7. Uncertainty

7.1 Lotteries

- An object is deterministic, if it has only one possible outcome (set of characteristics is given).
- A object is a lottery if it has a number of different possible outcomes (sets of characteristics) of which one is chosen at random.
- Sometimes the probabilities of the different outcomes of a lottery are known to a decision maker. We may call these objective probabilities.
- Sometimes a decision maker has only an opinion about the probabilities. We call his opinion his **beliefs**, and we say they are subjective probabilities.

- Here, we will not discuss the difference between uncertainty and risk or between objective and subjective probabilities.
- For now, let us think of lottery as an object in which all the outcomes and their probabilities are specified.

Example: Your aunt is going to give you a birthday present, which may be a horse, a cow, a doggy, an Alpha Romeo, or shoes. We may express the gift as a lottery:

$$\text{GIFT} = \begin{cases} 1 \text{ Horse} & \textit{with probability } .1 \\ 1 \text{ Cow} & \textit{with probability } .3 \\ 1 \text{ Doggy} & \textit{with probability } .2 \\ 1 \text{ Alpha} & \textit{with probability } .1 \\ 2 \text{ Shoes} & \textit{with probability } .3 \end{cases} .$$

DEFINITION 7.1. *A lottery is an object of the form*

$$x = \langle q,\ \alpha \rangle\,,$$

where q is a vector of outcomes, and α a vector of probabilities that the corresponding outcomes will be realized. The elements of α must sum to 1.

- There is no standardized notation for lotteries. For these notes I will write:

$$\textsf{GIFT} = \left\langle \begin{bmatrix} 1 \text{ Horse} \\ 1 \text{ Cow} \\ 1 \text{ Doggy} \\ 1 \text{ Alpha} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .1 \\ .3 \\ .2 \\ .1 \\ .3 \end{bmatrix} \right\rangle.$$

- A **simple lottery** has only deterministic outcomes.
- A **compound lottery** has some outcomes that are also lotteries.

- The outcome "horse" in the example is deterministic, because its characteristics are assumed to be determined (within the model).
  - In economic models, it is often convenient to construct lotteries from deterministic outcomes.
  - In real life, of course, no outcome is fully deterministic.
  - A horse is really a lottery. Among other things, we don't know when the beast will die.
- Compound lotteries can be reduced to simple lotteries by multiplying and adding appropriate probabilities.

- Example: Suppose

$$M = \left\langle \begin{bmatrix} L \\ 1 \text{ Doggy} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .5 \\ .1 \\ .4 \end{bmatrix} \right\rangle$$

with

$$L = \left\langle \begin{bmatrix} 1 \text{ Doggy} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .3 \\ .7 \end{bmatrix} \right\rangle.$$

Then M can be reduced to:

$$\tilde{M} = \left\langle \begin{bmatrix} 1 \text{ Doggy} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .5 \times .3 + .1 = .25 \\ .5 \times .7 + .4 = .75 \end{bmatrix} \right\rangle.$$

Why?

- The probabilities of the occurrence of the final (deterministic) outcomes of a compound lottery and the corresponding reduced lottery are the same.

7.2 Preferences over lotteries

- If an individual has preferences over lotteries, but cares only about the final probabilities of the various outcomes, then the individual should be indifferent between a compound lottery and the corresponding reduced lottery.

- Consequently, a complete set of preferences over all lotteries is implied by preferences specified over simple lotteries.

- Let $\mathcal{L}$ denote the set of all simple lotteries over a set of deterministic outcomes Q. More formally, $\mathcal{L} = \{\langle q, \ \alpha \rangle \mid q_i \in Q \text{ for all } i\}$.

- Example: suppose $Q = \{(q_1 \text{ Horses}, q_2 \text{ Cows}, q_3 \text{ Doggies}, q_4 \text{ Alphas}, q_5 \text{ Shoes}) \mid q_i \in \mathbb{R}_+\}$. Then, for

$$x = \left\langle \begin{bmatrix} 2 \text{ Shoes} \\ 1 \text{ Horse} \\ 1 \text{ Doggy} \end{bmatrix}, \begin{bmatrix} .7 \\ 0 \\ .3 \end{bmatrix} \right\rangle,$$

we have $x \in \mathcal{L}$.

- Furthermore, if

$$x' = \left\langle \begin{bmatrix} 1 \text{ Doggy} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .3 \\ .7 \end{bmatrix} \right\rangle,$$

we would like to say that $x = x'$.

