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TESTS FOR NONORIENTED GRAPHS

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A method is proposed for finding minimal tests for operability and failure diagnosis of arbitrary multiplicity in nonoriented graphs, using the method of branches and bounds. Lower and upper bounds are given on the number of test vectors for testing operability and failure diagnosis in non-oriented graphs.

In the present article we consider methods for constructing check and diagnostic tests for nonoriented graphs with two distinguished nodes (a terminal s and a terminal t). This kind of problem arises, for example, in the construction of tests for nonrepeating switching circuits.

I. Let there be given a connected nonoriented graph with two distinguished nodes s and t . We shall call test set of paths* (cutsets†) in the graph G (if it exists) a set of paths (cutsets) for which each branch belongs to at least one path (cutset). A test set of paths (cutsets) on the graph defines vectors entering into the test for checking operability of the corresponding nonrepeating switching circuit if only open circuits (only short circuits) of the contacts can occur.

Let there be given a set P of paths (cutsets). Then for any set of branches R it is possible to distinguish a maximal subset $P(R)$ of the set P such that each path (cutset) in $P(R)$ contains at least one branch of R . We shall call diagnostic set of paths (cutsets) for the localization of an l -fold open circuit (short circuit) a set P of paths (cutsets) such that for any two sets of branches R_1 and R_2 , containing not more than l elements each, the subsets $P(R_1)$ and $P(R_2)$ do not coincide. A diagnostic set of paths (cutsets) for the localization of an l -fold open circuit (short circuit) on the graph defines vectors occurring in the test for the localization of l -fold open circuits (short circuits) of the corresponding nonrepeating switching circuit.

We note that a test set of paths or cutsets, detecting single open or short circuits in the corresponding nonrepeating switching circuit, detect in this circuit open or short circuits of arbitrary multiplicity as well. Further we consider methods of construction of check and diagnostic tests and bounds are given on the number of vectors entering into them.

II. Let us consider the question of the construction of check sets of paths and cutsets and bounds on the number of vectors entering them. Let us number the nodes of the graph by $0, 1, 2, \dots, V(G)-1$ and the branches by $0, 1, 2, \dots, E(G)-1$, respectively, where the terminals s and t are assigned the numbers 0 and $V(G)-1$, respectively. Aside from this, we number all paths from terminal s to terminal t and all the cutsets.

We compose the path matrix $A^{(0)} = \|a_{ij}^{(0)}\|$ and the cutset matrix $A^{(1)} = \|a_{ij}^{(1)}\|$ of the graph in the following manner. The rows of matrix $A^{(0)}$ are paths, and

$$a_{ij}^{(0)} = \begin{cases} 1, & \text{if the } j\text{-th branch belongs to the } i\text{-th path,} \\ 0, & \text{otherwise.} \end{cases}$$

*By path in the graph G we understand a subgraph of G in which the terminals s and t are connected, where no subgraph of this subgraph has this property.

†By cutset in a graph G we understand a set of branches whose removal from the graph destroys the connectivity of terminals s and t , and no subset of this set has this property.

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TABLE 1

No. of row	No. of branch											$l_1^{(0)}$
	1	2	3	4	5	6	7	8	9	10	11	
1	0	0	0	1	1	0	1	1	1	1	1	7
2	0	0	1	0	0	1	1	1	1	1	1	7
3	1	1	1	1	1	0	0	0	0	0	0	5
4	1	1	0	0	0	1	0	0	0	0	0	3

TABLE 2

No. of row	No. of branch				$l_1^{(1)}$
	1	2	3	6	
3	1	1	1	0	3
4	1	1	0	1	3
2	0	0	1	1	2

TABLE 3

No. of row	No. of branch											$l_1^{(2)}$
	1	2	3	4	5	6	7	8	9	10	11	
1	1	1	1	0	1	1	1	1	1	1	1	7
2	1	1	1	1	1	0	0	0	0	0	0	5
3	1	1	0	0	0	1	0	0	0	0	0	3

TABLE 4

No. of row	No. of branch				$l_1^{(3)}$
	1	2	4	5	
3	1	1	1	1	4
4	1	1	0	0	2

Analogously, the rows of matrix $A^{(1)}$ are cutsets, and

$$a_{ij}^{(1)} = \begin{cases} 1, & \text{if the } j\text{-th branch belongs to the } i\text{-th cutset} \\ 0, & \text{otherwise.} \end{cases}$$

The path matrix may be constructed directly from the logical function realized by the given circuit (graph) by solution of systems of linear equations over the residue field modulo two [1] or by expansion in quasiminors of the correspondence matrix [2].

