Coset Error Detection in BIST Design

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***Abstract*–A finite-field algebraic description of a built-in self-test (BIST) design based on primitive feedback shift registers (LFSRs) implementing the test generator and the signature analyzer (SA) is presented. We show that the BIST schemes that use the TG and the SA with the same feedback polynomial detect errors which distort the output functions of the circuits-under-test for a set of the input vectors forming a coset of a subspace. We term this type of errors the *"single coset errors."* Signature schemes which detects the coset errors of multiplicity r are further described, (r<n, where n is the number of inputs). It is also shown that a BIST scheme based on TG and SA with feedback polynomials reciprocal of one another will have a poor error detection capability.**

***Index terms*– Built-in self-test, signature analysis, compaction of test responses multiple-input shift registers, error aliasing.**

I. Introduction

Built-in self-test (BIST) schemes have been widely used in a design for test of VLSI circuits [1], [2]. One of the ubiquitous scheme uses linear sequential circuits (e.g., the linear feedback shift registers, LFSRs or the cellular automata registers, CARs) with primitive feedback polynomials implementing the pseudorandom test pattern generator and the signature analyzer [1]-[3]. A test procedure for these schemes is to apply all possible nonzero input vectors generated by an LFSR with primitive polynomial in which the test responses are compacted into a signature using a multiple-input shift register, MISR (parallel-input LFSR).

The performance of a signature scheme is described in terms of the *aliasing* probability which is the probability that distorted test responses will be compacted into a correct signature. Research on the techniques that estimate the aliasing probability of the MISRs signature analyzers (with primitive or nonprimitive feedback polynomials) [4]-[6] have been reported where different error models such as the independent error model [5] and the 2^m-ary symmetric error model [6]. Since the MISR signature scheme is a linear operation aliasing occurs if only if errors are compacted into zero (signature of all zero), the characteristics and the statistics of errors plays an important factor in the estimation of the aliasing probability, or, in the design of an optimal signature scheme that minimizes the aliasing probability.

In this research a characteristic of errors which emerged from a fault analysis of a two-level AND-OR or a two-level AND-Exclusive-OR (AND-XOR) circuits called the *"single product-term"* errors is examined. Single product-term errors are the distortion in the output functions which can be expressed in terms of a single product term (cube). Furthermore, if a single cube error does not cover the zero minterm we called this type of errors the *"single coset"* errors since in this case the Boolean function is equal to one for a set of input vectors forming a coset of a linear subspace. Single product-term errors appear in two-level circuits from the fact that single faults occurring at the input of a product term either as a disappearance of an input to an AND gate (e.g., an input stuck-at one) or an addition of an input (e.g., an addition crosspoint in an AND array of a PLA) will result in an expansion or a shrinkage of the product term by one dimension, respectively. More, a product term could disappear from the output functions (e.g., due to a wordline stuck-at zero) or an additional product term could appear in the function (e.g., due to crosspoint fault in the OR array of a PLA). These types of defects will likely result in the single product-term errors, for instance, single crosspoint faults in a PLA with the exception that an expansion of faulty product-terms or an addition of product-term in an output function sometime results multiple product-term error function. However, for the case of the two-level AND-XOR circuits (Reed-Muller networks) [7] all single faults in the AND array and at the XOR-gate will result in single product-term errors.

We will show that the BIST schemes which are based on the test generation and the MISR signature analysis with the same feedback polynomial detect all single coset errors, and, detect almost all cases of single product-term errors. The single product-term error function also includes the case when the function is equal to one for a set of input vectors forming a coordinate linear subspace (coordinate subspace means that certain coordinates of the vectors in the subspace are fixed to a zero). In the next section an algebraic description of the BIST scheme is given. Detection of single coset error and, in general, single product-terms errors is then analyzed in Section 3. In Section 4 a signature scheme which detects multiple coset errors is presented. In Section 5 we show that the BIST that is based on the test generation and signature analysis in which the feedback polynomials are reciprocal of one another will have a poor error detection.

II. FINITE FIELD ALGEBRA AND MISR

Consider a BIST scheme that uses an LFSR and an MISR with the same primitive feedback polynomial, and, the circuit-under-test with n input lines and m output lines where n is assumed to be equal or greater than m. The input vectors and the output vectors can be considered as either n-dimensional vectors over $\{0, 1\}$ or elements in the Galois field of 2^n elements $GF(2^n)$, (for the case when m<n some components of the output vectors are assumed to be equal to zero). In this BIST scheme the input test vectors are applied to the circuit-under-test in the sequence

$$\{\alpha^t, t=0,1,...,2^n-2\}$$

where α is the primitive element of $GF(2^n)$ defined by the primitive feedback polynomial of the LFSR, and t is the moment of time.