- We can define a canonical (standardized) form for all lotteries in $\mathcal{L}$.
  - Let $\tilde{q}$ be a vector in which all outcomes in Q appear in a predetermined order.
  - If $x = \langle q, \alpha \rangle \in \mathcal{L}$, then, we can define its canonical form $\tilde{x} = \langle \tilde{q}, \tilde{\alpha} \rangle$ by reordering the vector α to match the order of $\tilde{q}$ and inserting $\tilde{\alpha}_i = 0$ when $\tilde{q}_i$ is not included in q.
  - We will say $x = x'$ when their canonical forms are the same.

- For example, if

$$\tilde{q} = (1 \text{ Horse}, 1 \text{ Cow}, 1 \text{ Doggy}, 1 \text{ Alpha}, 2 \text{ Shoes}),$$

then the standardized form of $\left\langle \begin{bmatrix} 1 \text{ Doggy} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} .3 \\ .7 \end{bmatrix} \right\rangle$ is

$$\left\langle \begin{bmatrix} 1 \text{ Horse} \\ 1 \text{ Cow} \\ 1 \text{ Doggy} \\ 1 \text{ Alpha} \\ 2 \text{ Shoes} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ .3 \\ 0 \\ .7 \end{bmatrix} \right\rangle.$$

- Note: MC omits the vector $\tilde{q}$ from their standardized notation. I find that to be confusing in many situations.

DEFINITION 7.2. *We say that a decision maker's preferences $\succsim$ on the space of lotteries $\mathcal{L}$ are rational if they are complete and transitive.*

- For $x, x' \in \mathcal{L}$ let $\alpha x + (1 - \alpha) x'$ with $\alpha \in [0, 1]$ denote the compound lottery $\left\langle \begin{bmatrix} x \\ x' \end{bmatrix}, \begin{bmatrix} \alpha \\ 1 - \alpha \end{bmatrix} \right\rangle$.

- In what follows, "$\succ$" means "strictly preferred." That is, $x \succ x'$ if $x \succsim x'$ and $x' \not\succsim x$.

DEFINITION 7.3. *The preferences $\succsim$ on $\mathcal{L}$ are continuous if for any x, x' and x'' in $\mathcal{L}$, the sets*

$$\{\alpha \mid x'' \succ \alpha x + (1-\alpha)\, x'\}$$

and

$$\{\alpha \mid \alpha x + (1-\alpha)\, x' \succ x''\}$$

are both open in $[0, 1]$ (i.e. contain no endpoints except possibly 0 or 1).

- This implies that if $x \succ x''$, the relation will remain true after sufficiently small changes in the probabilities of outcomes (including a change away from the probability of 0).

- This is similar to the idea of continuous preferences over commodity space: Preferences are continuous if sufficiently small changes in the quantities of goods do not change a strict preference relation [equivalent to formal definition].

DEFINITION 7.4. *The preferences $\succsim$ on $\mathcal{L}$ satisfy the independence axiom if for any x, x' and x'' in $\mathcal{L}$, with $x \succsim x'$, we have*

$$\alpha x + (1-\alpha)\, x'' \succsim \alpha x' + (1-\alpha)\, x''$$

for all $\alpha \in (0, 1)$.

- This says, for example, that if 1 Doggy $\succsim$ 1 Cow, then

$$\left\langle \begin{bmatrix} \text{1 Doggy} \\ \text{2 Shoes} \end{bmatrix}, \begin{bmatrix} .3 \\ .7 \end{bmatrix} \right\rangle \succsim \left\langle \begin{bmatrix} \text{1 Cow} \\ \text{2 Shoes} \end{bmatrix}, \begin{bmatrix} .3 \\ .7 \end{bmatrix} \right\rangle.$$

- Let $u : Q \to \mathbb{R}$ be a utility function defined on the set of outcomes of $\mathcal{L}$. Given a simple lottery $x = \langle q,\ \alpha\rangle$, we denote the vector of outcome utilities by $u(q)$.

DEFINITION 7.5. *The expected utility of the simple lottery $x = \langle q,\ \alpha\rangle$ is given by the inner product*

$$\mathrm{EU}[x] = \alpha u(q)\,.$$

[MC refers to outcome-utility u as Bernoulli utility and expected utility EU as von Neumann-Morgenstern expected utility. I will not bother with that terminology.]

- The expected utility of a compound lottery is given by the expected utility of the corresponding reduced lottery.

