UTILIZATION OF AUTOCORRELATION CHARACTERISTICS FOR THE REALIZATION OF SYSTEMS OF BOOLEAN FUNCTIONS

M. G. Karpovskii and É. S. Moskalev

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The class of autocorrelation characteristics of Boolean functions is considered. Questions are discussed concerning the utilization of these characteristics for the synthesis of circuits realizing systems of Boolean functions and the estimation of their complexity.

1. The solution of a number of problems arising in the synthesis of circuits realizing systems of Boolean functions and in the estimation of the complexity of their realization in various bases may be obtained relatively simply by the analysis of the autocorrelation characteristics of systems of Boolean functions. A method is described in [1] for solution by means of one of the autocorrelation characteristics of the problem of an optimal method of ordering the arguments of a system of Boolean functions from the viewpoint of the simplicity of realization of this system in its expansion in an orthogonal series in the system of Haar functions. In the present article we consider an entire class of autocorrelation characteristics of a system of Boolean functions and also consider questions of the utilization of these characteristics in the synthesis of circuits realizing systems of Boolean functions.

2. Let there be given a system of Boolean functions in the form of Table 1.

Table 1 defines k comb functions

$$y_j = f(x) \quad (j = 0, 1, \ldots, k-1), \tag{1}$$

where

$$x = \sum_{i=0}^{m-1} x_i 2^{-i-1}.$$

The autocorrelation modulo 2 function of the Boolean functions $y_j(x)$ defined in Table 1 or of the comb functions (1) is the comb function $B_{2,2}(\tau)$ ($\tau = 0.2^{-m}, \ldots, 1-2^{-m}$), defined in the following way:

$$B_{2,2}(0) = \sum_{j=0}^{k-1} \sum_{x=0}^{1-2^{-m}} y_j(x), B_{2,2}(\tau) = \sum_{j=0}^{k-1} \sum_{x=0}^{1-2^{-m}} y_j(x) y_j(x \ominus \tau); \ (\tau \neq 0), \bmod 2, \tag{2}$$

(Here and below $a \ominus b$ and the notation mod p at the right of an expression signifies the componentwise difference modulo p of vectors in the p-ary representation of the numbers a and b).

For k = 1, i.e., for one Boolean function, the autocorrelation modulo 2 of the function $B_{2,2}(\tau)$ differs from the ordinary autocorrelation function only in that in its calculation by formula (2) in place of ordinary subtraction, subtraction modulo 2 is used. A deeper analogy between autocorrelation modulo 2 and ordinary autocorrelation of functions follows from Theorem 1, formulated below.

We complete the comb functions $y_j(x)$ up to piecewise-constant functions $Y_j(x)$ in the following way:

$$Y_j(x) = y_j(x) \quad \text{for} \quad x \in [i2^{-m}, \ (i+1)2^{-m}]. \tag{3}$$

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TABLE 1

x_0	x_1	. . .	x_{m-1}	y_0	y_1	. . .	y_{k-1}

We expand further each of the piecewise-constant functions $Y_j(x)$ in an orthogonal Rademacher-Walsh series [2]. The i-th coefficient c_i of the expansion of $Y_j(x)$ in a Rademacher-Walsh series is calculated in the following way:

$$c_i = 2^{-m/2} \sum_{x=0}^{1-2^{-m}} Y_j(x) W_i(x),$$

where $W_i(x)$ is a function in the Rademacher-Walsh basis.

The number of terms of this series is always finite and does not exceed 2^m. The sequence of coefficients of the series is a comb function. We complete it to piecewise-constant, analogously to $y_j(x)$, and denote it by $W\{y_j\}$.

Theorem 1.

$$B_{2,2}(\tau) = 2^{m/2} \sum_{j=0}^{k-1} W\{(W\{y_j\})^2\}. \qquad (4)$$

(Theproof is given in [1].)

Theorem 1 establishes the connection between the autocorrelation modulo 2 function, defined by (2), and the expansion of the initial functions $y_j(x)$ in the Rademacher-Walsh basis, and defines a method for calculating $B_{2,2}(\tau)$ different from (2).

