Detection and Location of Input and Feedback Bridging Faults Among Input and Output Lines

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Abstract—**The study of short circuits between conducting paths (bridging faults) has become increasingly important. Yet very little work has been done in this area. In this paper, conditions for feedback bridging (short circuit) faults to generate oscillation and asynchronous behavior are given for short circuits among input lines and the primary output. The lower and upper bounds on the number of tests for detecting all feedback bridging faults are given. Necessary and sufficient conditions for the undetectability of input bridgings are presented. It is found that any test detecting single bridging fault *e* will also detect all multiple bridgings containing *e*. Complete test sets for locating either all input or all feedback bridgings of any multiplicities for networks implementing several classes of functions are given.**

Index Terms—**Asynchronous behavior, bridging faults, combinational networks, fault detection, fault location, multiple faults, oscillation, short circuit failures, single faults, test generation, undetectability.**

I. Introduction

The diagnosis of digital systems has become increasingly important in recent years. Unfortunately, most of the published research papers in the area of testing digital networks deal only with the stuck-at faults. A bridging (short circuit) fault is a fault in which two or more leads in the circuit are shorted together or, in other terms, wired together. There are few papers in the area of bridging faults [1]-[8]. This is partly because the research work on this topic became active only recently, and the treatment of bridging faults is much more complex than the treatment of stuck-at faults. Two types of bridging faults are considered in the literature—namely, the AND-type and OR-type bridging faults. The AND and OR types of bridging faults mean that two or more lines are short circuited to form AND and OR logical operations. The available techniques for bridging fault detection have been approached through the existing procedures for testing stuck-at faults since a great deal of work has already been reported on stuck-at faults.

II. Detection of Bridging Faults

In this section, we shall first present our results on the detection of feedback bridging (short circuit) faults between the primary output and the primary inputs in a single-output logic network. Then the results on the detection of bridging faults among the primary inputs shall be given. Only the AND-type bridgings will be considered here since the results may easily be modified for the OR-type bridgings.

Instead of considering the bridgings between two lines, we shall consider the general case of the bridging among the primary output and s primary input lines, called feedback bridging of multiplicity s. Similarly, an input bridging among s input lines is called an input bridging of the multiplicity s. Without loss of generality, we assume that for a network implementing function $F(x_1, x_2, \cdots, x_n)$, if the s input lines which are bridged together (either with the primary output or among themselves only) are known, then these lines are $x_1, x_2, \cdots, x_s$. $(Yx_1x_2\cdots x_s)$ and $(x_1x_2\cdots x_s)$ denote these feedback and input bridgings of multiplicity s, respectively.

A. Detection of Feedback Bridgings

Let us consider a combinational network implementing $F(x_1, x_2, \cdots, x_n)$. If the AND-type bridging fault exists between the primary output and s input lines $x_1, x_2, \cdots, x_s$, then the faulty primary output

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Y_e is equal to the AND function of the original output of the network and $x_1, x_2, \cdots, x_s$.

Each one of the first s primary inputs becomes $Yx_1x_2\cdots x_s$. This can be represented by the model shown in Fig. 1. Such a model will be used throughout the paper for feedback bridging faults.

The following definitions can be found in [8].

Definition 1: A circuit *oscillates* under a certain input combination (pattern) if the output of the circuit at the next instant is the complement of the current output, i.e., $Y^i = \overline{Y^{i-1}}$ where Y^i is the output at time i.

Definition 2: A circuit has *asynchronous behavior* under a certain input combination if the circuit is stable and the present output is a function of its previous inputs and $Y^i = Y^{i-1}$.

Theorem 1: Under feedback bridging $(Yx_1x_2\cdots x_s)$, any network N implementing $F(x_1, x_2, \cdots, x_n)$ oscillates if the binary input n-tuple $(x_1, \cdots, x_n)$ satisfies the following condition:

The following theorem is the generalization of Theorem 1 of [8] or Theorem 13 of [10].

$$x_1x_2\cdots x_sF(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) \times \overline{F}(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n) = 1; \quad (1)$$

N will have asynchronous behavior if

$$x_1x_2\cdots x_s\overline{F}(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) \times F(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n) = 1. \quad (2)$$

Proof:

i) Performing the Shannon expansion of $F(x_1, \cdots, x_n)$ s times $(1 \le s \le n)$, we obtain

$$\begin{aligned} F(x_1, \cdots, x_n) &= \overline{x}_1\overline{x}_2\cdots\overline{x}_sF(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) \\ &\quad + \overline{x}_1\overline{x}_2\cdots\overline{x}_{s-1}x_sF(0, 0, \cdots, 0, 1, x_{s+1}, \cdots, x_n) \\ &\quad + \cdots\cdots \\ &\quad + x_1x_2\cdots x_sF(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n). \end{aligned} \quad (3)$$

To obtain the equation for the network with a feedback bridging of multiplicity s, we substitute each $x_j (j = 1, 2, \cdots, s)$ by $Y^{i-1}x_1x_2\cdots x_s$:

$$Y^i = \overline{Y^{i-1}x_1x_2\cdots x_s}F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) + Y^{i-1}x_1x_2\cdots x_sF(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n). \quad (4)$$

From (1), we obtain

$$x_1x_2\cdots x_s = 1,$$

$$F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) = 1,$$

and

$$F(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n) = 0.$$

Substituting the above three equations into (4), we obtain $Y^i = \overline{Y^{i-1}}$; thus, the network oscillates.

ii) Substituting conditions generated from (2) into (4), we obtain $Y^i = Y^{i-1}$; hence, the circuit has asynchronous behavior.