PROPOSITION 7.1. *Any preference relation $\succsim$ defined by the expected utility of lotteries ($x \succsim x'$ whenever $\mathrm{EU}[x] \geq \mathrm{EU}[x']$) is rational, continuous and satisfies the independence axiom.*

- Prove this as an exercise.

PROPOSITION 7.2. *(Expected Utility Theorem) If the preferences $\succsim$ are continuous, rational and satisfy the independence axiom on a space of lotteries $\mathcal{L}$ over the outcomes Q, then it is possible to define a utility function $u : Q \to \mathbb{R}$ such that for any $x, x' \in \mathcal{L}$,*

$$x \succsim x' \text{ if and only if } \mathrm{EU}[x] \geq \mathrm{EU}[x']\,.$$

- I will not hold you responsible for the proof of this theorem, which appears on pp. 176-178 of MC.

- The best-known (and possibly the most important) critique of the use of expected utility for the determination of preferences over lotteries was published by Kahneman and Tversky (see D. Kahneman and A. Tversky, "Prospect theory: an analysis of decision under risk," *Econometrica*, 47:263–291, 1979.)
- A collection of articles on the subject is contained in: Daniel Kahneman, Paul Slovic, and Amos Tversky, [editors], *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge University Press, 1982.

Failure of expected utility theory.

- Which do you prefer?

$$x = \left\langle \begin{bmatrix} \$4000 \\ \$0 \end{bmatrix}, \begin{bmatrix} .80 \\ .20 \end{bmatrix} \right\rangle$$

or

$$x' = \$3000 \text{ (for certain)}$$

- Which do you prefer?

$$y = \left\langle \begin{bmatrix} \$4,000 \\ \$0 \end{bmatrix}, \begin{bmatrix} .20 \\ .80 \end{bmatrix} \right\rangle$$

or

$$y' = \left\langle \begin{bmatrix} \$3,000 \\ \$0 \end{bmatrix}, \begin{bmatrix} .25 \\ .75 \end{bmatrix} \right\rangle$$

- In experiments with students conducted by K&T, the results violated expected utility theory. This type of violation is called Allais' Paradox.
- Exercise: do the results violate the axiom of independence? Explain.

7.3 Risk Aversion

- In this section, we assume that all deterministic outcomes of lotteries are amounts of money drawn from an interval $Q \subseteq \mathbb{R}$ on the real line.
- Let $\mathcal{L}(Q)$ denote the set of simple lotteries over amounts in the interval Q.
- We will assume that $u : Q \to \mathbb{R}$ is an increasing utility function of money.

Definition 7.6. *In the special case of $Q \subseteq \mathbb{R}$, the* expected value *of the lottery $x = \langle q, \alpha\rangle$ is given by the inner product*

$$E[x] = \alpha q.$$

- Example: For

$$x = \left\langle \begin{bmatrix} \$4000 \\ \$0 \end{bmatrix}, \begin{bmatrix} .8 \\ .2 \end{bmatrix} \right\rangle$$

we have $E[x] = .8 \times 4000 + .2 \times 0 = 3200.$

- If the probability of the various outcomes is continuously distributed, then we can represent a lottery by $x = \langle Q, f \rangle$, where Q is the support of the probability density function, here represented by f.

- Example: Suppose Yuri has a chance of winning an amount uniformly distributed between 0 and 9.

  - This situation can be represented by the lottery

$$x = \langle [0, 9], f \rangle \text{ where } f(q) \equiv \frac{1}{9}.$$

  - With a continuum of outcomes, inner products generalize to integrals:

$$E[x] = \int_0^9 q f(q)\, dq = \frac{1}{9}\int_0^9 q\, dq = 4.5.$$

  - If $u(q) \equiv \sqrt{q}$ represents the utility of the deterministic outcomes, then expected utility in this case is given by

$$\mathrm{EU}[x] = \int_0^9 u(q) f(q)\, dq = \frac{1}{9}\int_0^9 \sqrt{q}\, dq = \frac{1}{9}\left(\frac{9^{3/2}}{3/2}\right) = 2.$$

- In the most general case, the probability of outcomes may be continuously distributed but have mass point (or atoms) as well.

