SPECTRAL TECHNIQUES FOR OFF-LINE TESTING AND DIAGNOSIS OF COMPUTER SYSTEMS*

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Abstract

In this paper we present some of our recent results on applications of spectral techniques over finite fields to the problems of testing and diagnosis of computer systems.

1. Testing By Space Compression Of Test Responses

Consider a computer system of (not necessarily identical) processing elements (PEs) (this system may be a computer board, multichip module, array processor or computer network). If the system has n output PEs and every PE has at most m output pins, then the response of the system at any given time can be represented as $y = (y_0, \dots, y_{n-1})$, where $y_i \in Z_q$, $q = 2^m$, Z_q is the field with $q = 2^m$ elements and $y \in Z_q^n$.

Let $E \subseteq Z_q^n$ be the set of all possible errors in the system such that as a result of the error $e \in E$ y is replaced by $y + e$.

Problems of compression of test responses for error detection can be formulated in the following way:

P1. Given $E \subseteq Z_q^n$, construct a transform $y \longmapsto H(y)$,

$$H(y) = \{H_j(y) | j = 0, 1, \dots, r-1\}, \qquad H_j(y) \in Z_q,$$

with minimal r such that

$$H(y+e) \neq H(y) \text{ for all } y \in Z_q^n \text{ and } e \in E.$$

P2. Given a distribution $P(E) = \{P_0, P_1, \dots, P_{M-1}\}$, where $M = q^n$ and $P_i = \text{Prob}\{e = i\}$, construct a transform $y \longmapsto H(y)$,

$$H(y) = \{H_j(y) | j = 0, 1, \dots, r-1\}, \qquad H_j(y) \in Z_q,$$

minimizing $\text{Prob}\{H(y+e) = H(y)\}$ (we assume that all $y \in Z_q^n$ are uniformly distributed).

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We note that $Z = H(y) \in Z_q^r$ $(Z = (Z_0, \ldots, Z_{r-1}), (Z_j = H_j(y),\ Z_j \in Z_q))$, and if $r \ll n$ by monitoring $Z \in Z_q^r$ instead of $y \in Z_q^n$ one can achieve a considerable reduction in overheads required for testing.

Let us consider the case $r = 1$. Select

$$Z = Z_1 = H_1(y_0, y_1, \ldots y_{n-1}) = \sum_{i=0}^{n-1} y_i \alpha^i, \tag{1}$$

where α is a primitive element in Z_q,and all the operations in (1) are in Z_q $(n \leq q-1)$. This approach is known as the signature analysis and is widcly used for off-line testing and for built-in self testing (BIST) [1, 2].

Computation of the signature, Z, can be implemented in n steps by the following recursive procedure:

$$Z^{(i-1)} = \alpha Z^{(i)} + y_{i-1} \qquad (i = n, n-1, n-2, \ldots, 1), \tag{2}$$

where

$$Z^n = 0, Z^{(i)} \in Z_q \text{ and } Z^{(0)} = Z.$$

This procedure can be implemented in hardware by an $m-$bit linear feedback shift register (LFSR) with parallel input and with the characteristic equation defined by (2) where y_{i-1} is the new input, $Z^{(i)}$ is the previous state and $Z^{(i-1)}$ is the new state.

The hardware implementation of an m-bit LFSR requires only m flip-flops and several XOR gates in the feedback loops [1].

We note that Z_1 defined by (1) is the first coefficient of the Fourier expansion of $y = (y_0, y_1, \ldots, y_{n-1})$ over Z_q [3, 4]. The network (LFSR) computing Z_1 can be considered as a network computing syndrome for the q-ary$(n, n-1)$ Reed Solomon code of length n with distance 2 [5]. In view of this one can see that (1) provides for the solution of **P1** for single errors, i.e. when $E = \{e \| \|e\| = 1\}$, where $\|e\|$ is the number of nonzero components in $e = (e_0, e_1, \ldots, e_{n-1})$ $(e_i \in Z_q)$.

Let us consider the efficiency of the signature analysis defined by (1) for solution of **P2**. In this case we assume that errors in different components of y are independent (components of e are independent), and all components of e have the same distribution

$$P(e) = (P_0, P_1, \ldots, P_{q-1})$$

$$(q = 2^m, P_i = \text{Prob}\{e_j = i \mid \text{for all } j = 0, 1, \ldots, n-1\}).$$

We will present below our results on probabilities $P_{AL} = \text{Prob}\{H_1(y+e) = H_1(y)\}$ (which are also known as masking or aliasing probabilities) for three important classes of error distributions which correspond to so called "symmetrical errors", "independent errors", and "errors of a given multiplicity".

For symmetrical errors we assume [6, 7, 8]:

$$P_i = \begin{cases} 1-p, & i = 0\,; \\ p(2^m - 1^{-1}, & i \neq 0\,. \end{cases} \quad \text{for some } 0 < p < 1 \tag{3}$$

In this case using results from [6, 10] it is possible to show that

$$\begin{aligned} P_{AL} &= \text{Prob}\{H(y+e) = H(y)\} \\ &= 2^{-m}[1 - 2^m(1-p)^n + (2^m - 1)(1 - 2^m p(2^m - 1)^{-1}). \end{aligned} \tag{4}$$

For independent errors [6, 7, 9] we consider every $e_j \in Z_q (q = 2^m)$ as an m-bit binary vector, and we assume that distortions in the binary components of these vectors are independent. If $\|i\|$ is the number ones in the binary representation of $i\,(i = 0, 1, \ldots 2^m - 1)$, then we have for independent errors

$$P_i = \binom{m}{\|i\|} p_b^{\|i\|}(1 - p_b)^{m - \|i\|}, \tag{5}$$

where p_b is a probability of a distortion of one bit, and for $n = c(2^m - 1)$ we have [6, 7]

$$P_{AL} = 2^{-m}[1 - (2^m - 1)(1 - 2p_b)^{cm2^{m-1}}] - (1 - p_b)^{c(2^m - 1)m}. \tag{6}$$

For errors of a given magnitude [6]

$$P_i = \begin{cases} 1 - p, & i = 0\,; \\ p, & i = a \qquad \text{for some } a \in Z_q - 0\,; \\ 0, & \text{otherwise}; \end{cases} \tag{7}$$

and for $n = 2^m - 1$

$$P_{AL} = 2^{-m}\left[1 - (2^m - 1)(1 - 2p)^{2^{m-1}}\right] - (1 - p)^{2^m - 1}. \tag{8}$$

For large n, (say, $n > 50$) for all three models $P_{AL} = 2^{-m}$. Analysis of aliasing probabilities for benchmark circuits [6] shows that for small n the independent error model predicts minimum aliasing, and the symmetrical model predicts aliasing more accurately than the other models.

