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Multiple Signature Analysis: A Framework for Built-In Self-Diagnostic

Mark G. Karpovsky, Saeed M. Chaudhry, and Lev B. Levitin

Research Laboratory for Design
and Testing of Computer Hardware
Department of Electrical, Computer
and Systems Engineering
Boston University
44 Cummington Street
Boston, MA 02215

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  - Fault Detecting and Locating Capabilities
- Probabilities of Fault Masking and Correct Fault Diagnosis
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Problem Motivation

Goals:

- Identification of faulty components in a board or system from space-time compressed responses.

- Analysis of fault-detecting and locating capabilities for space-time compressors.

- Estimation of fault-masking and correct fault diagnosis probabilities.

Assumption:

At most l components in the board may be faulty.

Approach:

Multiple signature analysis using nonbinary multiple error-correcting Reed-Solomon codes.

Multiple Signature Analysis Approach

Diagnostic Model:

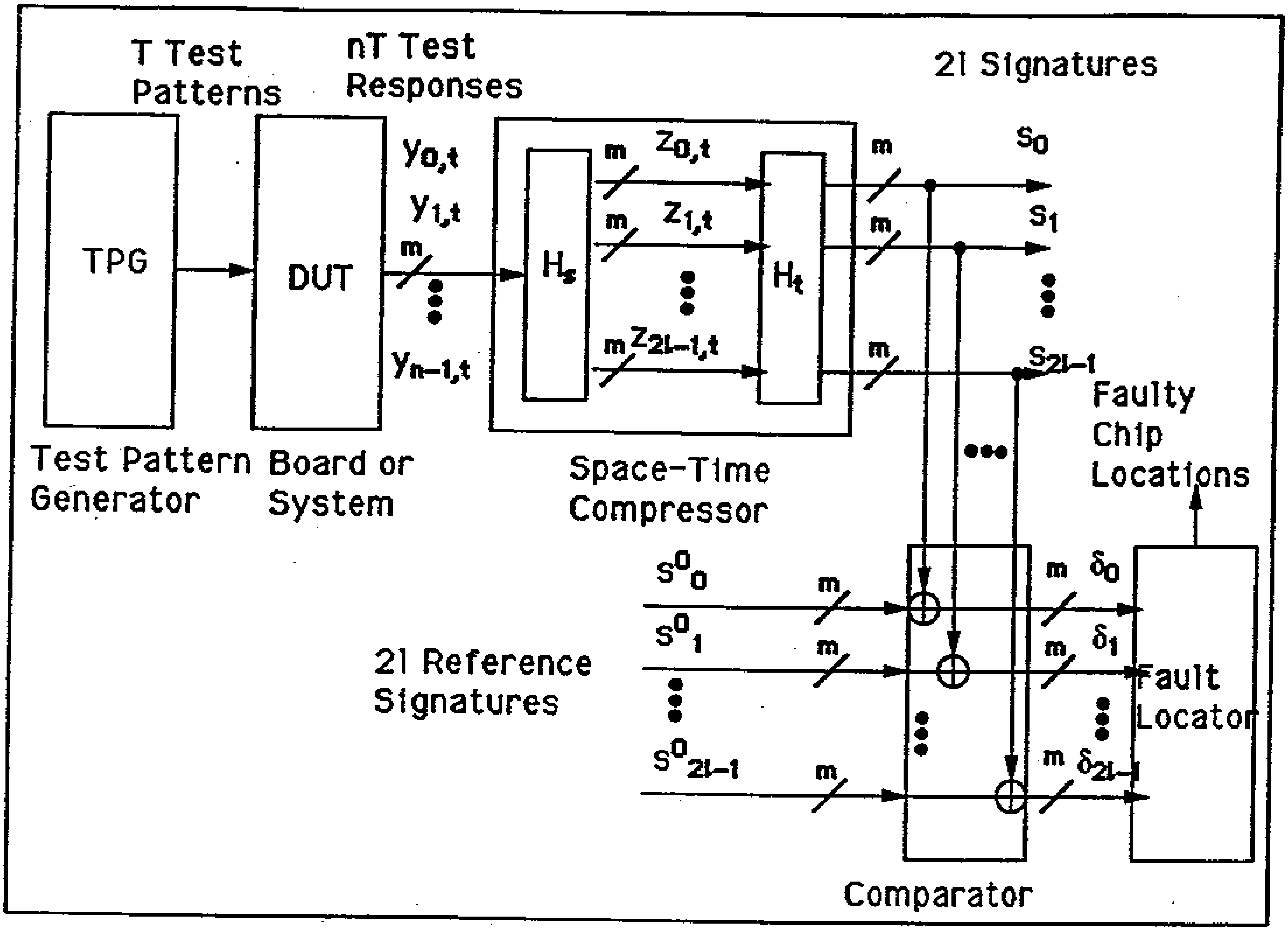
1. Application of pseudorandom test patterns onto the primary inputs of n chips on a board under test.

