# Soviet Pricing, Profits and Technological Choice

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It is an article of faith in Western economic thought that the only useful prices are scarcity prices, that is, marginal-cost prices which equate supply and demand. This idea receives its most vigorous support when the pricing of inputs is to be used as a means of bringing about productive efficiency. Even some socialists accepted this view. In his essay, "On the Economic Theory of Socialism", Oskar Lange [2, p. 94] wrote that "the rule to produce at the minimum average cost has no significance . . . unless prices represent the relative scarcity of the factors of production".

Official Soviet policy requires that prices of producers' goods be set so as to ensure a "profit for each normally functioning enterprise". Prices obtained under these circumstances reflect production costs associated with the entire range of production techniques in use, rather than those associated with only the "marginal" technique. These prices do not in general equal marginal costs (and they do not equate supply and demand) so it is hardly surprising that Western economists are sceptical about their efficacy. In particular, because prices of intermediate goods do not reflect their relative economic scarcity, one would expect the Soviet pricing policy to have an adverse impact on technological choice. Nevertheless, I believe that Soviet pricing policy does not inherently result in inefficient production. With the aid of a mathematical model designed to resemble certain aspects of the Soviet economic framework, I shall argue that the Soviet pricing policy will, under certain conditions, tend to promote the adoption of optimal production techniques.

# 1. PROFITS AND TECHNICAL CHOICE UNDER THE SOVIET ECONOMIC REFORM

In September 1965, Kosygin [1] announced the implementation of widespread reforms in the Soviet economic system. One of the most notable of the proposed reforms was the use of profits as a major success indicator for industrial firms. Material incentives were provided to encourage enterprise management to strive for increased profits. The reforms, however, did not represent anything akin to the adoption of a market system. Many restrictions remained on enterprise activities, and it was not clear what kind and how large a role profits would play in the post-reform period.

Prior to 1965, physical output quotas were assigned to each enterprise for a detailed assortment of goods. The reform promised to add some flexibility to the output quota system, but evidence suggests that no fundamental changes in this regard were actually implemented (see [4, pp. 36-42]). Officially, categories of goods for which output quotas were assigned were to be consolidated into broader categories, and most quotas were to be specified in the form of value of sales rather than physical output targets. But the freedom of enterprise management to use the size of profits (or potential profits) as an indicator for selecting an appropriate output mix remained limited.

Likewise, enterprise management does not have great flexibility in choosing its productive techniques. Key inputs, including capital and labour, are allotted in fixed quantities

by the planning authorities, and it may not be an easy matter for the enterprise management to have those input allotments changed to match a productive technique that the management desires to use.

But the planning authorities themselves are in a position to use profits as a guide to the distribution of output quotas (or sales quotas) among enterprises producing a given product. The total planned output of each commodity is probably not directly subject to considerations of profitability, but instead is largely determined by the planners' estimates of consumer tastes, by capital construction targets, and by planners' desire to have a reasonably consistent overall plan. However, profits could serve as a guide in deciding the fraction of the given total planned output of a commodity that should be assigned as an output quota to each enterprise producing that commodity. Any enterprise showing a profit from the production of some commodity in a given period could be assigned to produce an increased fraction of the total planned output of that commodity in the following period. On the other hand, the fraction of planned total output assigned to an enterprise showing a loss could be decreased. (Exceptions could be made for firms with economies of scale.) To the extent that the Soviet authorities treat profits as an important success indicator, we can expect the bureaucracy to shift quotas in this way.

Since the price of a given commodity is the same for all enterprises, the differences among enterprises in the profits accruing from the production of that commodity must be explained by the use of different productive techniques. This quota-shifting can be thought of as shifting away from unprofitable techniques for producing a commodity toward profitable techniques for producing it.

But if profits and losses do play an important role in the choice of production techniques, the question arises as to whether the use of profits in that role tends to promote economic efficiency. The answer to this question must depend on the nature of the prices of inputs and outputs used in computing profits.