From the definition of the path (cutset) matrix it follows that the test set of paths (cutsets) forms a complete system $L(A^{(0)})$ ($L(A^{(1)})$) of rows of the matrix $A^{(0)}$ ($A^{(1)}$), such that the submatrix of $A^{(0)}$ ($A^{(1)}$) formed by these rows does not contain a zero column.

Thus, the problem of finding a minimal check set of paths (cutsets) reduces to the problem of determining the shortest coverage, and can be solved by the method of branches and bounds* [3].

The lower bound on the number of rows of the shortest coverage at each s -th step ($s = 0, 1, \dots$) is constructed in the following way:

Let as the result of the construction of bounds for the preceding $s-1$ steps there be bound: rows $A_{i_1}^{(s)}, A_{i_2}^{(s)}, \dots, A_{i_t}^{(s)}$ enter into the shortest coverage, and rows $A_{j_1}^{(s)}, A_{j_2}^{(s)}, \dots, A_{j_{s-t-1}}^{(s)}$ do not enter it. We cross off from the initial matrix $A^{(0)}$ the rows $A_{i_1}^{(0)}, A_{i_2}^{(0)}, \dots, A_{i_t}^{(0)}, A_{j_1}^{(0)}, A_{j_2}^{(0)}, \dots, A_{j_{s-t-1}}^{(0)}$, and all columns with at least one one in the rows $A_{i_1}^{(0)}, A_{i_2}^{(0)}, \dots, A_{i_t}^{(0)}$. For the remaining matrix we denote by $l_1^{(s)}$ the number of ones in the l -th row and shall assume that $l_1^{(s)} \geq l_1^{(s+1)}$. Then the lower bound $a(s)$ of the number of rows of the shortest coverage is defined by the equality

$$a(s) = t + r, \quad (1)$$

where r is the minimum number defined by the condition $\sum_{l=1}^r l_1^{(s)} \geq E_s(G)$, $E_s(G)$ is the number of columns not

crossed off at the s -th step in the initial matrix $A^{(0)}$ ($E_0(G) = E(G)$). We note that in the tree of choice of variant these branches are not to be examined for which the rows not entering into the coverage after crossing off contain at least one zero column. An analogous procedure is carried out on matrix $A^{(1)}$.

*The realization of the method of branches and bounds on M-20 requires two hundred instructions. The time for finding the shortest coverage for a 45×3000 matrix is 25 min.