- Example: Suppose an person has a 10% chance of winning 2, a 40% chance of winning 5, and a 50% chance of winning an amount uniformly distributed between 0 and 9.
  - We can represent this by the lottery

$$x = \left\langle \begin{bmatrix} 2 \\ 5 \\ [0,9] \end{bmatrix}, \begin{bmatrix} .1 \\ .4 \\ .5 \times f(q) = 1/18 \end{bmatrix} \right\rangle.$$

  - In this case

$$E[x] = (.1 \times 2) + (.4 \times 5) + \frac{1}{18}\int_0^9 q dq = .2 + 2 + 2.25 = 4.45.$$

  - This terminology is a bit cumbersome, however. A more elegant way to represent lotteries over a continuum of outcomes is with cumulative distribution functions (cdf's).

DEFINITION 7.7. *Suppose Q is an interval on the real line. The cumulative probability distribution function $F : Q \to [0,1]$ is given by*

$$F(q) \equiv \Pr(x \leq q)$$

where x is a random element drawn from Q.

- In the case of

$$x = \left\langle \begin{bmatrix} 2 \\ 5 \\ [0,9] \end{bmatrix}, \begin{bmatrix} .1 \\ .4 \\ f(q) = .5/9 \end{bmatrix} \right\rangle,$$

  - the cdf of outcomes is given by

$$F(q) = \begin{cases} \frac{q}{18} & \text{for } 0 \leq q < 2 \\ \frac{q}{18} + .1 & \text{for } 2 \leq q < 5 \\ \frac{q}{18} + .5 & \text{for } 5 \leq q \leq 9 \end{cases},$$

- the lottery can be described by

$$x = \langle [0, 9], F \rangle,$$

- the expected value is given by

$$E[x] = \int_0^9 q dF,$$

- and expected utility is given by

$$\text{EU}[x] = \int_0^9 u(q)\, dF.$$

- However, the process of calculating the expected value and expected utility of x is the same as above:
  - dF changes to $f(q)\, dq$ for q at which F is differentiable,
  - and the discrete probabilities $F(q) - F(q^-)$ have to be applied to those q that are points of discontinuity.

PROPOSITION 7.3. *Let $\mathcal{L}(Q)$ be the space of lotteries over an interval of outcomes $Q \subseteq \mathbb{R}$. Then any lottery $x \in \mathcal{L}(Q)$ can be represented in the form $x = \langle Q,\ F \rangle$ where F is a cdf with support Q.*

DEFINITION 7.8. *A person with a given utility function u is* risk neutral *on the space of lotteries $\mathcal{L}(Q)$, if for all $x \in \mathcal{L}(Q)$, $\text{EU}[x] = u(E[x])$.*

- A person is risk neutral, if for all lotteries, expected utility is equal to the utility of the expected value.

- We will say that a lottery is risky if no outcome occurs with probability *1*.

DEFINITION 7.9. *A person with a given utility function u is* risk averse *on the space of lotteries $\mathcal{L}(Q)$, if for all risky $x \in \mathcal{L}(Q)$, $\mathrm{EU}[x] < u(E[x])$.*

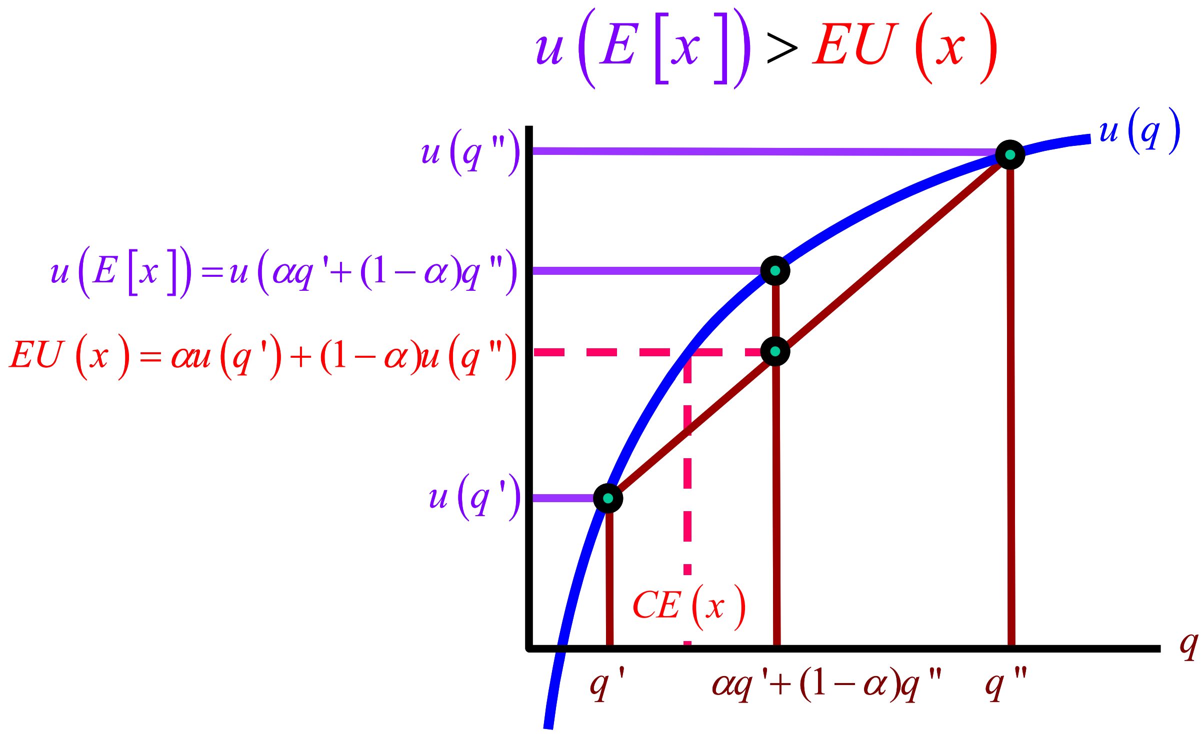
- If a person is risk averse, expected utility is less than the utility of the expected value of all risky lotteries.