# 2. INDUSTRY-WIDE AVERAGE COST PRICING AND ECONOMIC EFFICIENCY

The official Soviet statement that prices be set so as to secure a profit for every normally functioning enterprise has been interpreted to mean that the price level ought to allow the "average" firm producing a given commodity to earn a normal profit. In other words, the price of each commodity is to be set equal to the industry-wide average cost (including a normal profit) of producing the commodity [5, p. 464]. Such prices are not market-clearing prices, so that the level of demand must be considered in setting the total output quota for the production of each commodity.

With prices of inputs and outputs set equal to industry-wide average costs, we must ask what the size of profits really indicates. The traditional answer would be that because the prices are non-scarcity prices, profits mean relatively little. As a means of disputing this position it would be useful to show that when a firm earns positive economic profits from the production of a commodity, the productive technique it uses is "better" than average. If this were the case, it would make perfect sense to expand that firm's output quota relative to the quota of a firm earning a negative profit. But consider the following example.

A number of firms are manufacturing an industrial refrigeration unit with certain performance characteristics. Some of these firms are using copper tubing in the unit, while others use plastic tubing. Because sub-optimal production techniques are used by the plastics industry, the industry-wide average cost of producing plastic tubing is higher than the industry-wide average cost of producing copper tubing, and consequently, plastic tubing is more expensive than copper tubing. Therefore, the firms making the refrigeration unit with copper will earn a profit, while those using plastic will show a loss, and output quotas of the former will be expanded relative to those of the latter. Unfortunately, these shifts may be in the wrong direction. If optimal production techniques had been used in the manufacture of both copper and plastic tubing, plastic tubing might have been less

expensive than copper. In this case, the technique using plastic tubing would be the optimal technique. Since the plastics industry may be using these refrigeration units in its own production processes, a shift away from the optimal technique for producing refrigeration units may cause technology in plastic tubing production to move even further from the optimum, and so on. The trouble here is this: prices determined by sub-optimal techniques lead to expanding the use of techniques that are relatively costly in terms of the correct shadow prices.

It would seem, then, that industry-wide average cost pricing does not lead to an optimal technology in a simple, direct way. The tendency to move toward an optimum exists for subtle reasons. Although industry-wide average cost prices do not reflect true opportunity costs of commodities, they do measure the average amount of the primary factors of production embodied in each commodity given the current pattern of technique usage. Therefore, if this pattern changes in such a way as to lower some prices without raising any others, it follows that the new pattern of techniques as a whole must be more efficient than the old pattern, even though it is not possible to say that any individual techniques are better than any others in a meaningful sense. An important part of the analysis below is the demonstration that in a linear model designed to resemble the Soviet economic framework, shifting output quotas toward profitable production techniques does lower industry-wide average cost prices. This is followed by a proof that in the model presented, convergence to an optimal technology occurs and is achieved in a finite number of time periods. The model described below is closely related to the activity analysis algorithms of Malinvaud [3] and Weitzman [6]. This relationship is discussed in the appendix.

# 3. GLOSSARY OF SYMBOLS

Matrices and vectors are designated by upper-case letters; corresponding lower-case letters not otherwise defined represent elements of those matrices and vectors. Throughout the paper i, j and k are used to index the type of input, the type of output, and the technique of production used, respectively.

- $l_{jk}$  direct labour input coefficient for producing commodity j by technique k
- $l_j$  average direct labour input coefficient for producing commodity j
- L vector of average direct labour input coefficients
- m number of techniques available for producing each commodity
- n number of industries
- P vector of industry-wide average cost prices
- $\pi_{ik}$  profits per unit output of commodity j produced by technique k
- $Q_{jk}$  vector of input coefficients for producing commodity j by technique k
- $q_{ij}$  average input coefficient for the use of input i to produce commodity j
- Q technology matrix (matrix whose elements are  $q_{ij}$ )
- $s_{ik}$  fraction of output j produced by technique k
- S pattern of technique usage (matrix whose elements are  $s_{ik}$ )
- *τ* maximum phasing-out time.

