Thursday, Dec 9, Lecture 24
Extensive-Form Dynamic Games

IMPORTANT:

Read ALL the recent announcements on the course website!
Your course grade could depend on knowing what's in them.

Normal-Form and Extensive-Form Games

- So far, we've described games with a matrix in which each row or column represents a player's strategy: the ***normal-form game***.
- But to find a ***subgame-perfect Nash equilibrium*** with time-consistent strategies,…
- We need a different game structure: the ***extensive-form game***.
- We will use the sequential Battle of the Sexes as an example of how to build an extensive-form game.

The sequential Battle of the Sexes in normal form

- ***Vanesa moves first***: she buys a ticket *either* for the football match *or* for the opera.
- She shows Miguel her ticket, so he knows what she has done.
- ***Then Miguel moves:*** he buys his ticket *either* for the football match *or* for the opera.
- The game in normal form:

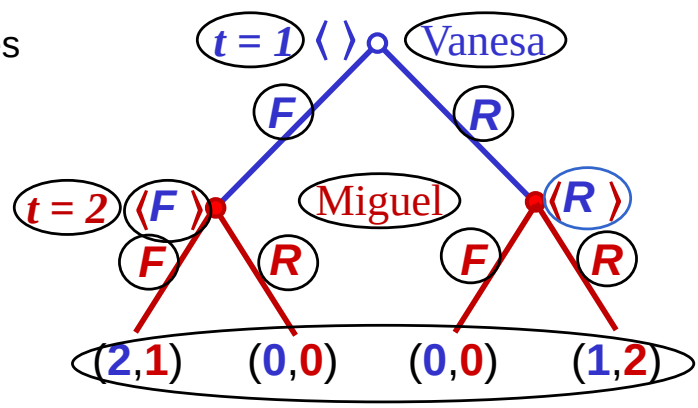
Vanesa \ Miguel	Always F	Copy	Oppo-site	Always R
F	2, 1	2, 1	0, 0	0, 0
R	0, 0	1, 2	0, 0	1, 2

Extensive-Form Games

- Extensive-form games are described with a ***game tree***.
- Each level of the tree designates
  - a time period

    and
  - the player who has a turn to move in that time period.
- Each ***branch*** of the tree describes an ***action*** the player can choose.
- Each ***node*** (where branches meet) describes what a player knows before she moves.

The Sequential Battle of the Sexes

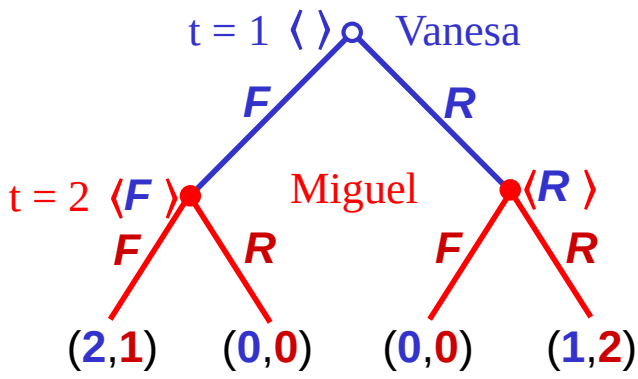


- Each player's payoffs are given at the bottom of the tree.
- A ***strategy*** is a complete plan that states what action a player should take at every one of the nodes. [Described later.]

Comparison: Normal Form vs. Extensive Form

- Can you see the connection between the two forms?

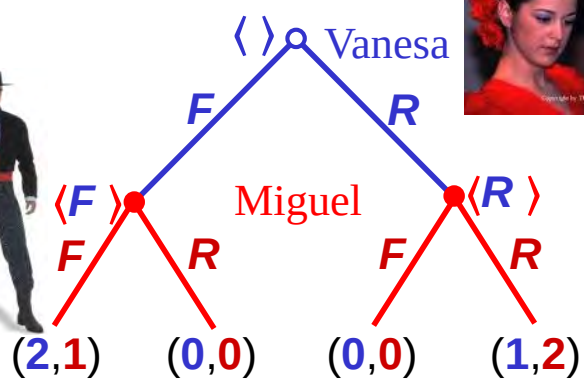
Vanesa \ Miguel	Always F	Copy	Oppo-site	Always R
F	2, 1	2, 1	0, 0	0, 0
R	0, 0	1, 2	0, 0	1, 2



- For example, what does Miguel's strategy ***Always R*** mean in the extensive form?
- We'll see the answer in the next few minutes.