2. Space and time compression of test responses into $2l$ space-time signatures where l is the number of faulty chips to be located.

3. Identification of faulty chips by analyzing distortions in the space-time signatures.

Multiple Signature Analysis Approach

Built-In Self-Diagnostic:



Space-Time Compression of Test Responses

Test Responses:

Test responses obtained from n chips of a board under test with respect to T test patterns are

$$Y = \begin{bmatrix} y_{0,0} & y_{0,1} & \cdots & y_{0,T-1} \\ y_{1,0} & y_{1,1} & \cdots & y_{1,T-1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n-1,0} & y_{n-1,1} & \cdots & y_{n-1,T-1} \end{bmatrix},$$

where $y_{i,t} \in GF(2^m)$ is a (m-bit) chip test response from the chip number i, $0 \leq i \leq n-1$, at time moment t (m is the number of output pins in a chip).

Space-Time Compression of Test Responses

Space Compression:

Space compression based on a check matrix H_s for the $[n, n-2l, 2l+1]$ l-error-correcting Reed-Solomon code is

$$Z = [z_{j,t}] = H_s Y,$$

where

$$H_s = \begin{bmatrix} 1 & \alpha & \cdots & \alpha^{n-1} \\ 1 & \alpha^2 & \cdots & \alpha^{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{2l} & \cdots & \alpha^{2l(n-1)} \end{bmatrix},$$

$z_{j,t} = \sum_{i=0}^{n-1} y_{i,t}\alpha^{(j+1)i}$, $0 \leq j \leq 2l-1$, $0 \leq t \leq T-1$, and α is a primitive in $GF(2^m)$, that is $\alpha^i \neq \alpha^j$ $i \neq j$, $i, j = 0, 1, \ldots, 2^m - 2$.

Space-Time Compression of Test Responses

Time Compression:

Time compression based on a check matrix H_t for the $[T, T-1, 2]$ Reed-Solomon code is

$$\begin{aligned} s = (s_0, \cdots, s_{2l-1})^{tr} &= ZH_t^{tr}, \\ &= H_s Y H_t^{tr}, \end{aligned}$$

where

$$H_t = \left[\alpha^{T-1} \alpha^{T-2} \cdots 1\right],$$

and $s_j = \sum_{t=0}^{T-1} z_{j,t} \alpha^{T-1-t}$, $0 \leq j \leq 2l-1$.