The state transition of MISR signature analyzer can be described by a linear first order difference equation over $GF(2^n)$ as

$$s(t+1) = \alpha s(t) + z(t),\ 0 \le t \le 2^n-2; \tag{1}$$

where s(t) is the content of the MISR and z(t) is the test test response at moment t; the addition is defined in $GF(2^n)$; s(t), s(t+1) (the next state) and z(t) are elements in $GF(2^n)$.

The signature S, that is, the state of the MISR after all 2^n-1 responses were shifted into the MISR is given by

$$S = \alpha^{2^n-1}s(0)+\alpha^{2^n-2}z(0)+\alpha^{2^n-3}z(1)+...+\alpha z(2^n-3)+z(2^n-2); \tag{2}$$

where s(0) is the initial state of the MISR which can be pre-computed such that the expected fault-free signature will be equal to zero [9].

Let ΔS be the signature of errors, that is, the distortion in the expected signature, then, for errors represented by a function

$$e(x):\ x\in GF(2^n) \rightarrow e(x)\in GF(2^n)$$

we have,

$$\Delta S= \alpha^{2^n-2}e(1)+\alpha^{2^n-3}e(\alpha)+\alpha^{2^n-4}e(\alpha^2)+...+\alpha e(\alpha^{2^n-3})+e(\alpha^{2^n-2})$$

$$= \alpha^{-1}[e(1)+\alpha^{-1}e(\alpha)+...+\alpha^{-(2^n-3)}e(\alpha^{2^n-3})+\alpha^{-(2^n-2)}e(\alpha^{2^n-2})], \tag{3}$$

where $\alpha^{-(2^n-2)} = \alpha^{-1}$ (the power of α is modulo 2^n-1).

Error e(x) is detected if and only if $\Delta S = 0$.

Let us define a single coset error as

$$e(x) = \begin{cases} \varepsilon;\ x\in \{p+V_d\};\ p\notin V_d; \\ 0;\ \text{elsewhere}; \end{cases} \tag{4}$$

where $\varepsilon\in GF(2^n)$ is a fixed error pattern (vector) appearing at the outputs of the circuit-under-test and $\{V_d+p\}$ denotes a coset of a d-dimensional subspace V_d with the coset leader p, $p\in GF(2^n)$, ($d\le n$).

III. SINGLE COSET ERROR DETECTION

Consider the coset errors defined in (4). Then, from (3) these errors are detected by the BIST scheme where the test generation and signature analyzer have the same feedback polynomial if and only if the sum of the inverses of the elements in the coset, denoted by $A(p+V_d)$, is not equal to zero. The single coset detection property for the scheme will be stated in Theorem 1. A proof of Theorem 1, however, requires the following lemma.

Lemma 1: Let V_d be a subset of $GF(2^n)$ ($GF(2^n)$ is also an n-dimensional vector space over $\{0,1\}$, V_n), then

$$\prod_{i\in\{p+V_d\}} i + \prod_{i\in\{q+V_d\}} i = \prod_{i\in\{p+q+V_d\}} i \tag{5}$$

where the products are defined in $GF(2^n)$.

In other words, (5) states that a set of the products of all elements in every equivalent class of $GF(2^n)$ induced by V_d, (the coset partition) forms an (n-d)-dimensional subspace in $GF(2^n)$.

Proof: By induction:

Basis: d=1, $V_1=\{0, v\}$, $p(p+v) + q(q+v) = (p+q)(p+q+v)$.

Hypothesis: Assume (5) holds we show for the case of d+1. Since $V_{d+1} = V_d \cup \{r+V_d\}$, where $\{r+V_d\}$ is a coset of V_d, $r \notin V_d$, we have,

$$\prod_{i\in V_{d+1}}(p+i) + \prod_{i\in V_{d+1}}(q+i) = \prod_{i\in V_d}(p+i)^2(r+i)$$

$$+ \prod_{i \in V_d} (q+i)^2 (r+i)$$

$$= \prod_{i \in V_d} (p+q+i)^2 (r+i) \;=\; \prod_{i \in V_{d+1}} (p+q+i) . \quad \text{Q.E.D.}$$

Theorem 1: Let V_d^* denote the set of all nonzero elements in V_d, then,

$$A(\{p+V_d\}) = \frac{\prod_{i \in V_d^*} i}{\prod_{i \in \{p+V_d\}} i} \tag{6}$$

Since, ΔS for a single coset error is given by

$$\Delta S = \varepsilon\alpha^{-1} A(\{p+V_d\}), \tag{7}$$

therefore, these errors are always detected by the BIST scheme.