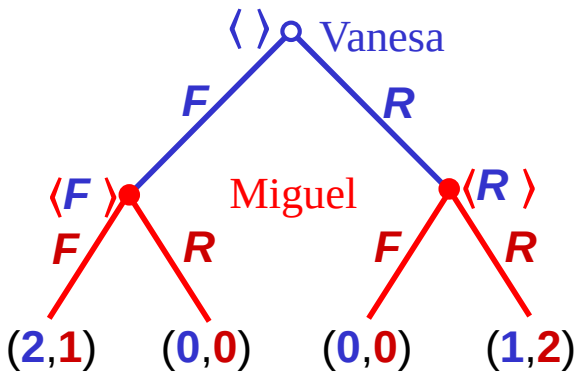
Battle of the Sexes in Extensive Form

- Vanesa moves first.
  - She has beliefs but no information.
  - She can choose football…
  - …or choose opera
- Then it is Miguel's turn.
  - He looks at Vanesa's ticket.
    - He sees football…
    - …or he sees opera.
  - If he sees football,
    - He can choose football…
    - …or choose opera.
    - If he chooses football, Vanesa gets **2** and he gets **1**.
    - If he chooses opera, Vanesa gets **0** and he gets **0**.
  - If he sees opera,
    - He can choose football…
    - …or choose opera.
    - If he chooses football, Vanesa gets **0** and he gets **0**.
    - If he chooses opera, Vanesa gets **1** and he gets **2**.



Clicker Question

According to the game tree on the right, how much does ***Vanesa*** get if she chooses ***R*** and ***Miguel*** follows his own self interest?



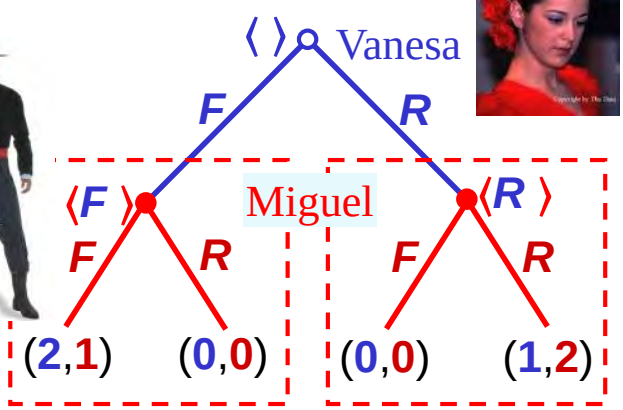
a. 2

b. 1

c. 0

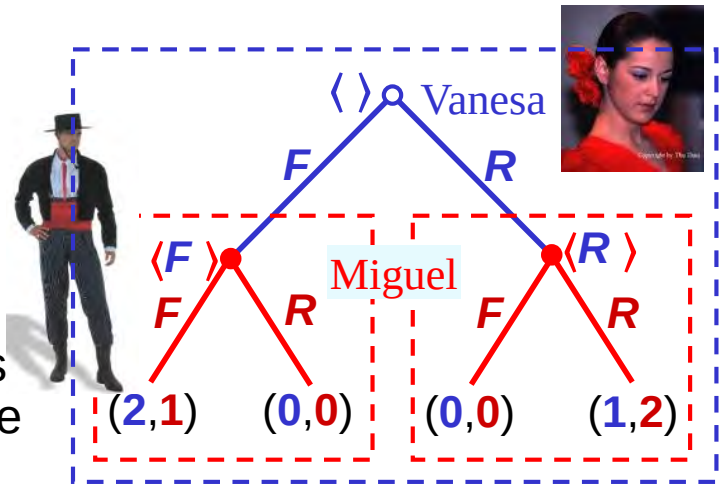
d. More information needed

Subgame-Perfect Equilibrium



- We will show that in a ***subgame-perfect equilibrium***, all strategies are time-consistent,…
- …so that no one wants to change his strategy during the game.
- For this purpose, we break the game into subgames.
- Miguel has two subgames:
  - ⟨***F*** ⟩ and ⟨***R*** ⟩
  - Each of Miguel's subgames corresponds to the game he faces after he finds out what Vanesa did.