Corollary 1: Under feedback bridging (Yx_1), any network N implementing $F(x_1, x_2, \cdots, x_n)$ oscillates if the binary input n-tuple $(x_1, x_2, \cdots, x_n)$ satisfies

$$x_1\overline{F}(1, x_2, \cdots, x_n)F(0, x_2, \cdots, x_n) = 1; \quad (5)$$

N has asynchronous behavior if

$$x_1F(1, x_2, \cdots, x_n)\overline{F}(0, x_2, \cdots, x_n) = 1. \quad (6)$$

Example 1: In Fig. 2, $F = \overline{x_1x_2x_3}x_4 + x_4x_5x_6$. Since

$$F(0, 0, 1, 1, 0, 0) = 1$$

$$F(1, 1, 1, 1, 0, 0) = 0,$$

(1) is satisfied if $(x_1, x_2, x_3, x_4, x_5, x_6) = (1, 1, 1, 1, 0, 0)$ for $s = 2$ and bridging (Yx_1x_2). Therefore, the network oscillates when the

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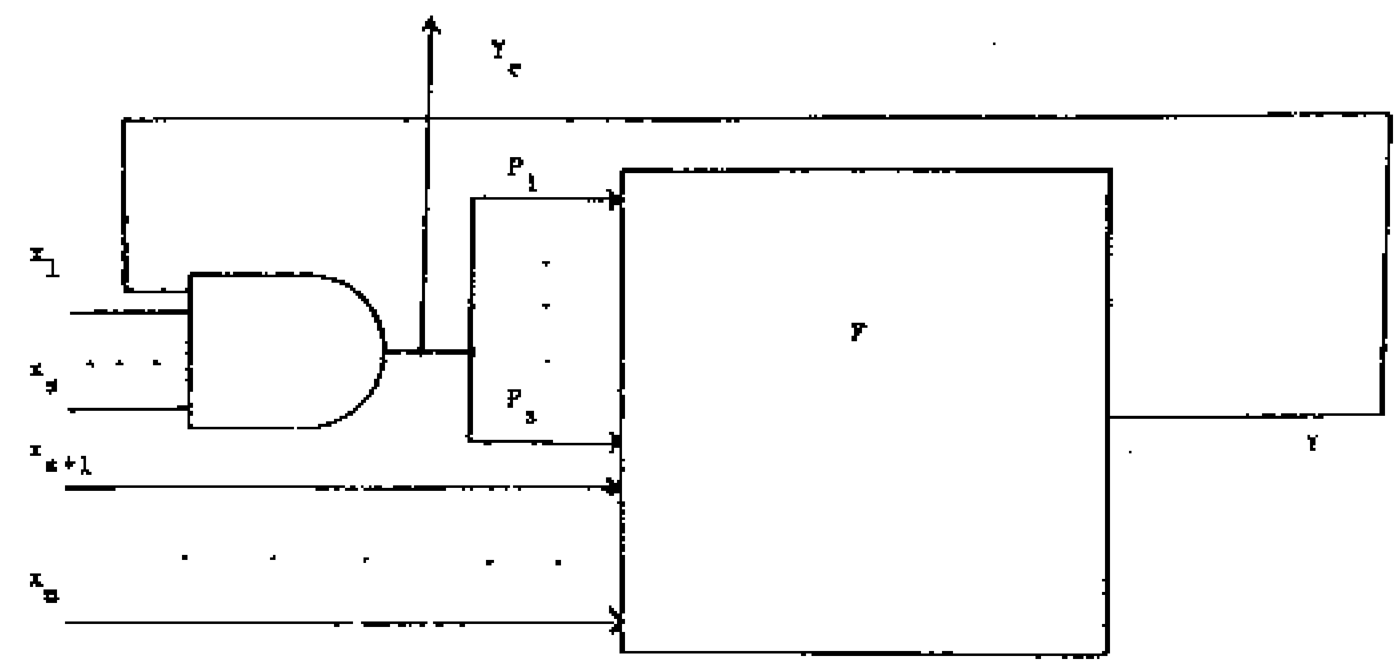

Fig. 1. Logical model of feedback bridging ($Yx_1 \cdots x_s$).