DEFINITION 7.10. *The* certainty equivalent $\mathrm{CE}(x)$ *of a lottery x is a deterministic outcome with the property that $u(\mathrm{CE}(x)) = \mathrm{EU}[x]$*

PROPOSITION 7.4. *Suppose a person with a utility function u is an expected utility maximizer. Then the following three statements are equivalent:*

- *the person is risk averse.*
- *u is strictly concave*
- *$\mathrm{CE}(x) < E[x]$ for all risky lotteries x.*

PROOF. See graph. ■



- In what follows, assume that $u(q)$ be a twice differentiable utility function on Q.

DEFINITION 7.11. *The* coefficient of absolute risk aversion $r_A(q)$ *at outcome* q *is given by*

$$r_A(q) \equiv -\frac{u''(q)}{u'(q)}.$$

- Consider the utility function $u(q) \equiv 1 - e^{-aq}$. We have

$$u'(q) = ae^{-aq}$$

and

$$u''(q) = -a^2 e^{-aq},$$

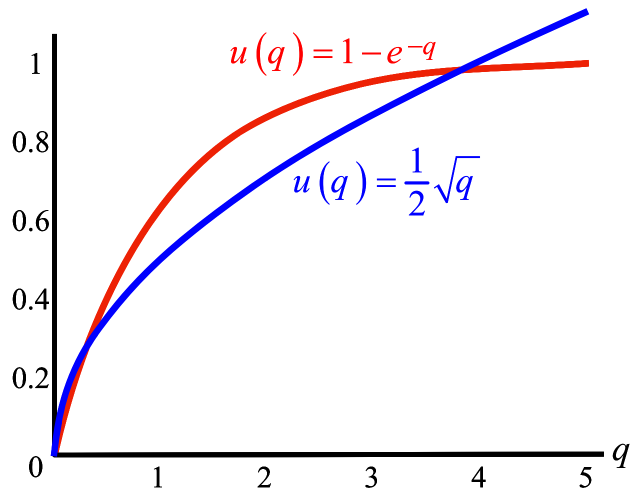
so that

$$r_A(q) \equiv -\frac{u''(q)}{u'(q)} = -\frac{-a^2 e^{-aq}}{ae^{-aq}} = a,$$

a constant that is independent of the outcome q.

- A person with the utility function $u(q) \equiv 1 - e^{-aq}$ is said to have constant absolute risk aversion a.

The graph displays two utility functions: the red one implies constant absolute risk aversion, the blue one implies constant relative risk aversion.