Let us consider now the case $r > 1$. In this case we select the following system of r q-ary functions

$$Z_j = H_j(y_0, \ldots, y_{n-1}) = \sum_{i=0}^{n-1} y_i \alpha^{ji}, \qquad (j = 0, \ldots, r - 1) \tag{9}$$

where α is a primitive in Z_q, $q = 2^m$. This approach is known as the multisignature analysis, and it was also used for off-line testing and diagnosis [11, 12, 13]. The hardware implementation of this approach requires r m-bit LFSRs with characteristic equations

$$Z^{(i-1)} = \alpha^j Z^{(i)} + y_{i-1}, \quad Z^{(n)} = 0, \quad Z^{(0)} = Z_j, \quad j = 0, 1, \ldots, r - 1. \tag{10}$$

In this case Z_0, Z_1, …, Z_{r-1} defined by (9) are the first r coefficients of the Fourier expansion of $y = (y_0, y_1, \ldots, y_{n-1})$ over Z_q. The network computing Z_0, Z_1, …, Z_{r-1}

(rm—-bit LFSRs) computes syndromes for the q-ary $(n, n-r)$ Reed-Solomon codes of length n with distance $r+1$. Thus, (9) provides for the minimal solution of **P1** for the case when at most r components of $e = (e_0, e_1, \ldots, e_{n-1})$ are not equal to 0, i.e. at most r components of $y = (y_0, \ldots, y_{n-1})$ are distorted $(E = \{e \,|\, 0 < \|e\| \leq r\})$. For large n we have for the above error models $P_{AL} \longmapsto 2^{-m} = q^{-r}$ for all e such that $\|e\| > r$. ($P_{AL} = 0$ for $0 < \|e\| \leq r$).

Let us consider a more general case when the set, E, of errors is defined by the topology of the system. Let X be the set $\{X_0, X_1, \ldots, X_{N-1}\}$ of N processing elements. Consider a digraph G having X as a set of vertices and a set $U = \{U_0, U_1, \ldots, U_{M-1}\}$ of directed edges (m-bit communication links) between vertices of G. We shall also assume that the graph has no cycles and all n output vertices are reachable from at least one input vertex. Let $y = (y_0, y_1, \ldots, y_{n-1}) \in Z_q^n$ be an output vector for the system represented by the graph G, where $y_i \in Z_q$ is an output of the corresponding output vertex $(i = 0, 1, \ldots, n-1)$.The problem to be considered is the problem of error detection under the assumption of single vertex failures in the graph G. A failure in the graph (system of processing elements) refers to a physical malfunction that cause an undesired event. We consider a fault in the graph which alter its output value to $\widetilde{y} = (\widetilde{y}_0, \widetilde{y}_1, \ldots, \widetilde{y}_{n-1})$, where $\widetilde{y}_i \in Z_q$. The error in the graph's output y can be characterized by the error vector $e = (e_0, e_1, \ldots, e_{n-1})$ where $e_i = \widetilde{y}_i + y_i$ for $i = 0, 1, \ldots, n-1$.

Let us define an error set $E(G)$ characterized by the underlying graph G. In our definition of an error set we assume that at most one vertex or any number of incoming edges to this vertex may fail and a fault in the graph manifests itself by distorting all successor vertices' outputs, i.e. error propagates along a directed path. Let $E_j = \{(e_0^{(j)}, e_1^{(j)}, \ldots, e_n^{(j)})\}$ $(j = 0, \ldots, N-1)$ be a set of error patterns corresponding to a fault in vertex X_j, where $e_i^{(j)} \in Z_q - 0$, if there exists a directed path from X_j to an output and $e_i^{(j)} = 0$ otherwise. The set $E\,(G) = \cup_{j=0}^{N-1} E_j$ of all possible error patterns corresponding to all single vertex failures in G called the error set for G. To illustrate the above definition let us now consider the p-ary star network topology (see Fig.). For this p-ary star, the single central processing element (root) is connected to all others, $N = p+1$ and $n = p$. Due to single vertex of processing element failures we have the following nonzero errors in the p-ary star:

$$E\,(G) = \left\{ \begin{array}{c} (e_0, e_1, \ldots, e_{p-1}, \\ (e_0, 0, 0, 0, \ldots, 0), \\ (0, e_1, 0, 0, \ldots, 0), \\ \vdots, \\ (0, 0, e_2, 0, \ldots, 0), \end{array} \right\}, \tag{11}$$

where $e_i \in Z_q - 0$. Thus, we have $|E\,(G)| = (q-1)^p + p\,(q-1)$ nonzero error vectors for p-ary star over Z_q.

Let $Z = Hy$, where H is a $(r \times n)q$-ary transform matrix over Z_q $(y \in Z_q^n, Z \in Z_q^r)$. Consider a graph, $G\,(E)$, having the error set $E\,(G)$ $(0 \notin E\,(G))$ as a set of vertices and $U = \{(E_i, E_j)\,|E_i, E_j \in E\,(G)\,,\, E_i + E_j \in E\,(G)\}$ as a set of edges.

