x_0 x_1 \quad (k = 0,0)$; 2. $f = \bar{x}_1 \oplus x_0 \oplus x_0 \bar{x}_1 \quad (k = 0,1)$;
3. $f = 1 \oplus \bar{x}_0 x_1 \quad (k = 1,0)$; 4. $f = 1 \oplus \bar{x}_0 \oplus \bar{x}_0 \bar{x}_1 \quad (k = 1,1)$.

Relations between $f(X)$, and $\hat{f}_k(W)$ are given in [11], however, these relations are not convenient for mathematical manipulation. We now define RM expansions as special type of linear transforms (similar to the Walsh-Hadamard [12] or the Discrete Fourier transforms), and investigate their properties useful for fault detection.

Definition. Let $X = (x_0, x_1, .., x_{n-1})$ and $W = (w_0, w_1, .., w_{n-1})$. Then the Reed-Muller (RM) functions are defined as

$$R_X(W) = \textstyle\prod_{i=0}^{n-1} x_i^{w_i}; \qquad (0^0 = 1^0 = 1,\ 0^1 = 0). \tag{2}$$

2.1 Reed-Muller Transforms

The relation between $f(X)$ and $\hat{f}_0(W)$ for 0^{th} polarity is given by

$$\hat{f}_0(W) = \bigoplus_{X=0}^{N} R_W(X)\, f(X) \;=\; \bigoplus_{X \subseteq W} f(X)\,, \tag{3}$$

$$f(X) = \bigoplus_{W=0}^{N} R_X(W)\, \hat{f}_0(W) \;=\; \bigoplus_{W \subseteq X} \hat{f}(W)\,, \tag{4}$$

where $\bigoplus$ represents modulo 2 addition, $N = 2^n - 1$ and $X \subseteq W$ denotes X is a descendant of W ($X \subseteq W$ if and only if $x_i \leq w_i$ for all i). In the case of 0^{th} polarity '0' is dropped from $\hat{f}_0(W)$.

For the generalized RM transform with polarity k,

$$\hat{f}_k(W) = \bigoplus_{X=0}^{N} R_W(X \oplus k)\, f(X)\,, \tag{5}$$

$$f(X) = \bigoplus_{W=0}^{N} R_{X \oplus k}(W)\, \hat{f}_k(W) \tag{6}$$

where $X \oplus k$ represents the componentwise (dyadic) addition.

From now onwards, $\hat{f}(W) \triangleq \hat{f}_0(W)$ is reffered to as an W^{th} RM spectral coefficient and the complete set of coefficients as the RM spectrum. As in the case of fast WH matrices, RM matrices have recursive structure [14] given below.

$$[R]^{(n)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \otimes [R]^{(n-1)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{(n)}\,, \tag{7}$$

where $\otimes$ is the Kronecker product and $[R]^{(n)}$ is the $2^n \times 2^n$ Reed-Muller matrix for the case of 0^{th} polarity. Let $f = [f(0),\, f(1),\, \ldots,\, f(N)]^T$, where T stands for matrix transpose. Then $\hat{f} = [R]^n f$. From (2) it can be easily shown that

$$\bigoplus_{W=0}^{N} R_X(W)\, R_W(Y) = \delta_{X,Y}\ , \tag{8}$$

where $\delta_{X,Y} = 1$, if $X = Y$, $\delta_{X,Y} = 0$ otherwise. From (8)

$$[R]^{(n)} \cdot [R]^{(n)} = [I]\,, \tag{9}$$

where $[I]$ is the identity matrix.

Computation of $\hat{f}$ by (7) requires [15, 16] at most $n.2^{n-1}$ modulo 2 operations and to obtain $\hat{f}_k$, for all $k \in \{0, 1, .., N\}$, N iterations of 2^{n-1} modulo 2 additions need to be performed. The approach in [17] for obtaining $\hat{f}_k$ from $\hat{f}$ is based on folding of the Reed-Muller map introduced in [16]. An efficient matrix multiplication technique for construction of $\hat{f}_k$, $(k = 1, .., N)$ from $\hat{f}$ is presented in [18, 19].

2.2 Some Properties Of RM Spectra

In this subsection we present some properties of RM transforms which will be used in Sections 4-6 for detection of stuck-at and bridging faults. Complete list of RM transform properties and their proofs can be found in [18, 19]. From now on we shall consider only 0^{th} polarity RM spectra, but all the results obtained for the 0^{th} polarity can be easily generalized for any polarity.

From (3) and (4) one can find composite spectra of two functions f and ϕ, namely, $f \oplus \phi$, $f \wedge \phi$, and $f \vee \phi$, where $\vee$ and $\wedge$ denote componentwise OR and AND operations.

1. LINEARITY: Let $\varphi(X) = f(X) \oplus \phi(X)$, then

$$\hat{\varphi}(W) = \hat{f}(W) \oplus \hat{\phi}(W)\,. \tag{10}$$

2. Let $\varphi(X) = f(X) \wedge \phi(X)$, then

$$\hat{\varphi}(W) = \bigoplus_{U \vee V = W} \hat{f}(U).\hat{\phi}(V)\ . \tag{11}$$

3. Let $\varphi(X) = f(X) \vee \phi(X)$, then

$$\hat{\varphi}(W) = \hat{f}(W) \oplus \hat{\phi}(W) \oplus \bigoplus_{U \vee V = W} \hat{f}(U)\, \hat{\phi}(V) \ . \tag{12}$$

4. CONVOLUTION THEOREM: If $\hat{\varphi}(W) = \hat{f}(W) \wedge \hat{\phi}(W)$, then

$$\varphi(X) = \bigoplus_{Y \vee Z = X} f(Y).\phi(Z) \ . \tag{13}$$

It is worth noting that in the convolution theorem (13) the operation between Y and Z is componentwise OR where as for the Walsh-Hadamard transform this operation is componentwise modulo 2 addition.

In the next section we describe fault detection techniques based on verification of a subset of RM spetral coefficients called signature for a device under test (DUT). If any one of the RM spectral coefficients in the signature differs from the corresponding coefficient for the fault-free network, then the DUT is said to be faulty.

3. DETECTION OF INPUT (TERMINAL) STUCK-AT AND BRIDGING FAULTS BY REED-MULLER SPECTRA

Fault models considered by many authors for general combinational networks consist of stuck-at and bridging faults. The inter-connecting lines between integrated chips are often more unreliable than the internal lines and hence some authors have concentrated their efforts in fault detection at input and output lines of a network [2,3,4,9,10]. We will use the following fault models.