# 4. THE TECHNOLOGY

In this model we assume that there are n industries, each producing one homogeneous output. Each industry j has available to it m techniques for the production of its output. We assume that these techniques are linearly homogeneous and use all inputs in fixed proportions. The kth technique for producing output j can be completely described

by the pair,  $(Q_{jk}, l_{jk})$ .  $Q_{jk}$  is an *n*-dimensional vector whose *i*th component is the quantity of commodity *i* required as an input per unit output of commodity *j*, and  $l_{jk}$  is the labour required per unit output *j* using technique *k*. Although we will not do so here, the interpretation of  $Q_{jk}$  may be generalized to include depreciation and rental of required stocks, provided that the depreciation and rental rates are assumed to remain constant at all levels of production. Likewise,  $l_{jk}$  may be defined to include primary factors other than labour if the relative prices of these factors remain fixed. If there is more than one factor, and the relative prices of these factors vary, then the choice of an optimal technology will depend on demand (i.e. the non-substitution property is lost) and the mathematical proofs below will not work.

Many of the techniques for producing output *j* may be in use simultaneously. For the sake of simplicity, assume that each firm in the *j*-producing industry is using exactly one of these techniques and that the firm is tied to its technique in the short run. It is not required, here, that each firm use a different technique, several firms, and, at the extreme, all firms in an industry, may use the same technique.

The total output of an industry, and the output of each firm in the industry, is assigned by the planning authorities in the form of output quotas. This assignment implicitly defines what we shall call the pattern of technique usage: the fraction of the total output of each commodity being produced by each productive technique. In what follows, the pattern of technique usage is denoted by the  $n \times m$  matrix S, whose elements  $s_{jk}$  represent the fraction of the output of industry j to be produced by technique k. In formal terms, we define S to be a pattern of technique usage if the following conditions are met:

- 1. Each row of S refers to a commodity, and the elements in that row refer to the techniques for producing that commodity;
- 2.  $0 \le s_{ik} \le 1$ , for all j and k; and
- 3. for all j,  $\Sigma_k s_{ik} = 1$ .

#### 5. PRODUCTION COSTS

We define the industry-wide average production cost of a unit of a commodity recursively as the industry-wide average production cost of required inputs plus the cost of the labour used. (The unit cost of labour is defined to be 1.) Of course, the industry-wide average production cost of any commodity depends upon the pattern of technique usage. Despite the fact that each technique is linear, these average costs are in no sense the same as marginal costs or shadow prices, since use of the most efficient set of techniques (either on the average or on the margin) is not assumed.

Formally, we implicitly define the vector of industry-wide average production costs P by the equation

$$p_{i} = P(\Sigma_{k} s_{jk} Q_{jk}) + \Sigma_{k} s_{jk} l_{jk}, \qquad \dots (1)$$

where  $p_j$  represents the jth component of P. If we allow Q to denote the matrix whose jth column is  $\sum_k s_{jk} Q_{jk}$  and L to denote the vector whose jth element is  $\sum_k s_{jk} l_{jk}$ , then (1) implies the vector equation

We shall refer to Q as the technology matrix associated with S. Each element  $q_{ij}$  of Q measures the average input-flow of commodity i required to produce a unit of commodity j given the pattern of technique usage S. Likewise, each element  $l_j$  of the vector L measures the average input of labour required to produce a unit of commodity j. At the outset, it should be noted that only certain patterns of technique usage have finite non-negative production costs associated with them. We shall call such patterns productive patterns. Formally, a pattern of technique usage S is defined to be productive if, given S, there exists a vector  $P \ge 0$  which satisfies equation (2).