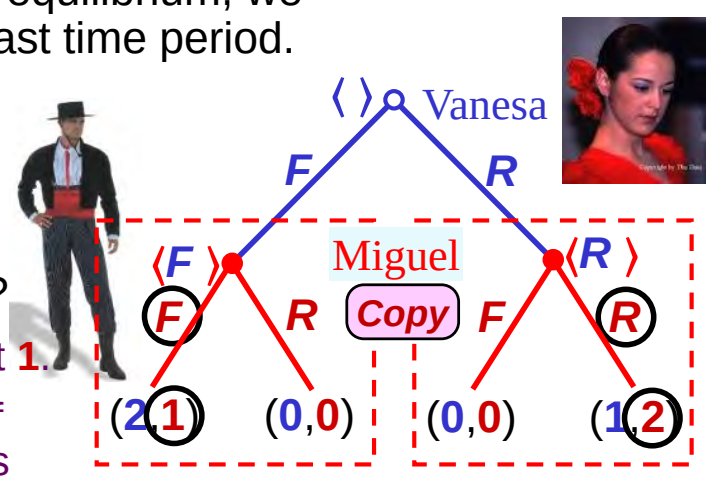
- Vanesa has one subgame ⟨ ⟩, the whole thing.
- She moves at the start of the game…
- …and what Miguel does afterwards will determine her payoff.



- Players get to their subgames when it's their turn during the game.
- An equilibrium is ***subgame-perfect***, if and only if it creates a Nash equilibrium in ***every subgame***.
- This means that the players always follow their best responses during the game.

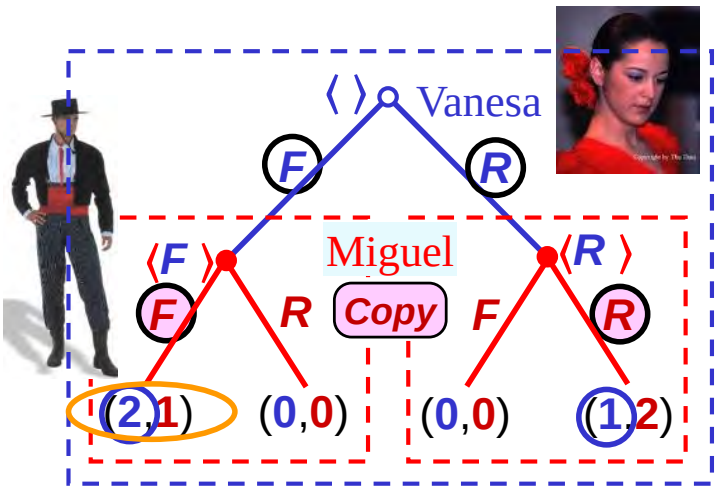
Finding the Subgame-Perfect Equilibrium

- To find a subgame-perfect equilibrium, we work backwards from the last time period.
- This method is called ***backwards induction***.
- What is Miguel's best response in subgame ⟨*F* ⟩?
  - He would choose ***F*** and get **1**.
  - ***F*** is the Nash equilibrium of subgame ⟨*F* ⟩ (because ***F*** is his best response).
- What is Miguel's best response in subgame ⟨*R*⟩?
  - He would choose ***R*** and get **2**.
  - ***R*** is the Nash equilibrium of subgame ⟨*R* ⟩.



