

INVESTIGATION OF A BIOLOGICAL SYSTEM AS AN AUTOMATON WITH ERROR CORRECTION

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The authors describe the simulation of a biological system of motion control (which maintains human vertical posture at the same time) by methods of the theory of finite automata. The capacity of the resultant mathematical model (finite automaton) for self-correction of errors is analyzed.

One of the mathematical models (MM) of data processing systems is the model of a finite automaton (FA) [1]. There have been attempts to apply the theory of finite automata to the analysis and synthesis of simple biomechanical systems [2]. In this paper, the object we have chosen to simulate by methods of finite-automata theory is the human system of motion control while the human being is maintaining a vertical posture.

1. It is pointed in [3] that a muscle has differing responses to the same signal from the central nervous system (CNS) depending on the state, length, and tension of the muscle. This indicates that the system in question has a "memory" in the sense that a subsequent state depends not only on the input signal (control signal from the CNS) but also on the preceding state. The state of a FA which simulates a biological system (BS) has been estimated from the tonograms (TG) of the muscles involved in maintaining vertical postures. Tonograms have been recorded by the method of continuous local tonometry [4], where the transverse muscle hardness was measured; this yielded an indirect estimate for muscle tension (since under low effort these parameters are proportional to one another).

It is known [5] that interference electromyograms (EMG) carry information on the control signals from the CNS. These signals are input signals for the corresponding muscles, since the muscles change state accordingly. Thus the input signal for the FA being simulated was evaluated from the ensemble of EMG of the corresponding muscles.

It is natural to take the output signal of the BS under investigation to be the position of the projection of the general center of gravity (GCG) of a human being onto the horizontal support plane. This parameter has been recorded by a tensometric stabilograph.

For the amount of data to be processed not to exceed the capacities of advanced computers, it is necessary to reduce the number of variables of the BS. Hence we used a special training suit with a head support to eliminate mobility in all joints above the knee (Fig. 1). For observation we chose six muscles which are most actively involved in maintaining vertical posture. To construct our MM we continuously recorded the parameters of a standing human being that were chosen for observations.

2. Simultaneous recording of six EMG, six TG, and two components (horizontal and vertical) of the projection of the GCG we used a special multichannel device (Fig. 2) consisting of two arrays, a measurement array and a recording array. The measurement array contains 12 four-channel U-34 A biopotential amplifiers (PBA), 18 TU-12M tensometric amplifiers, and 24 operational amplifiers (OA) of a MN-7 simulation device. The EMG signals were picked up by means of skin electrodes with a three-point (with subsequent addition) electrode-point pickup and were amplified by the PBA. It was possible to obtain the average ("envelope") of the EMG. To measure the TG and projection of the GCG, we used tensometric sensors whose signals were amplified by the TU-12M.

The biomechanical picture of the motion under consideration is reflected in changes in the joint angles, which were also measured by means of tension sensors. The rates of change of the corresponding parameters and their acceleration were determined by differentiation for which the OA were used.

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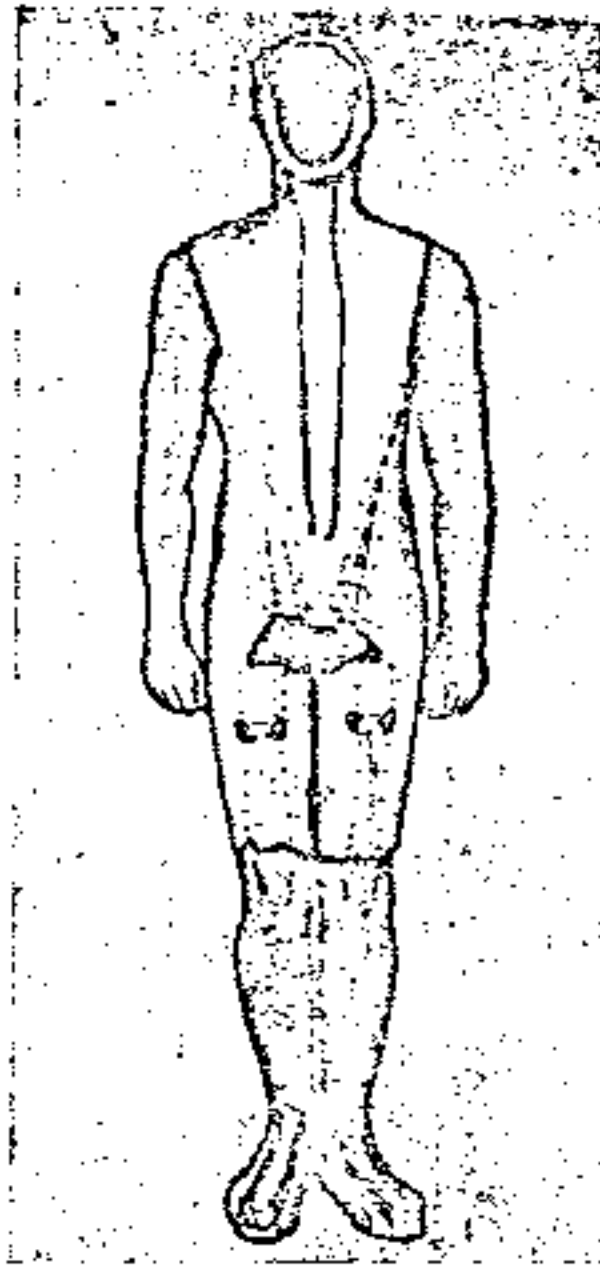