In order to obtain desired results, we must assume that some labour services are embodied directly or indirectly in every commodity produced, regardless of the production techniques used. In other words, it is assumed that labour is used either in producing the commodity itself or in producing some input used directly or indirectly in producing the commodity. Since the *j*th column of the matrix  $Q^t$  represents the inputs for the inputs, etc., required t stages back in producing commodity j, and since labour must enter at some stage, it follows that for each commodity j there is an integer t such that the jth component of  $LQ^t$  is positive.

We shall make much use of the following lemma, which is proved by Weitzman (see his Lemma 2 in [6, pp. 418-19]). It is stronger than the standard theorem of this type in that L need not be strictly positive.

**Lemma 1** (Weitzman). Let Q be any technology matrix and L the associated vector of direct labour inputs, and assume that some labour services are embodied (directly or indirectly) in every commodity produced. If there exists, a vector of non-negative prices P such that  $P \ge PQ + L$ , then the technology matrix Q is productive, i.e.  $(I-Q)^{-1}$  exists and is non-negative.

**Corollary.** If S is productive, then the associated technology matrix Q is productive, and the associated vector of industry-wide average production costs is given by

$$P = L(I - Q)^{-1}.$$
 ...(3)

P is unique and strictly positive.

**Proof.** Since S is productive, equation (2) must be satisfied for some P, so that by Lemma 1, Q must be productive. Thus equation (2) may be solved for P, and equation (3) results. It follows also that  $P = \sum_{0}^{\infty} LQ^{t}$  which must be positive in every component by our assumption that every commodity embodies labour.

# 6. SOVIET-TYPE PRICING AND TECHNICAL CHOICE

Three behavioural rules are incorporated into this model which are intended to be consistent with the general framework of Soviet-type economies. These rules are descriptive rather than normative, and while they lead to the adoption of optimal production techniques in this model, there are other sets of rules which would certainly achieve the same results more quickly. The behavioural rules are the following:

- 1. The price of each commodity is set equal to its average cost of production throughout the producing industry. This implies that the vector of average production costs P, given by equation (3), is also the vector of selling prices.
- 2. Each time period, the pattern of technique usage is revised as follows. If, during the previous period, a technique for producing a certain commodity proved to be profitable (or potentially profitable, if the technique wasn't actually in use), then the fraction of the commodity produced by that technique may not be decreased. If the technique was unprofitable, then the fraction of the commodity produced by that technique may not be increased. In addition, if there are any profitable (or potentially profitable) techniques for producing a given commodity, the fraction associated with at least one profitable technique for producing that commodity must be increased. (Note that while it merely requires good bookkeeping to differentiate profitable from unprofitable techniques for those techniques actually in use, it requires research—and institutions designed for that purpose—to discover potentially profitable techniques which are not currently in use.)

Let  $\pi_{jk}$  denote the profit per unit of production of commodity j earned by technique k, i.e.

$$\pi_{ik} = p_i - PQ_{ik} - l_{ik}.$$
 ...(4)

Let S represent the pattern of technique usage for one period, and let S' represent the pattern for the next period. Then, for each j, the second behaviourial rule is given by the following formal relationship:

$$s'_{jk} \ge s_{jk}$$
 when  $\pi_{jk} > 0$   
 $s'_{jk} \le s_{jk}$  when  $\pi_{jk} \le 0$  ...(5)  
 $s'_{jk} > s_{jk}$  for some  $\hat{k}$  unless  $\pi_{jk} \le 0$  for all  $k$ .

If a single profitable technique produced the entire output of some commodity, then the pattern modification procedure (5) could not be implemented. But this situation cannot arise, since, given industry-wide average cost pricing, any one technique producing the entire output of a commodity would earn zero profits.

3. Techniques which continually operate at a loss must eventually be phased out of use. Formally, there is a maximum phasing-out time  $\tau$ . If for  $\tau$  consecutive periods  $\pi_{jk} < 0$ , then  $s_{jk} = 0$  at the end of the  $\tau$  periods. In this model, the phasing-out time is the only pace-setter for improvements in the technology. Presumably, the smaller  $\tau$  is, the faster convergence will occur. The size of  $\tau$  is constrained from below by the rate at which the use of profitable techniques can be expanded.