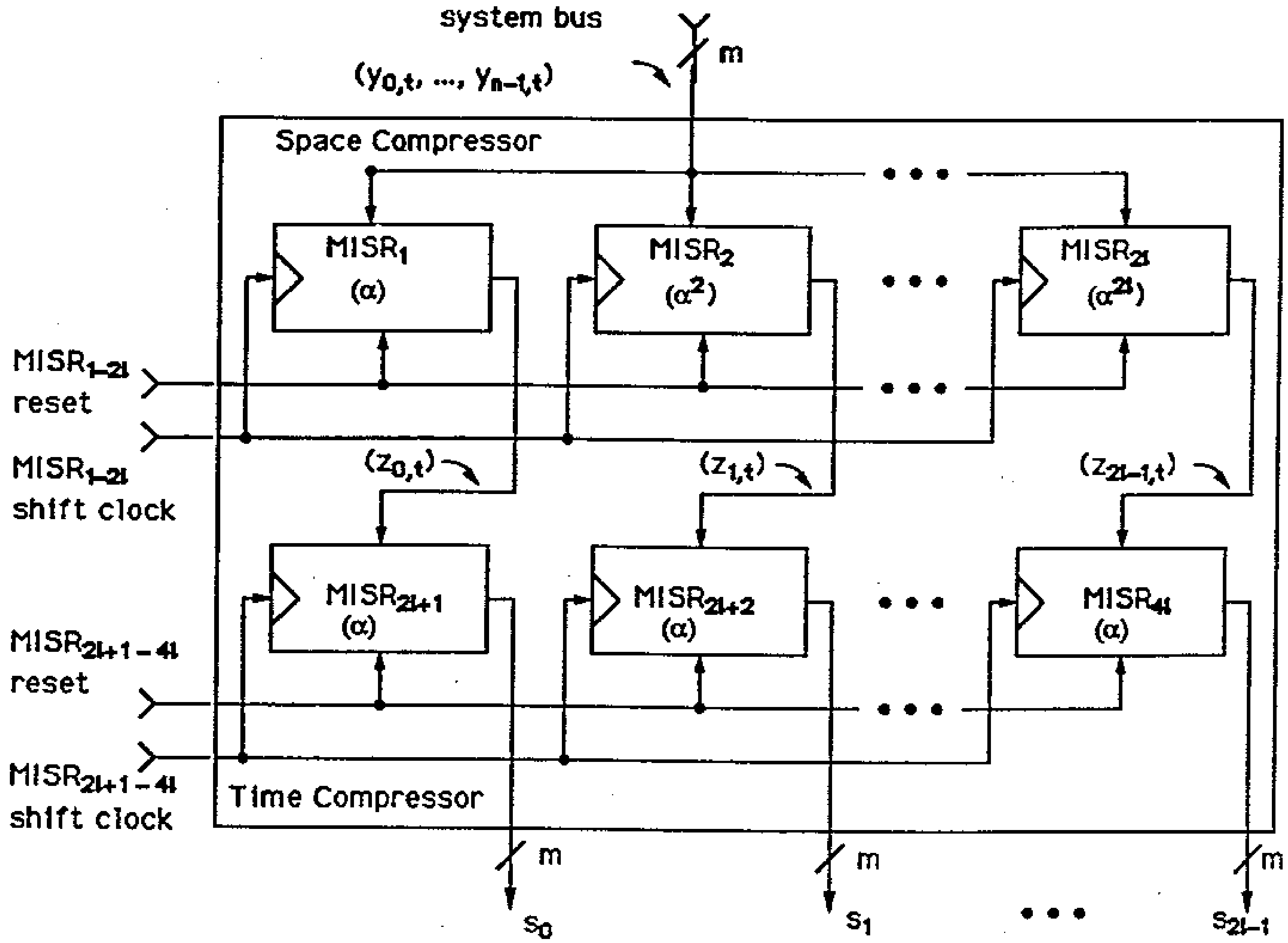
Space-Time Compression of Test Responses

Structure of a Space-Time Compressor:

Space compression by H_s requires $2l$ MISRs with feedback polynomials corresponding to α, $\alpha^2, \ldots, \alpha^{2l}$ and time compression is performed by $2l$ identical MISRs with feedback polynomial corresponding to α.

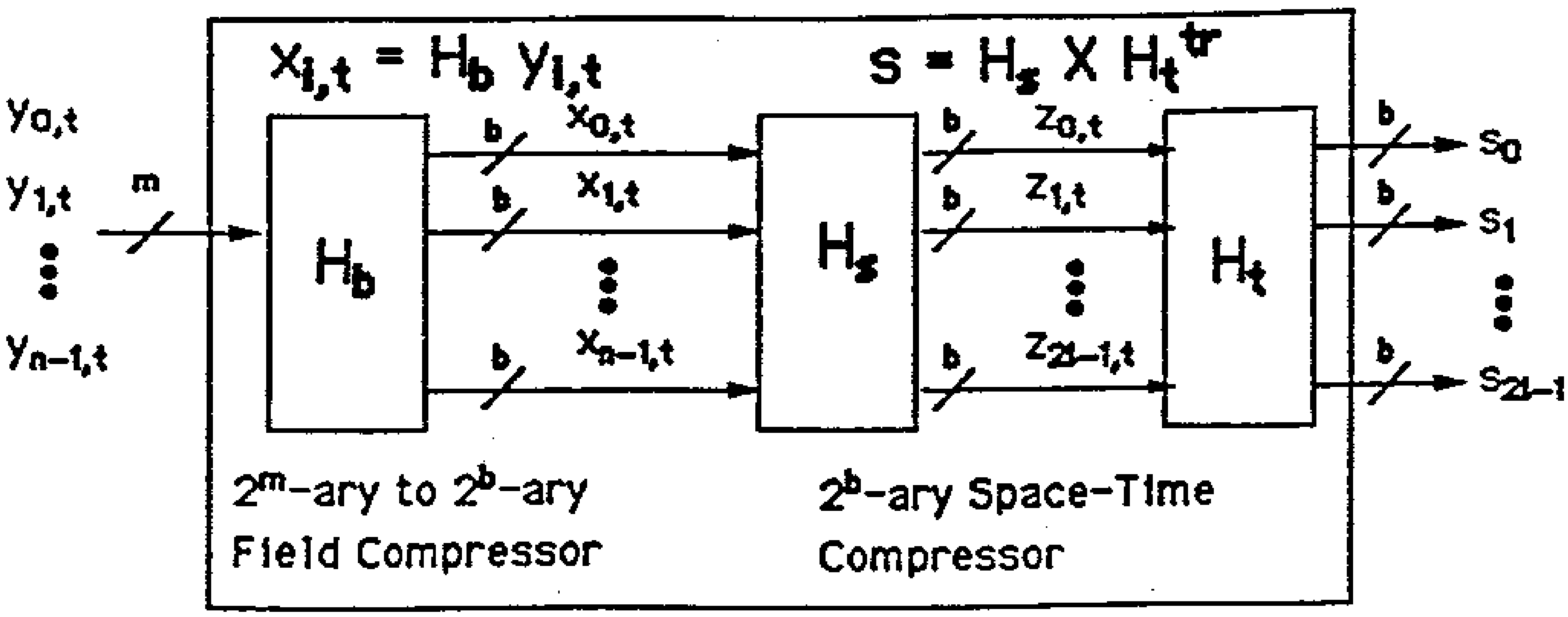
Space-Time Compression of Test Responses