Stuck-at-faults (s-a-f): A line h_i in a combinational network is said to be at s-a-0(1) if that line becomes permanently fixed at the binary value 0 (1), as a result of a fault. This fault is denoted by $h_i/0(1)$.

Bridging-fault: If two lines h_i and h_j are shorted and if the resulting signal after the short is an AND operation, then the corresponding AND bridging fault is denoted by $(h_i h_j)_*$. Similarly one can introduce OR bridging $(h_i h_j)_+$.

From now on a function implemented by a faulty device is denoted as f^*. All the theorems and corollaries presented in this section are proved for 0^{th} polarity, but they can be easily shown to be true for any polarity.

The following theorem presents a condition on RM coefficients for detection of s-a-fs at the input lines.

Theorem 1 : *A stuck-at fault at input line x_i of a combinational network implementing $f(x_0,\ x_1,\ \ldots,\ x_{n-1})$ can be detected by verification of $\hat{f}(W)$ if and only if for the fault free network $\hat{f}(W) = 1$, and $W = (w_0, .., w_{i-1}, 1, w_{i+1}, .., w_{n-1})$.*

Proof: From (3)

$$\hat{f}^*(W) = \bigoplus_{X=0}^{N} R_W(X) f^*(X) \ = \ \bigoplus_{X=0}^{N} w_0^{x_0} \ldots 1^{x_i} \ldots w_{n-1}^{x_{n-1}}\ f^*(X).$$

Since $(w_0^{x_0}, .., w_{i-1}^{x_{i-1}}, 1^0, w_{i+1}^{x_{i+1}}, .., w_{n-1}^{x_{n-1}}) \ = \ (w_0^{x_0}, .., w_{i-1}^{x_{i-1}}, 1^1, w_{i+1}^{x_{i+1}}, .., w_{n-1}^{x_{n-1}})$ and

$$f^*(x_0, .., x_{i-1}, 0, x_{i+1}, .., x_{n-1}) \ = \ f^*(x_0, .., x_{i-1}, 1, x_{i+1}, .., x_{n-1}),$$

the above sum modulo 2 over all X will be zero.

Fault detection by parity checking given in [7. 8] computes spectral coefficient $\hat{f}(N) = \hat{f}(2^n - 1)$ for fault detection. and hence parity checking method is a sub-class of RM spectral techniques.

Corollary 1 : *All multiple input s-a-fs are detected by verification of $\hat{f}(1, 1, \ldots, 1)$ if and only if $\hat{f}(1, 1, \ldots, 1) = 1$ for the fault free network.*

From the above theorem and corollary the following theorem is obvious.

Theorem 2 : *For any combinational network all single or multiple terminal s-a-fs can be detected by observing at most n RM coefficients, where n is the number of input lines.*

The following theorem provides a necessary and sufficient condition for detection of input bridging faults.

Theorem 3 : *The AND bridging fault between two input lines x_i and x_j in a combinational network implementing $f(x_0, \ldots, x_{n-1})$ is detected by verification of $\hat{f}(w_0, .., w_{n-1})$ if and only if $\hat{f}(w_0, .., w_{n-1}) = 1$ for the fault free network and $w_i \neq w_j$.*

Proof: From (3)

$$\hat{f}^*(W) = \bigoplus_{X=0}^{N} R_W(X) f^*(X) \;=\; \bigoplus_{X=0}^{N} w_0^{x_0} ... w_i^{x_i} ... w_j^{x_j} ... w_{n-1}^{x_{n-1}}\, f^*(X).$$

Since $w_i \neq w_j$, let $w_i = 0$ and $w_j = 1$. (A similar proof can be used for $w_i = 1, w_j = 0$). Then for AND bridging $(x_i x_j)$,

$$\hat{f}^*(W) = \bigoplus_{X=0}^{N} w_0^{x_0} ... 0^{x_i} ... 1^{x_j} ... w_{n-1}^{x_{n-1}}\, f^*(X) = 0$$

since $(w_0^{x_0}, .., \overset{i}{0^0}, .., \overset{j}{1^1}, .., w_{n-1}^{x_{n-1}}) = (w_0^{x_0}, .., \overset{i}{0^0}, .., \overset{j}{1^0}, .., w_{n-1}^{x_{n-1}})$, and after the fault

$$f^*(x_0, .., \overset{i}{0}, .., \overset{j}{0}, .., x_{n-1}) \;=\; f^*(x_0, .., \overset{i}{0}, .., \overset{j}{1}, .., x_{n-1}).$$

The following theorem combines the above results.

Theorem 4 : *The lower and upper bounds on the minimal number of spectral coefficients $|S_{BF}|$ required to detect all input bridging faults for any combinational network with n input lines is given by*

$$\lceil log_2 n \rceil \;\leq\; |S_{BF}| \;\leq\; n-1$$

Proof: Let $\{\hat{f}(W_1), ..., \hat{f}(W_T)\}$, $T = |S_{BF}|$ and S_{BF} is the minimal set of RM coefficients required for detection of all input bridgings. Construct a $(T \times n)$ binary matrix with rows $W_1, ..., W_T$. Then all columns in the matrix are different and $T \geq \lceil log_2 n \rceil$. The upper bound is obtained directly from Theorem 3 .

The following example illustrates fault detection by verification of RM coefficients.

Example 2 : *Let $f(x_0 x_1 x_2) = x_0 x_1 \vee x_1 x_2 \vee x_0 x_2$, then the spectra of the fault free and* *faulty functions for $x_1/0$ and $(x_0x_1)_*$ are shown below.*

x_0	x_1	x_2	f	$\hat{f}$	$f^*(x_1/0)$	$\hat{f}^*(x_1/0)$	$f^*((x_0x_1)_*)$	$\hat{f}^*((x_0x_1)_*)$
w_0	w_1	w_3						
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	0	0
1	1	0	1	1	0	0	1	1
1	1	1	1	0	1	0	1	0

Faults $x_1/0$ and $(x_0x_1)_*$ may be detected by verification of $\hat{f}^*(0,1,1)$.

Combining Theorems 2 and 3 we have the following result.