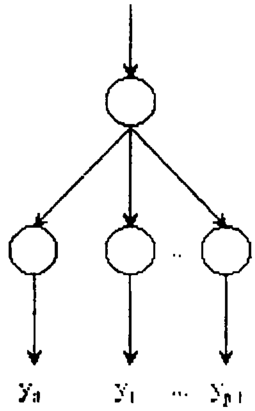

Figure 1: p-ary Star Network Topology

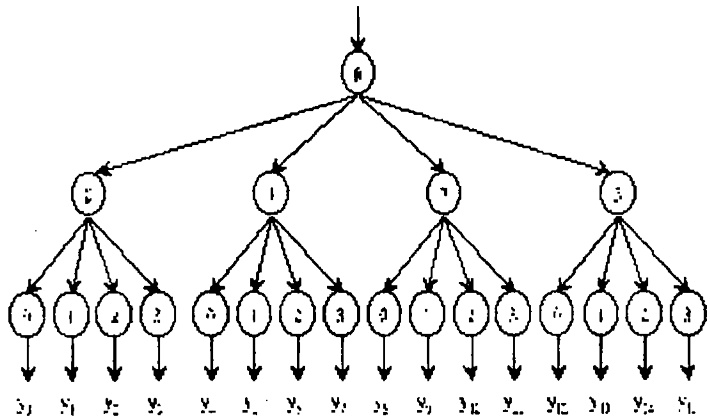

Figure 2: 4-ary Full Tree of Height h=3

Denote by $\chi(E)$ the chromatic number for $G(E)$. ($\chi(E)$ is a minimal number of colors required to color the vertices of $G(E)$ in such a way that no two neighboring vertices have the same color. Techniques for graph coloring with lower and upper bounds for $\chi(E)$ can be found e.g. in [14]). Then using results from [14, 15] we have for a minimal number $r_{\min}$ of transform coefficients required for detection of $E(G)$:

$$\min(\eta_1, \eta_2) \leq r_{\min} \leq \lceil \log_q (|E(G)|(q-1)+1) \rceil \tag{12}$$

where $\eta_1 = n - \lfloor \log_q (q^n - |E(G)|) \rfloor$, and $\eta_2 = \lceil \log_q (\chi(E)+1) \rceil$.

We will present now solutions for the data compression problem **P1** for the two important topologies: trees and Fast Fourier Transform (FFT) networks.

Let T_h be a p-ary full tree of height h ($p \geq 2$, $h \geq 2$). The height of the tree is the length of a longest path from the root to any leaf. (Here we assume that input vertex is the root and the output vertices are $n = p^{h-1}$ leaves of the tree). Then

$$|E(G)| = \sum_{i=0}^{h-1} p^i (q-1)^{p^{h-1-i}}. \tag{13}$$

For example, 4-ary tree T_3 is represented in Fig. .

For this tree the error set $E\left(G\right)$ is:

$$\left\{\begin{array}{llllllllllllllll}
(e_0, & e_1, & e_2, & e_3, & e_4, & e_5, & e_6, & e_7, & e_8, & e_9, & e_{10}, & e_{11}, & e_{12}, & e_{13}, & e_{14}, & e_{15}),\\
(e_0, & e_1, & e_2, & e_3, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0),\\
(0, & 0, & 0, & 0, & e_4, & e_5, & e_6, & e_7, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0),\\
(0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & e_8, & e_9, & e_{10}, & e_{11}, & 0, & 0, & 0, & 0),\\
(0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & e_{12}, & e_{13}, & e_{14}, & e_{15}),\\
(e_0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0),\\
(0, & e_1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0)\\
\multicolumn{16}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots}\\
(0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & e_{15}),
\end{array}\right\} \tag{14}$$

where $e_i \in Z_q - 0$. It is possible to show ([15, 16]), that for a p-ary full tree with $h \leq \left(\log_p\left(1+(q-1)^{-1}\right)\right)$ for any p, $r_{\min} = h$ and

$$H = H^{(h)} = \begin{bmatrix} \overbrace{H^{(h-1)}\ H^{(h-1)}\ H^{(h-1)}\ \dots H^{(h-1)}}^{p} \\ W \end{bmatrix},$$

where W is a row of one 1 followed by $n-1 = p^h - 1$ zeros, and

$$H^{(2)} = \begin{bmatrix} 1\,1\,1\,1\ \dots\ 1 \\ 1\,0\,0\,0\ \dots\ 0 \end{bmatrix}. \tag{15}$$

For example, for T_3 represented by Fig. $(p = 4, h = 3)$ we have

$$H = H^{(3)} = \begin{bmatrix} 1111\ 1111\ 1111\ 1111 \\ 1000\ 1000\ 1000\ 1000 \\ 1000\ 0000\ 0000\ 0000 \end{bmatrix}.$$

For the n-point FFT, there are $N = n\log_2 n$ vertices interconnected with $\log_2 n$ levels of butterfly structures, e.g., the graph for the 8-point FFT with decimation-in-frequency (DIF) is shown in Fig..

If we include input fanout branches as possible source of errors, then there are $n\left(\log_2 n + 1\right)$ single faults in the n-point FFT graph. Due to these single faults we have the following nonzero errors for the 8-point FFT graph of Fig. :

$$E(G) = \left\{\begin{array}{llllllll}
(e_0, & e_1, & e_2, & e_3, & e_4, & e_5, & e_6, & e_7),\\
(e_0, & e_1, & e_2, & e_3, & 0, & 0, & 0, & 0),\\
(0, & 0, & 0, & 0, & e_4, & e_5, & e_6, & e_7),\\
(e_0, & e_1, & 0, & 0, & 0, & 0, & 0, & 0),\\
(0, & 0, & e_2, & e_3, & 0, & 0, & 0, & 0),\\
(0, & 0, & 0, & 0, & e_4, & e_5, & 0, & 0),\\
(0, & 0, & 0, & 0, & 0, & 0, & e_6, & e_7),\\
(e_0, & 0, & 0, & 0, & 0, & 0, & 0, & 0),\\
(0, & e_1, & 0, & 0, & 0, & 0, & 0, & 0),\\
\multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots}\\
(0, & 0, & 0, & 0, & 0, & 0, & 0, & e_7),
\end{array}\right\}$$

where $e_i \in Z_q - 0$ and $|E(G)| = \sum_{i=0}^{t} 2^i (q-1)^{2^{t-i}}$, $n = 2^t$.

