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On a Covering Radius of (n,k) Codes

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Abstract

Let $t(V)$ be the covering radius of a code V, $V_{n,k}$ be the set of all linear binary (n,k) codes and

$$t(n,k) = \min_{V \in V_{n,k}} t(V).$$

Denote $L(n,k)$ the minimal number of arithmetical operations for computing $t(V)$ for any $V \in V_{n,k}$.

In this paper we present Upper bounds for functions $t(n,k)$ and $L(n,k)$.

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The lower bound for t(n,k) follows from the sphere-packing condition

$$\sum_{i=0}^{t(n,k)} \binom{n}{i} \geq 2^{n-k} \quad . \tag{1}$$

We note that

$$t(n,0)=n,\ t(n,n)=0,\ t(n,1) = \lfloor n/2 \rfloor \ , \tag{2}$$

$$t(n,k) = 1 \text{ iff } n - \lceil \log_2 n \rceil \leq k \leq n-1 \ , \tag{3}$$

$t(2^m-1,\ 2^m-m-1) = 1$, and using uniformly packed codes

$$t\ (r(2^m-1),\ r\ (2^m-1)\ -2m) = 2, \tag{4}$$

$$t\ (r(2^m+1)\ ,\ r(2^m+1)\ -2m)\ =2,\ t((2^m-1)(2^{2m}-1),\ (2^m-1)(2^{2m}-1)-3m)=2(m\geq 1), \tag{5}$$

$$t(2^m-2,\ 2^m-m-2)=2. \tag{6}$$

We note also that

$$t(n,k) \leq t(n+1,\ k) \leq t(n,k) + 1. \tag{7}$$

Let us present now some upper bounds for t(n,k).

<u>Theorem 1</u>, $t(n+2^m-1,\ k+2^m-m-1) \leq t(n,k) +1.$ (8)

<u>Theorem 2</u>, Let k(n,d) is the maximal k such that there exists an (n,k (n,d)) code with the distance d. Then

$$t(n,k(n,d)) \leq d-1. \tag{9}$$

<u>Theorem 3,</u> Let V be an (n,k) code with a distance $d \lesssim 3$ and the covering radius $t(V) = t(n,k)$. Then

$$t(n+2,k) \leq t(n,k) + 1. \quad (10)$$

<u>Theorem 4,</u> If $k \geq \sum_{i=1}^{q} 2^{m_i} - \sum_{i=1}^{q} m_i - q$ where m_i are integers $(i=1,\ldots,q)$, then

$$t(n,k) \leq \lfloor 0.5\,(n-k-\sum_{i=1}^{q} m_i)\rfloor + q. \quad (11)$$

<u>Corollary 1</u>

(i) $t(n,k) \leq \lfloor 0.5\,(n-k)\rfloor$; (12)

(ii) If $k \geq 2^m - m - 1$, then $t(n,k) \leq \lfloor 0.5(n-k-m)\rfloor + 1$; (13)

(iii) If $k \geq q(2^m - m - 1)$, then $t(n,k) \leq \lfloor 0.5(n-k-qm)\rfloor + q$. (14)

<u>Example 1.</u> Taking q=2, m=5 we have from (13) and (1) $t(62,52) = 2$.

Values of $t(n,k)$ for $n \leq 32$, $k \leq 26$ have been computed. Denote $L(n,k)$ the minimal number of arithmetical operations for the computation of $t(V)$ for any $V \in V_{n,k}$.

<u>Theorem 5,</u> For $n \to \infty$

$$L(n,k) \lesssim 2^{n-k}(n-k)\log_2\left(n-k-H^{-1}\left(\frac{n-k}{n}\right)\right), \quad (15)$$

where $H^{-1}(\alpha)$ is the inverse for $H(\alpha) = -\alpha\log_2\alpha - (1-\alpha)\log_2(1-\alpha)$.

Other estimations on the complexity of computation for a covering radius of (n,k) codes are given in [1].

References

M. Karpovsky, "Weight Distribution of Translates, Covering Radius and Perfect Codes Correcting Errors of the Given Weights", IEEE Trans. Inf. Theory, July, 1981.