Theorem 5 : *For any irredundant combinational network implementing $f(x_0, x_1, \ldots, x_{n-1})$ there exists a set S with at most n RM coefficients, such that all detectable single or multiple terminal s-a-fs and single input AND bridging faults can be detected by verification of $\hat{f}(W)$ for all $W \in S$.*

4. HARDWARE AND TIME COMPLEXITIES FOR DETECTION OF TERMINAL FAULTS BY VERIFICATION OF RM-COEFFICIENTS

In Section 3 we described fault detection by verification of RM coefficients. In this section, hardware and time complexities for computing RM coefficients are given and estimations on the additional hardware required for testing any combinational network are provided. Time complexity for testing all input multiple s-a-fs is shown to be $\lceil 1.25n \rceil$, for almost all networks.

4.1 Hardware Complexity

Testing of a network for any fault involves verification of a set of RM coefficients. Since $\hat{f}^*(W)$ for any W depends on the modulo 2 sum of the output responses of the network, it requires only one T flip-flop to compute $\hat{f}^*(W)$. Inputs to the T flip-flop can be controlled by a NOR gate whose inputs, depending on the RM coefficient being computed, can be determined as follows. We have from (2) and (3)

$$\hat{f}(W) = \bigoplus_{X=0}^{N} w_0^{x_0} \ldots w_i^{x_i} \ldots w_{n-1}^{x_{n-1}} \, f(X)$$

but $w_i^{x_i} = w_i \vee \bar{x}_i = \bar{x}_i$, if $w_i = 0$; and $w_i^{x_i} = 1$ if $w_i = 1$. Therefore

$$\hat{f}(W) = \bigoplus_{X=0}^{N} \prod_{i:w_i=0} \bar{x}_i \, f(X) = \bigoplus_{X=0} \overline{\bigvee_{i:w_i=0} x_i} \, f(X) .$$

For example, let $n = 3$ then

$$\hat{f}(010) = \bigoplus_{X=0}^{N} \bar{x}_0 \bar{x}_2 \, f(X) = \bigoplus_{X=0}^{N} \overline{x_0 \vee x_2} \, f(X)$$

hence the inputs to the NOR gate are x_0 and x_2.

The test circuit for the general case is shown in Fig 1. (If more than one RM coefficient has to be computed, then the corresponding NOR gates and T flip-flops have to be added.) Thus, instead of an n bit counter and a network for coefficient selection as in the case of WH coefficients [4], RM coefficients require only one T flip-flop and a couple of gates. The following theorem provides an estimation on a number of RM coefficients required for detection of all multiple input s-a-fs for almost all networks.

Theorem 6 : *For almost all combinational networks as $n \longrightarrow \infty$ there exist two spectral coefficients which would detect all input s-a-fs with any multiplicity.*

Proof: Consider any coefficient $\hat{f}(W_i)$, then

$$prob \, \{ \hat{f}(W_i) = 1 \} = prob \, \{ \hat{f}(\bar{W}_i) = 1 \} = 0.5 \ \ and$$

$$prob \, \{ \hat{f}(W_i) \neq 1 \ or \ \hat{f}(\bar{W}_i) \neq 1 \} = 0.75 ,$$

Let $\| W_i \|$ denote the number of ones in W. Then we have for the probability that there exists W_i, with $\| W_i \| = \lfloor n/2 \rfloor$, such that $\hat{f}(W_i) \neq 1$ or $\hat{f}(\bar{W}_i) \neq 1$:

$$prob \, \{ (\hat{f}(W_i) \neq 1 \ or \ \hat{f}(\bar{W}_i) \neq 1), \ \| W_i \| = \frac{n}{2} \} = \left(\frac{3}{4}\right)^{\frac{1}{2} \cdot \binom{n}{\lfloor \frac{n}{2} \rfloor}} ,$$

which goes to zero as n increases. Hence only two coefficients are sufficient for almost all combinational networks.

From the above theorem, the additional hardware required for testing almost all combinational networks for input s-a-fs is two T flip-flops, two $\lfloor n/2 \rfloor$ input NOR gates and two 2-input AND gates. When compared to the n bit counter and a coefficient selection network required to compute every WH spectral coefficient [4], hardware required by the RM spectral technique is very small.

4.2 Time Complexity

Time complexity (time required to test the network) is equal to the total time necessary for computing all spectral coefficients of the spectral signature. The time complexity depends on the spectral coefficients being computed. From (3)

$$\hat{f}(W) = \bigoplus_{X=0}^{N} R_W(X) f(X) = \bigoplus_{X \subseteq W} f(X)\,,$$

where $X \subseteq W$ means $x_i \leq w_i$ for all i, and hence the fewer 1's in W the less time required to compute W. The time complexity to compute $\hat{f}(W)$ is

$$\textit{time complexity} \;=\; 2^{\|W\|}\,. \tag{14}$$

The following theorem provides an estimation on the time complexity required for testing almost all combinational networks.

Theorem 7 : *For almost all combinational networks as* $n \longrightarrow \infty$ *the number of RM coefficients required to detect all multiple terminal s-a-fs is equal to* $\lceil 0.75n \rceil$ *and the time complexity is equal to* $\lceil 1.25n \rceil$ *where* n *is the number of input variables.*

Proof is given in the appendix.

We have performed a statistical analysis using computer simulation to substantiate the above theorem and found that the results of the simulation are very close to the results presented in Theorem 7. For simulation purposes 50 randomly generated functions are considered for each value of n. The corresponding optimal RM spectral signatures for these functions are constructed and the time complexities to compute these signatures are found by (14). Average testing time is computed by averaging time complexities for all 50 networks and similarly standard deviation is found. We summarize the computer simulation results in Table 1.

Time complexities for detection of input s-a-fs by RM spectral techniques for some of the standard computer hardware components are presented in Table-2. (In case a device has multiple outputs the EXOR sum of all the outputs have been used for observation).

5. DETECTION OF INTERNAL STUCK-AT-FAULTS

This section deals with detection of internal s-a-fs in combinational networks. Theorem 9 presents an estimation on a number of test patterns to detect any fault, from a given fault set E, depending only on the maximum number of RM coefficients that are distorted by any fault from E. This result is independent of the architecture of the network and the type of fault (s-a-f or bridging fault) and where the fault occurs.

5.1 Detection of Internal Faults by Verification of RM Spectral Coefficients

The following theorem deals with detection of internal faults in a general fanout-free network using RM coefficients.