It follows from (2) and (4) that the autocorrelation modulo 2 differs for k = 1 from the ordinary autocorrelation function in that it is not connected with the initial function $y_j(x)$ by the discrete Laplace transform [3] but by the Rademacher-Walsh transform.

Example 1. A system of two Boolean functions y_0, y_1 of the four arguments x_0, x_1, x_2, x_3, the functions $W\{y_j\}$ and $(W\{y_j\})^2$, and the autocorrelation modulo 2 of the function $B_{2,2}(\tau)$ are presented in Table 2.

3. Let us consider the question of using the autocorrelation modulo 2 of the function $B_{2,2}(\tau)$ to simplify the realization of a prescribed system of Boolean functions. A method is described in [1] for realizing a system of Boolean functions, based on the piecewise-continuous function $Y(x) = \sum_{j=1}^{k-1} Y_j(x) 2^j$ in an orthogonal series with ...te number of terms. The block diagram of the device realizing Y(x) consists of the basis function generator, ...nsion coefficients storage block, a multiplier, and accumulator for calculating the sum of the series. It is use... to take the system of Haar functions as the basis functions, since the basis functions generator is very simply ...ized and the need for the multiplier drops out. It follows from (1) that the form of Y(x), and therefore the num... of nonzero coefficients of the series depends on the method of ordering the arguments $(x_0, x_1, \ldots, x_{m-1})$. In this ...ction the problem arises of the optimal ordering of the arguments for which the number of nonzero coefficients ... series is minimized. In [1] a more general problem is solved of finding the optimal linear modulo 2 nonde...rate transform of the vector of arguments $x = (x_0, x_1, \ldots, x_{m-1})$ for which the number of nonzero coefficients ... series is minimized. The problem of seeking such a transform reduces essentially to finding the maximum of ...correlation modulo 2 of function $B_{2,2}(\tau)$.

...e use of the autocorrelation modulo 2 of the function $B_{2,2}(\tau)$ is useful not only to simplify the realization ...scribed system of Boolean functions by means of expansion in orthogonal series, but also in the realization ... of Boolean functions in the classical bases "AND, OR, NOT," "AND, NOT, sum mod 2," etc.

TABLE 2

$x_3x_2x_1x_0$	y_0y_1	$4W(y_0)$	$4W(y_1)$	$16(W(y_0))^2$	$16(W(y_1))^2$	$W((W(y_0))^2)$	$W_1((W(y_1))^2)$	$B_{2,2}(\tau)$	$y_0^{(\sigma)}$	$y_1^{(\sigma)}$
0 0 0 0	0 0	5	5	25	25	5	5	10	0	0
0 0 0 1	0 0	1	—1	1	1	2	0	2	0	1
0 0 1 0	1 1	3	—1	9	1	0	0	0	1	0
0 0 1 1	1 0	—1	1	1	1	2	2	4	1	0
0 1 0 0	0 0	—3	—1	9	1	2	0	2	0	0
0 1 0 1	0 1	—3	1	9	1	2	2	4	0	0
0 1 1 0	1 0	—1	—3	1	9	0	2	2	0	0
0 1 1 1	0 0	—1	—1	1	1	0	2	2	0	0
1 0 0 0	1 0	1	—1	1	1	2	2	4	1	0
1 0 0 1	0 0	1	1	1	1	2	2	4	0	1
1 0 1 0	0 0	—1	1	1	1	2	0	2	1	1
1 0 1 1	1 1	—1	3	1	9	0	2	4	0	1
1 1 0 0	0 0	1	—3	1	9	0	2	2	0	0
1 1 0 1	0 1	—3	—1	9	1	2	0	2	0	0
1 1 1 0	0 1	3	3	9	9	2	2	4	1	1
1 1 1 1	0 0	—1	—3	1	9	0	2	2	0	0