- To see why the definition of absolute risk aversion makes sense, consider a person with utility $u(q) \equiv 1 - e^{-aq}$ and wealth w. Suppose that person is offered a \$1 even bet in which he has a 50% chance of either winning or losing \$1.
  - If he takes the bet, he is facing the lottery

$$x = \left\langle \begin{bmatrix} w+1 \\ w-1 \end{bmatrix}, \begin{bmatrix} .5 \\ .5 \end{bmatrix} \right\rangle$$

  - We have

$$u(w+1) = 1 - e^{-a(w+1)} = 1 - e^{-a}e^{-aw}$$

and

$$u(w-1) = 1 - e^{-a(w-1)} = 1 - e^{a}e^{-aw},$$

so that

$$\mathrm{EU}(x) = 1 - \left(.5e^{a} + .5e^{-a}\right) e^{-aw} = 1 - e^{-a(w-c)},$$

where

$$c = \frac{\log(.5e^{a} + .5e^{-a})}{a}.$$

- The certainty equivalent of the lottery x is $w - c$, so that the person would have to be paid c to induce her to take the bet.
- The fact that c is the same for all values of w means that the person's aversion to a gamble of a fixed absolute size is the same whatever her wealth. This justifies the name "constant absolute risk aversion."
- Note: for $a = 1$, $c = \log\big(.5e + .5e^{-1}\big) = .43$

Definition 7.12. *The* coefficient of relative risk aversion $r_R(q)$ *at outcome* q *is given by*

$$r_R(q) \equiv -\frac{qu''(q)}{u'(q)}.$$

- A person with the utility function

$$u(q) = \frac{1}{1-r} q^{1-r}$$

is said to have constant relative risk aversion r.

- *Exercise: prove that the term "constant relative risk aversion" applies to this utility function, and show that the name is justified.*

- Recall that $\langle Q, F\rangle$ denotes a lottery on the set of outcomes Q with probabilities given by the cdf F,

- $\mathrm{EU}[\langle Q, F\rangle \mid u]$ denotes the expected utility of $\langle Q, F\rangle$ given a utility function u defined on the space of outcomes.

DEFINITION 7.13. *Let F and G be two cdfs defined on Q. We say that F (first-order) stochastically dominates G if $\mathrm{EU}[\langle Q, F\rangle \mid u] \geq \mathrm{EU}[\langle Q, G\rangle \mid u]$ for every monotonically-increasing utility function u on outcomes. That is, F stochastically dominates G if for all increasing u,*

$$\int_Q u(q)dF \geq \int_Q u(q)dG$$

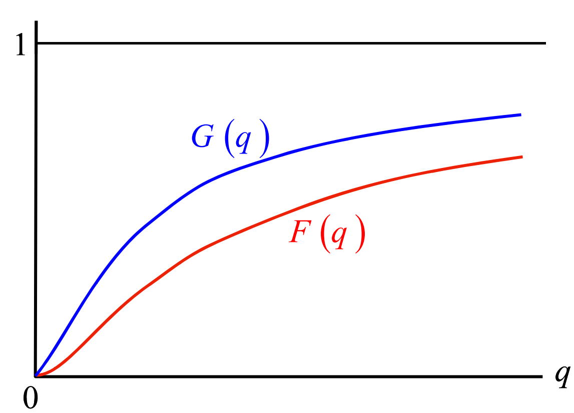
- If F stochastically dominates G then every expected-utility maximizer would prefer the lottery defined by F to that defined by G, independently of her underlying utility on outcomes.
  - Risk aversion, for example, could have no effect on the choice between F and G.

PROPOSITION 7.5. *F (first-order) stochastically dominates G if and only if $F(q) \leq G(q)$ for all q.*

PROOF. See MC, p. 195 for a formal, but partial proof. ∎

- Keep in mind that F and G are cdfs, which means that they must be monotonically increasing functions with values between *0* and *1* and converging to *1* on the right.

- In the graph below, F stochastically dominates G.



- **The proposition says that if F stochastically dominates G, then F can be obtained from G by moving probability from lower to higher values of q.**
  - **For example**

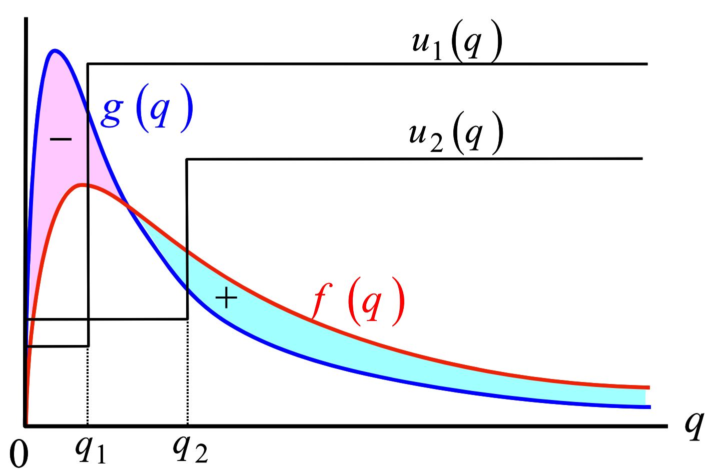
$$\left\langle \begin{bmatrix} 10 \\ 3 \end{bmatrix}, \begin{bmatrix} .8 \\ .2 \end{bmatrix} \right\rangle \text{ dominates } \left\langle \begin{bmatrix} 10 \\ 3 \end{bmatrix}, \begin{bmatrix} .6 \\ .4 \end{bmatrix} \right\rangle$$