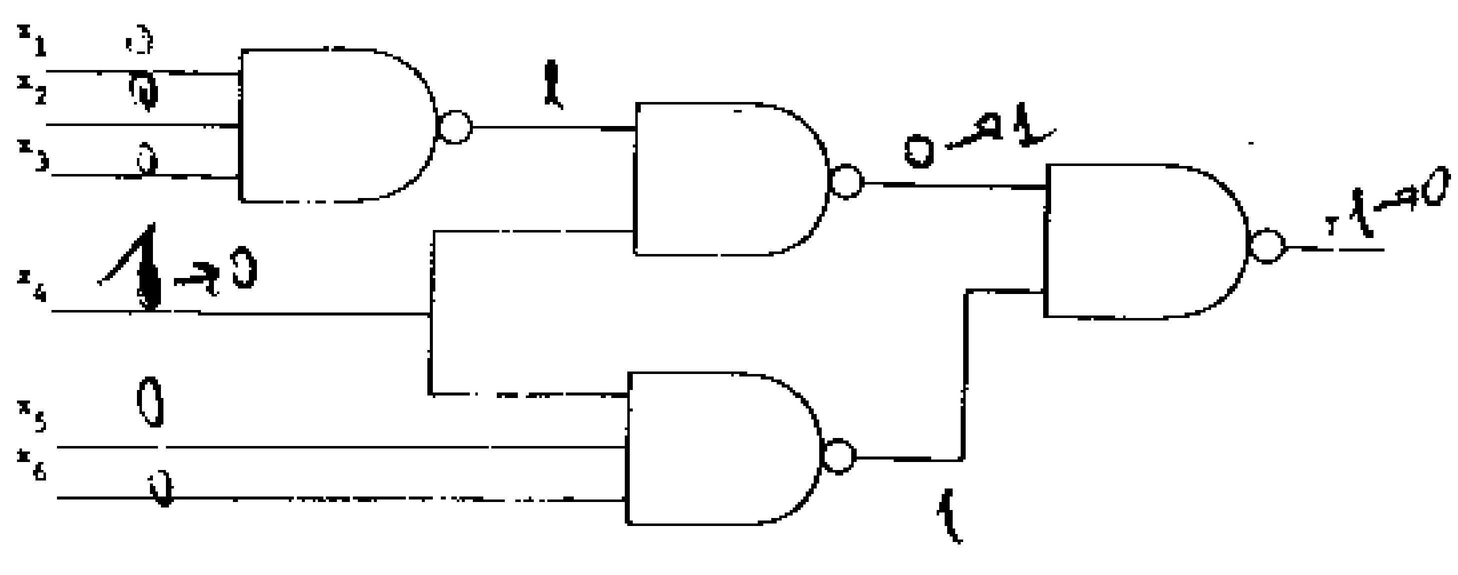

Fig. 2. A NAND network with a fan-out $Y = x_1x_2x_3x_4 + x_4x_5x_6$.

AND-type bridging exists between the output and both x_1 and x_2.

Note that for the network to oscillate, the total number of inversions in the feedback loop must be odd. If the number of inversions in the feedback loop is even, bridging faults in the network can be detected by utilizing the asynchronous behavior property. If (2) is satisfied, then

$$F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) = 0,$$

which means that the circuit output can be reset to 0 as long as the first s input variables are 0's. After resetting the output to 0, if we apply an input pattern v such that $F(v) = 1$, then from the model shown in Fig. 1, the output response to v is 1 for the fault-free network and 0 for the network with feedback bridging of a multiplicity s.

Let us now examine the model of Fig. 1 in the case of asynchronous behavior more carefully. Suppose the output is reset to 0 by applying a test pattern u such that $F(u) = 0$ (if the output is equal 1 for this test pattern, then the fault is detected). Now if another pattern v is applied and $F(v) = 1$ for fault-free circuit, then for a circuit with feedback bridging ($Yx_1x_2 \cdots x_s$), the first s components of vector v becomes 0's. Two cases can occur.

Case 1: $F(\underbrace{0,0,\cdots,0}_{s},x_{s+1},\cdots,x_n) = 0.$

The feedback bridging is detected since the good circuit produces a 1 and the faulty circuit produces a 0.

Case 2: $F(\underbrace{0,0,\cdots,0}_{s},x_{s+1},\cdots,x_n) = 1.$

According to the model shown in Fig. 1, the output Y_e will be 0 for a short time and then becomes 1. So a 0-pulse ($1 \to 0 \to 1$ transition) is obtained, [the duration of this pulse will be determined by the delay in a network implementing $F(x_1, \cdots, x_n)$]. Again the fault is detected by such a 0-pulse. This capturing can easily be done by using an edge-triggered flip-flop in a monitoring device.

For example, let is consider a 2-bit adder which accepts inputs (x_1,x_2) and (y_1,y_2) and produces the sum (z_1,z_2,z_3). Let us first apply $(x_1,x_2) = (y_1,y_2) = (0,0)$ and obtain $(z_1,z_2,z_3) = (0,0,0)$. Next we apply $(x_1,x_2) = (y_1,y_2) = (1,1)$ to the adder, then the fault-free adder should produce $(z_1,z_2,z_3) = (1,1,0)$. If there is a feedback bridging between z_2 and x_2, then according to Fig. 1 the input (x_1,x_2) will become $(1,0)$ [instead of $(1,1)$] which produces the faulty output $(z_1,z_2,z_3) = (1,0,1)$. On the other hand, if there is a feedback bridging between z_1 and x_1 then the input (x_1,x_2) will become $(0,1)$ which produces the faulty output $(1,0,0)$. Now z_1 becomes 1 and according to the model shown in Fig. 1, (x_1,x_2) becomes $(1,1)$ which yields $(z_1,z_2,z_3) = (1,1,0)$. Thus the faulty output pattern $(1,0,0)$ only appears for a short time but can be captured since the output of the fault-free circuit has the $(0,0,0) \to (1,1,0)$ transition but the faulty circuit has the $(0,0,0) \to (1,0,0) \to (1,1,0)$ transitions.

Example 2: In Fig. 2, since

$$F(x_1, x_2, x_3, 0, 0, 0) = 0$$

and

$$F(x_1, x_2, x_3, 1, 1, 1) = 1,$$

(2) is satisfied if $x_4 = x_5 = x_6 = 1$ for the bridging ($Yx_4x_5x_6$) of multiplicity 3, and ($Yx_4x_5x_6$) can be detected by applying $x_4 = x_5 = x_6 = 0$ followed by $x_4 = x_5 = x_6 = 1$ (each of the rest of the variables can be 0 or 1).

Remark 1: From Theorem 1, if there exists $x_{s+1}, \cdots, x_n \in \{0, 1\}$ such that $F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n) \neq F(1, 1, \cdots, 1, x_{s+1}, \cdots, x_n)$, then either (1) or (2) is satisfied for an input pattern with the first s variables equal to 1's. Therefore, the bridging ($Yx_1x_2 \cdots x_s$) can be detected by observing the oscillation or asynchronous behavior of the faulty network.

It is interesting to note that sometimes the same test pattern can detect oscillation for the bridging of the given multiplicity s, but cannot detect bridgings of multiplicity less than s. Formally speaking, if test pattern t satisfies (1) [or (2)] for the bridging ($Yx_1 \cdots x_s$), then t not necessarily satisfies (1) [or (2)] for a bridging ($Yx_1 \cdots x_{s-q}$)($q = 1, \cdots, s-1$).