Proof: By induction:
Basis: d=1, $A(\{p+V_1\}) = \frac{1}{p+v} + \frac{1}{p} = \frac{v}{(p+v)p}$ where $V_1=\{0,v\}$.
Hypothesis: Assume (6) holds we show for the case of d+1. Since a coset of V_{d+1} is a union of two distinct cosets of V_d where V_d is a subset of V_{d+1}, we have, $A(\{p+V_{d+1}\}) = A(\{q+V_d\})+A(\{r+V_d\})$. Substituting $A(\{q+V_d\})$ and $A(\{r+V_d\})$ by (6) and, using Lemma 1, the hypothesis is shown for the case of d+1. Q.E.D.

The following theorem gives an upper bound on the probability that $A(V_d^*)$, the sum of the inverses of the nonzero elements in a coordinate subspace V_d is equal to zero.

Theorem 2: Let $V_d=\text{span}\{y^{h_i}, i=0,...,d-1\}$, $h_i\in \{0,..., n-1\}$, be a coordinate subspace where y is an indeterminate of the polynomial representation of vectors, then, the probability that $A(V_d^*) = 0$, is upperbounded by $\frac{1}{n-d+1}$.

Proof: Since $V_d=V_{d-1}\cup\{p+V_{d-1}\}$ where $V_d \supset V_{d-1}$, so that, $A(V_d^*) = A(V_{d-1}^*) + A(\{p+V_{d-1}\})$. Moreover, if $A(V_{d-1}^*)=0$, then, $A(V_d^*)=A(\{p+V_{d-1}\})\neq 0$, hence, $A(V_d^*) = 0$ if and only if $A(V_{d-1}^*)=A(\{p+V_{d-1}\})$. Substituting $A(\{p+V_{d-1}\})$ by (6), and if $\exists$ the coset $\{p+V_{d-1}\}$ satisfying $A(V_{d-1}^*)=A(\{p+V_{d-1}\})$, by Lemma 1, that coset is unique. Therefore, there may exist at most one coset such that $A(V_d^*)=0$ for a given $V_d \supset V_{d-1}$.

For V_d is a coordinate subspace, an upper bound on the probability that $A(V_d^*)=0$ is equal to the probability that for any $V_d \supset V_{d-1}$, for instance, $V_{d-1} = \text{span}\{y^{h_i}, i=1,...,d-1\}$ $\exists$ $p=y^{h_0}$, $h_0 \notin \{h_1,...,h_{d-1}\}$, such that $A(V_d^* \cup \{p+V_{d-1}\}) = 0$. This upper bound is given by $\frac{1}{n-d+1}$. Q.E.D.

Theorem 3: (A Lower Bound on Detection of Single Product-Term Error). Consider the single product-term errors e(x) defined similarly to the single coset errors (4), however, in this case p of $\{p+V_d\}$ may be an element of V_d, that is, e(x) can now be described in general as a single product-term Boolean function. The detection probability of a single product-term error is lowerbounded by

$$P_{det} \geq 1 - \frac{1}{(n-d+1)2^{n-d}} . \tag{9}$$

Proof: The probability that a single product-term error will be a subspace can be estimated as $\frac{1}{2^{n-d}}$ and by Theorem 2, (9) is shown. Q.E.D.

IV. Multiple Coset Error Detection

A signature scheme which is viable for detection of coset errors with multiplicity r, $r\leq n$, can be constructed similar to the check matrix of the Reed-Solomon codes [8]. The scheme consists of r signatures $\{S_i, i=0,1, ...,r-1\}$ obtained separately using r MISRs. The ith MISR implements a first-order linear difference equation over $GF(2^n)$ of the form

$$s_i(t+1) = \alpha^{2^i} s_i(t) + z(t), \; 0\leq t\leq 2^n-2. \tag{10}$$

An MISR which implements the above difference equation can be constructed by writing out the present states $s_i(t)$ in its polynomial form where the coefficients are the contents of the MISR flip-flops, then, multiplying the polynomial by x^{2^i} modulo the primitive polynomial used in defining the field $GF(2^n)$. The coefficients of the product determine the next states for the correponding flip-flops.

The following theorem summarizes the required condition for a detection of multiple coset errors which is defined as

$$e(x) = \begin{cases} e_0;\ x \in C_0; \\ e_1;\ x \in C_1; \\ \vdots \\ e_{r-1};\ x \in C_{r-1}; \\ 0;\ \text{elsewhere} \end{cases} \quad (12)$$

where C_i denotes ith coset and $C_i \cap C_j = \varnothing$, $i \neq j$.