The burden of this section is to show that with industry-wide average cost pricing (3), the pattern modification procedure (5) will generate patterns of technique usage which converge to an optimal pattern in a finite number of steps. An optimal pattern of technique usage is defined as a pattern which simultaneously minimizes the average production cost of every commodity. Lemma 2 shows that a pattern is optimal if no technique is profitable (or potentially profitable) at prevailing prices, and Lemma 3 supplies the propositions which are necessary to the demonstration that such a no-profit situation is reached in a finite number of steps.

**Lemma 2.** Suppose that for the pattern of technique usage  $S^*$  and associated price vector  $P^*$ , we have  $\pi_{jk}^* \leq 0$  for all jk, i.e.

$$p_j^* \leq P^*Q_{jk} + l_{jk}.$$

Then for any productive pattern of technique usage S and associated price vector P,  $P^* \leq P$ .

*Proof.* Choose any productive pattern S. We have

$$\Sigma_k s_{jk} p_j^* \leq \Sigma_k s_{jk} (P^* Q_{jk} + l_{jk})$$

and since  $\Sigma_k s_{ik} = 1$ , it follows that

$$p_j^* \leq P^*(\Sigma_k S_{jk} Q_{jk}) + \Sigma_k S_{jk} l_{jk}.$$

Since this equation is valid for every j, we have, by the definitions of Q and L, that  $P^* \leq P^*Q + L$ , or

$$P^*(I-Q) \le L. \tag{6}$$

Because S was assumed to be productive, the corollary to Lemma 1 implies  $(I-Q)^{-1}$  exists and is non-negative. Therefore, equation (6) implies that  $P^* \leq L(I-Q)^{-1}$ . But  $L(I-Q)^{-1}$  is exactly the price vector P associated with pattern S, and the lemma is proved.

- **Lemma 3.** Let S be a productive pattern of technique usage and assume that prices are set equal to average costs P as given by (3). Let S' be a pattern of technique usage for the following period satisfying the conditions of pattern modification (5). Then
  - (a) S' must be productive.
  - (b)  $P' \leq P$ , where P' is the vector of average production costs associated with S'.
  - (c) If industry j has a profitable technique at prices P, then  $p'_i < p_i$ .

*Proof.* Consider the product  $(s'_{jk} - s_{jk})\pi_{jk}$ . By (5) the two factors in this product can never have opposite signs, so that  $(s'_{jk} - s_{jk})\pi_{jk} \ge 0$  for all k. It follows that

$$\Sigma_k(s'_{jk}-s_{jk})\pi_{jk}\geq 0$$

so that

$$\sum_{k} s'_{ik} \pi_{ik} - \sum_{k} s_{ik} \pi_{ik} \ge 0. \qquad \dots (7)$$

But given industry-wide average cost pricing, industry-wide average profit  $\Sigma_k s_{jk} \pi_{jk}$ , must be zero. Therefore, (7) becomes

$$\Sigma_k s'_{ik} \pi_{ik} \ge 0, \qquad \dots (8)$$

i.e. the average profit given the new pattern of technique usage and the old prices is non-negative. Thus, prices must be at least as great as average costs; i.e.

$$P \ge PQ' + L'. \qquad \dots (9)$$

Therefore, by Lemma 1,  $(I-Q')^{-1}$  exists and is non-negative.

We set  $P' = L'(I - Q')^{-1}$ . We have that P' = P'Q' + L', so that P' must be a vector of average-costs associated with the pattern of technique usage S'. Thus S' is productive and (a) is proved. From (9) we have that  $P(I - Q') \ge L'$ , and since  $(I - Q')^{-1}$  is nonnegative, it follows that  $P \ge L'(I - Q')^{-1} = P'$  and (b) is proved.