Example 3: Let $n = 4$, $F(0, 1, 0, 0) = F(1, 1, 0, 0) = 0$, and $F(0, 0, 0, 0) = 1$. Then from (1), $t = (1, 1, 0, 0)$ generates the oscillation in the case of the fault (Yx_1x_2), but t does not satisfy (1) in the case of the fault (Yx_1) since $1 \cdot F(0, 1, 0, 0) \cdot \overline{F}(1, 1, 0, 0) = 0$.

Theorem 1 is devoted to the conditions of the oscillation or of the asynchronous behavior and to the detection of the given feedback bridgings. The following theorem will be devoted to the case when we do not know which input lines are bridged and only the multiplicity s of a fault is given. Let $|x|$ denote the number of 1's in the binary n-tuple $x = (x_1, \cdots, x_n)$; then

$$|x| = \sum_{i=1}^{n} x_i \quad \text{and} \quad F(|x| = i) = F(x)\big|_{|x|=i}. \tag{7}$$

We note that, as it follows from the model of a faulty network (see Fig. 1), if t is a single test pattern generating the oscillation for all possible bridgings of the given multiplicity s ($1 \le s \le n$), then $t = \mathbf{1} = (1, \cdots, 1)$.

Theorem 2:

i) The single test vector $\mathbf{1} = (1, 1, \cdots, 1)$ detects all possible feedback bridgings of the given multiplicity s by oscillation in a network realizing $F(x_1, x_2, \cdots, x_n)$ if

$$F(|x| = n) = 0,$$
$$F(|x| = n - s) = 1. \tag{8}$$

ii) The test sequence $(R, \mathbf{1})$ detects all possible feedback bridgings of the given multiplicity s by asynchronous behavior in a network realizing $F(x_1, x_2, \cdots, x_n)$ if

$$F(|x| = n) = 1$$
$$F(|x| = n - s) = 0 \tag{9}$$

where R is the input pattern which resets the output to 0.

Proof:

i) If (8) is satisfied, then for any bridging of multiplicity s, test pattern $\mathbf{1}$ satisfies (1) since $1 \cdot 1 \cdots 1 \cdot F(0, 0, \cdots, 0, 1, \cdots, 1)\,\overline{F}(1, 1, \cdots, 1, 1, \cdots, 1) = 1$, and by Theorem 1, the network oscillates.

ii) Let R be an input pattern such that $F(R) = 0$. If we apply R and the output is 1, there must be a fault in the network and the fault is detected. If the output is 0 and we apply, after R, the test

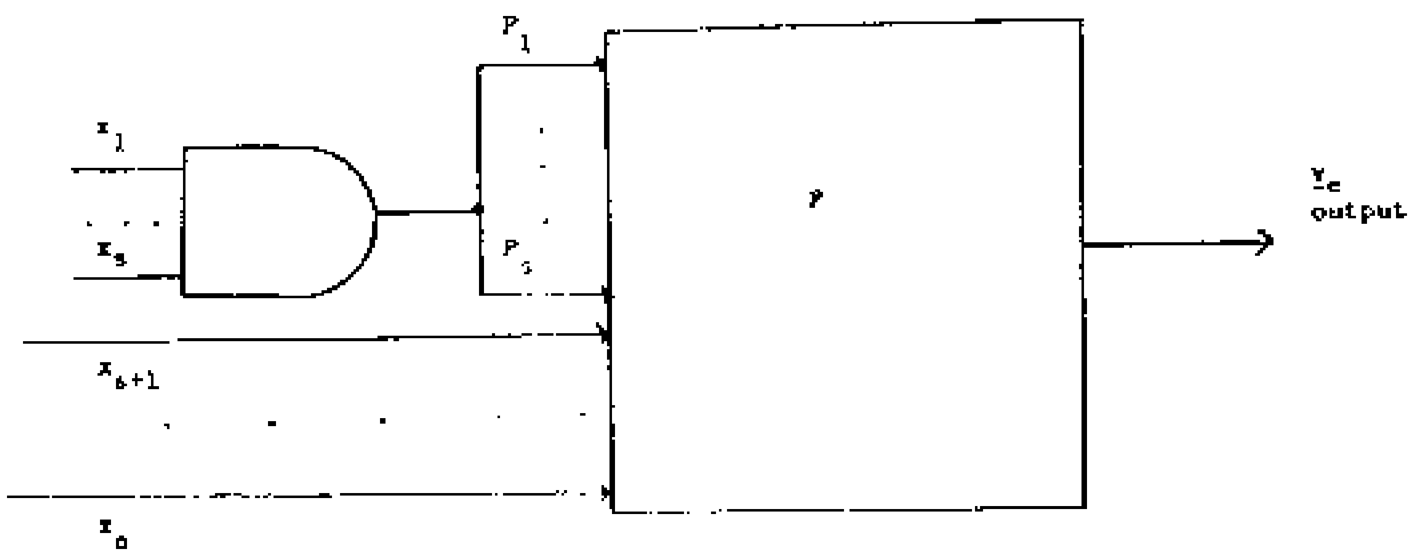

Fig. 3. Logical model of input bridging ($x_1 \cdots x_s$).

pattern **1**, then from (9), the output of the fault-free network will be 1. For any feedback bridging of multiplicity s, since s inputs will be "ANDed" with 0, only $n - s$ inputs are 1's. From (9), the output will be 0.

Example 4:

i) For $F = \oplus_{i=1}^{n} x_i$ and n = even, $\mathbf{1} = (1, 1, \cdots, 1)$ generates the oscillation for any feedback bridging of odd multiplicity.

ii) For an n-input AND gate, test sequence $(\mathbf{0}, \mathbf{1})$ detects any feedback bridging of any multiplicity.