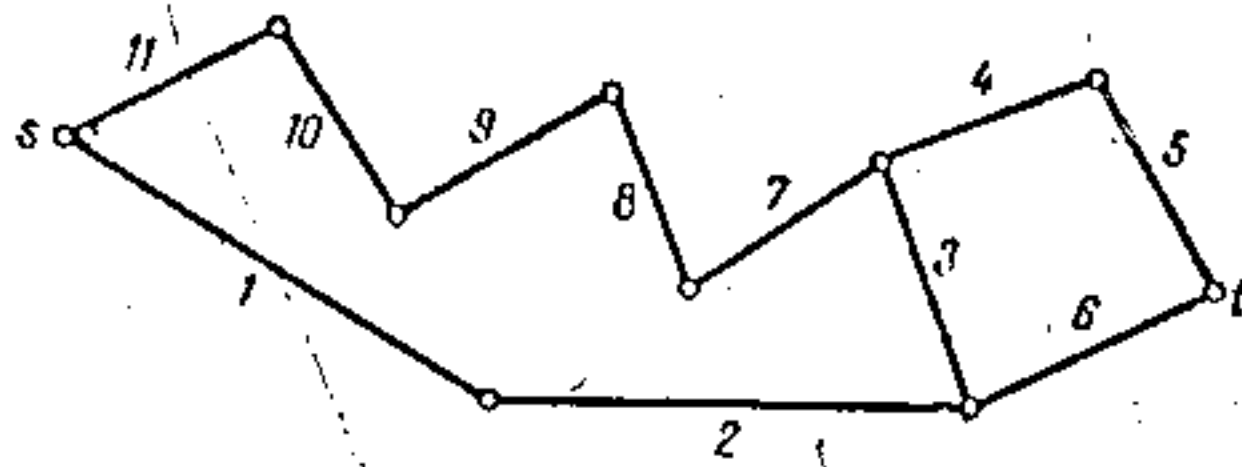


Fig. 1

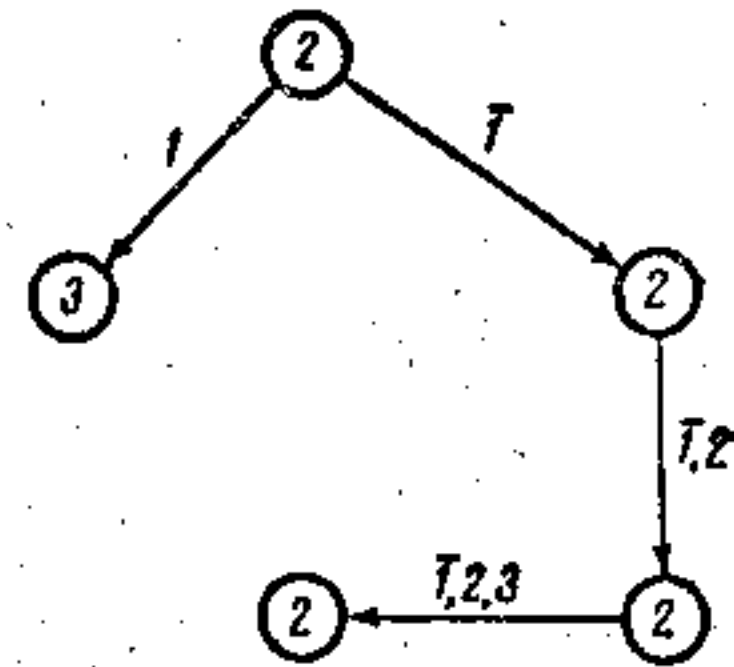


Fig. 2

Example 1. Find the shortest coverage for the path matrix (Table 1) (Fig. 1, graph G).

1. We write down the number of ones $l_i^{(0)}$ in each i -th row of matrix $A^{(0)}$ (Table 1). Since $l_1^{(0)} + l_2^{(0)} \geq E_0(G) = E(G) = 11$, we have $a(0) = 2$.

2. We assume that the first row enters the sought coverage. We cross off from matrix $A^{(0)}$ the first row and the columns containing one in the first row, and order the rows of the matrix obtained by the number of ones. For the matrix obtained (Table 2) we write $l_i^{(1)}$. Since $l_1^{(1)} + l_2^{(1)} \geq E_1(G) = 4$, we have $a(1) = 1 + 2 = 3$.

3. Since $a(1) > a(0)$, we consider a variant where the first row does not enter the shortest coverage. We cross off from matrix $A^{(0)}$ the first row, and for the matrix obtained (Table 3) we write down $l_i^{(2)}$. Since $l_1^{(2)} + l_2^{(2)} \geq E_2(G) = 11$, we have $a(2) = 2$. Further we shall consider the case where the first row does not enter the coverage.

4. Assume that the first row does not and the second row does enter the sought coverage. We cross off from the initial matrix $A^{(0)}$ the first and second rows and the columns containing ones in the second row. For the matrix obtained (Table 4) we write down $l_i^{(3)}$. Since $E_3(G) = 4$ and $l_2^{(3)} = 4$, the sought coverage consists of the second and third rows. For the given matrix $A^{(0)}$ the tree of choice of variants is given in Fig. 2, the nodes of the tree are assigned the corresponding lower bounds on the coverage rows, and the branches the number of rows.