The recursive construction for matrices H for DIF FFT topologies is given by

$$H = H^{(n)} = \begin{bmatrix} H^{(n/2)} H^{(n/2)} \\ W \end{bmatrix} \qquad W = [100\ldots0] \quad H^{(2)} = \begin{bmatrix} 11 \\ 10 \end{bmatrix} \tag{16}$$

For example, for the 8-point DIF FFT graph represented by Fig. :

$$H = H^{(8)} = \begin{bmatrix} 11\ 11\ 11\ 11 \\ 10\ 10\ 10\ 10 \\ 10\ 00\ 10\ 00 \\ 10\ 00\ 00\ 00 \end{bmatrix}.$$

(A construction similar to (16) can be used for FFTs with decimation-in-time [16]).

Thus for a n-point FFT we have

$$r_{\min} = \log_2 n + 1. \tag{17}$$

The flow graph of the Walsh-Hadamrd Transform (WHT) [3, 4] differs from the flow graphs of the FFT only in the values of twiddle factors (± 1 for WHT). Thus, solution (16) is valid also for networks computing WHT. Solutions for multidimensional FFTs and for Fast Chrestenson Transforms can be found in [16].

To conclude this section we note that elements of matrices H for trees and FFTs are zeros and ones only.

2. Robust Quadratic Transforms for Testing of Computer Systems.

In the previous section we considered linear data compressors $Z = Hy$, ($Z \in Z_q^r$, $y \in Z_q^n$). For any linear transform we have for the probability, $P_{AL}(e)$, of masking (aliasing probability) for a given error pattern e

$$P_{AL}(e) = \text{Prob}\{H(y+e) = H(y)\} = \text{Prob}\{He = 0\} = \begin{cases} 1, & e \in \text{ Kern } H; \\ 0, & e \notin \text{ Kern } .H \end{cases} \tag{18}$$

Thus, performances of linear compressors are very sensitive to error distributions

$$P = \{P_0, P_1, \ldots, P_{M-1}\} \ (M = q^n,\ P_i = \text{Prob}\{e = i\}).$$

We will describe in this section a class of nonlinear quadratic *robust* transforms, such that their performances do not depend on error distributions. We will show that for optimal robust transforms described by r quadratic q-ary functions, we have

$$P_{AL}(e) = q^{-r} \quad \text{for any } e \neq 0.$$

Suppose $n = 2rs$. Consider the following nonlinear mapping $Z_q^n \longmapsto Z_q^r$

$$Z = (Z_0, \ldots, Z_{r-1}) = H(y_0, \ldots, y_{n-1}) = Y_0 Y_1 + Y_2 Y_3 + \ldots + Y_{2s-2} Y_{2s-1} \tag{19}$$

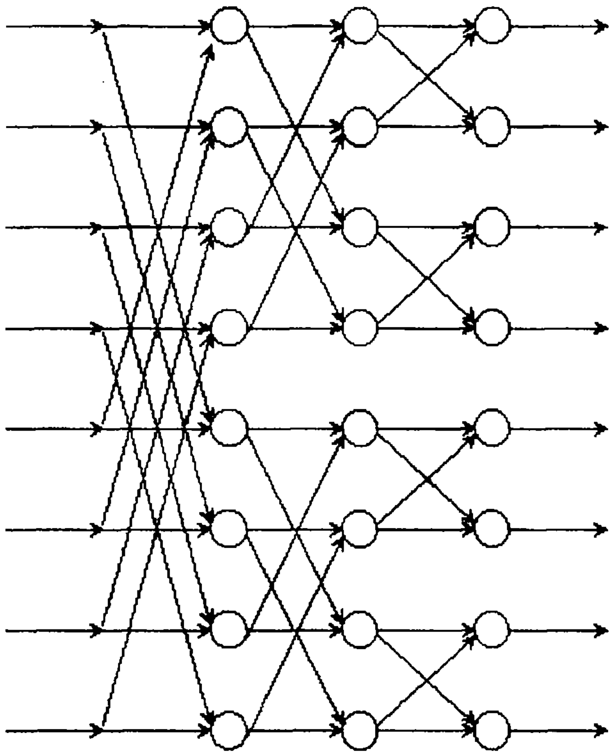

Figure 3: Eight-Point DIF FFT Network Topology

where $Y_i = (y_{ir}, \ldots, y_{(i+1)r-1})$ $(i = 0, \ldots, 2s-1)$, and all operations in the quadratic form (19) are in Z_q^r.

For $q = 2$ and $r = 1$ these quadratic forms are known as bent functions [3, 5, 18, 19, 20, 21]. In this case $n = 2s$, $Y_i = y_i$ and

$$Z = y_0 y_1 + y_2 y_3 + \ldots + y_{n-2} y_{n-1}, \tag{20}$$

and we have for the autocorrelation for bent functions

$$B(e) = \sum_{y \in Z_2^n} H(y)\, H(y+e) = \begin{cases} 2^{n-1} - 2^{n/2-1}, & e = 0\,; \\ 2^{n-2} - 2^{n/2-1}, & e \neq 0\,. \end{cases} \tag{21}$$

Thus $B(e) = \mathrm{Const}_{e \neq 0}$. (We note that $B(e)$ can be computed by applying twice the Walsh-Hadamard Transform using the Wiener-Khinchin theorem [3].)