Theorem 2 deals with the case when the multiplicity of bridgings is known. The following result is for the general case where the multiplicity of the bridgings is unknown.

Theorem 3: Let $N_{df}(n)$ be the minimum number of tests for detecting feedback bridgings of any multiplicity in any network implementing a function F of n variables (the subscripts d and f denote "detection" and "feedback," respectively). Then

$$1 \leq N_{df}(n) \leq n. \tag{10}$$

Proof:

i) *Lower Bound:* If $F(\mathbf{0}) = 1$, then $\mathbf{0}$ is the single test pattern for detecting all feedback bridgings.

ii) *Upper Bound:* There are only three cases to consider.

Case 1: If F contains a minterm with $\bar{x}_i$ for each $i = 1, 2, \cdots, n$, then the test pattern t corresponding to this minterm can detect a feedback bridging (of any multiplicity) involving x_i. Therefore, the set of n such test patterns will detect all feedback bridgings.

Case 2: If F does not contain *any* minterm with $\bar{x}_i$ for some i, then all minterms of F must contain x_i. In general, F may not contain any minterm with the complements of several variables. Without loss of generality, we can express F as

$$F = x_1 x_2 \cdots x_q F(1, 1, \cdots, 1, x_{q+1}, \cdots, x_n)$$

where $F(1, 1, \cdots, 1, x_{q+1}, \cdots, x_n)$ contains at least one minterm with $\bar{x}_j$ for every $j = q + 1, \cdots, n$. There are $n - q$ such minterms. The test patterns corresponding to these $n - q$ minterms will be used for detecting the feedback bridgings. Let $t^j = (t_1^j, t_2^j, \cdots, t_n^j)$ be a test pattern with $t_i^j = 1$ for all $i = 1, 2, \cdots, q$ and $t_i^j = 0$ for $i = j$ where $j \in \{q + 1, q + 2, \cdots, n\}$. Let R be a test pattern such that $F(R) = 0$. Then the test sequence $(R, t^{q+1}, t^{q+2}, \cdots, t^n)$ such that $F(t^{q+1}) = F(t^{q+2}) = \cdots = F(t^n) = 1$ detects any feedback bridging since the fault-free network will produce the sequence $(0, 1, 1, \cdots, 1)$, and the output sequence of the faulty network will contain at least two zeros. For this case, $N_{df}(n) \leq n - q + 1 \leq n$.

Case 3: If $q = n$, i.e., F does not contain *any* minterm with $\bar{x}_i$ for all i, then $F = x_1 x_2 \cdots x_n$, and test sequence $(\mathbf{0}, \mathbf{1})$ will detect all feedback bridgings.

Remark 2: Since 50 percent of functions have the property $F(\mathbf{0}) = 1$, feedback bridgings in networks implementing half of the functions can be detected by applying only one test pattern, $\mathbf{0}$.

Example 5: Let F be the following threshold function:

$$F(x_1, x_2, \cdots, x_n) = \begin{cases} 1 & \text{when } |x| \geq n - 1, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Then the sequence $T_n = ((0, 1, 1, \cdots, 1), (1, 0, 1, \cdots, 1), \cdots, (1, 1, \cdots, 1, 0))$ detects all feedback bridgings. For example, $(0, 1, \cdots, 1)$ will detect all feedback bridgings involving x_1 and any number of other input lines. In fact, as we shall see in Section III, for this function, T_n can *locate* any feedback bridging as long as the output is set to a 1 before T_n is applied.

Remark 3: Let t be a test pattern (input binary n-tuple) and let $\bar{t}$ be its complement. If for a function F, $F(t) = F(\bar{t}) = 1$, then in a network realizing F, any feedback bridging of any multiplicity can be detected by applying just two test patterns: t and $\bar{t}$. To see this, let $|t| = n - k$; then without loss of generality, we can permute the variables of F such that the first k components have 0's in t and the last $n - k$ components of $\bar{t}$ are 0's. Any feedback bridging involving at least one of the first k (last $n - k$) input lines will be detected by $t(\bar{t})$ since the fault-free circuit will produce a 1 and a faulty circuit will produce a 0.

The following theorem is the generalization of Theorem 3, Remarks 2 and 3.

Theorem 4: For the function $F(x_1, x_2, \cdots, x_n)$, any sequence of input patterns $(t^1, t^2, \cdots, t^N)$ such that $F(t^i) = 1$ $(i = 1, 2, \cdots, N)$ and $\prod_{i=1}^{N} t^i = \mathbf{0}$ ($\prod_{i=1}^{N} t^i$ is the componentwise multiplication of vectors t^i) is the test sequence for detecting all possible feedback bridgings in any network implementing $F(x_1, x_2, \cdots, x_n)$.

Proof: If the kth component of t^i is a 0, then any feedback bridging (of any multiplicity) involving x_k will be detected by t^i since the fault-free and faulty circuits will produce 1 and a 0 at the output, respectively. $\prod_{i=1}^{N} t^i = \mathbf{0}$ guarantees that this case is true for all variables. Q.E.D.

Example 6: For function $F(x_1, x_2, x_3, x_4)$ such that $F(0, 1, 1, 1) = F(1, 0, 0, 1) = F(1, 1, 0, 0) = 1$, $((0, 1, 1, 1), (1, 0, 0, 1), (1, 1, 1, 0))$ is the test sequence for detecting all feedback bridgings since $(0, 1, 1, 1) \cdot (1, 0, 0, 1) \cdot (1, 1, 0, 0) = (0, 0, 0, 0)$. Test pattern $(0, 1, 1, 1)$ detects the feedback bridging between x_1 and the output $(1, 0, 0, 1)$ detects the feedback bridgings between the output and x_2, x_3. The bridging between x_4 and the output is detected by $(1, 1, 1, 0)$.