Let us consider the question of using $B_{2,2}(\tau)$ to simplify the realization of a prescribed system of Boolean functions, for example, in the basis "AND, OR, NOT." As the complexity criterion of the system of Boolean functions (1) we take the number $N_2(y)=\sum_{j=0}^{k-1} N_2(y_j)$, where $N_2(y_j)$ is the number of nonordered pairs $(x^{(1)}, x^{(2)})$ of merging vectors of arguments of the given function $y_j(x)$, i.e., such pairs of vectors of the arguments that $y_j(x^{(1)}) = y_j(x^{(2)}) \neq 0$ and $x^{(1)} \ominus x^{(2)} = (\underbrace{0 \ldots 01}_{s} 0 \ldots 0)$, mod 2 $(s = 1, 2, \ldots, m)$. This criterion and a similar criterion for functions in multivalued logic are used here only as the criterion of complexity of the system of Boolean functions itself, but not as the criterion of complexity of the circuit realizing it. This criterion $N_2(y_j)$ is used, in particular, in [4, 5] (in these publications this criterion for the function $y_j(x)$ is called the sum of weights of the productive functions $y_j(x)$ with respect to the variables $x_0, x_1, \ldots, x_{m-1}$). An experimental test of this criterion shows that in the majority of cases it reflects fairly simply the complexity of the circuit realizing $y_j(x)$. In particular, it can be shown that the number of nonintersecting pairs of merging vectors of the arguments is equal to $\frac{1}{4}\sqrt{\frac{2^{m+1}}{m}N_2(y_j)}$ for almost all functions $y_j(x)$ of m arguments (i.e., the fraction of functions for which this proposition is valid tends to unity as $m \to \infty$). Consequently, at least $\frac{1}{2}\sqrt{\frac{2^{m+1}}{m}N_2(y_j)}$ constituents of unity of the function $y_j(x)$ are covered by $\frac{1}{4}\sqrt{\frac{2^{m+1}}{m}N_2(y_j)}$ implicants of length $m-1$.

Let us consider the set Ξ of linear modulo 2 nondegenerate transforms of the vector $\mathbf{x} = (x_0, x_1, \ldots, x_{m-1})$. Let $\sigma = \|\sigma_{ij}\| \in \Xi$ be a nondegenerate $m \times m$ matrix over GF(2). Then a device realizing the given system $y_j(x)$ $(j = 0, 1, \ldots, k-1)$ will be constructed in the form of the block diagram given in the figure, where σ is a block realizing the linear transform of the argument $z = \sigma \otimes x$ (the symbol $\otimes$ and the notation mod p to the right of the expression denote the product of matrices over GF(p)), $y^{(\sigma)}$ is a block realizing the function defined by the relation

$$y^{(\sigma)}(\sigma \otimes x) = y(x), \bmod 2. \tag{5}$$

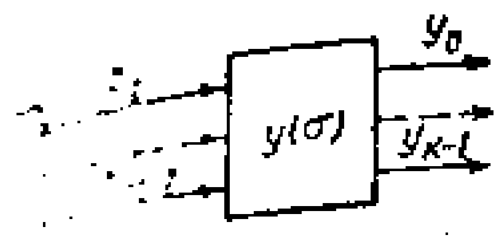

Fig. 1

Here and further y, $y(\sigma)$ are k-element vector Boolean functions, and y(x) is defined by (1).

It can be shown that the block σ may be realized for any σ by not more than $^3/_2 m\,]m/\log_2 n[- m$ adders modulo 2. (Here]a[denotes the nearest integer not smaller than a.) Therefore the complexity criterion for the block ... the figure will be taken to be the complexity of the function $y(\sigma)$, i.e., $N_2'(y(\sigma))$. Let us consider the ... finding an optimal nondegenerate linear transform of the arguments $\sigma \in \Xi$, for which the quantity $N_2(y(\sigma))$ ... minimum:

$$N_2(y^{(\tilde{\sigma})}) = \min_{\sigma \in \Xi} N_2(y^{(\sigma)}), \quad \mod 2. \tag{6}$$

... $T = \|\tau_{ij}\|$, where $\tau_{ij} \in \{0, 1\}$ (i, j = 0, 1, ..., m−1). We put

$$B_{2,2}(T) = \sum_{j=0}^{m-1} B_{2,2}\left(\sum_{i=0}^{m-1} \tau_{ij} 2^{-i-1}\right). \tag{7}$$

Theorem 2. Let

$$\max_{|T|_1 \neq 0} B_{2,2}(T) = B_{2,2}(\tilde{T}), \tag{8}$$

Then

$$\tilde{\sigma} \otimes \tilde{T} = E, \mod 2. \tag{9}$$

(Here and below $|T|_p$ is the determinant of T over GF(p), E is the unit matrix.)