Space-Time Compressor:



Space-Time Compression of Test Responses

Field Compressor:



Fault Detecting and Locating Capabilities

Let $E = [e_{i,t}]$ where $e_{i,t} = y_{i,t} \oplus y^0_{i,t} \in GF(2^m)$, $0 \leq i \leq n-1$, $0 \leq t \leq T-1$ are error manifestations due to faulty chips on board under test and $y^0_{i,t}$ are the correct chip responses.

Then distortions $\delta = (\delta_0, \ldots, \delta_{2l-1})^{tr}$ in space-time signatures are

$$\begin{aligned}\delta = s \oplus s^0 &= H_s Y H_t^{tr} \oplus H_s Y^0 H_t^{tr}, \\ &= H_s E H_t^{tr},\end{aligned}$$

where s^0 are precomputed space-time signatures for the fault free response Y^0.

Fault Detecting and Locating Capabilities

Now let $e = (e_0, \ldots, e_{n-1})^{tr}$ denote the time compressed error vector as

$$e = EH_t^{tr},$$

where $e_i = \sum_{t=0}^{T-1} e_{i,t}\alpha^{T-1-t}$, $0 \leq i \leq n-1$.

Then distortions δ in space-time signatures are

$$\delta = H_s e,$$

where $\delta_j = \sum_{i=0}^{n-1} e_i \alpha^{(j+1)i}$, $0 \leq j \leq 2l-1$.

Since, the minimum distance of the space compression code is $2l+1$, up to $2l$ errors in e can be detected and up to l errors in e can be located.

Probabilities of Fault-Masking and Diagnosis

Chip-independent Error Model:

There are two components involved in error $e_{i,t} = y_{i,t} \oplus y^0_{i,t}$, *temporal* and *spatial*.

For the *temporal* aspect, we assume that errors at two different times are not correlated (*temporal independence*).

For the *spatial* aspect, we assume that each chip output at a given time is distorted independently of other chips (*spatial independence*) and for each chip we assume 2^m-ary symmetric error model.

Probabilities of Fault-Masking and Diagnosis

Fault Masking Probability:

$$P_{fm} = \text{Pr}\{\delta = 0\} - \text{Pr}\{E = 0\},$$

Let w be the probability that chip i is faulty and p_T be the conditional probability that e_i (ith component of time compressed error vector e) is nonzero given chip i is faulty. Then

$$\text{Pr}\{\delta = 0\} = \sum_{i=0}^{n} A_i \left(\frac{p_T w}{q-1}\right)^i (1 - p_T w)^{n-i}.$$

where n is the number of chips on board, A_i are the number of codewords of weight i in the space compression code, $q = 2^m$ and m is the number of output pins per chip.

