Almost Exhaustive Testing for Combinational Devices*

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Consider a combinational device with n inputs where each output implements a Boolean function of at most s input variables ($s<n$). The problem of exhaustive testing of almost all such devices can be formulated as follows: construct a matrix $T(\epsilon, n, s)$ (rows of $T(\epsilon, n, s)$ are test patterns) with n columns such that all 2^s possible binary vectors appear in each s columns of the matrix, except a small fraction ϵ of all possible s-tuples. A test with $T(\epsilon, n, s)$ as a test matrix is called (ϵ, s)-exhaustive.

The concept of (ϵ, s)- exhaustive tests is a generalization of that of s-exhaustive tests (for $\epsilon = 0$), that have been recently considered in a number of papers [1,2,3,4]. A sequence of tests $T(\epsilon, n, s)$ is called asymptotically s-exhaustive, if $\epsilon \to 0$ with $n \to \infty$.

Considerations based on coding theory can be used to approach the problem of constructing efficient (ϵ,s)-exhaustive tests. For instance, it is obvious that columns of an s-exhaustive test is a subset of a code with the minimum distance 2^{s-1} between codewords and the maximum distance $|T(0,n,s)| - 2^{s-1}$, where $|T(0,n,s)|$ is the number of rows in $T(0,n,s)$.

Let $f(\varepsilon,n,s)$ be the minimum number of test patterns in an (ε,s)- exhaustive test and $\varphi(n,s) = f(0,n,s)$.

In this paper upper and lower bounds for $f(\varepsilon, n,s)$ are presented, and constructions of suboptimal (ε,s)-exhaustive tests are suggested. Bounds for the number of test patterns in asymptotically s-exhaustive sequences of tests are also derived.

The main results obtained are summarized below.

<u>Theorem 1</u> For $n \gg s$ and $\varepsilon > 0$

$$f(\varepsilon, n, s) \leq \varphi(r,s) \quad ,$$

where
$$r = \left\lceil \frac{s(s-1)n}{s(s-1)-2n\ln(1-\varepsilon)} \right\rceil . \tag{1}$$

It follows from Theorem 1 that for any fixed ε and s,

$$\lim_{n\to\infty} f(\varepsilon,n,s) \leq \varphi\left(-\frac{s(s-1)}{2\ln(1-\varepsilon)} , s\right) < \infty .$$

<u>Theorem 2</u> (Lower bound)

$$f(\varepsilon, n, s) \geq 2^{s-2}\log_2 \frac{n-s+2}{(n-s+1)\varepsilon + 1} . \tag{2}$$

For $n-s \to \infty$ $\qquad f(\varepsilon, n, s) \gtrsim -2^{s-2}\log_2\varepsilon$.

<u>Theorem 3</u> (Constructive upper bound)

$$f(\varepsilon,n,s) \leq \frac{(2^s-1)\log_2\varepsilon}{\log_2\left[1-(s!)\binom{2^s-1}{s}^{-1}\prod_{i=0}^{s-1}(2^s-2^i)\right]} \tag{3}$$

for $\log_2 n \leq s \leq cn$ $\quad (0<c<1)$.

For $n \to \infty$ we obtain from (3)

$$f(\varepsilon, n, s) \lesssim c\, 2^{s} \log_2 \varepsilon^{-1} \quad , \tag{4}$$

where $c = -(\log_2 0.711211\ldots) = 2.034\ldots$

A non-constructive upper bound, based on probabilistic reasoning, is given by Theorem 4.

<u>Theorem 4</u>

$$f(\varepsilon,n,s) \le \frac{\log_2 \left(\varepsilon + \binom{n}{s}^{-1}\right) - s}{\log_2 (1-2^{-s})} \quad . \tag{5}$$

Denote by $\Psi(n,s)$ the number of test patterns in tests which belong to an asymptotically s-exhaustive sequence of tests.

<u>Theorem 5</u>

1) For any asymptotically s-exhaustive sequence of tests, there exists a function $\Delta(n-s)$, such that $\Delta(n-s) \to \infty$ when $n-s \to \infty$ and

$$\Psi(n,s) \gtrsim 2^{s} \; (n-s) \; . \tag{6}$$

2) For any $\Delta(n)$, such that $\Delta(n) \to \infty$ when $n \to \infty$, there exists an asymptotically s-exhaustive sequence of tests such that

$$\Psi\,(n,s) < 2^{s} \Delta(n) \tag{7}$$

Theorem 5 shows that $\Psi(n,s)$ can grow substantially slower with n than $\varphi(n,s)$.

References

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