Error Defection in Combinational Networks by Use of Codes Based on Hadamard Matrices*

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Abstract

It is well known that the problem of optimal test generation for VLSI circuits is NP-hard. To overcome this difficulty, a new approach to error detection is suggested. The approach is based on the idea of universal tests which are able to detect all the faults from a given set of faults F in almost all circuits from a given set C. (The probability P(T,F,C) that all the faults from F will be detected by the tests T in a circuit randomly chosen from C, approaches one when the number of inputs in the circuit tends to infinity.) The lower bound on the probability P(T,F,C) is found. Universal sequences of tests are constructed for detection of all input stuck-at and bridging faults in a wide range of multiplicities.

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Consider the set C of combinational circuits with m inputs and k outputs, ((m,k)-circuits) where each circuit is a realization of k (not necessarily distinct) Boolean functions of m input variables, and exactly one circuit corresponds to any possible ordered k-tuple of Boolean functions. Obviously, $|C|=2^{k\cdot 2^m}$. Denote by $F=F(m,k)=\{f_w\}(w=0,1\ldots,W;\ W=|F|-1)$ a set of faults in an (m,k)-circuit. Consider a test $T=T(m,N)$ which is a sequence of N test patterns (m-dimensional binary vectors t_g, $g=1,\ldots,N$) applied to the inputs of an (m,k)-circuit, chosen randomly from C with probability $|C|^{-1}$. Denote by $P(T,F,C)$ the probability that the test T detects all the errors from F in a circuit randomly chosen from C.

Def. 1. A sequence of tests $T=T(m,N)$ is called universal for detection of all faults from set $F=F(m,k)$, if

$$\lim_{m\to\infty} P(T,F,C)=1 \quad . \tag{1}$$

Denote by $\alpha(T,F)$ a lower bound over all faults from F on the fraction of test patterns from T which are distorted by a fault from F. Assume that there exists $\lim_{m\to\infty}\alpha(T,F)=\alpha$. Let $d_r(\hat{T})$ be the minimum Hamming distance between test patterns in a test T (i.e., between rows of a test matrix $\hat{T}$).

<u>Theorem 1.</u> Let F be a set of any input stuck-at and/or bridging faults of multiplicity at most ℓ. Then for any test T with $d_r(\hat{T}) \geq 2\ell+1$

$$P(T,F,C) \geq (1-2^{-\alpha Nk})^{W} \quad . \tag{2}$$

Let n be the minimum number such that $n \geq m$ and there exists a binary Hadamard matrix $\hat{A}_n$ of order n [1]. We use the conjecture (proved for all n<268) that an Hadamard matrix exists for all n which are multiples of 4. Consider an $(n \times m)$-matr $\hat{A}$ whose columns coincide with first m columns of $\hat{A}_n$. Let a_h, $h=1,2,\ldots,n$ be the h-th row of the matrix $\hat{A}$. Design a test $T=T_{st}$ as follows:

$$t_{2g-1}=a_g, t_{2g}=\bar{a}_g \quad (g=1,2,\ldots,\frac{N}{2}) , \tag{3}$$

where $N=|T| \leq 2n$ and $\bar{a}_g$ is the complement (negation) of a_g.

<u>Theorem 2.</u> Let $F=F_{st}$ be the set of all input stuck-at faults of multiplicity at most ℓ ($\ell \leq \frac{m}{4}-1$). Then tests T_{st} defined by (3) form a universal sequence of tests, if

$$N=2\left\lceil \frac{1}{k}\left(\ell og_2 \sum_{i=1}^{\ell} 2^i \binom{m}{i} + \varepsilon(m)\right)\right\rceil , \tag{4}$$

where $\varepsilon(m)$ is any function, such that $\varepsilon(m) \to \infty$ when $m \to \infty$.

<u>Corollary 1.</u> (i). The minimum number of test patterns in a universal sequence of tests for detection of all faults from F_{st} is

$$N(F_{st},m,k) \lesssim 2\left\lceil \frac{1}{k} \ell og_2 \sum_{i=1}^{\ell} 2^i \binom{m}{i}\right\rceil . \tag{5}$$

(ii). For detection of all single input stuck-at faults ($\ell=1$):

$$N(F_{st},m,k) \sim 2\left\lceil \frac{1}{k} \ell og_2 m\right\rceil . \tag{6}$$

(We denote $a(m) \lesssim b(m)$, if $\lim_{m\to\infty} \frac{a(m)}{b(m)} \leq 1$;

$a(m) \sim b(m)$, if $a(m) \lesssim b(m)$ and $b(m) \lesssim a(m)$.)

Consider now detection of input bridging faults. Generate a test $T=T_{br}(m,N)$ in the following way. Let $2^{K-1} < m \le 2^K$. The first K rows of $\hat{T}_{br}$ are formed by binary numbers from 0 to m-1 written as columns (these rows are the first K rows of a matrix consisting of the first m columns of the binary Hadamard matrix $\hat{A}_{2^K}$. All the other rows of $\hat{T}_{br}$ are binary linear combinations of the first K rows. Thus, the columns of $\hat{T}_{br}$ are codewords of a linear error-correcting code. Denote by $n(m,d)$ the length of a shortest linear binary code $C(m,d)$ with the minimum Hamming distance d which contains at least m code words. The upper and lower bounds on $n(m,d)$ are well known [1]. Let the columns of $\hat{T}_{br}$ be the first me codewords of $C(m,d)$.

<u>Theorem 3.</u> Let F_{br} be the set of all input bridging faults of multiplicity at most ℓ ($\ell \le \frac{m}{5} - 1$). Then the tests T_{br} described above form a universal sequence of tests for detection of all the faults from F_{br} if

$$d = \left\lceil \frac{1}{k} \left(\log_2 \sum_{i=2}^{\ell} \binom{m}{i} + \varepsilon(m)\right) \right\rceil . \tag{7}$$

<u>Corollary 2.</u> (i). The minimum number of test patterns in a universal sequence of tests for detection of all faults from F_{br} is

$$N(F_{br},m,k) \lesssim n\left(m, \left\lceil \frac{1}{k} \log_2 \sum_{i=2}^{\ell} \binom{m}{i} \right\rceil \right). \tag{8}$$

(ii). If $\frac{\ell}{k} \to 0$, then

$$N(F_{br},m,k) \sim \log_2 m. \tag{9}$$

It can be shown that tests of the same structure can be used for universal identification of faults from F_{st} and F_{br}, but N (in case of stuck-at faults), and d (in case of bridging faults) must be twice as large as for fault detection.

The results presented here are generalizations of earlier results, published in [2,3].

References.

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