Theorem 2 gives a simple method for finding the optimal linear transform of the arguments $\tilde{\sigma}$. For this we construct the autocorrelation function modulo 2 and find m linearly independent references over GF(2) such that ... sum of values of $B_{2,2}(\tau)$ in these references be maximal. These m references can be found in the following recurrent way. If i references (i = 0, 1, 2, ..., m−1) $\tilde{\tau}_0, \tilde{\tau}_1, \ldots, \tilde{\tau}_{i-1}$ have been found, then the (i+1)-st reference $\tilde{\tau}_i$ ... found from the condition

$$B_{2,2}(\tilde{\tau}_i)\max_{\tau \in L_i} B_{2,2}(\tau),$$

where L_i is the set of all linear modulo 2 combinations of vectors $\underbrace{00\ldots0}$ $\tilde{\tau}_0, \tilde{\tau}_1, \ldots, \tilde{\tau}_{i-1}$. The $\tilde{\tau}_0, \tilde{\tau}_1, \ldots, \tilde{\tau}_{m-1}$ ... thereby found from the column T. The matrix of the optimal transform is found from T by means of formula (9).

For the example considered above $N_2(y) = 4$

$$\tilde{T} = \begin{Vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{Vmatrix}, \quad \tilde{\sigma} = \begin{Vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{Vmatrix}.$$

The functions $y_0^{(\tilde{\sigma})}$, $y_1^{(\tilde{\sigma})}$ are given in Table 2, where $N_2(y^{(\tilde{\sigma})}) = 8$.

Thus, if k = 1, then in the Boolean function $y^{(\tilde{\sigma})}$ $B_{2,2}(\tilde{\tau}_s)$ constituents of unity can be merged by the s-th argument, which in the general case leads to a simpler realization than for the initial function y. Thus, for example, if $B_{2,2}(\tilde{\tau}_0) = B_{2,2}(\tilde{\tau}_1) = \ldots = B_{2,2}(\tilde{\tau}_i) = N$, where N is the number of constituents of unity of the initial Boolean function it can be shown that as $m \to \infty$ the complexity of the circuit realizing $y^{(\tilde{\sigma})}(L^{(\tilde{\sigma})})$) is bounded by the quantity

$$L(y^{(\tilde{\sigma})}) \sim \frac{2^{m-i}}{m-i}\rho, \tag{10}$$

where ρ is the most complex two-input element of "AND," "OR,"

TABLE 3

x_0	x_1	y	$B_{3,3}$	$y^{(\tilde{\sigma})}$
0	0	1	6	1
0	1	0	0	1
0	2	1	0	1
1	0	1	0	1
1	1	1	6	1
1	2	0	0	1
2	0	0	0	0
2	1	1	0	0
2	2	1	6	0

The method described above for finding the optimal linear transform of arguments $\tilde{\sigma}$ by means of the analysis of the autocorrelation functions $B_{2,2}(\tau)$ was programmed on M-20. The corresponding program contains about 250 instructions and for a system of five Boolean functions of 10 arguments it takes not more than 5 min. to find $\tilde{\sigma}$.

4. Let us extend the method considered above for finding the optimal linear transform of arguments to the case of p-ary (p prime) logic.