Fig. 1. General view of special restraining suit.

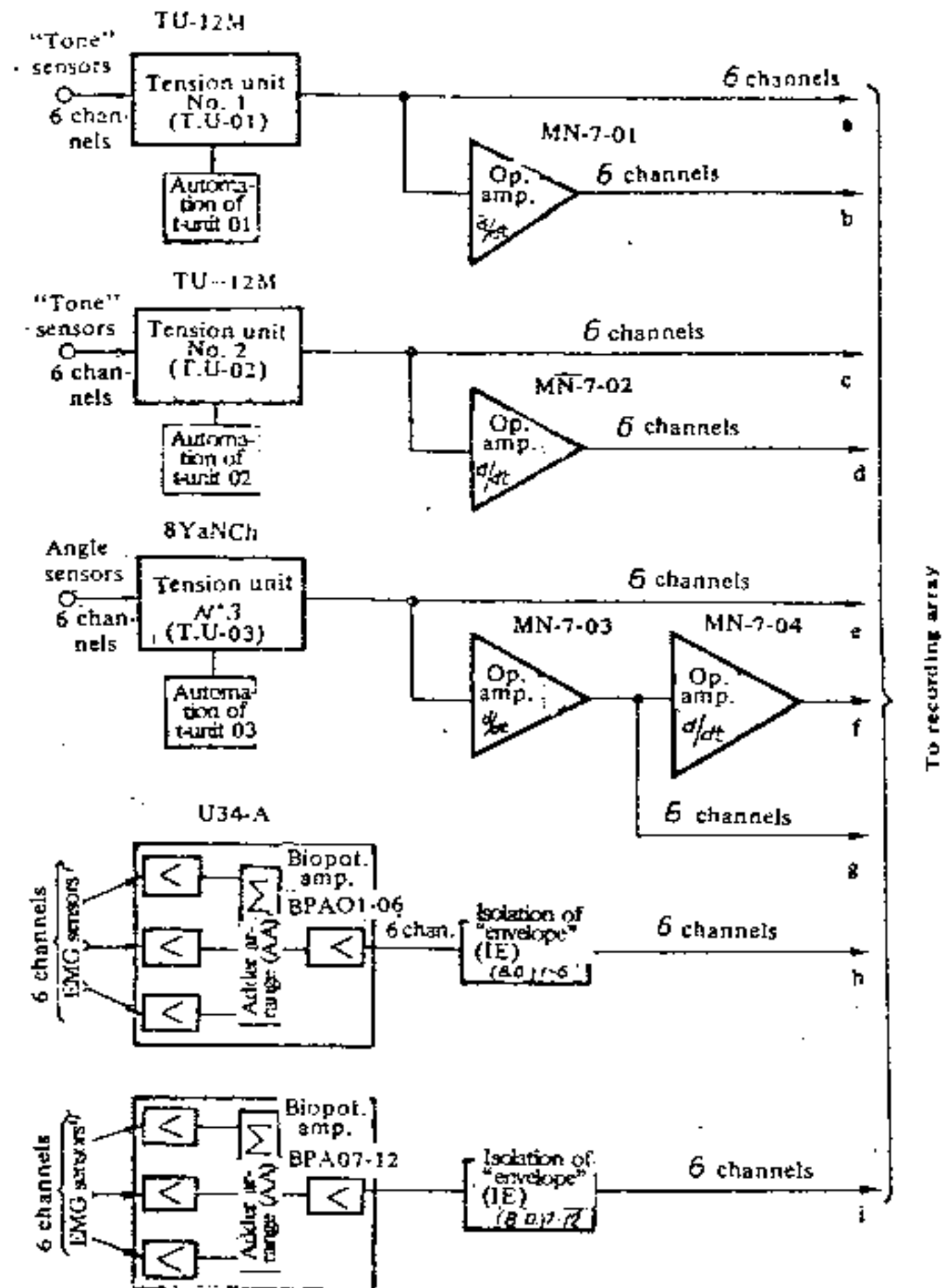


Fig. 2. Block diagram of device (measurement array).

The recording array made it possible to record signals on magnetic tape in analog and digital form. In the first case we used the 16-channel recorder of the Uran system. A specially developed device was used for recording in digital code and subsequent computer input.

3. To analyze a BS in terms of the theory of finite automata, it is necessary to quantize the processes being recorded. Given our computer capacity for processing experimental results, we selected three quantization levels for each process. The levels were fixed such that they divided the interval of change of the probability distribution density into three equal parts.

The output signals were taken to be the two projections of the-GCG and the derivatives of the components, which were quantized with respect to two levels. As discrete time instants of the FA, we took the instants of change of the levels of the quantized processes.

The quantization algorithm was implemented on a M-20 computer for 12 five-minute experiments with a BS. As a result we obtained tapes of the input signals, FA states, and output signals [7]; the over-all tape length was 75,000. The mathematical expectation of the discreteness interval was 50 msec.

We are then faced with the problem of constructing a transition graph for the FA from its tapes. The BS under study responds stochastically to the input signals and, moreover, the experimental results contain measurement and quantization errors. Thus a finite probabilistic automaton (FPA)  $A_{pr}$  [8] describing the functioning of a BS, once it is in state  $a_k$  can change under an input signal  $x_s$  to any state  $a_l$  and have any output signal  $y_t$  with probability  $P(a_l, y_t / a_k, x_s)$ .