Since  $P' \leq P$  we have

$$P' = P'Q' + L' \le PQ' + L'$$

so that

$$p_j' \leq \Sigma_k s_{jk}'(PQ_{jk} + l_{jk}) = \Sigma_k s_{jk}'(p_j - \pi_{jk}).$$

Recalling that  $\Sigma_k s'_{ik} = 1$  and  $\Sigma_k s_{ik} \pi_{ik} = 0$ , we have

$$p_{i} - p'_{i} \ge \Sigma_{k} s'_{ik} \pi_{ik} = \Sigma_{k} (s'_{ik} - s_{ik}) \pi_{ik}.$$
 ...(10)

If we assume that industry j has a profitable technique at prices P, then (5) implies that the right-hand side of (10) is strictly positive, and (c) is proved.  $\parallel$ 

**Theorem.** Let the following conditions be considered as given:

- (i) an initial productive pattern of technique usage;
- (ii) industry-wide average cost pricing as defined by equation (3);
- (iii) the modification each period of the pattern of technique usage according to rules (5); and
- (iv) a maximum phasing-out time of  $\tau$  periods for techniques showing continual losses.

Then the pattern of technique usage will converge to an optimal pattern in a finite number of periods with industry-wide average production costs decreasing monotonically.

*Proof.* Let S(0), S(1), ..., S(t)... be the sequence of patterns of technique usage generated by (5). By Lemma 3, each S(t) is productive and the sequence of corresponding prices, P(0), P(1), ..., P(t), ... is monotonically decreasing. Since each P(t) must be non-negative, the sequence  $\{P(t)\}$  must converge. Let  $\overline{P}$  be its limit. If for some T,  $P(T) = \overline{P}$ , monotonicity implies that  $P(t) = \overline{P}$  for all t > T. From Lemma 3 (c) it follows that no technique is profitable given prices  $\overline{P}$  and by Lemma 2, S(T) must be optimal. Thus, if the limit prices  $\overline{P}$  are actually achieved (i.e. for some T,  $P(T) = \overline{P}$ , then the theorem is true.

It remains only to rule out the possibility that the limit prices  $\overline{P}$  are not achieved. Let  $\overline{\pi}_{jk}$  be the profits associated with technique jk at prices  $\overline{P}$ .

The theorem follows from the proof of the following four propositions.

- (a) If  $\bar{\pi}_{ik}$ <0, technique jk is phased-out in finite time.
- (b) If  $\bar{\pi}_{jk} > 0$ , technique jk is phased-out in finite time. (Actually it will follow from this theorem that  $\bar{\pi}_{jk} \leq 0$  for all jk.)

- (c) There exists an integer T such that for all jk,  $s(T)_{jk} > 0$  implies  $\bar{\pi}_{jk} = 0$ .
- (d)  $P(T) = \overline{P}$ .

Given  $\bar{\pi}_{jk} < 0$ , there is an integer  $t_0$  such that  $\pi(t)_{jk} < 0$  for  $t > t_0$ . It follows that for  $t > t_0 + \tau$  (where  $\tau$  is the maximum phasing-out time)  $s(t)_{jk} = 0$  and (a) is proved.

Proposition (b) is a consequence of (a) and the fact that average profits must be identically zero in every time period. Given  $\bar{\pi}_{jk} > 0$ , there is a  $\delta > 0$  and an integer  $t_0$  such that  $\pi(t)_{jk} > \delta$  if  $t > t_0$ . Suppose  $\hat{j}\hat{k}$  is not phased out in finite time. Then there exists  $t_1 > t_0$  such that  $s(t_1)_{jk} > 0$ . By (a), for any  $\varepsilon > 0$ , we can choose an integer  $t_2 > t_1$  such that  $\pi(t_2)_{jk} > -\varepsilon$  for any k such that  $s(t_2)_{jk} > 0$ . Set  $\varepsilon$  equal to  $s(t_1)_{jk}\delta$ . We have

$$\sum_{k} s(t_2)_{jk} \pi(t_2)_{jk} > s(t_2)_{jk} \delta + (1 - s(t_2)_{jk})(-s(t_1)_{jk} \delta) = \left[ s(t_2)_{jk} - s(t_1)_{jk} \right] \delta + s(t_1)_{jk} s(t_2)_{jk} \delta.$$