Let p be the conditional probability that $e_{i,t}$ is nonzero given chip i is faulty. Then

$$\text{Pr}\{E = 0\} = (w(1-p)^T + 1 - w)^n.$$

Probabilities of Fault-Masking and Diagnosis

Fault Masking Probability:

The conditional probability p_T that e_i is not equal to zero when T test patterns are applied is

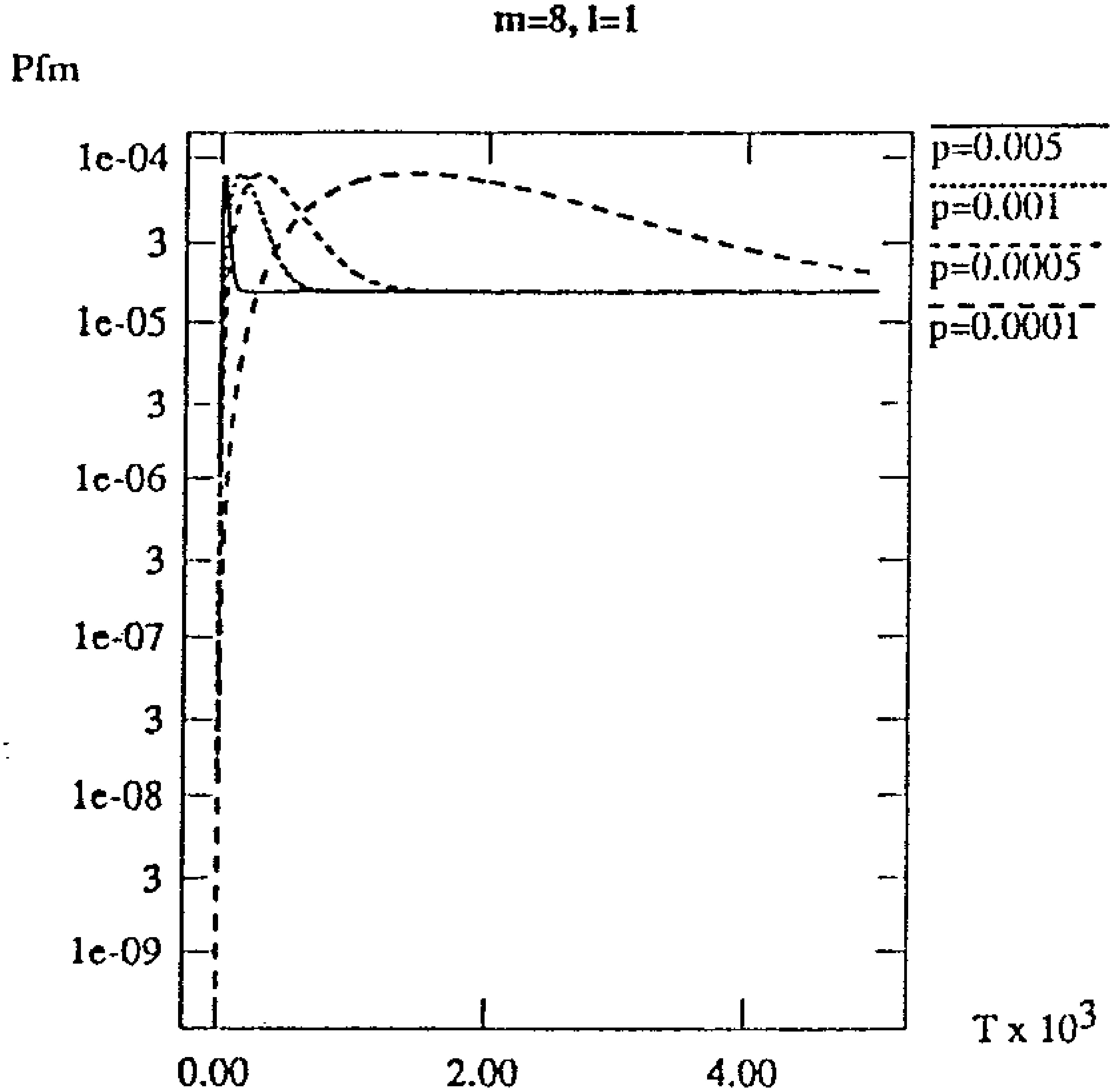
$$p_T = 1 - \left(\frac{p}{q-1}\right) p_{T-1} - (1-p)(1-p_{T-1}).$$