The probabilistic mapping (PM) of the set of input words onto the set of output words for a FPA  $A_{pr}$  in state  $a_k$  has a section point  $\lambda$  [8] for words of length  $d$  if for any

$x^{(d)}$  from the set  $\{X^{(d)}\}$  of all possible words of length  $d$  in the alphabet of input signals  $X$  there exists a word  $y_v^{(d)}$  from the set  $\{Y^{(d)}\}$  of all possible words of length  $d$  in the alphabet of output signals  $Y$  such that

$$P(Y_v^{(d)}|X^{(d)}) \geq 1 > P(Y^{(d)}|X^{(d)}) \quad (1)$$

for all  $Y^{(d)} \neq Y_v^{(d)}$ .

A deterministic automaton  $A_{det}$  and a FPA  $A_{pr}$  having identical input and output alphabets are called  $P_d$ -equivalent if the PM induced by  $A_{pr}$  has a section point  $\lambda$  for all words of length  $c \leq d$  and the conditional partial mapping of the subset of input words  $\{X^{(c)}\}$  ( $c \leq d$ ) onto the subset of output words  $\{Y^{(c)}\}$  ( $c \leq d$ ) coincides with the mapping induced by  $A_{det}$  for all words of length  $c \leq d$ .

A deterministic MM of a BS can be represented as a FA  $A_{det}$  for which the  $P_d$ -equivalent FPA  $A_{pr}$  is specified by the tapes obtained in experiments with the BS. In comparing  $A_{det}$  and  $A_{pr}$  we naturally encounter the problem of evaluating their "similarity." The similarity measure is  $M[f_y] = \sum_{x^{(d)}} P(Y^{(d)}|X^{(d)}) P(X^{(d)})$  for the output function;  $M[f_a] = \sum_{x^{(d)}} P(a^{(d)}|X^{(d)}) P(X^{(d)})$  for the transition function, where  $P(Y^{(d)}|X^{(d)})$  and  $P(a^{(d)}|X^{(d)})$  are the probabilities of appearance of an output signal  $Y^{(d)}$  and of state  $a^{(d)}$  respectively when an input word  $X^{(d)}$  of length  $d$  is fed to the FA input.