Theorem 8 : *For any fanout-free network with n primary inputs there exists a set of at most n RM coefficients which detects all multiple input s-a-fs and all single internal s-a-fs.*

Proof: Since the network is fanout-free there are exactly n paths from the inputs to the output. Consider a path p_i starting from input line x_i. Let X_i be the input vector which sensitizes path p_i such that $\| X_i \|$ is minimal and $f(X_i) = 1$. From (3), $\hat{f}(X_i) = 1$ and if any line h_i on the path p_i is at s-a-0 or s-a-1 then $\hat{f}^*(X_i) = 0$ detecting the fault.

The following results are concerned with detection of internal s-a-0 faults in networks implemented by AND-OR two level structures. The s-a-1 faults in AND-OR networks are detected by verification of $\hat{f}^*(W)$ for any $\hat{f}(W) = 1$ and $W \neq (0,0,...,0)$.

Lemma 1 : *If in an irredundant AND-OR network implementing a Boolean function on n variables, all AND gates have n inputs, then only one RM coefficient $\hat{f}(1,1,...,1)$ is sufficient for detection of all single internal s-a-0 faults.*

Proof: Clearly

$$\hat{f}(1,1,..,1) = \bigoplus_{X=0}^{N} f(X)$$

is a parity function and any single internal s-a-0 fault changes the parity detecting the fault.

Corollary 2 : *For any threshold function*

$$f(X) = \begin{cases} 1, & if \ \sum_{i=0}^{n-1} x_i \ \geq \ t\,; \\ 0, & if \ \sum_{i=0}^{n-1} x_i \ < \ t\,, \end{cases}$$

implemented by a two level AND-OR network only one RM coefficient is sufficient to detect all single internal s-a-0 faults.

Proof: Every product term in f consists of exactly t variables. If any AND gate in an AND-OR structure realizing a product term, say $p_i = x_1 x_2 ... x_t$, is s-a-0, then only one $f(X_i)$, where $X_i = (x_1, x_2, .., x_t, 0, .., 0)$ will be zero and RM coefficient $\hat{f}^*(1,1,...,1)$ will be distorted. Hence $\hat{f}(1,1,...,1)$ would detect all single s-a-0 faults.

5.2 Estimations on Number of Test Patterns for Detection of Internal Stuck-at Faults

Let E be a set of faults in a given network. For $e \in E$ denote by $A_k(e)$ a number of coefficients in the RM expansion with polarity k, which are distorted due to the fault e. Denote also

$$A_k \ = \ max \ A_k(e) \ \ for \ all \ \ e \in E\,.$$

We present an upper bound on A_k in Lemmas 2 - 4. A relation between A_k and the minimal number of test patterns to detect all faults in E is given in Theorem 9. The corresponding tests T have the following standard structure

$$T = T_r = \{X \mid \ d(\bar{k}, X) \ \leq \ r\},$$

where r depends on A_k and $d(\bar{k}, X)$ is the Hamming distance between X and $\bar{k} = (\bar{k}_0, \bar{k}_1, ..., \bar{k}_{n-1})$. This standard structure of test sets results in a simple test generation procedure.

The following three lemmas provide estimations on a number of RM coefficients that may be distorted when the gate implementing a product term is faulty. The estimations are provided on the assumption that a function is realized as a sum of product terms. Since s-a-1 faults at the outputs of AND gates can be easily detected, only s-a-0 faults are considered. It is shown that the number of RM coefficients that may be distorted by the s-a-0 fault at the output of an AND gate is directly proportional to the number of variables appearing in the negated form in the product term in 0^{th} polarity spectra. General result for any polarity k is presented in Lemma 2.

Lemma 2 : *Suppose $f(x_0, \ldots, x_{n-1})$ is implemented by a 2-level AND-OR network. If the output of an AND gate implementing a product term $P = \dot{x}_0, \ldots, \dot{x}_{n-1}$, $\dot{x}_i \in \{x_i, \bar{x}_i, d\}$ (d stands for "don't care") is s-a-0, then the number A_k of RM coefficients in $\hat{f}_k$, $k \in \{0, 1, \ldots, N\}$, that may be distorted by the fault is upper bounded by*

$$A_k \leq 2^{n - \|p \oplus k\|}, \tag{15}$$

where $k = (k_0, \ldots, k_{n-1})$ is the polarity of the RM transform and p is obtained from P by replacing x_i by 1 and $\bar{x}_i$ by 0 in P and $\| p \oplus k \|$ is the number of 1's in $p \oplus k$ $(d \oplus k_j = 0,\ k_j \in \{0, 1\}$.)

Proof: From (5)

$$\hat{f}_k(W) = \bigoplus_{X=0}^{N} R_W(X \oplus k)\, f(X) = \bigoplus_{X \oplus k \subseteq W} f(X).$$

Clearly, if $\| X \oplus k \| = m$, then there exists 2^{n-m} W's which satisfy $X \oplus k \subseteq W$. Now if $\hat{f}_k(W) \neq \hat{f}_k^*(W)$ for a given W, then there exists an X such that $X \oplus k \subseteq W$ and $P(X) = 1$. Without loss of generality, suppose that

$$P = x_0 x_1 \ldots x_{i-1} \bar{x}_i \ldots \bar{x}_{j-1} \quad \Longrightarrow \quad p = \overbrace{11\ldots1}^{i}\overbrace{00\ldots0}^{j-i}\overbrace{dd\ldots d}^{n-j} = p_0 p_1 \ldots p_{n-1}.$$

If $s \leq j-1$ and $k_s \oplus p_s = 0$, then for all $W = (w_0, w_1, \ldots, w_{n-1})$ with $w_s = 0$ or $w_s = 1$, $\hat{f}(W)$ may be distorted by P s-a-0. If $k_s \oplus p_s = 1$, then only $\hat{f}(W)$ with $w_s = 1$ may be distorted by P s-a-0. If $s > j-1$, then for all $W = (p_0 \oplus k_0, p_1 \oplus k_1, \ldots, p_{j-1} \oplus k_{j-1}, w_j, \ldots, w_{n-1})$ with $w_s = 0$ or $w_s = 1$, $\hat{f}(W)$ may be distorted by P s-a-0. Thus, we have for a total number of RM coefficients that may be distorted by the fault

$$A_k \leq 2^{j - \|p \oplus k\| + n - j} = 2^{n - \|p \oplus k\|}$$

In particular if $k = 0$, the number of RM coefficients that may be affected by a s-a-0 fault at the output of an AND gate realizing P is upper bounded by $2^{n-\|P\|}$, where $\| P \|$ is the number of variables appearing in P in their true form.