III. We now present bounds of the minimal number of vectors forming a test set of paths $A_0(G)$ and cutsets $A_1(G)$.

Let $V(G)$ be the number of nodes of the graph G ; $E(G)$ the number of branches; $\beta(G)$ the cyclomatic number; $p_i(G)$ the degree of the i -th node of the graph G [4].

Theorem 1. For the test set of paths

$$\max \left\{ \max \left[\frac{p_i(G)}{2} \left[\cdot, p_i(G), p_{V(G)-1} \right] \right] \right\} \leq A_0(G) \leq \beta(G) + 1, \quad (2)$$

where $\left[\frac{p_i(G)}{2} \right]$ is the nearest integer not smaller than $\frac{p_i(G)}{2}$.

The inequality $A_0(G) \leq \beta(G) + 1$ follows from the rank of $A^{(0)}$ over the residue field modulo two being equal to $\beta(G) + 1$ [1].

Theorem 2. For the test set of cutsets

$$\max \left\{ \left\lceil \log_2 \frac{V^2(G)}{V^2(G) - 2E(G)} \right\rceil \left[\cdot, \frac{E(G)}{\beta(G) + 1} \right] \right\} \leq A_1(G) \leq V(G) - 1. \quad (3)$$

The proof of the theorem is given in Appendix 1. We note that the bound $\left\lceil \log_2 \frac{V^2(G)}{V^2(G) - 2E(G)} \right\rceil$ is effectively used for fairly dense graphs, i.e., with high density of ones in the incidence matrix [4], and the bound

$\left[\frac{E(G)}{\beta(G) + 1} \right] \leq A_1(G)$ for graphs of low density.

All of the bounds defined by formulas (2), (3), are exact, in the sense that for each of the bounds there exists a nonempty class of graphs on which these bounds are attained. Aside from this, the upper bounds on $A_0(G)$ and $A_1(G)$ are satisfied not only for minimal, but for redundant test sets of paths or cutsets (a path or cutset is redundant in a given test set if all the branches of this path or cutset belong to other paths or cutsets of the given set).

Example 2. A complete graph G with n nodes:

$$V(G) = n, \quad E(G) = \frac{n(n-1)}{2}, \quad \beta(G) = \frac{1}{2}(n^2 - 3n + 2).$$

Then, from (2), (3)

$$n-1 \leq A_0(G) \leq \frac{1}{2}(n^2 - 3n + 4), \quad (4)$$

$$\lceil \log_2 n \rceil \leq A_1(G) \leq n-1. \quad (5)$$

The lower bounds, defined by (4), (5), are attained for complete graphs.

Example 3. For the graph G with $n-1$ nodes, successively connected by branches we have $V(G) = n$, $E(G) = n-1$, $\beta(G) = 0$. Then $1 \leq A_0(G) \leq 1$, $n-1 \leq A_1(G) \leq n-1$; i.e., in this case both the upper and lower bounds are attained.

IV. Let us now consider the question of the construction and bounds on the number of vectors for diagnostic sets of paths and cutsets.

If in a graph, i.e., in the corresponding nonrepeating contact network, both open and short circuits are possible, then we shall call such failures symmetrical. If only open or only short circuits are possible, then we shall call such failures asymmetrical. Further we consider methods for constructing and estimating the number of vectors in diagnostic sets for the localization of asymmetrical failures in graphs.

For the existence of a diagnostic set for the localization of l open (short) circuits it is necessary that the degree of any node of the graph, except the terminals t and s , be not less than $l+2$ (the length of any loop in the graph be not less than $l+2$).

In this connection the question arises of the existence of an l_{\max} , such that there does not exist any graph for which it would be possible to construct a diagnostic set for the localization of an l -fold symmetrical failure for $l \geq l_{\max}$.

Theorem 3. In any plane graph there exists a symmetrical failure of multiplicity two or more ($l_{\max} = 2$), for which a diagnostic test for its localization does not exist.