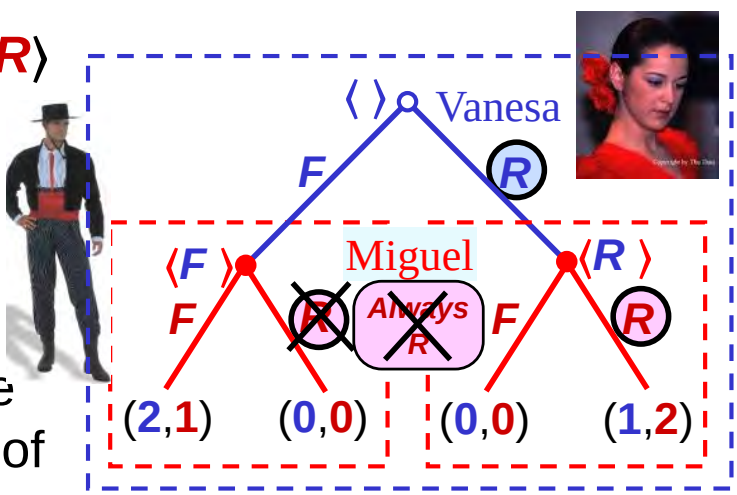
- If we look at ***the two subgames together***, we can see Miguel's equilibrium strategy (complete plan).
- His equilibrium strategy is ***Copy***. Why?

- Vanesa can predict that if Miguel is rational, his strategy must be ***Copy***.
- So, what is Vanesa's best response in her subgame ⟨ ⟩?
  - If she chooses ***F***, Miguel will choose ***F***, and she will get **2**.
  - But if she chooses ***R***, Miguel will choose ***R***, and she will get **1**.
- So Vanesa's Nash equilibrium strategy is ***F***.



- ⟨***F***, ***Copy***⟩ is a unique subgame-perfect [time-consistent] equilibrium.
  - ⟨***F***, ***Copy***⟩ creates a Nash equilibrium in every subgame.
  - In ⟨***F***, ***Copy***⟩, Vanesa gets **2**; Miguel gets **1**.

- Note that ⟨***R***, ***Always R***⟩ is NOT a subgame-perfect equilibrium,…
- …because to be subgame perfect every strategy must be a best response in ***all*** of the subgames.



- in Miguel's subgame ⟨***F*** ⟩, the strategy ***R*** is not a best response or a Nash equilibrium strategy.
- If Vanesa had chosen ***F***, Miguel would not choose ***R***.
- ***Always R*** is NOT time-consistent.

Clicker Question

In the extensive-form dynamic game between Vanesa and Miguel, if both players are rational,

a. Vanesa can accurately predict what Miguel would do.
b. Miguel must decide what to do without knowing what Vanesa has done.
c. Coordination is difficult.
d. Both players have the same number of strategies.

Commitment versus Information

- In the subgame-perfect equilibrium of the sequential Battle of the Sexes, Vanesa moves first.
- Vanesa can make a prediction, but she has no information.
- When Miguel moves, he already knows what Vanesa had done.
- Miguel has the information advantage, yet Vanesa gets ***2*** and Miguel gets only ***1***. Why?
- Vanesa gets her way, because she makes a ***commitment*** by choosing ***F*** before Miguel gets to move.
- In business and in life, commitment is a big advantage.
- But in other settings, ***information*** may prove to give a bigger advantage.

Example: Pedestrian Crossing

- You are crossing Comm Ave. You can choose to either ***cross Comm Ave without waiting (C)*** or ***wait for cars to pass (W)***.
- The driver on the road can either ***stop for pedestrians (S)*** or ***keep going (G)***.
- You prefer ⟨***C***, ***S***⟩, but the driver prefers ⟨***W***, ***G***⟩.
- ⟨***C***, ***G***⟩ has terrible negative payoffs for you and the driver (you are dead, and the driver is in prison).
- If you move first, you can step off the curb and commit to crossing the road ***C***, force the driver to stop ***S***, and obtain your preferred result ⟨***C***, ***S***⟩.
- What would you do in real life ☺?

As an exercise, I suggest that you draw the game tree, insert reasonable payoffs, and find the subgame perfect equilibrium.