Solving the above difference equation with initial condition $p_1 = p$ we have

$$p_T = 1 - q^{-1}\left[1 + (q-1)\left(1 - \frac{qp}{q-1}\right)^T\right].$$

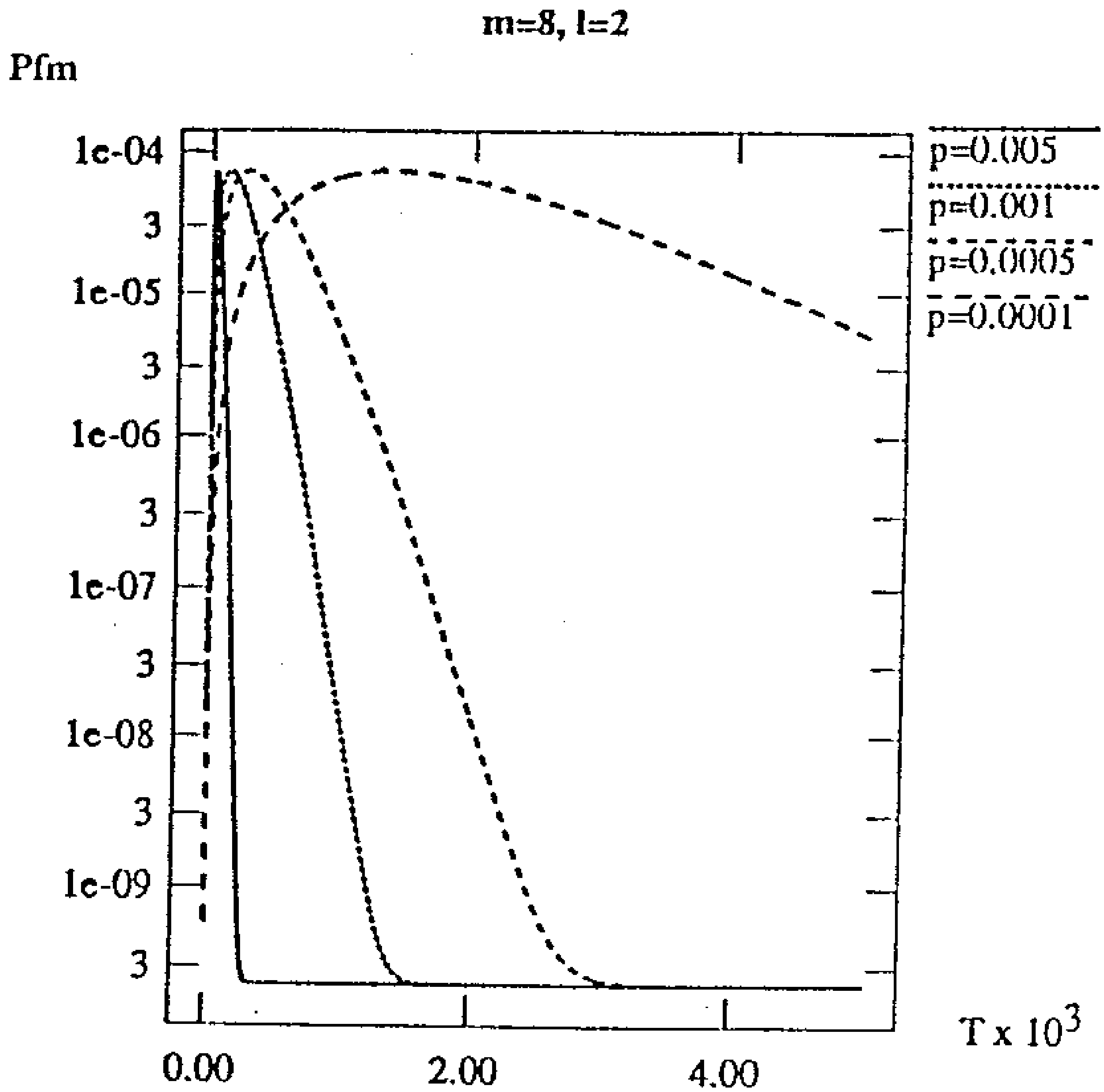
Probabilities of Fault-Masking and Diagnosis

Fault Masking Probability:



Probabilities of Fault-Masking and Diagnosis

Fault Masking Probability:



Probabilities of Fault-Masking and Diagnosis

Fault Masking Probability:

$$P_{fm} = \sum_{i=0}^{n} A_i \left(\frac{p_T w}{q-1}\right)^i (1 - p_T w)^{n-i} - (w(1-p)^T + 1 - w)^n$$

This result is a generalization of several known results. For example, for $n = 1$, MISR compression becomes a special case of the space-time compressor.

$$P_{fm} = q^{-1} \left[1 + (q-1)\left(1 - \frac{qp}{q-1}\right)^T\right] - (1-p)^T.$$