For any linear function $f(y_0, \ldots, y_{n-1})$ $(y_i \in Z_2)$ such that

$$f(y_0, \ldots, y_{n-1}) = \begin{cases} 1, & (y_0, \ldots, y_{n-1}) \in V \\ 0, & (y_0, \ldots, y_{n-1}) \notin V \end{cases},$$

where V is a subspace of Z_2^n, we have for its autocorrelation

$$B(e) = \sum_{y \in Z_2^n} f(y)\, f(y+e) = \begin{cases} |V|, & e \in V \\ 0, & e \notin V \end{cases}. \tag{22}$$

Since for bent functions defined by (20), we have $B(e) = \mathrm{Const}_{e\neq 0}$, one can say that bent functions are at the maximal distance from any linear function [5].

Let us introduce a system of characteristic functions $h_i(y)$ $(i = 0, \ldots, q^r - 1)$ for non-repeatative quadratic form H over Z_q^r defined by (19):

$$h_i(y) = 1 \text{ iff } H(y) = i. \tag{23}$$

Autocorrelation functions $B_i(e)$ for $h_i(y)$ can be defined as

$$B_i(e) = \sum_{y\in Z_q^n} h_i(y)h_i(y+e) = |\{y|H(y) = H(y+e) = i\}|. \tag{24}$$

(Autocorrelation functions $B_i(e)$ can be computed by the Wiener-Khinchin theorem using the Chrestenson transform over Z_q^r [3, 4].)

It is possible to show [22, 23] that for $i \neq 0$

$$B_i(e) = \begin{cases} q^{2rs-r} - q^{rs-r}, & e = 0; \\ q^{2rs-r} \pm q^{rs-r}, & e \neq 0; \end{cases} \tag{25}$$

and we have for the total autocorrelation

$$B(e) = \sum_i B_i(e) = |\{y|H(y) = H(y+e)\}| = \begin{cases} q^{2rs}, & e = 0; \\ q^{2rs-r}, & e \neq 0; \end{cases} \tag{26}$$

where $H(y)$ is defined by (19).

Thus, quadratic forms defined by (19) have the flat total autocorrelation and compressors implementing these forms are robust with a probability of masking any error

$$P_{AL}(e) = q^{-r} = 2^{-mr}. \tag{27}$$

Let C be a quadratic q-ary error-detecting code defined as $y \in C$ iff

$$H(y) = Y_0Y_1 + \ldots + Y_{2s-2}Y_{2s-1} = 1, \tag{28}$$

where $Y_i = \left(y_{ir}, \ldots, y_{(i+1)r-1}\right)$ and all operations in (28) are in Z_q^r. Then C is a code with the length $n = 2rs$ and the number of codewords $|C| = q^{2sr-r} - q^{sr-r}$. These codes provide for an equal protection against all possible errors for large q or large n and are optimal for the minimax criterion on error detection [22]. For a given block size, n, and the number of codewords, $|C|$, these codes minimize $\max\limits_{e\neq 0} Q(e)$ where $Q(e)$ is the conditional error masking probability given the error pattern e. For these codes for any $e \neq 0$

$$\frac{\left(q^{2rs-2r} - q^{rs-r}\right)}{\left(q^{2rs-r} - q^{rs-r}\right)} \leq Q(e) \leq \frac{\left(q^{2rs-2r} + q^{rs-r}\right)}{\left(q^{2rs-r} + q^{rs-r}\right)} \tag{29}$$

and

$$Q(e) \sim q^{-r} \tag{30}$$

for large n or large q. Simple encoding and decoding procedures for these codes are presented in [22].

3. Soft Decision Diagnosis.

Let $y = (y_0, y_1, \ldots, y_{n-1})$, $y_i \in Z_q$, $q = 2^m$ be a test response for a system-under-test. As a result of faults in the system y may be distorted into $y + e$, where $e = (e_0, e_1, \ldots, e_{n-1})$, $e_i \in Z_q$, $e \in E(G)$, and the set of errors, $E(G)$, is defined by the topology of the system. Denote

$$\operatorname{supp}(e) = (\operatorname{supp}(e_0), \ldots, \operatorname{supp}(e_{n-1})),$$

where

$$\operatorname{supp}(e_i) = \begin{cases} 1, & e_i \neq 0; \\ 0, & e_i = 0. \end{cases}$$

The soft decision diagnosis problem can be formulated in the following way. For a given $E(G) \subseteq Z_q^n$ construct a transform $Z = H(y)$ over Z_q

$$Z_j = H_j(y_0, \ldots, y_{n-1}) \quad (j = 0, 1, \ldots, r-1) \tag{31}$$

with minimal r such that

$$H\left(y + e^{(1)}\right) \neq H\left(y + e^{(2)}\right) \tag{32}$$

for any $e^{(1)}, e^{(2)} \in E(G)$ and $\operatorname{supp}\left(e^{(1)}\right) \neq \operatorname{supp}\left(e^{(2)}\right)$. We will consider the case when H is linear. Then (32) can be written as:

$$He^{(1)} \neq He^{(2)} \tag{33}$$

for any $e^{(1)}, e^{(2)} \in E(G)$, $\operatorname{supp}\left(e^{(1)}\right) \neq \operatorname{supp}\left(e^{(2)}\right)$ and all operations are in Z_q. We will call $H(y)$ a fault-free signature, $H(y+e)$ a faulty signature and $S = He$ a syndrome of error e. If (33) is satisfied, then the location, supp(e), of an error e can be computed by analysis of the distortion $H(y) + H(y+e)$ in the fault-free signature. The bounds on a minimal number of spectral coefficients in the transforms satisfying (33) can be obtained from bound (12) for error detection by replacing the error set $E(G)$ with $E(G) \cup \{e^{(1)} + e^{(2)} \mid \operatorname{supp}\left(e^{(1)}\right) \neq \operatorname{supp}\left(e^{(2)}\right); e^{(1)}, e^{(2)} \in E(G)\}$. We note that any matrix H satisfying (33) is a parity check matrix of a linear q-ary $(n, n-r)$ code locating the error set $E(G)$. We note also that a linear $(n, n-r)$ code with a parity check matrix H locating an error set $E(G)$, $0 \notin E(G)$ corrects $E(G)$ if and only if for every distinct $e^{(1)}, e^{(2)} \in E(G)$ with $\operatorname{supp}\left(e^{(1)}\right) = \operatorname{supp}\left(e^{(2)}\right)$, there exists a pair $e^{(3)}, e^{(4)} \in E(G) \cup 0$ with $\operatorname{supp}\left(e^{(3)}\right) \neq \operatorname{supp}\left(e^{(4)}\right)$ such that $e^{(1)} + e^{(2)} = e^{(3)} + e^{(4)}$.