Lemma 3 : *For a two-level AND-OR network if $f = \bigvee_{j=1}^{N_p} P_j$, where $P_j \wedge P_s = \emptyset$ for all j, s the number of RM coefficients A $(k = 0)$ that may be distorted by a s-a-0 at the output of an AND gate realizing P_i is upper bounded by*

$$A \leq \ max\ 2^{|P_i| - \|P_i\|} = \ max\ 2^{\text{no. of negated variables in } P_i}, \quad for\ all\ i.$$

where $|P_i|$ denotes the number of variables in P_i and $\| P_i \|$ denotes the number of variables present in true form in P_i.

Proof: Since $P_i \wedge P_j = \emptyset$, for all i, j, function f can be expressed as $\bigoplus_{i=1}^{N_p} P_i$, and from (10) we have

$$\hat{f} = \bigoplus_{i=1}^{N_p} \hat{P}_i .$$

Clearly, if P_i is s-a-0 then from the above formula the number of RM coefficients that may be distorted will be equal to $| \hat{P}_i |$. Now we will estimate $| \hat{P}_i |$. Without loss of generality, let $P_i = \bar{x}_0 ... \bar{x}_{i-1} x_i ... x_{j-1} dd..d$. From (3) we have

$$\hat{P}_i(W) = \bigoplus_{X \subseteq W} P_i(X) .$$

For any vector $W = \overbrace{0, 0, ..., 0}^{i}, \overbrace{1, 1, ..., 1}^{j-i}, d, d, ..., d$, there are even number of X's such that $X \subseteq W$ and $P_i(X) = 1$ and hence $\hat{P}_i(W) = 0$. Now, for any vector $W = \overbrace{d, d, ..., d}^{i}, \overbrace{1, 1, ..., 1}^{j-i}, 0, 0, ..., 0$, there is only one $X \subseteq W$, such that $P_i(X) = 1$ and hence $\hat{P}_i(W) = 1$. Since there are $2^{|P_i| - \|P_i\|}$ W's, such that $\hat{f}(W)$ is distorted by the fault and $A = 2^{|P_i| - \|P_i\|}$.

In Lemma 3 product terms are disjoint. The following lemma generalizes the results of Lemma 3 for any two-level AND-OR network.

Lemma 4 : *For a two-level AND-OR network if $f = \bigvee_{j=1}^{N_p} P_j$, then the number of RM coefficients A that may be distorted by s-a-0 at the output of an AND gate realizing P_i*

is at most

$$A \leq \sum_{j=1}^{m} 2^{|P^f_{i_j}| - \| P^f_{i_j} \|} = \sum_{j=1}^{m} 2^{\textit{no. of negated variables in } P^f_{i_j}} \tag{16}$$

where $P^f_i = \{X \mid P_i(X) = 1,\ P_j(X) = 0 \ \mathit{if}\ j \neq i\} = \bigcup_{l=1}^{m} P^f_{i_l}$
and $P^f_{i_j} \cap P_l = \emptyset$, $P^f_{i_j} \cap P^f_{i_q} = \emptyset$ *for all* i, j, l *and* q.

Proof follows directly from Lemma 3 .

In the next couple of paragraphs we recall some results from coding theory which we use for derivation of a relation between A and the number of test patterns required to test a fault, which distorts atmost A RM coefficients.

Consider the vectors $v_0, v_1, v_2, \ldots, v_n$ of length 2^n and all products of r or fewer vectors from $\{v_1, v_2, \ldots, v_n\}$ at a time which form the basis for Reed-Muller codes of the r^{th} order [21]. They can be arranged in a matrix form V, with $\sum_{i=0}^{r} \begin{pmatrix} n \\ i \end{pmatrix}$ rows and 2^n columns. It can be easily seen that the set of rows of V is the subset of the set of rows in $[R]^{(n)}$ (see (7)). Hence, from (4), components of $V.\hat{f}$ are values of $f(X)$ for $\| X \| \geq n - r$.

For example, for $n = 3$ and $r = 2$

$$V.\hat{f} = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_1 v_2 \\ v_1 v_3 \\ v_2 v_3 \end{bmatrix} .\hat{f} = \begin{bmatrix} 1111 & 1111 \\ 1111 & 0000 \\ 1100 & 1100 \\ 1010 & 1010 \\ 1100 & 0000 \\ 1010 & 0000 \\ 1000 & 1000 \end{bmatrix} \begin{bmatrix} \hat{f}(000) \\ \hat{f}(001) \\ \hat{f}(010) \\ \hat{f}(011) \\ \hat{f}(100) \\ \hat{f}(101) \\ \hat{f}(110) \\ \hat{f}(111) \end{bmatrix} = \begin{bmatrix} f(111) \\ f(011) \\ f(101) \\ f(110) \\ f(001) \\ f(010) \\ f(100) \end{bmatrix}$$

The following theorem gives an estimation on a number of test patterns required to test a network.

Theorem 9 : *If any single or multiple fault in a network implementing f causes distortion of at most A spectral coefficients in the RM expansion with $k = 0$, then a minimal number of test patterns required to detect all single or multiple faults in the network is* *upper bounded by*

$$\sum_{i=0}^{\lceil log_2(A+1)\rceil - 1} \binom{n}{i}. \tag{17}$$

Proof: Let us take as test patterns all X's such that $\| X \| \geq n - r$, where $r = \lceil log_2(A+1) \rceil - 1$. It is known from coding theory [20], that any $2^{r+1} - 1$ columns in V are linearly independent, since V is the matrix whose rows are basis vectors of the r^{th} order Reed-Mulller code and hence $2^{r+1} - 1$ errors in $\hat{f}$ can be detected by the code with check matrix V. Computation of r and the number of rows in V yields (17).