Paths in the transition graph formed by input signals that are encountered with frequency  $f \geq \delta$  will be termed  $\delta$ -essential paths. Fixing  $\delta$  and choosing transition belonging only to  $\delta$ -essential paths on the graph of  $A_{det}$ , we can construct the MM of the BS as an asynchronous FA. This FA is weakly determined, since a convenient form for assigning it in the computer memory is the set of tetrads of the form  $(x_i^{i+1}, a_i^{i+1}, y_i^{i+1})$  (42-digit vectors) that define the arcs and nodes of the transition graph.

The basis of the algorithm for constructing the MM is sequential search on the FA tapes and successive accumulation of differing tetrads. Then on the set of nodes of a 42-dimensional cube of tetrads we mark the nodes belonging to  $\delta$ -essential paths; the remaining nodes are eliminated from further consideration. We should note that the BS in question the number of tetrads that do not lie on  $\delta$ -essential paths is relatively small (less than 5% of the total number of tetrads for  $\delta = 0.001$  and  $d = 25$ ).

The MM constructed by this algorithm is an asynchronous FA having 140 states, 315 input signals and 36 output signals; the FA graph contains 3500 arcs. For input words of length  $d = 25$  the following estimates were obtained:  $M[f_y] = 0.87$ ;  $M[f_a] = 0.85$ . Since these values are close to unity, we can advance the hypothesis that there are deterministic mechanisms for data processing in the BS under study.

4. Analysis of the MM that has been constructed reveals certain data processing mechanisms in our BS. Let us briefly deal with the possibility of error correction. We may assume that the functional synergy of the vertical posture [9] is somewhat redundant even under our experimental conditions. This means that certain combinations of muscular tension function identically in controlling the position of the projection of the GCG. These combinations define sets of compatible states [10] of the model for an incompletely defined FA. We used the Gill method to find the maximum sets of compatible states (MSCS) for our FA. The number of such sets was 418, and the mathematical expectation of the number of elements in the MSCS was 9.

For each MSCS  $A_i$  ( $i = 1, 2, \dots, 418$ ) we constructed the average distance  $b_i$ , defined by the formula

$$b_i = 2 \left[ \frac{\sum_{n>m} q_{nm}}{N_i(N_i-1)} \right]$$

where  $q_{nm}$  is the Hamming distance [12] between the 12-digit vectors of the FA states that