Assume Table 1 prescribed k functions of p-ary logic. We construct analogously to (1) the functions $y_j(x)$ ($j = 0, 1, \ldots, k-1$), putting

$$x = \sum_{i=0}^{m-1} x_i p^{-i-1}. \tag{11}$$

The autocorrelation function modulo p of the system of p-ary logic functions is the function $B_{p,p}(\tau)$, defined in the following manner:

$$B_{p,p}(0) = \sum_{j=0}^{k-1} \sum_{x=0}^{1-p^{-m}} y_j(x),$$

$$B_{p,p}(\tau) = \sum_{j=0}^{k-1} \sum_{x=0}^{1-p^{-m}} y_j(x) y_j(x \ominus \tau) \ldots y_j(x \ominus \underbrace{\tau \ominus \tau \ominus \ldots \ominus \tau}_{p-1}), \bmod p \tag{12}$$

$$(\tau \neq 0), \bmod p.$$

As the criterion of complexity of the system of p-ary logic functions (analogously to the functions of binary logic), we take the number $N_p(y) = \sum_{j=0}^{k-1} N_p(y_j)$, where $N_p(y_j)$ is the number of unordered sets of p-vectors ($x^{(1)}$, $x^{(2)}, \ldots, x^{(p)}$) of the arguments for the functions $y_j(x)$ such that $y_j(x^{(1)}) = y_j(x^{(2)}) = \ldots = y_j(x^{(p)}) = 0$ and $x^{(i-1)} \ominus x^{(i-1)} = (\underbrace{0 \ldots 010}_{s} \ldots 0) \bmod p$

$$(s = 1, 2, \ldots, m;\ i = 2, 3, \ldots, p).$$

We shall realize the p-ary logic function, as before, in the form of the block diagram of the figure. Only here $\sigma_{ij} \in \{0, 1, \ldots, p-1\}$ and the block σ are realized by adders modulo p. Then, as before, the problem arises of finding $\tilde{\sigma}$ for which $N_p(y^{(\sigma)})$ attains a minimum.

Let $T = \|\tau_{ij}\|$ ($\tau_{ij} \in \{0, 1, \ldots, p-1\}$; $i, j = 0, 1, \ldots, m-1$). We put

$$B_{p,p}(T) = \sum_{j=0}^{m-1} B_{p,p}\left(\sum_{i=0}^{m-1} \tau_{ij} p^{-i-1}\right). \tag{13}$$

Then, analogously to Theorem 2, it can be shown that if

$$\max_{|T|_p \neq 0} B_{p,p}(T) = B_{p,p}(T), \tag{14}$$

then

$$\sigma \otimes T = E, \bmod p. \tag{15}$$

TABLE 4

x	y	$B_{11,1}$	$y^{(\tilde{\sigma})}$
0	1	6	1
1	0	3	1
2	1	1	1
3	1	4	1
4	0	5	1
5	0	2	1
6	1	2	0
7	1	5	0
8	0	4	0
9	0	1	0
10	1	3	0

Formula (15) gives a method for finding the optimal linear transform of the arguments for a system of functions in p-ary logic.

Example 2. Consider the case p = 3 (Table 3).

We have

$$N_3(y) = 0,$$

$$T = \begin{Vmatrix} 1 & 1 \\ 0 & 1 \end{Vmatrix}, \quad \tilde{\sigma} = \begin{Vmatrix} 1 & 2 \\ 0 & 1 \end{Vmatrix},$$

$$N_3(y^{(\tilde{\sigma})}) = 2.$$

5. Let us generalize the concept introduced above of autocorrelation functions modulo p. We shall consider the class $\{B_{p,q}\}$ of autocorrelation functions defined in the following way:

$$B_{f,p}(\tau) = \sum_{j=0}^{k-1} \sum_{x=0}^{1-p^{-m}} y_j(x)\, y_j(x \ominus \tau) \ldots y_j(x \ominus \underbrace{\tau \ominus \tau \ominus \ldots \ominus \tau}_{-1}), \ \text{mod}\ p. \tag{16}$$

Here we shall consider that $p \geq q$, since for $p < q$

$$B_{p,q}(\tau) = B_{p,p}(\tau) \quad (\tau = 0, p^{-m}, \ldots, 1 - p^{-m}). \tag{17}$$

The function $B_{p,q}(\tau)$ is the cross-correlation function of q successive shifts of the initial function, modulo p.

The most important functions in the class $\{B_{p,q}\}$ for practical application are the functions $\{B_{p,2}\}$ and, in particular, for m = 1, $B_{n,2}$ and $B_{\infty,2}$, where n is the number of references on which the function is prescribed. The function $B_{n,2}$ coincides with the cyclic autocorrelation function, while $y(x) \in \{0, 1\}$ (Sherman codes [7]) or $y(x) \in \{0, 1, \ldots, p-1\}$ (Frank codes [8]).