Proof. If for the plane graph G $\varphi_j(G)$ ($j = 2, 3, \dots$) is the number of closed regions bounded by exactly j branches, then

$$[\min p_i(G)] V(G) \leq \sum_{j=2}^{V(G)} j \varphi_j(G), \quad (6)$$

$$2E(G) = \sum_{j=2}^{V(G)} j \varphi_j(G). \quad (7)$$

If $\Gamma(G)$ is the number of closed regions of the graph G , then

$$\Gamma(G) = \sum_{j=2}^{V(G)} \varphi_j(G) \quad (8)$$

and, furthermore, by Euler's formula [4]

$$V(G) - E(G) + \Gamma(G) = 1. \quad (9)$$

TABLE 5

l	1	2	3	4	6
V_{\min}	4	8	26	62	302
V_{\max}	8	24	336	31440	37320

From relations (6)-(9) we have

$$2 \min p_l(G) \leq \sum_{j=2}^{V(G)} (2j - (j-2) \min p_l(G)) \varphi_j(G). \quad (10)$$

Since for the existence of diagnostic sets for the localization of a double open circuit it is necessary that $p_1(G) \geq 4$, we have

$$\sum_{j=2}^{V(G)} (8 - 2j) \varphi_j(G) \geq 8 \quad (11)$$

and, consequently, at least one of the numbers $\varphi_2(G)$, $\varphi_3(G)$ is positive; i.e., there exists in the graph a cycle of length 2 or 3 and, therefore, it is impossible to construct a diagnostic set for the localization of short circuits of multiplicity two or more. Thus, for plane graphs $l_{\max} = 2$.

However, if we consider the class of connected nonoriented loopfree graphs, l_{\max} does not exist. This follows from Theorem 4.

Theorem 4 [5]. If $r \geq 3$, $m \geq 2$ are two arbitrary natural numbers and A_2, A_3, \dots, A_m are $m-1$ positive natural numbers, then there always exists a nonoriented connected loopfree graph G for which $p_l(G) = r$ ($l = 1, 2, \dots, V(G)$) and G contains exactly A_j ($j = 1, 2, \dots, V(G)$) cycles of length j .

For the minimal number of nodes $V(l_1, l_2)$ of the graph in which $p_l(G) = l_1 - 2$ ($l = 1, 2, \dots, V(G)$) and minimal length of cycle equal to $l_2 - 2$, i.e., a graph for which it is possible to construct diagnostic sets, localizing l_2 short circuits and l_1 open circuits, there holds the inequality

$$1 + \frac{(l_1 + 2)}{l_1} (\sqrt{(l_1 + 1)^{l_2 + 1}} - 1) \leq V(l_1, l_2) \leq 4 \left(\left(\frac{1}{l_1} (l_1 + 1)^{l_2 + 1} - 1 \right) - 1 \right) \quad (12)$$

for l_2 odd, and

$$2 \frac{1}{l_1} (\sqrt{(l_1 + 1)^{l_2} (l_1 + 1)} - 1) \leq V(l_1, l_2) \leq 8 \left(\frac{1}{l_1} (l_1 + 1)^{l_2} - 1 \right) - 1 \quad (13)$$

for l_2 even.

Upper and lower bounds are given in Table 5 for the minimal number of nodes in the graph for $l_1 = l_2 = l$.

Let us now consider methods for finding and estimating the number of vectors in diagnostic sets for the localization of failures.

We call check matrix $H = \|h_{ij}\|$ for open (short) circuit the matrix whose rows are paths (cutsets)

$$h_{ij} = \begin{cases} 1, & \text{if the } i\text{-th path (cutset) passes through the } j\text{-th branch,} \\ 0, & \text{otherwise} \end{cases}$$

The set of rows of the check matrix is a subset of the rows of matrix $A^{(0)}(A^{(1)})$. The rank over the residue field modulo 2 for the check matrix is equal to $B(G) + 1 (V(G) - 1)$. We shall call error vector the vector $e = (e_1, \dots, e_{E(G)})$, where

$$e_i = \begin{cases} 1, & \text{if a failure has occurred in the } i\text{-th branch,} \\ 0, & \text{otherwise.} \end{cases}$$

Then, as is the case in the detection and correction of errors in the asymmetric communication channel by means of error-correcting codes [6], in order that the rows of the check matrix form a test set, it is necessary and sufficient that for any e

$$H \times e = 0, \quad (14)$$

and for the rows of H to form a diagnostic set for localization of a set of failures $T(G)$ it is necessary and sufficient that for any two error vectors $e^{(i)}, e^{(j)} \in T(G) (e^{(i)} \neq e^{(j)})$

$$H \times e^{(i)} \neq H \times e^{(j)} \quad (15)$$

(Here the symbol \times signifies that in multiplication of the matrices the summation operation is substituted by disjunction.)