Matching Pennies: Static Version

		Esther	
		H	***T***
Eva	***H***	1, −1	−1, 1
	T	−1, 1	1, −1

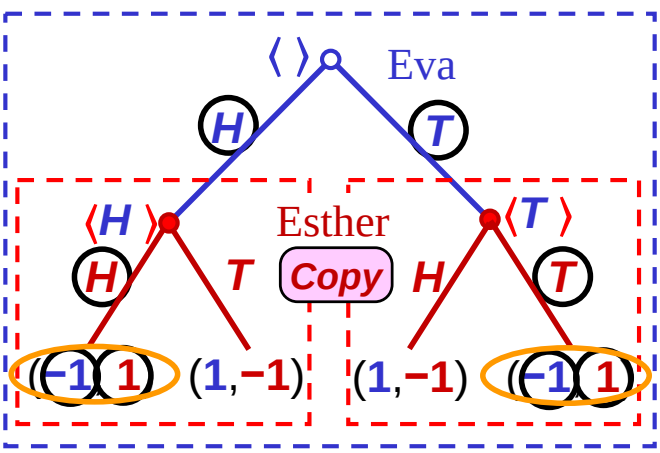
- Remember "Matching Pennies", the offense vs. defense game?
- ***Eva*** and ***Esther*** simultaneously put a penny on the table. *(Each **chooses** heads or tails—they don't flip the coin.)*
- If ***Esther*** matches ***Eva*** (both heads or both tails), then ***Eva*** pays ***Esther*** $1.
- But if ***Esther*** fails to match ***Eva*** (one is heads, one is tails) ***Esther*** pays ***Eva*** $1
- The game has no Nash equilibrium with pure (nonrandom) strategies.

Matching Pennies: Dynamic Version

- Now suppose that ***Eva*** moves first.
- ***Esther*** sees ***Eva***'s move, then she moves.
- ***Esther*** wants to match ***Eva***'s move.
- Which player has the advantage, ***Eva*** or ***Esther***?
- We analyze the extensive-form game.

Matching Pennies in Extensive Form

- What does ***Esther*** do in her subgames?
  - Esther uses strategy ***Copy***.
- What does Eva do in her subgame?



  - If ***Eva*** chooses ***H***, she gets ***−1***.
  - If ***Eva*** chooses ***T***, she gets ***−1***.
  - Both ***H*** and ***T*** are best responses (although both are bad).
- Two subgame-perfect equilibria: ⟨***H***, ***Copy***⟩ and ⟨***T***, ***Copy***⟩

- In both subgame-perfect equilibria, ***Eva***, who moves first, gets ***−1***,…
- and ***Esther***, who moves second, gets ***+1***.
- Even though Eva has the power of ***commitment***, she loses,…
- …and Esther, who has more ***information***, wins.
- In games of offense versus defense, information seems more important than commitment.
- **Example:** Microsoft waits for another company to build a software application and uses its idea.

Dynamic Cournot Duopoly

(Stackelberg Competition)

- Remember the static game between ***L'Eau*** and ***N'Eau***?
  - Demand curve was $Q_D = 120 - P$.
  - Cost was given by $AC \equiv MC \equiv 0$.
  - ***L'Eau*** sets q_L and ***N'Eau*** sets q_N at the same time.
  - ***L'Eau***'s best response to q_N is $\hat{q}_L = \frac{1}{2}(120 - q_N)$,…
  - …and ***N'Eau***'s is $\hat{q}_N = \frac{1}{2}(120 - q_L)$.
  - Equilibrium: $q_L^* = 40$, $q_N^* = 40$, $P = 40$.
  - **Profits:** $Y_L = Y_N = 1600$, **CS = 3200.** Why?

- Now suppose ***L'Eau*** sets q_L **first**.
- ***N'Eau*** sees q_L, and then he sets q_N based on the value of q_L.
- What will happen? Will the results change?
- $q_L^* = 60$, $q_N^* = 30$, $Y_L = 1800$, $Y_N = 900$, $CS = 4050$.
  - Can you derive these results? *[NOT required for exam]*
- ***L'Eau***, the first firm, will have greater profits than ***N'Eau***,…
- …because, in this game, commitment is more important than information is.

Clicker Question

Which is more important for a player in a dynamic game, commitment or information?

a. commitment
b. information
c. commitment for some games, information for other games
d. neither is important

Thank you!
I enjoyed teaching this course.

End of Lecture 24