For the autocorrelation functions modulo p of the functions $B_{p,2}$ there exists a connection with the initial function y(x) in terms of the corresponding spectral transforms, analogous to the Walsh or Laplace transforms.

Let us consider the system of Chrestenson functions $\{\chi_k^{(p)}\}$, defined for arbitrary fixed prime p thus:

$$x = \sum_{r=0}^{m-1} x_r p^{-r-1}, \quad k = \sum_{r=0}^{m-1} k_r p^{-r-1}, \tag{18}$$

$$\chi_k^{(p)}(x) = \exp\left(\frac{2\pi}{p} i \sum_{r=0}^{m-1} x_r k_r\right). \tag{19}$$

The system of functions $\{\chi_k^{(p)}\}$ is complete, orthogonal, and for p = 2 coincides with the system of Walsh functions. The functions $\{\chi_k^{(p)}\}$ are complex functions of the real variable and form a group of characters of the p^m-dimensional p-ary vectors, i.e., there exists an isomorphism [2] between the multiplicative group of $\{\chi_k^{(p)}\}$ $(k = 0, p^{-m}, \ldots, 1-p^{-m})$ and the group of p-ary p^m-dimensional vectors with respect to the operation of componentwise addition modulo p. This makes it possible to realize a system of p-ary logic functions by expansion in orthogonal series in the Chrestenson basis, analogous to the realization of a system of Boolean functions by expansion in a series in Walsh and Haar bases.

The piecewise-constant function obtained by completion of the sequence of coefficients of the expansion of ... in the basis $\{\chi_k^{(p)}\}$ is denoted by $\chi^{(p)}\{y_j\}$, and the complex-conjugate function by $\overline{\chi^{(p)}\{y_j\}}$.

Theorem 3.

$$B_{p,2}(\tau) = p^{m/2} \sum_{j=0}^{k-1} \chi^{(p)} \{\chi^{(p)}\{y_j\}\, \overline{\chi^{(p)}\{y_j\}}\}. \tag{20}$$

For $p < 2$ Theorem 3 is the generalization of Theorem 1 and gives a method for calculating $B_{p,2}(\tau)$ [illegible] out from (16).

The cyclic autocorrelation function $B_{n,2}$ can be used to minimize the number of discontinuities $|\Delta y(x)|$ of the Boolean function

$$|\Delta y(x)| = \sum_{x=0}^{n-1} (y(x) - y(x \ominus 1)), \ \mathrm{mod}\, n, \tag{21}$$

which can, for example, lead to a simplification of the realization of y(x) in a threshold basis.

As before, we shall assume that the circuit realizing y(x) is constructed according to the block diagram of the figure, except that the block σ realizes a linear transformation modulo n of the arguments. Then the optimal linear transformation modulo n $\tilde{\sigma}$, for which $|\Delta y^{(\sigma)}_{(x)}|$ attains a minimum, can be defined from the relation

$$\tilde{\sigma}\tilde{\tau} \equiv 1, \cdot \ \mathrm{mod}\, n, \tag{22}$$

where

$$B_{n,2}(\tilde{\tau}) = \max_{\tau \neq 0} B_{n,2}(\tau). \tag{23}$$

This relation always possesses a solution for the highest common divisor $(\tilde{\tau}, n) = 1$ and, in particular, if n is prime (since $\tilde{\tau} < n$).

Example 3. $n = 11$, $|\Delta y(x)| = 6$, $\tilde{\tau} = 7$, $\tilde{\sigma} = 8$, $|\Delta y^{(\sigma)}(x)| = 2$.

Thus we have considered here the class of autocorrelation characteristics of systems of logical functions. The connections between these characteristics and the spectral Walsh and Chrestenson transforms for the initial logical functions are exhibited. Methods are proposed for solving the problem of determining the optimal nondegenerate linear transform of arguments both for systems of Boolean functions and for systems in multi-valued logic. The proposed methods are based on the analysis of various autocorrelation characteristics of the prescribed system of logical functions.

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