Thus, to localize an l -fold asymmetric error it is necessary and sufficient that the syndromes [6] of all errors be distinguishable, i.e., for any $c_i, d_i \in \{0, 1\}$

$$c_1 H_{i_1} \vee c_2 H_{i_2} \vee \dots \vee c_l H_{i_l} \neq d_1 H_{j_1} \vee \dots \vee d_l H_{j_l} \quad (16)$$

(Here H_{i_s}, H_{j_k} are columns of the matrix H ; \vee denotes componentwise disjunction of columns, where not all $c_i d_i$ are simultaneously equal to zero).

The lower bound for the number $A(G, l)$ of vectors in the diagnostic set, localizing l asymmetrical failures, is analogous to the Hamming bound for error correction codes [6].

Theorem 5. To localize l -fold open circuits in the graph G , the number of vectors forming a diagnostic set of paths satisfies the inequality

$$A_0(G, l) \geq \frac{\log_2 C_{E(G)}^l}{R(p)}, \quad (17)$$

where

$$R(p) = -p \log_2 p - (1-p) \log_2 (1-p), \quad p = \min \left(\frac{C_{V(G)}^l}{C_{E(G)}^l}, \frac{1}{2} \right). \quad (18)$$

The bound (17) is also valid for the number $A_1(G, l)$ of vectors in the diagnostic set of cutsets for the localization of l -fold short circuits, where in (17) $p = \min \left(\frac{C_{V(G)-1}^l}{C_{E(G)}^l}, \frac{1}{2} \right)$. (The proof of the theorem is given in Appendix 2.)

The check matrix H may be constructed from the path matrix $A^{(0)}$ (cutset matrix $A^{(1)}$) by means of the method of branches and bounds.

Let us consider the most frequently encountered practical cases of localization of single asymmetrical failures. Here condition (16) takes the form

$$c_i H_i \neq c_j H_j \quad (i \neq j, i, j = 1, 2, \dots, E(G)). \quad (19)$$

Using the method of branches and bounds* the lower bound $a(s)$ on the number of vectors of the diagnostic set at the s -th step for t fixed rows of matrix H is found in the following way.

Let t fixed rows form a matrix H_t and for it the maximum number of identical columns be equal to $\rho(H_t)$. Then by way of lower bound it is possible to take, for example,

$$a_s = t + \lceil \log_2 \rho(H_t) \rceil. \quad (20)$$

(The bound $a(0)$ is constructed by formulas (17), (18).)

Another way of finding a diagnostic set, localizing a single failure, is given by Theorem 6.

Theorem 6. Let the matrices $A^{(0)}$ and $A^{(1)}$ not contain identical columns and a zero column. Then

$$A_0(G, 1) \leq \beta(G) + 1, \quad (21)$$

$$A_1(G, 1) \leq V(G) - 1. \quad (22)$$

Indeed, it is possible to take as the matrix H to localize a single open (short) circuit, any system of $\beta(G) + 1$ ($V(G) - 1$) linearly independent rows of the matrix $A^{(0)}$ ($A^{(1)}$).

It follows from (21), (22) that to localize a single symmetrical failure the number $A(G, 1)$ of vectors forming a diagnostic test satisfies the inequality

$$A(G, 1) \leq E(G) + 1. \quad (23)$$

We note that the methods described above for constructing check and diagnostic sets and the lower bounds (2), (3), (17), (18) may be used both for nonoriented and oriented graphs, which correspond to relay-contact-rectifier circuits.

APPENDIX 1

Proof of Theorem 2

1. The right side of inequality (3) follows from the rank of matrix $A^{(1)}$ over the field of residues modulo two being equal to $V(G) - 1$.