B. Detection of Input Bridgings

When the bridging lines are known, without loss of generality, we can assume that the input bridging exists between the first s input lines. The logical model for the AND-type bridging of multiplicity s among lines $x_1, x_2, \cdots, x_s$ in a network implementing $F(x_1, x_2, \cdots, x_n)$ is given in Fig. 3. The next theorem presents the necessary and sufficient conditions for undetectability of an input bridging of any given multiplicity.

Theorem 5: For any network implementing $F(x_1, x_2, \cdots, x_n)$, the bridging between input lines $x_1, x_2, \cdots, x_s$ is undetectable if and only if for every $x_{s+1}, x_{s+2}, \cdots, x_n \in \{0, 1\}$ and every $A = (a_1, a_2, \cdots, a_s) \neq \mathbf{0}, \mathbf{1}$ where $\mathbf{0} = \underbrace{(0, 0, \cdots, 0)}_{s}$ and $\mathbf{1} = \underbrace{(1, 1, \cdots, 1)}_{s}$,

$$F(a_1, a_2, \cdots, a_s, x_{s+1}, \cdots, x_n) = F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n). \tag{12}$$

Proof: $A \neq \mathbf{0}, \mathbf{1}$ means that there exists at least one $a_i = 0$ where $i \in \{1, 2, \cdots, s\}$. If there is bridging between $x_1, x_2, \cdots, x_s$, the output of the faulty network will be $F(0, 0, \cdots, 0, x_{s+1}, \cdots, x_n)$ which, according to (12), is the same as the output of the fault-free network. Hence, the fault is undetected. Conversely, if the bridging is undetectable, (12) holds.

Example 7: $F(x_1, x_2, x_3) = \bar{x}_1(\bar{x}_2 + \bar{x}_3) + x_1 x_2 x_3$. The bridging between x_2 and x_3 is undetectable since $F(x_1, 0, 1) = F(x_1, 1, 0) = F(x_1, 0, 0) = \bar{x}_1$ and (12) is satisfied.

Corollary 2: An input pattern $t = (t_1, t_2, \cdots, t_n)$ detects the bridging $(x_1 x_2 \cdots x_s)$ if and only if $(t_1, t_2, \cdots, t_s) \neq \mathbf{0}, \mathbf{1}$ and

$$F(0, 0, \cdots, 0, t_{s+1}, \cdots, t_n) \neq F(t_1, t_2, \cdots, t_n). \tag{13}$$

Example 8: In Fig. 2, since $F(x_1, x_2, x_3, x_4, 0, 0) = F(x_1, x_2, x_3, x_4, 0, 1) = F(x_1, x_2, x_3, x_4, 1, 0) = \bar{x}_1 x_2 x_3 \cdot x_4$, the bridging between x_5 and x_6 is undetectable. However, since $F(x_1, x_2, x_3, 0, 0, x_6) = 0$ and $F(x_1, x_2, x_3, 1, 0, x_6) = \bar{x}_1 x_2 x_3$, the bridging between x_4 and x_5 can be detected by $(0, 0, 0, 1, 0, 0)$.

Corollary 2 provides us with a simple method for detecting a given input bridging by finding an input vector satisfying (13).

Example 9: Let $F(x_1, \cdots, x_n) = 0$ if $|x| = 0$ and $F(x_1, \cdots, x_n) = 1$ if $|x| = 1$ (e.g., $F = \Sigma_{i=1}^n x_i$), then by Corollary 1, the test sequence $T_1 = ((1, 0, \cdots, 0), (0, 1, 0, \cdots, 0), \cdots, (0, 0, \cdots, 0, 1))$ detects all input bridgings of all multiplicities.

It is well known that two stuck-at faults may compensate each other [9]. It is interesting to see that the situation is different for bridging faults. The next theorem states that two AND-type (or OR-type) input bridgings *cannot* compensate each other.
each other.

Theorem 6: For a network implementing $F(x_1, x_2, \cdots, x_n)$, if there does not exist any test for detecting the double input bridging $e_1 = (x_1 x_2 \cdots x_s)$ and $e_2 = (x_{s+1} \cdots x_{s+q})$ $(s + q \le n)$, then no test pattern can detect either bridging.

Proof: For $A = (a_1, a_2, \cdots, a_s)$, $B = (b_1, b_2, \cdots, b_q)$ with a_i, $b_j \in \{0, 1\}$, we denote

$$F_{A,B} = F(a_1, a_2, \cdots, a_s, b_1, b_2, \cdots, b_q, x_{s+q+1}, \cdots, x_n).$$

If the double AND-type bridging $(x_1 x_2 \cdots x_s)$ and $(x_{s+1} x_{s+2} \cdots x_{s+q})$ cannot be detected by any test pattern, then using the same arguments as in the proof of Theorem 5, we have the following.

$$\text{1) For every } A, B \ne \mathbf{0}, \mathbf{1}, F_{A,B} = F_{\mathbf{0},\mathbf{0}} \tag{14}$$

where $F_{A,B}$ is the fault-free output and $F_{\mathbf{0},\mathbf{0}}$ is the faulty output with the aforementioned double bridging of multiplicities s and q.

$$\text{2) For } A = \mathbf{0}, \text{ we have } F_{\mathbf{0},B} = F_{\mathbf{0},\mathbf{0}}. \tag{15}$$

From (14) and (15), we obtain for $A \ne \mathbf{1}$

$$F_{A,B} = F_{\mathbf{0},B} \qquad \text{where } B \ne \mathbf{1}. \tag{16}$$

$$\text{3) For } B = \mathbf{1}, \text{ we have } F_{A,\mathbf{1}} = F_{\mathbf{0},\mathbf{1}}, \quad \text{for } A \ne \mathbf{1}. \tag{17}$$

From (16) and (17), we see that for every B and $A \ne \mathbf{0,1}$

$$F_{A,B} = F_{\mathbf{0},B}. \tag{18}$$

Therefore, by Theorem 5, the bridging $(x_1 x_2 \cdots x_s)$ cannot be detected. Similarly, we can prove that $F_{A,B} = F_{A,\mathbf{0}}$ for every A and $B \ne \mathbf{0}, \mathbf{1}$ and the bridging $(x_{s+1} x_{s+2} \cdots x_q)$ cannot be detected either.