We note that the above condition is a necessary and sufficient condition on the set $E(G)$ such that if and only if this condition is satisfied any linear code locating

$E(G)$ will also correct $E(G)$. For example, for the p-ary star with $p = 5$ and $q = 2^2$,

$$E(G) = \{(e_0, e_1, e_2, e_3, e_4), (e_0, 0, 0, 0, 0), (0, e_1, 0, 0, 0), (0, 0, e_2, 0, 0), (0, 0, 0, e_3, 0), (0, 0, 0, 0, e_4)\},$$

where $e_i \in \{1 = 01,\ \alpha = 10,\ \alpha^2 = 11\}$. A code locating $E(G)$ does not guarantee error correction since for two errors $e^{(1)} = (1, 1, 1, 1, 1)$, $e^{(2)} = (1, 1, \alpha, \alpha, \alpha)$ there is no pair $e^{(3)}, e^{(4)}$ with different support such that $e^{(1)} + e^{(2)} = e^{(3)} + e^{(4)} = (0, 0, \alpha^2, \alpha^2, \alpha^2)$ (note that $\alpha^2 + \alpha + 1 = 0$).

Using the above arguments one can see that any code over Z_q locating up to l independent errors ($E(G) = \{e \mid 0 < \|e\| \leq 2l\}$) can also correct l errors. The same is also true for codes locating burst errors.

Let us now consider the case when at most l components in $y = (y_0, \dots, y_{n-1})$ may be distorted ($E = \{e \mid 0 < \|e\| \leq l\}$). In this case H can be taken as the check matrix of the q-ary Reed-Solomon code of length n [5]

$$H = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{n-1} \\ 1 & \alpha^2 & \alpha^4 & \dots & \alpha^{2(n-1)} \\ & & \ddots & & \\ 1 & \alpha^{r-1} & \alpha^{2(r-1)} & \dots & \alpha^{(r-1)(n-1)} \end{bmatrix}, \tag{34}$$

where α is primitive in Z_q, $n \leq q - 1$ and $r = 2l$. For this matrix any r columns are linearly independent over Z_q. Computing $Z = Hy$, where H is defined by (34), requires n clocks and r LFSRs with parallel input. An error locating procedure for this case identifies supp(e) for a given He. This procedure is based on the Euclidean algorithm [12] and has a complexity $L = O(lm) + O(\log_2 n)$. (L is the number of equivalent two-input gates, $q = 2^m$.) For example, for $n = 100, m = 32$ and $l = 5$ we have $L \simeq 28,000$ [12]. We note also that H defined by (34) with $r = 2l$ can be used for locating of more than l errors. It was shown in [13] that if $\|e\| = t > l$ then the fraction w of localizable errors with $\|e\| = t,\ l < t < 2l$, is lowerbounded by

$$w \geq (1 - q^{-1})^{\binom{n}{t} - 1} \sim e^{-\binom{n}{t} q^{-1}}. \tag{35}$$

For example, if $t = 5, r = 6, n = 100, q = 2^{32}$ ($m = 32$), then $w = 0.979$. Thus, by allowing a small fraction of errors not to be located, one can reduce substantially the required redundancy from $r = 2l$ to $r = l + 1$.

Let us consider now the error-locating problem when the original system is the full p-ary tree of height h $\left(n = p^{h-1}\right)$ (the 4-ary full tree of height $h = 3$ is presented in Fig. , and the error set for this tree is given by (14)).

The following recursive procedure can be used for construction of $H = H^h$ for p-ary trees [16].

$$H^{(h)} = \begin{bmatrix} H^{(h-1)} & H^{(h-1)} & \dots & & H^{(h-1)} & & \\ 11\dots1 & \alpha\alpha\dots\alpha & \dots & \alpha^{p-1} & \alpha^{p-1} & \dots & \alpha^{p-1} \\ 10\dots0 & 00\dots0 & \dots & 0 & 0 & \dots & 0 \\ 00\dots0 & 10\dots0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} \tag{36}$$

where

$$H^{(2)} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{p-1} \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \end{bmatrix}_{4\times p} \tag{37}$$

$4 \leq p < q$, and α is primitive in Z_q. Thus, for any p-ary, $(4 \leq p < q)$ tree $r = 3h - 2$. For the case of $p = 3$ one can also use the above construction (36) with $H^{(2)} = I_3$ (I_3 is the identity matrix of 3×3). Therefore, for the p-ary tree with $p = 3$, $q > p$, $r = 3h - 3$.

For binary trees over Z_q the following recursive construction can be used:

$$H^{(h)} = \begin{bmatrix} H^{(h-1)} & H^{(h-1)} \\ 10\dots0 & 00\dots0 \\ 00\dots0 & 10\dots0 \end{bmatrix}, \tag{38}$$

where $H^{(2)} = I_2$. Thus, for the binary tree over Z_q of height h, $r = 2h - 2$. The complexity L for the syndrome computing network implementing $H^{(h)}y$ in terms of numbers adders and multipliers in Z_q is [16]:

$$L = \left(2\left(p^{h-1} - p\right) + h\,(p-1)\right) L_{\otimes} + \left((h-1)\,(p-1)\right) L_{\otimes}, \tag{39}$$

where $L_{\otimes}$ is the complexity of a multiplier that multiplies a field element from Z_q by a fixed element from the same field. This network for $h = 3$ is represented in Fig.