Example 3 *: Let f is a 9 or more out of 12 function, ie.,*

$$f(X) = \begin{cases} 1, & if \quad \| X \| \geq 9\,; \\ 0, & otherwise. \end{cases} \qquad (n = 12)$$

It is clear that every product term P_i consists of nine variables and P_i^l (see Lemma 4) consists of 9 variables in direct form and 3 variables in negated form. If any single fault affects at most one product term then by Lemma 4, with $m = 1$ we have $A = 2^3$, $r = 3$ and a number of test patterns required to test stuck-at-0 faults at outputs of AND gates of the network is at most

$$T_3 = \sum_{i=0}^{3} \binom{n}{i} = 299.$$

In general, for a given $k \neq 0$ the set of test patterns forms the Hamming ball of radius r with center $\bar{k}$, which results in a simple test generation procedure. (It is easy to show [19] that a gate count for the corresponding test generator in this case is proportional to rn).

6. DETECTION OF STUCK-AT AND CONTACT FAULTS IN PLA's:

In this section we consider the problem of stuck-at and contact faults in PLAs. We establish that if a test set T_r, described in Section 4, detects all s-a-0 faults at the outputs of AND gates (AND array), then the test set T_{r+1} would detect all contact faults. An upper bound on the number of test patterns required for detection of all single s-a-fs and contact faults (crosspoint) is presented in Theorem 10.

A design for PLAs with universal test set was presented in [22] and the number of test patterns is shown to be of order $n + N_p$, where n is the number of input lines and N_p is the number of product terms. A testable design for PLAs is considered in [23] and design for self checking PLA is presented in [24]. In both the designs it is assumed that the product terms are mutually disjoint.

We assume that product terms are mutually disjoint (this assumption has been widely used in literature on PLA testing [22-24]), and show in this case that PLAs can be tested for all single cross point faults and internal s-a-fs by small universal test sets.

The following fault model will be considered.

Contact or Crosspoint faults: These faults are classified as follows.

- **Shrinkage fault:** An erroneous cantact in the AND array causes a prime implicant to gain a literal and thus include either half of its original minterms or none.
- **Growth fault:** A missing contact in the AND array causes a prime implicant to loose a literal and hence include twice as many minterms.
- **Disappearance fault:** A missing contact in the OR array causes a prime implicant to be dropped from the corresponding output function.
- **Appearance fault:** An erroneous contact in the OR array causes a prime implicant to be added to the corresponding output function.

In the following lemma an estimation on the number of RM coefficients that may be distorted by a contact fault is presented. This estimation will be used for finding an upper bound on a minimal number of test patterns using Theorem 9.

Lemma 5 *: Any contact fault may result in distortion of at most max* $2.2^{|P_i|-\|P_i\|}$*, for all* i*, RM coefficients, if* $P_i \wedge P_j = \emptyset$*, for all* $i, j \neq i$*, where* $|P_i|$ *denotes the number of variables present in* P_i *and* $\|P_i\|$ *denotes the number of variables present in* P_i *in the true form.*

Proof: i. Shrinkage fault: Without loss of generality let $P_i = x_0 \ldots x_{a-1}\bar{x}_a \ldots \bar{x}_{a+b-1}$. Then due to a shrinkage fault P_i is replaced by $P_i' = x_0 \ldots x_{a-1}\bar{x}_a \ldots \bar{x}_{a+b-1}x_{a+b+c}$ or $P_i'' = x_0 \ldots x_{a-1}\bar{x}_a \ldots \bar{x}_{a+b-1}\bar{x}_{a+b+c}$ for some $c > 0$. Since $P_i \wedge P_j = \emptyset$, for all i, j, it can be easily shown that $P_i' \wedge P_j = \emptyset$, and $P_i'' \wedge P_j = \emptyset$ for all $j \neq i$. If $f = P_1 \vee \ldots \vee P_{N_p}$, then the faulty function f^* can be represented as

$$f^* = P_1 \oplus \ldots \oplus P_{i-1} \oplus P_i' \oplus P_{i+1} \oplus P_{N_p} \quad or \quad f^* = P_1 \oplus \ldots \oplus P_{i-1} \oplus P_i'' \oplus P_{i+1} \oplus P_{N_p},$$

But $P_i' = P_i \oplus \bar{x}_{a+b+c}P_i$ and $P_i'' = P_i \oplus x_{a+b+c}P_i$. Hence

$$f^* = f \oplus \bar{x}_{a+b+c}P_i \;\; or \;\; f^* = f \oplus x_{a+b+c}P_i \; .$$

Thus, by (10) the number of RM coefficients that may be affected by a shrinkage fault is equal to a maximal number of coefficients in RM expansions for product terms P_i' and P_i''. From Lemma 3 these numbers are upper bounded by

$$max \; 2^{|\bar{x}_{a+b+c}P_i| - \|\bar{x}_{a+b+c}P_i\|} = max \; 2.2^{|P_i| - \|P_i\|} \; , \quad for \; all \; i.$$

Hence, a maximum number of RM coefficients that may be distorted by a shrinkage fault in product term is upper bounded by $max\, 2.2^{|P_i| - \|P_i\|}$ for all i

ii. Growth fault: In this case it can be easily shown that the maximum number of RM coefficients that may be distorted by a growth fault as $max \frac{1}{2} 2^{|P_i| - \|P_i\|}$, for all i.

Disappearance or appearance fault: In this case a product term is dropped or added to a function. In either case the number of RM coefficients that may be distorted is upper bounded by $max 2^{|P_i| - \|P_i\|}$, for all i.

Hence, the maximum number of RM coefficients that may be distorted by a contact fault is upper bounded by $max 2.2^{|P_i| - \|P_i\|}$, for all i.

From Theorem 9 and Lemma 5, a number of test patterns required to detect all single stuck-at-0 faults at outputs of AND gates, and all single contact faults is at most $\sum_{i=0}^{r+1} \begin{pmatrix} n \\ i \end{pmatrix}$, where $r = max \; (| \; P_i \; | \; - \; \| \; P_i \; \|)$ for all i. The corresponding test $T_{r+1} = \{X \mid \| X \| \geq n - r - 1\}$ contain all vectors with at most $r + 1$ zeros.

Every test pattern which detects the s-a-0 fault at the output of an AND gate also detects $x_i/0$ $(x_i/1)$ if x_i is an input to this AND gate and x_i appears in its true (negated) form in the corresponding product term. Thus, a test set which detects all single s-a-0 faults at outputs of AND gates also detects n input stuck-at faults (out of $2n$ possible input stuck-at faults). Hence, at most n more additional test patterns are required to detect all single input stuck-at faults. Similarly, at most q additional test patterns are required to detect stuck-at-1 faults at outputs of AND gates and all output stuck-at-1 $(f_i/1)$, where q is the number of outputs in a PLA under test.