Example 10: In Fig. 2, since there is no test pattern which can detect the double bridging $(x_1 x_2 x_3)$ and $(x_5 x_6)$, it is impossible to find a test pattern to detect either bridging. Since test pattern (0, 0, 0, 1, 0, 0) detects bridging $(x_3 x_4)$, it can also detect $(x_3 x_4 x_5)$ or the double bridging $(x_3 x_4)$ and $(x_5 x_6)$.

III. Location of Feedback and Input Bridgings

An input test sequence T *locates* a set E of bridgings if and only if the output sequences for all bridgings from E under the input sequence T are all distinct and all different from the output sequence for the fault-free network.

A. Location of Feedback Bridgings

In this subsection, we shall assume that for the locating of AND-type bridgings, output Y in Fig. 1 is set to 1 (for OR-type bridgings, the network output is reset to 0) before applying a fault-location test sequence.

Once we set signal Y in Fig. 1 to 1, we then apply an input sequence such that the fault-free circuit will produce the sequence of 1's. If there is a bridging between Y and, say, x_i, then based on the model shown in Fig. 1, the output sequence of the faulty network will be the same as the input sequence of the component x_i. Then, if we make sure that input sequences for x_i's for all $i (i = 1, \cdots, n)$ are *all* distinct and are not equal to $(1, 1, \cdots, 1)$ (fault-free output sequence), we can locate all bridgings $e_i = (Yx_i)$. Based on the above observation, the following theorem is established, assuming that the network output is preset to logic 1.

Theorem 7: Let $T_s = (t^1, t^2, \cdots, t^N)$ be a sequence of input patterns such that $F(t^i) = 1$ for all $i = 1, 2, \cdots, N$, and in the binary matrix (T_s) with rows $t^1, t^2, \cdots, t^N$, all componentwise multiplications of at most s columns are different and not equal to 1. Then T_s is a test sequence locating all feedback bridgings of multiplicity at most s.

Proof: Without loss of generality, we may assume that the bridging is $e = (Yx_1 x_2 \cdots x_i)$ where $i \in \{1, 2, \cdots, s\}$. Then the output sequence of the faulty network with the input sequence T_s will be

$$(t_1^1 t_2^1 \cdots t_i^1 F(t^1), \cdots, t_1^N t_2^N \cdots t_i^N F(t^N)) = (t_1^1 t_2^1 \cdots t_i^1, \cdots, t_1^N t_2^N \cdots t_i^N). \tag{19}$$

Since the above output sequence is distinct for each different bridging and not equal to $(1, 1, \cdots, 1)$ (the output sequence for the fault-free networks), T_s locates feedback bridgings of multiplicity at most s.

Example 11: Consider the network shown in Fig. 4, $F(x_1, x_2, x_3, x_4, x_5) = x_1 + x_2 x_4 + x_2 x_5 + x_3 x_5 + x_3 x_4$.

Let

$$(T) = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Since columns of (T) are distinct and $F(1, 0, 0, 0, 1) = F(0, 1, 0, 1, 0) = F(0, 0, 1, 1, 1) = 1$, T will locate any single feedback bridging.

Example 12: Let $F(x_1, \cdots, x_n) = 1$ if $|x| = n - 1$. Then $T_n = ((0, 1, 1, \cdots, 1, 1), (1, 0, 1, \cdots, 1, 1), \cdots, (1, 1, 1, \cdots, 1, 0))$ is the test sequence locating all feedback bridgings of all multiplicities.

We note that if T_s is the test sequence for the location of all feedback bridgings of multiplicity at most s, then matrix (T_s) is similar to the matrix of cutsets (or paths) locating bridgings (or open circuits) of multiplicity at most s in the nonoriented graphs or unate contact networks [7]. (For graphs, we need that all componentwise logical additions (instead of componentwise multiplications) of at most s columns are different. The problem of the construction of the matrix (T_s) for graphs was considered in [7].)

The condition of Theorem 7 for the matrix (T_s) gives us the possibility to estimate the minimal number $N_{l,f}(n, s)$ of tests for the location of all feedback bridgings of the multiplicity at most s in any network with n input lines (l and f stand for "location" and "feedback," respectively).

Corollary 3:

$$\lfloor \log_2 \sum_{i=0}^{s} \binom{n}{i} \rfloor \le N_{l,f}(n, s) \le \sum_{i=0}^{s} \binom{n}{i}. \tag{20}$$

($\lfloor a \rfloor$ is the least integer greater or equal to a.)

Proof: Let $\delta = N_{l,f}(n, s)$ and $T_s = (t^1, t^2, \cdots, t^\delta)$ be the minimal test sequence for locating all feedback bridgings of the multiplicity at most s in a network with n input lines and let (T_s) be the corresponding $\delta \times n$ matrix. Then by Theorem 7, $t^i \ne \mathbf{0}, \mathbf{1}$ where $i = 1, 2, \cdots, \delta$. Let $(\bar{T}_s)$ be the expanded matrix with columns of matrix (T_s) and all possible componentwise multiplications of at most s columns of matrix (T_s). Then the dimensions of $(\bar{T}_s)$ are $\delta \times \Sigma_{i=0}^s \binom{n}{i}$. Since all columns of $\bar{T}_s$ are different and $t^i \ne \mathbf{0}, \mathbf{1}$, we have $N_{l,f}(n, s) \le \Sigma_{i=0}^s \binom{n}{i}$. In order to provide $\Sigma_{i=0}^n \binom{n}{i}$ distinct columns, we need at least $\lfloor \log_2 \Sigma_{i=0}^s \binom{n}{i} \rfloor$ rows. Q.E.D.