and

$$\left\langle \begin{bmatrix} 10 \\ 3 \end{bmatrix}, \begin{bmatrix} .6 \\ .4 \end{bmatrix} \right\rangle \text{ dominates } \left\langle \begin{bmatrix} 9 \\ 3 \end{bmatrix}, \begin{bmatrix} .6 \\ .4 \end{bmatrix} \right\rangle$$

  - ***Exercise: What are the cdfs for the above lotteries? Show that they satisfy the proposition.***

- **Compare the density functions f and g below, which correspond (roughly) to the cdfs of the previous graph.**



- **f dominates g because f is created from g by shifting probability to the right.**
- **First-order stochastic dominance requires that in the graph above, the blue area to the right of any q be larger than the purple area.**

- Consider two-step utility functions like $u_1(q)$ and $u_2(q)$.
- We have
$$\int u_1(q)f(q)dq \geq \int u_1(q)g(q)dq$$
because f concentrates more probability where u_1 is high (and less where u_1 is low) than g does. The same is true for $u_2(q)$ or for any two-step utility.
- This means that $\mathrm{EU}[\langle Q, F\rangle] \geq \mathrm{EU}[\langle Q, G\rangle]$ for any two-step utility function.
- But any utility function is the limit of the sum of two-step (increasing) utility functions, and ...

- Exercise: Demonstrate that if $u = \sum_1^\infty u_i$, then $\mathrm{EU}[x \mid u] = \sum_1^\infty \mathrm{EU}[x \mid u_i]$.
- Consequently, $\mathrm{EU}[\langle Q, F\rangle \mid u] \geq \mathrm{EU}[\langle Q, G\rangle \mid u]$ for any utility function u.

Definition 7.14. ***Let F and G be two cdfs with support $\mathbb{R}_+$ and the same mean. We say that F second-order stochastically dominates G if $\mathrm{EU}[\langle \mathbb{R}_+, F\rangle \mid u] \geq \mathrm{EU}[\langle \mathbb{R}_+, G\rangle \mid u]$ for every monotonically-increasing, concave utility function u on outcomes in $\mathbb{R}_+$. That is, F stochastically dominates G if for all increasing and concave u,***

$$\int_0^\infty u(q)\,dF \geq \int_0^\infty u(q)\,dG$$

- **Rather than use the cumbersome phrase "F second-order stochastically dominates G" we will say that G is riskier than F**
- **If G is riskier than F, then every risk-averse expected-utility maximizer would prefer the lottery defined by F to that defined by G.**

Proposition 7.6. ***Let F be a cdf on $\mathbb{R}_+$ with bounded support. Then the mean μ of F is given by the area above its graph and below the horizontal line at 1, i.e. by***

$$\mu = \int (1 - F(q))\,dq$$

Proof. **The mean is defined by**

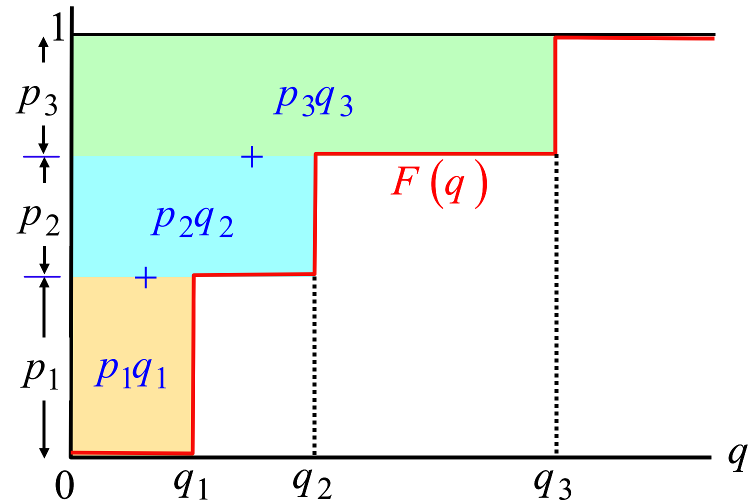
$$\mu \equiv \int q\,dF$$

Let q_m be the upper bound of the support of F. Applying integration by parts, we have