The error locating procedure for tree errors is very simple. Let us denote

$$S = H^{(h)}y = \begin{bmatrix} S^{h-1} \\ S_1^h \\ S_2^h \\ S_3^h \end{bmatrix}, \tag{40}$$

where S^{h-1} are syndromes due to the $\left[H^{(h-1)}H^{(h-1)}\dots H^{(h-1)}\right]$ part of $H^{(h)}$ (see (36)) and S_1^h, S_2^h, S_3^h are syndromes for the last three rows of the parity check matrix $H^{(h)}$. Let S^1 denote the syndrome for the all 1 row. The location algorithm to find a faulty vertex is described as follows:

1. If $S_i = 0,\ i = 1, 2, \dots, 3h-2$: no error, end.
2. Let $j = h$
3. If both $S_2^j \neq 0$ and $S_3^j \neq 0$: error location is the root of the tree of height j, end.
4. For $j > 2$, if either $S_2^j = 0$ or $S_3^j = 0$: error location in the subtree k, $0 \leq k \leq p-1$, where $\alpha^k = S_1^j / S_2^{j-1}$; for $j = 2$, if either $S_2^2 = 0$ or $S_2^3 = 0$: error location is in vertex (leaf) k, $0 \leq k \leq p-1$, where $\alpha^k = S_1^2 / S_2^1$.

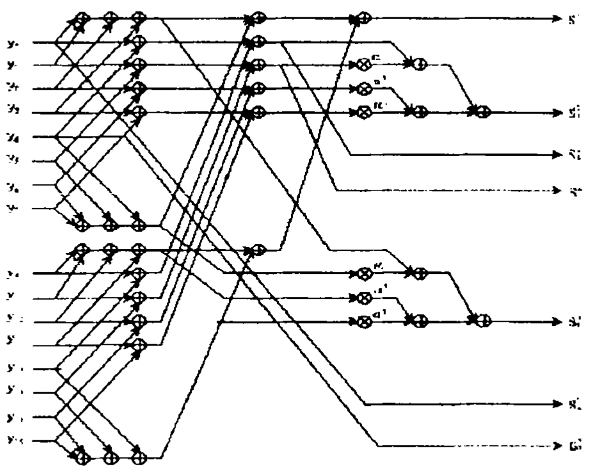

Figure 4: Network for Computing $S = H^{(3)}y$

5. Repeat step 3 and 4 for the tree of height $j = j - 1$.
6. End.

The transform matrix H_3 for the 4-ary tree of height $h = 3$ is

$$H^{(3)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 & \alpha^3 & 1 & \alpha & \alpha^2 & \alpha^3 & 1 & \alpha & \alpha^2 & \alpha^3 & 1 & \alpha & \alpha^2 & \alpha^3 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ ! & 1 & 1 & 1 & \alpha & \alpha & \alpha & \alpha & \alpha^2 & \alpha^2 & \alpha^2 & \alpha^2 & \alpha^3 & \alpha^3 & \alpha^3 & \alpha^3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

Suppose that a root of a subtree 1 (see Fig.) is faulty, $q = 8$, $\alpha^3 + \alpha + 1 = 0$ and

$$e = \left(0, 0, 0, 0, \alpha, \alpha, 1, \alpha^3, 0, 0, 0, 0, 0, 0, 0, 0\right). \tag{42}$$

Then

$$S = H^{(3)}e = \left(\alpha, \alpha^5, \alpha, \alpha, \alpha^2, 0, \alpha\right). \tag{43}$$

This yields $S_2^3 = 0$, therefore the error is in the subtree 1 since $S_1^3/S_2^3 = \alpha^2/\alpha = \alpha$, $i = 1$. Since $S_2^2 \neq 0$ and $S_2^3 \neq 0$, error is in the root of subtree 1 of height 2.

We note that the number, r, of rows in $H^{(h)}$ for p-ary trees of height h can be drastically decreased (from $r = 3h - 2$ to $r = h$) if we allow a small probability of misdiagnosis. In this case one can take [16]

$$H^{(h)} = \begin{bmatrix} H^{(h-1)} \; H^{(h-1)} & \ldots & H^{(h-1)} \\ 11\ldots1 \;\; \alpha\alpha\ldots\alpha & \ldots & \alpha^{p-1}\alpha^{p-1}\ldots\alpha^{p-1} \end{bmatrix}, \tag{44}$$

where

$$H^{(2)} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{n-1} \end{bmatrix}, \quad n = p < q.$$

The number of rows in $H^{(h)}$, defined by (44), is equal to h. The location procedure in this case is very simple, and the probability of correct diagnosis for this procedure can be estimated as [16]

$$w = 1 - \left(1 - pq^{-1}\right)^{h-1} \tag{45}$$

which is small for large q.

4. Hard Decision Diagnosis.

The hard decision diagnosis problem can be formulated in the following way. For a given $E(G) \subseteq Z_q^n$ construct a **binary** matrix H with minimal number of rows such that for any $e^{(1)}, e^{(2)} \in E(G)$ with

$$\operatorname{supp}\left(e^{(1)}\right) \neq \operatorname{supp}\left(e^{(2)}\right), \quad \operatorname{supp}\left(He^{(1)}\right) \neq \operatorname{supp}\left(He^{(2)}\right),$$

$$\text{where} \quad \operatorname{supp}\left(Z = Z_0, \dots, Z_{r-1}\right) = \left(\operatorname{supp}\left(Z_0\right), \dots, \operatorname{supp}\left(Z_{r-1}\right)\right). \tag{46}$$

The hard decision approach allows identification of errors e by analyzing supports of their syndromes $S = He$. Magnitudes of distortions in components of a syndrome are not important for the hard decision diagnosis. The identification of locations of errors is possible if there is a one-to-one mapping

$$E(G) \leftrightarrow \{\operatorname{supp}(He) \mid e \in E(G)\}.$$

In general, this approach requires more rows in H (more observation points for the compressed response $Z = Hy$) but the decoding procedure is very simple and has a straightforward hardware implementation.