These results are summarized in the following theorem.

Theorem 10 : *For any PLA with n input variables, with $P_i \wedge P_j = \emptyset$ for all i, j and q output functions, the number of test patterns required to detect all single stuck-at faults and all single contact faults is upper bounded by*

$$\sum_{i=0}^{r+1} \binom{n}{i} + n + q$$

where $r = max\ (| P_i | - \| P_i \|)$ for all i.

Example 4 : *Let $f_1 = P_1 \vee P_2 \vee P_3 \vee P_4 = \bar{x}_0 x_1 x_2 x_4 \vee \bar{x}_0 x_1 x_3 x_4 \vee x_0 x_1 x_2 \bar{x}_3 \vee x_0 x_1 \bar{x}_2 x_4$, $f_2 = P_5 \vee P_6 \vee P_7 \vee P_8 = \bar{x}_0 x_2 x_3 x_4 \vee \bar{x}_0 x_1 x_2 x_3 \vee x_0 \bar{x}_1 x_3 x_4 \vee x_0 x_2 x_3 \bar{x}_4$, and $f_3 = P_1 \vee P_3 \vee P_5 \vee P_7$.*

Though P_1, P_2, P_5, P_6 are not disjoint. from Lemma 4 we have $| P_i' | - \| P_i' \| = 1$ for all $i \in \{1, 2, 5, 6\}$. where P_i' is defined as in Lemma 4 and $| P_j | - \| P_j \| = 1$ for all $j \in \{3, 4, 7, 8\}$. hence $r = 1$. the number of test patterns required for detection of all single contact faults is $| T_2 | = \sum_{i=0}^{r+1} \binom{5}{i} = 16$ and the test set T_2 consisting of all vectors with at most two zeros can be used for testing of this PLA. Since every variable is present in direct and negated forms. all input stuck-at faults are detected by T_2. It is also easy to check that all ouput stuck-at faults are detected by T_2. Thus. T_2 with 16 test patterns can be used for testing of all single stuck-at and contact faults in this PLA.

APPENDIX

Proof of Theorem 7: The proof consists of two parts.

i). It will be shown that there exist B_1 and B_2 such that

$$\| \bigvee_{W \in B_1 \cup B_2} W \| = n \, ,$$

$$B_1 = \{W \mid \| W \| = 1,\ W_i \wedge W_j = (0, 0, ..., 0),\ \| \bigvee_{W \in B_1} W \| = \lfloor n/2 \rfloor\},$$

$$prob\ \{\hat{f}(W) = 1\ \ for\ all\ W \in B_1\} \longrightarrow 1\ as\ n \longrightarrow \infty$$

and

$$B_2 = \{W \mid \| W \| = 2,\ W_i \wedge W_j = (0, 0, ..., 0),\ \| \bigvee_{W \in B_2} W \| = \lceil n/2 \rceil\},$$

$$prob\ \{\hat{f}(W) = 2 \ \ for\ all\ W \in B_2\} \longrightarrow 1\ as\ n \longrightarrow \infty$$

ii). From i) it will be shown that Theorem 7 is true.

i): First we note that $prob\ \{\hat{f}(W) = 1\ \ \} = 0.5$ for any W. Hence, with the probability converging to 1 as $n \longrightarrow \infty$ there exists a set B_1 of $\lfloor n/2 \rfloor$ RM coefficients $\hat{f}(W)$ with $\| W \| = 1$ which are sufficient for detection of stuck-at faults at the corresponding $\lfloor n/2 \rfloor$ input lines. (If $W_i = (0,0,...,0,\overset{i}{1},0,...,0)$. and $\hat{f}(W_i) = 1$. then by Theorem 1 the stuck-at fault at input line x_i can be detected by verification of $\hat{f}(W_i)$).

Stuck-at faults at the remaining $\lceil n/2 \rceil$ input lines will be detected by verification of $\hat{f}(W)$ where $\| W \| = 2$. It will now be shown that such a set of W exists with probability converging to 1 as $n \longrightarrow \infty$. Let us denote $\lceil n/2 \rceil = m$.

Consider B_2 such that for any $W \in B_2$, $\| W \| = 2$.

$\| \bigvee_{W \in B_2} W \| = m$ and for any $W_i, W_j \in B_2$, $W_i \wedge W_j = (0,0,...,0)$.

Then

$$prob\ \{\hat{f}(W) = 1\ for\ all\ W \in B_2\} \geq 2^{-m/2}$$

Let $P = \{B_2\}$, the set of all B_2 satisfying the above definition. Then

$$prob\ \{\exists\ B_2 \in P\} \geq 1 - (1 - 2^{-m/2})^Q$$

where $Q = | P |$, and

$$Q \ \leq \ \left(\lfloor \frac{m}{2} \rfloor !\right)^{-1} \binom{m}{2} \binom{m-2}{2} \cdots \binom{2}{2} = \left(\lfloor \frac{m}{2} \rfloor !\right)^{-1} \frac{m!}{2^{m/2}}.$$

Using Sterling's formula one can get $Q \simeq m^{m/2} e^{-m/2} \sqrt{2}$, and

$$\lim_{m \to \infty} (1 - 2^{-m/2})^Q = 0 \ \ iff \ \ \frac{Q}{2^{m/2}} \to \infty\ as\ \ m \to \infty\ .$$

Clearly

$$\frac{Q}{2^{m/2}} = \frac{m^{m/2}\sqrt{2}}{e^{m/2}\, 2^{m/2}} \to \infty\ as\ \ m \to \infty\ .$$

This implies that there exists at least one set B_2 which is sufficient for detection of stuck-at faults at the remaining $\lceil n/2 \rceil = m$ input lines.

ii). The total number of coefficients to be observed is $| B_1 | + | B_2 | \leq \lfloor n/2 \rfloor + \lceil \frac{1}{2} \lceil n/2 \rceil \rceil \simeq$

$3n/4$. Hence $3n/4$ memory cells are sufficient to store these coefficients. The time complexity required to compute these RM coefficients can be estimated as follows. Suppose

$$W_i = (0,0,...,1,...,0).\, Then\ \hat{f}(W_i) = \begin{cases} f(0,...,1,...,0), & if \quad f(0,0,...,0)=0; \\ 1 \oplus f(0,...,1,...,0), & if \quad f(0,0,...,0)=1. \end{cases}$$

In both cases only $(0,...,\overset{i}{1},...,0)$ has to be applied to the network and the ouput of the T flip flop or its complement is taken as $\hat{f}(0,...,1,...,0)$. Since there are $\lfloor n/2 \rfloor$ RM coefficients with $\| W \| = 1$ only $\lfloor n/2 \rfloor$ steps are required. Similarly, for the case when $\| W \| = 2$, 3n/4 steps are required. Thus, the time complexity for detection of all multiple terminal s-a-fs is 1.25n .