$$\int_0^{q_m} q\,dF = qF(q)]_{q_m} - \int_0^{q_m} F(q)\,dq = \int_0^{q_m} (1 - F(q))\,dq.$$

■

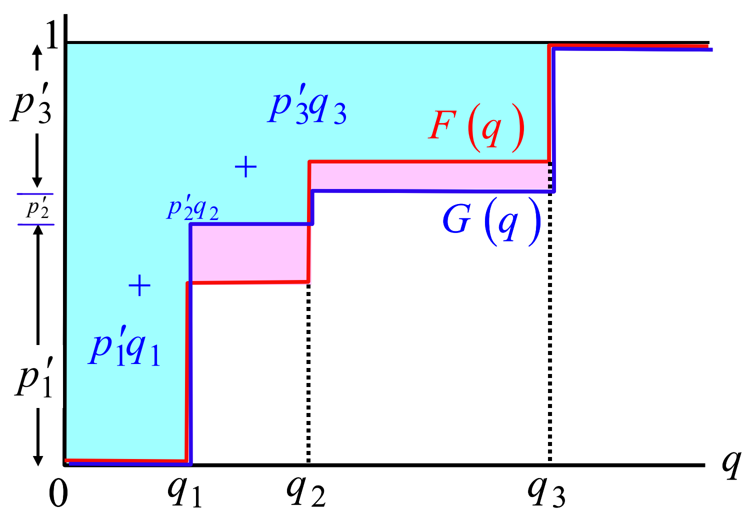
- **To see why this proposition makes sense, look at the graph below, which pictures a cdf for a discrete distribution on outcomes q_1, q_2 and q_3.**

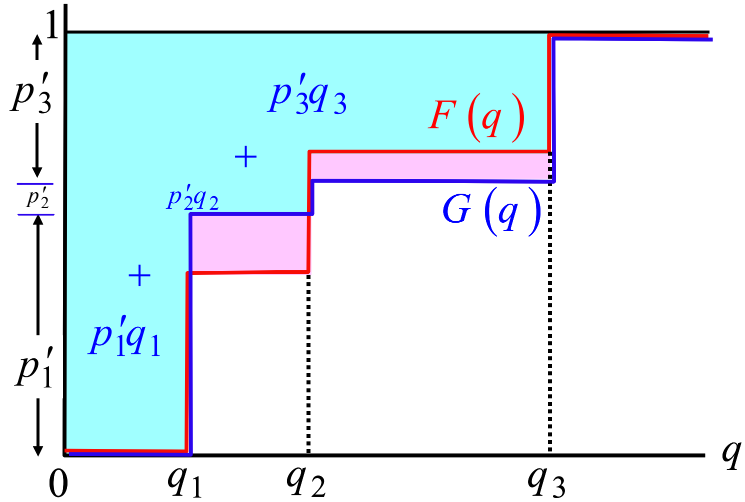


- **Because the cdf for continuous distributions is a limit of cdfs for discrete distributions as the number of points increases, the proposition carries over to continuous distributions as well.**

- **In the graph below, the area above F has been reduced to the left of q_2 and an equal area has been added to the right of q_2 in order to construct the cdf G.**

- **Because the area above G is the same as the area above F, both cdfs have the same mean.**





- **The graph demonstrates that G is riskier than F because the probability on q_2 has been reduced and shifted to the more extreme points q_1 and q_3.**

- **But note that**

$$\int_0^{q_2} F(q)\, dq \leq \int_0^{q_2} G(q)\, dq.$$

- **This generalizes as follows:**

PROPOSITION 7.7. ***Let F and G be two cdfs with support $\mathbb{R}_+$ and the same mean. Then F second-order stochastically dominates G if and only if for all $\bar{q} > 0$,***

$$\int_0^{\bar{q}} F(q)\, dq \leq \int_0^{\bar{q}} G(q)\, dq.$$

- **See MC, pp. 197-199, for a somewhat more formal treatment of the problem.**