Let $H \otimes \operatorname{supp}(e)$ denote the Boolean multiplication of an $(r \times n)$ binary matrix H by an n-bit binary vector $\operatorname{supp}(e) = (\operatorname{supp}(e_0), \dots, \operatorname{supp}(e_{n-1}))$ with the addition being replaced by logical summation (OR). Then with probability $1 - q^{-1}$ if

$$H \otimes \operatorname{supp}\left(e^{(1)}\right) \neq H \otimes \operatorname{supp}\left(e^{(2)}\right), \tag{47}$$

then

$$He^{(1)} \neq He^{(1)}; \; e^{(1)}, e^{(2)} \in E(G) \subseteq Z_q^n$$

and

$$\operatorname{supp}\left(e^{(1)}\right) \neq \operatorname{supp}\left(e^{(2)}\right).$$

Let us consider now the hard decision diagnosis problem for the case of l independent errors ($E = \{e \mid 0 < \|e\| \leq l\}$). We note that in this case any check matrix of an *l-th* order binary superimposed code [24, 25, 26, 27] can be chosen as the transform matrix H.

A binary superimposed code of order l consists of a set of codewords such that componentwise Boolean sum (OR) of any l codewords differs from every other componentwise sum of l or fewer codewords. Thus, in view of (47), any check matrix of an $l-th$ order linear superimposed code can be chosen as a hard decision diagnostic matrix H. Since there are $|E| = \sum_{i=0}^{l}\binom{n}{i}$ different locations of l errors, we have the following lower bound on the minimal number $r = r(n,l)$ of the required spectral coefficients (rows in H)

$$r(n,l) \geq \lceil \log_2 \sum_{i=0}^{l} \binom{n}{l} \rceil . \tag{48}$$

For the case of single errors ($l = \|e\| = 1$), one can take as H any binary matrix with different nonzero columns. Thus

$$r(n,l) = \lceil \log_2 (n+1) \rceil . \tag{49}$$

The case of multiple errors ($l > 1$) is not as simple and it is difficult even to estimate $r = r(n,l)$. Several good constructions of check matrices for linear superimposed codes can be found in [24, 25, 26].

A hardware implementation of the hard decision diagnostic algorithms for $l = 1, n = 100, q = 2^{32}$ requires about 10,000 equivalent two-input gates [27].

Let us consider now a general case of hard decision diagnosis, when the error set E(G) is defined by the topology G of the system. Let N be the number of processing elements in the systems (nodes in G) and d is the length (number of nodes) of the longest path in G. Then we have the following attainable bounds on minimal number of rows in H

$$\max(\lceil \log_2 (N+1) \rceil, d) \leq r \leq n. \tag{50}$$

These bounds can be improved if we have additional information about the topology of the system.

Let $N(j)$ be the number of paths of length at least j and $M(j)$ be the number of paths of length at least j which do not have any endpoints in common. The following two lower bounds on r have been proven in [28, 29]:

$$\sum_{i=1}^{r-j+1} \binom{r}{i} \geq N(j), \tag{51}$$

$$\sum_{i=1}^{\lceil (r-j)2^{-1} \rceil} \binom{r}{i} + \sum_{i=\lceil (r+j)2^{-1} \rceil}^{r} \binom{r}{i} \geq M(j). \tag{52}$$

Lower bounds (51), (52) are valid for all values of $j = 1, 2, \ldots, d$.

For the p-ary full tree of height h we have $d = h, n = p^{h-1}, N = \frac{p^h-1}{p-1}$ and $N(d) = p^{d-1}$. Thus, by (50), (51) we have the following lower bounds for the minimal

d	2	3	4	5	6	7
r(d)	2	4	5	6	8	9-10

d	8	9	10	11	12
r(d)	10-11	12	13-14	14-16	16-17

Table 1: Minimal Numbers of Spectral Coefficients Required for Hard Decision Diagnosis of Binary Trees

number $r = r\,(d)$ of spectral coefficients required for hard diagnosis of p-ary trees of height $h = d$.

$$r\,(d) \geq \lceil \log_2 \left(\frac{p^{d-1} - 1}{p - 1} + 1 \right) \rceil \tag{53}$$

and

$$\sum_{i=1}^{r(d)-d+1} \binom{r(d)}{i} \geq p^{d-1}. \tag{54}$$

Solving (54) for $d \gg 1$ we have asymptotically

$$r\,(d) > \begin{cases} 1.29\,(d-1)\,, & p = 2; \\ 1.64\,(d-1)\,, & p = 3; \\ (\log_2 p)\,(d-1)\,, & p \geq 4. \end{cases} \tag{55}$$

We note, that for $p \geq 4$ and $d \gg 1$ (53) gives better lower bounds than (55). The best known upper bounds on r for binary trees [29] are given by

$$r\,(d) \leq \begin{cases} 1.5\,(d-1)\,, & d = 4r + 1; \\ \lceil 1.5\,(d-1) \rceil + 1, & otherwise. \end{cases} \tag{56}$$

Values of $r\,(d)$ for binary trees with $N < 2^{12}$ are given in the Table 1:

Optimal or near optimal constructions for $\left(r\,(d) \times p^{d-1}\right)$ transform matrices H for p-ary trees, together with lower and upper bounds for $r\,(d)$ for different $p > 2$ and $N = \frac{p^d-1}{p-1} < 5,000$ can be found in [28, 29]. The gap between bounds is small.

Constructions for optimal or near optimal transforms for hard diagnosis for two dimensional meshes and multidimensional cube topologies can be found in [29].

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