Furthermore,  $s(t_2)_{jk} > s(t_1)_{jk}$  by (5). Thus  $\sum_k s(t_2)_{jk} \pi(t_2)_{jk} > 0$ , a contradiction since this sum represents average profits and must equal zero. The supposition is therefore false and (b) follows.

Proposition (c) follows directly from (a) and (b) and the fact that there are a finite, number of techniques.

Proposition (d) is implied by the fact that for this T we have  $s(T)_{jk}\bar{\pi}_{jk}=0$ . Therefore, for each j,

$$\begin{split} 0 &= \Sigma_k s(T)_{jk} \overline{\pi}_{jk} = \Sigma_k s(T)_{jk} (\overline{p}_j - \overline{P}Q_{jk} - l_{jk}) \\ &= \overline{p}_j - \overline{P}\Sigma_k s(T)_{jk} Q_{jk} - \Sigma_k s(T)_{jk} l_{jk}. \end{split}$$

Thus  $\overline{P} = \overline{P}Q(T) + L(T)$ , so that  $\overline{P}$  must be the unique price vector associated with S(T) In short,  $\overline{P} = P(T)$ , and (d) and the theorem are proved.

# 7. SUMMARY AND CONCLUSION

In the Soviet Union prices are based on industry-wide average production costs (including normal profits). Such prices will not automatically balance supply and demand, nor will they lead producers to select socially optimal output assortments. We have shown, however, that when profits are used to guide the choice of productive techniques in a linear model, industry-wide average cost pricing does lead eventually to the selection of an optimal technology. This suggests that profits (or potential profits) based on industry-wide average cost prices can serve the Soviet authorities as a useful measure of the comparative efficiency of different productive techniques. In this framework, scarcity pricing is not a requirement for efficient technological choice.

#### APPENDIX: THE MODEL AND THE LITERATURE

From an abstract point of view, the model constructed and explored here is a generalization of models that have been examined in the activity analysis literature by Malinvaud [3, Part IV] and by Weitzman [6], under the heading "Activity Iteration".

Unlike the present model, those of Malinvaud and Weitzman were not intended to describe the dynamics of any existing economy. Instead, they were intended to explore the properties of certain methods for producing optimal plans. Their models postulate an economy which produces a fixed number of goods. At any stage in the planning process, exactly one production activity is selected for each of these goods, and prices are set equal to the cost of production given the use of the selected activities. In the next stage of the planning process a new set of production activities (one activity for each good) is selected in accordance with the profits determined by the prices of the previous stage.

In the model presented here, many techniques for producing the same good may operate simultaneously in any convex combination. Nevertheless, this model retains many of the characteristics of the Malinvaud and Weitzman models. In particular,

the result that in this model industry-wide average cost prices do not rise when output quotas are shifted toward profitable techniques has a genesis parallel to the analogous result for Weitzman's model, and in its demonstration (Lemma 3 (a) and 3 (b)), I borrowed freely of Weitzman's methods and made use of one of his results (stated in Lemma 1).

In this particular model there is no reason to expect convergence unless sufficient movement toward profitable techniques is assumed to occur in each time period. The most natural assumption of this sort, that the use of at least one profitable technique in each industry increases by at least a certain fixed amount, seems to permit non-convergent oscillation in the pattern of technique usage. However, the requirement that techniques operating at a continual loss have to be phased out of use in finite time, proved to be sufficient to guarantee convergence and that it occurs in a finite number of time periods.

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