For $T = 1$, a multiplex MISR consisting of $d - 1 = 2l$ MISRs becomes a special case of the space-time compressor.

$$P_{fm} = \sum_{i=1}^{n} A_i \left(\frac{p}{q-1}\right)^i (1-p)^{n-i}.$$

Probabilities of Fault-Masking and Diagnosis

Correct Fault Diagnosis Probability:

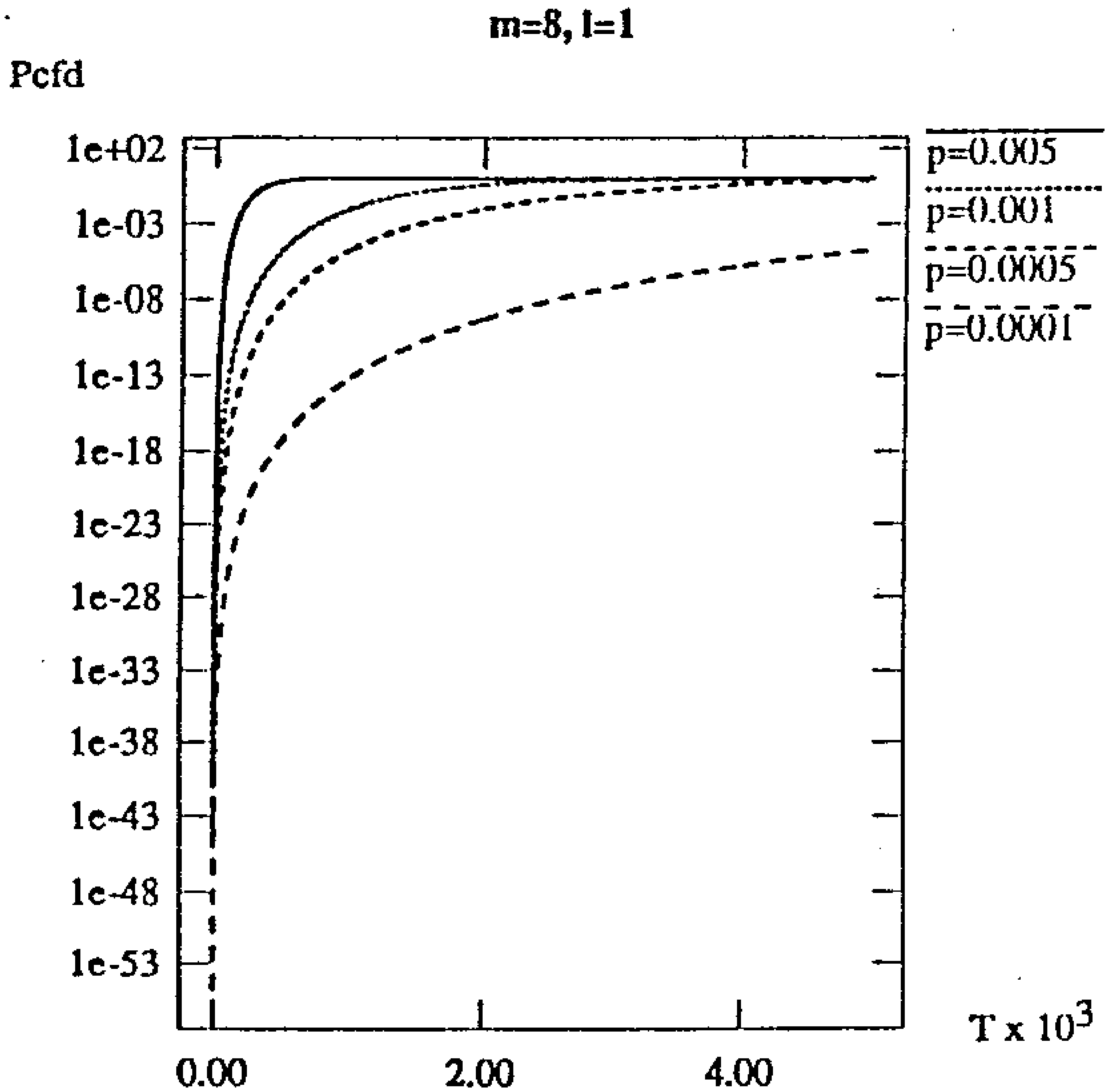
The probability of correct fault diagnosis, P_{cfd}, for a bounded distance fault-locator is the probability that the fault locator output corresponds to correct fault locations.