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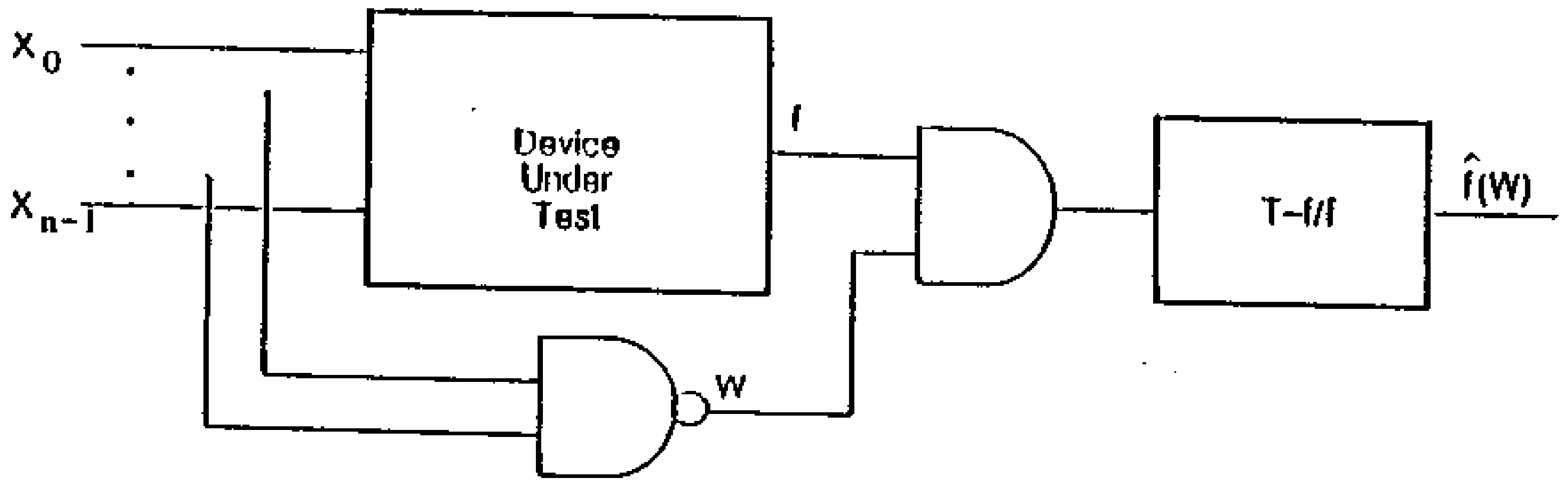
X_0
X_{n-1}
Device Under Test
f
W
T-f/f
$\hat{f}(W)$

Figure 1: Network for computing RM spectral coefficient.

n	Av. testing time	Std. dev.
8	11.96	1.28
16	22.54	1.66
24	32.12	1.33
32	42.22	1.56

Table 1: Average Testing times for detection of all input faults.

S.No.	*Function*	*time complexity*
1.	*parity checker* $f(X) = x_0 \oplus ... \oplus x_{n-1}$	$2n$
2.	*OR function* $f(X) = x_0 \vee ... \vee x_{n-1}$	$2n$
3.	*NOR function* $f(X) = \overline{x_0 \vee ... \vee x_{n-1}}$	$2n$
4.	*Quadratic function* $f(x) = x_0x_1 \oplus ... \oplus x_{n-2}x_{n-1}$	$2n$
5.	*Match detector* $f(XY) = 1 \; iff \; X = Y$	$2n$
6.	*Threshold detector* $(t = 2)$; $f(X) = 1 \; iff \; \lVert X \rVert = 2$	$2n$
7.	$f(X) = S_2(X) = 1 \; iff \; \lVert X \rVert = 2$	$2n$
8.	*Threshold function* $(t = c)$; $f(X) = 1 \; iff \; \lVert X \rVert = c$	$\lceil \frac{n}{c} \rceil 2^c$
9.	$f(X) = S_c(X) = 1 \; iff \; \lVert X \rVert = c$	$\lceil \frac{n}{c} \rceil 2^c$
10.	*ANDfunction* $f(X) = x_0 ... x_{n-1}$	2^n
11.	*NAND function* $\overline{x_0 ... x_{n-1}}$	2^n
12.	*comparator* $X \leq Y \Rightarrow 1$	$3n$
13.	*Multiplexer* $f(y_0, ..., y_{2^n-1}, x_0 ..., x_{n-1}) = y_i \; iff \; X = i$	$\sum_{i=0}^{n-1} \begin{pmatrix} n \\ i \end{pmatrix} 2^{i+1}$
14.	$f(X) = X^i$	$n + 1$
15.	$f(X) = X^i + 1$	$n + 2$
16.	$f(X) = X + 1$	$n + 2$
17.	$f(X, Y) = X + Y$ *(adder)*	$n + 1$
18.	$f(X, Y) = X.Y$ *(multiplier)*	$1.5n + 1$
19.	$\odot$ *shift left/right* $\left\{ \begin{array}{l} f(X) = (x_1, ..., x_{n-1}, x_0) \\ f(X) = (x_{n-1}, x_0, ..., x_{n-2}) \end{array} \right.$	$n + 1$
20.	*shift left/right* $\left\{ \begin{array}{l} f(X) = (x_1, ..., x_{n-1}, 0) \\ f(X) = (0, x_0, ..., x_{n-2}) \end{array} \right.$	$n + 2$

($\lceil m \rceil$ denotes the least integer greater or equal to m).

Table 2: Time Complexities For Standard Components For Detection Of Input S-A-Fs With Any Multiplicity