It follows from Theorem 7 and (20) that the test sequences T_1 and T_n from Examples 9 and 12 are minimal for location of all possible feedback bridgings. We note also that these test sequences T_1 and T_n with good error-locating capabilities have a cyclic structure, and the test generators for these test sequences may be implemented by an n-bit end-around (circular) shift register.

B. Location of Input Bridgings

For a network with n input lines, the number of input bridgings of multiplicity at most s is $\Sigma_{i=2}^s \binom{n}{i}$ (see Fig. 2). Thus, similar to (20),

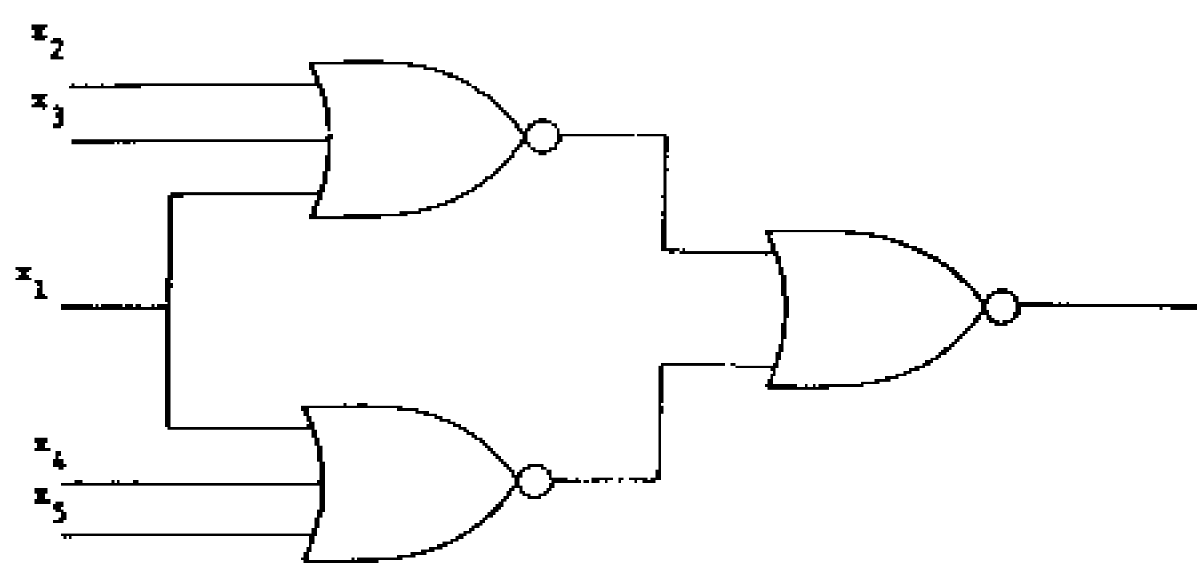

Fig. 4. The logic network for Example 11 (location of feedback bridging faults).

we have for the minimal number $N_{1,i}(n, s)$ of tests for the location of all input bridgings of the multiplicity at most s in any network with n input lines

$$\lfloor \log_2 \left(1 + \sum_{i=2}^{s} \binom{n}{i}\right) \rfloor \le N_{1,i}(n, s). \tag{21}$$

Corollary 4:

i) If $F(0) = 0, F(|x| = 1) = 1$, then the test sequence $T_1 = ((1, 0, 0, \cdots, 0, 0), (0, 1, 0, \cdots, 0, 0), \cdots, (0, 0, 0, \cdots, 1, 0), (0, 0, 0, \cdots, 0, 1))$ locates all input bridgings of all multiplicities.

ii) If $F(|x| = n - 1) = 1, F(|x| = n - l) = 0 (l = 2, \cdots, s)$, then $T_n = ((0, 1, 1, \cdots, 1, 1), (1, 0, 1, \cdots, 1, 1), \cdots, (1, 1, 1, \cdots, 1, 0))$ locates all input bridgings of multiplicity at most s.

Proof:

i) For input bridging $(x_1 x_2 \cdots x_s)$ of multiplicity s, the output for test sequence T_1 has 0's at positions $1, 2, \cdots, s$ and 1's at all other positions. Thus, $Y_e(T_1)$ for all input bridgings are different, not equal to **1**, and T_1 locates all input bridgings of all multiplicities.

ii) The proof is similar to the proof for i).

Combining the above results with the results of the previous subsection, we obtain the following corollary.

Corollary 5:

i) If $F(0) = 0, F(|x| = 1) = 1$, then T_1 locates all input bridgings and all single feedback bridgings.

ii) If $F(|x| = n - 1) = 1$, $F(|x| = n - l) = 0$ $(l = 2, \cdots, s)$, then T_n locates all input bridgings of multiplicity at most s and all feedback bridgings of all multiplicities.

Summarizing the results of the last two sections, we may say that the location of feedback and input bridgings may be carried out for the most practical cases in a reasonably short time.

Conclusion

The results presented in this correspondence have been applied for generating complete test sets for detecting input, output, and feedback bridgings as well as stuck-at faults at the input and output pins of standard digital components including shift registers, counters, decoders, multiplexers, adders/subtractors, multipliers, dividers and RAM [11]. Future unsolved problems in this area are also given in [11].

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