Theorem 4: Error e(x) defined in (12) is detected for arbitrary values of $\{\varepsilon_0,..., \varepsilon_{r-1}\}$ if and only if the sum of the inverses of the elements in each coset are different for $i=0,...,r-1$.

Proof: Since the sum of the inverses of the elements where each raised to the power 2^i, $0 \le i \le n-1$, is equal to the sum of the inverses raised to the power 2^i, ($a^{2^i} + b^{2^i} = (a+b)^{2^i}$; $a,b \in GF(2^n)$). The r signatures of e(x) are then given by the system of linear equation

$$\frac{1}{\alpha}\begin{bmatrix} A_0 & A_1 & \dots A_{r-1} \\ A_0^2 & A_1^2 & \dots A_{r-1}^2 \\ & \vdots & \\ A_0^{2^{r-1}} & A_1^{2^{r-1}} & \dots A_{r-1}^{2^{r-1}} \end{bmatrix}\begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_{r-1} \end{bmatrix} = \begin{bmatrix} \Delta S_0 \\ \Delta S_1 \\ \vdots \\ \Delta S_{r-1} \end{bmatrix}; \quad (13)$$

which are linearly independent if and only if $A_i \neq A_j$, $i \neq j$, and the theorem is proved. Q.E.D.

V. Poor Error Detection by the Reciprocal Polynomial MISR

The state transition of the MISR in which the feedback polynomial is the reciprocal polynomial, $x^n f(\frac{1}{x})$, of the test generator feedback polynomial f(x) is described by

$$s(t+1) = \alpha^{-1}s(t) + z(t),\ 0 \le t \le 2n\text{-}2; \quad (14)$$

where $s(t), z(t) \in GF(2^n)$ (the $GF(2^n)$ constructed by the primitive polynomial f(x)) [9]. The signature is then expressed by

$$S = \alpha^{-(2^n-1)}s(0) + \alpha^{-(2^n-2)}z(0) + \alpha^{-(2^n-3)}z(1) + ... + \alpha^{-1}z(2^n-3) + z(2^n-2). \quad (15)$$

For the pre-computed initial state s(0) such that the expected signature S will be equal to zero, we have the signature of error e(x) expressed as

$$\Delta S = \alpha[e(1) + \alpha e(\alpha) + ... + \alpha^{2^n-3}e(\alpha^{2^n-3}) + \alpha^{2^n-2}e(\alpha^{2^n-2})]. \quad (16)$$

Theorem 5: The BIST scheme which uses MISR with the feedback polynomial reciprocal to the test generation polynomial does not detect any error e(x) which contains only two or higher dimensional cubes in its Reed-Muller canonical representation.

Proof: Consider the Reed-Muller canonical form [7], [8] of an error e(x), that is, the modulo-two sum of the products (cubes), where without loss of generality, using the variable in the direct form. Thus, e(x), is in general, a multiple coset error where the cosets are represented by the product terms of the expansion. From (16) we see that the signature of e(x) is a sum of the elements in the different cosets weighted with the associated error magnitude ε_i. Since the sum of the elements in a coset containing two or more elements is equal to zero, therefore, the signatures for the components in the Reed-Muller expansion of e(x) are all equal to zero if e(x) contains only cubes of two or higher dimension. Q.E.D.

Fault simulation had been conducted on the *7485 4-bit Comparator* (11 inputs, 3 outputs and 162 single stuck-at faults), a *4-bit carry-lookahead adder* (9 inputs, 5 outputs, 121 faults), and the *74181 4-bit ALU* (14 inputs, 8 outputs, and 330 faults) to illustrate a poor fault detection capability of the reciprocal MISR BIST scheme. The results were that the scheme did not detect any error (0% fault coverage) produced by the faults simulated in all three circuits. (For the case of BIST scheme with the same feedback polynomials the fault coverages were 75%, 95% and 99%, respectively).

VI. Conclusions

The notion of coset errors has been introduced which is related to the Reed-Muller expansion of the error function. With the analysis of MISR using finite field algebra a formula expressing the sum of inverses of the elements in the coset in terms of products was derived which implies the detection of single coset errors. With this algebraic analysis it also follows that the reciprocal MISR BIST scheme will have a poor error detection.

Representation of errors in terms of the modulo-two sum of cubes in error detection analysis based on algebra can be extended into a construction of t-coset error correcting scheme. The results reported in this paper can be the ground work for a research and development project of self-repaired Reed-Muller circuits where single coset error can be identified in the test mode and corrected during the normal operating.

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