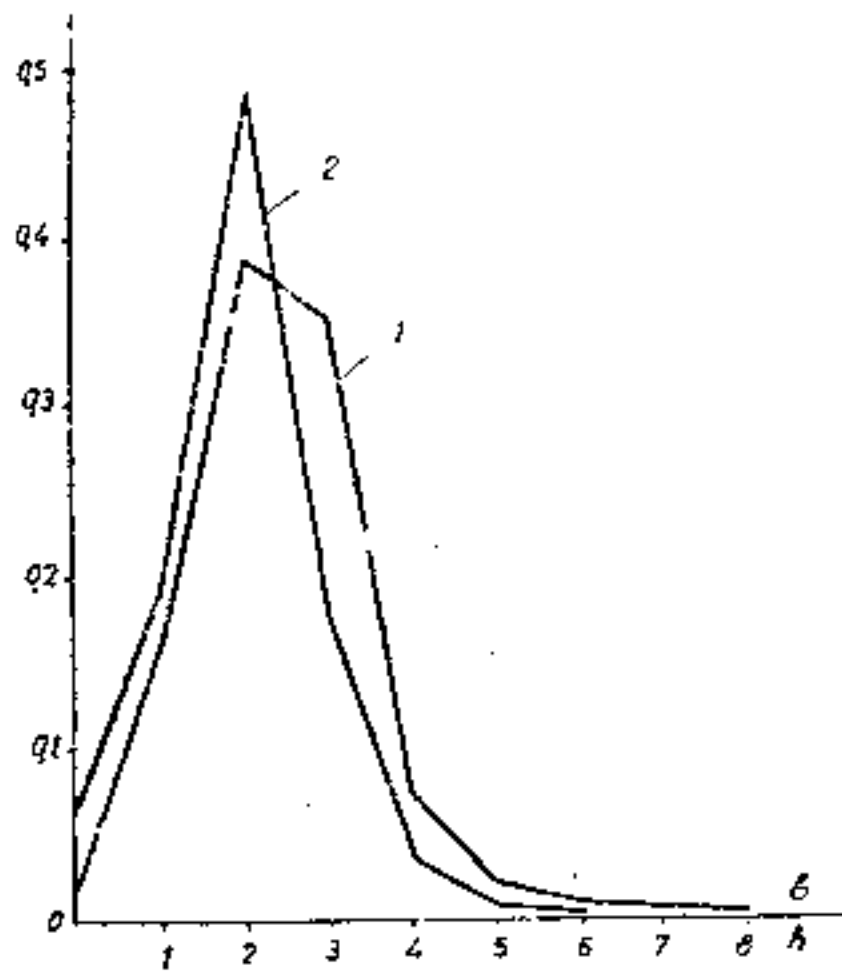


Fig. 3. Empirical distribution densities of MSCS: 1) with respect to  $b$ ; 2) with respect to  $h$ .