For the chip-independent error model

$$P_{cfd} = \sum_{i=0}^{l} \binom{n}{i} (p_T w)^i (1 - p_T w)^{n-i}.$$

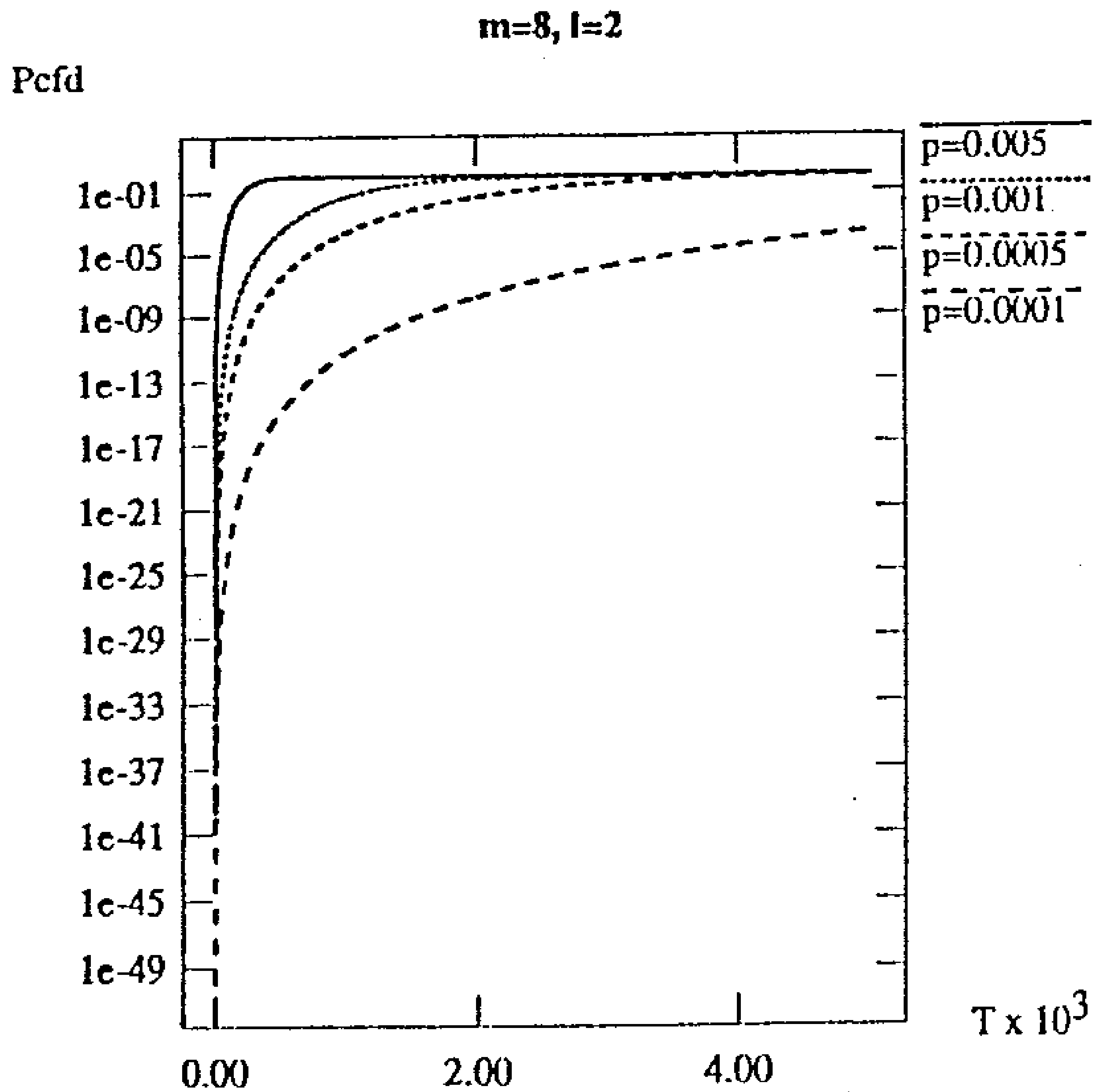
Probabilities of Fault-Masking and Diagnosis

Correct Fault Diagnosis Probability:



Probabilities of Fault-Masking and Diagnosis

Correct Fault Diagnosis Probability:



Conclusions

1. Up to l faulty chips can be identified using $2l$ space-time signatures.

2. Probability of fault masking is 2^{-2lm}.