2. We shall prove the inequality

$$\lceil \log_2 \frac{V^2(G)}{V^2(G) - 2E(G)} \rceil \leq A_1(G). \quad (A.1)$$

We assign to each cutset dividing the graph G into subgraphs G_1 and G_2 a vector of length $V(G)$ whose j -th component is equal to zero if for the given cutset the j -th node belongs to G_1 , and one if it belongs to G_2 . Then for any system of cutsets it is possible to construct a matrix whose rows are the vectors constructed above. Here the branch (sk) belongs to this system of cutsets if the s -th and k -th columns are distinct.

Let α_i be the number of columns of the matrix, constituting the binary expansion of i read from top to bottom:

$$\sum_{(i)} \alpha_i = V(G). \quad (A.2)$$

*The problem of constructing a diagnostic set of paths (cutsets), localizing an l -fold failure, can be reduced to the problem of finding a check set, but the number of columns in the path (cutset) matrix increases with $E(G)$ to

$\sum_{k \in G_1} C_{k(G_1)}$ columns. (The additional columns are formed as the componentwise sums modulo two of not more than l columns of the initial matrix).

Then, if the matrix consists of m rows, the number of branches corresponding to cutsets is not more

$$\text{than } \frac{1}{2} \sum_{i=1}^m \alpha_i \alpha_j.$$

Considering (A.2), this value does not exceed

$$\frac{2^m - 1}{2^{m+1}} V^2(G).$$

(This bound is attained for $\alpha_i = V(G)/2^m$ for any i and number of distinct columns in the matrix 2^m .)

Since to detect errors each branch must belong to at least one cutset, we have

$$\frac{2^{A_1(G)} - 1}{2^{A_1(G)+1}} V^2(G) \geq E(G).$$

Hence follows (A.1).

3. The inequality

$$\left] \frac{E(G)}{\beta(G)+1} \left[\leq A_1(G) \quad (\text{A.3})$$

will be proved in the following way. For any i -th ($i = 1, 2, \dots, A_1(G)$) cutset

$$E(G) = E(G_1) + E(G_2) + \Delta_i$$

where Δ_i is the number of branches belonging to the i -th cutset.

Since $E(G_1) + E(G_2) \geq V(G_1) + V(G_2) - 2 = E(G) - \beta(G) - 1$, then for any $i \Delta_i \geq \beta(G) + 1$, whence follows (A.3).

APPENDIX 2

Proof of Theorem 5

Let there be given a matrix H of dimensions $A_0(G, l) \times E(G)$ for a diagnostic set, localizing l -fold open (short) circuits. Let us consider the matrix $H^{(l)}$, whose columns are all possible disjunctions $\bigvee c_s H_{is}$ (where $c_s \neq 0$, $H_{is} = H_{iq}$, $s, r, q \in \{1, 2, \dots, l\}$; $i_s \in \{1, 2, \dots, E(G)\}$, H_{is} are the columns of the matrix H).

The matrix $H^{(l)}$ has dimensions $A_0(G, l) \times C_{E(G)}^l$ ($A_1(G, l) \times C_{E(G)}^l$). It follows from (16) that the columns of $H^{(l)}$ do not coincide with each other; i.e., the system of vectors forming the diagnostic set must contain not less than $\log_2 C_{E(G)}^l$ bits of information. Since for any row of matrix H the number of zeros does not exceed $\beta(G) = E(G) - V(G) + 1$ ($V(G) - 1$), for $H^{(l)}$ the number of zeros in a row does not exceed $C_{\beta(G)}^l (C_{V(G)-1}^l)$. Consequently, each row contains not more than $R(p)$ bits of information, where

$$p = \min \left(\frac{C_{\beta(G)}^l}{C_{E(G)}^l}, \frac{1}{2} \right) \quad \left(p = \min \left(\frac{C_{V(G)-1}^l}{C_{E(G)}^l}, \frac{1}{2} \right) \right).$$

Hence follows (17).

LITERATURE CITED

1. L. Lefgren, "The solution of problems of realizability by irredundant circuits," *Cybernetic Coll.* [Russian translation], No. 5 (1962).
2. G. N. Povarov, "On the structural theory of communication networks," in: *Problems of Information Transmission*, No. 4 [in Russian], Izd. AN SSSR (1959).