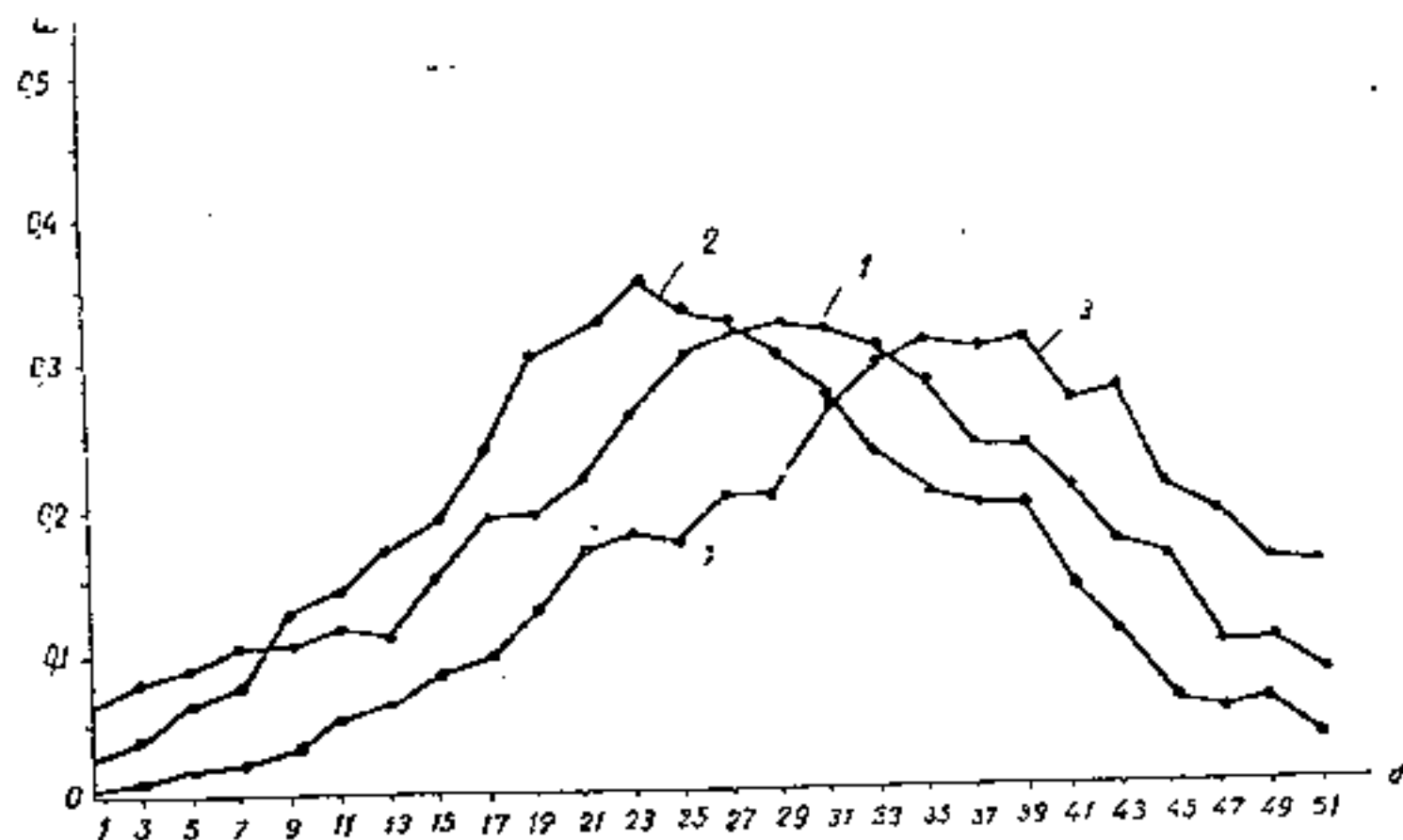


Fig. 4. Error correction in time; values of  $q$ , the error multiplicity are 1) 1; 2) 2; 3)  $>2$ .

appear in  $A_i$ ;  $N_i$  is the number of elements in  $A_i$ ; and  $]B[$  is the most proximate integer to  $B$ .

For  $A_i$  we find the subset  $\{A_i^k\}$  of MSCS that intersects  $A_i$ . Similarly, we define  $b_i^k$  for the set  $A_i^k$ . The quantity  $n_i = \left\lfloor \frac{\sum_{k=1}^{c_i} b_i^k}{c_i} \right\rfloor$ , where  $c_i$  is the number of sets  $A_i^k$ , characterizes the degree to which  $A_i^k$  intersects the remaining MSCS (Fig. 3).

We define the correctable errors in the states of the FA under consideration as follows. The state  $a_i^*$  corresponding to  $A_i$  will be called basic, while the remaining states of this MSCS will be called distorted. Transition of the automaton from a state  $a_m^*$  to a basic state  $a_l^*$  will be called a correct transition, while a transition to a distorted state  $a_l^k$  will be called an incorrect transition that arises as a result of some noise. Since  $a_l^*$  and  $a_l^k$  belong to the same MSCS,  $A_l$ , these correct and incorrect transitions yield the same output signal. Therefore we will assume that in this case the FA corrects an error of multiplicity  $e_i^k$  [12], where  $e_i^k$  is the Hamming distance between  $a_j^k$  and  $a_j^*$ .

The expectation of the distribution of  $b$  with respect to the MSCS is equal to three, and this corresponds to the presence on average of single correctable errors in the mathematic model. As can be seen from Fig. 3, most of the MSCS intersect one another; this intersection of compatible states frequency makes possible a substantial decrease in the amount of hardware required to synthesize reliable devices [11].

Let us consider errors of higher multiplicity in the MM and correction in time of these errors. Assume that an input word  $X^{(d)}$  of length  $d$  carries the FA from states  $a_j^*$  and  $a_l^k$  into states belonging to one MSCS. Then an error is corrected in exactly  $d$  cycles by the input word  $X^{(d)}$ . We denote by  $\omega_j^{(d)}$  the ratio of the number of words that correct errors of multiplicity  $e$  in state  $a_j^k$  of the automaton in exactly  $d$  cycles to the total number of input words of length  $d$ . Figure 4 shows the ratio  $\omega_j^{(d)}$  as averaged on the set of all basic states  $a_j^*$ . We should note that double errors are corrected on average in a shorter number of cycles than single errors. Arbitrary time errors are corrected on average over 50 cycles (Fig. 4, curve 3). Therefore, we may assume that the FA under consideration is self-adjusting.

We used a M-20 computer to construct and analyze the MM for the BS. The number of instructions in all the programs was around 3500.

5. In this paper we have considered a method of investigating certain data proc

algorithms in a BS for controlling human motion while maintaining a vertical posture, by simulating this system as a finite automaton. Analysis of the automaton can help us to understand the principles which are used in biological systems for reliable control of a large number of variables. For example, the weak determinacy of the algorithm and strong intersection of the sets of compatible states indicate that control actions (as evaluated from bioelectrical muscle activity) are important in changing the system in question from one state to another (the states being defined by combinations of muscle tension). The presence of single correctable errors indicates that the system remains operable under certain major perturbations. Further investigations of the properties of this mathematical model may be useful in the study of the mechanism of appearance of errors in systems, and this of interest in constructing reliable discrete data processing devices.

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