Tuesday, Dec 7, Lecture 23
Offense vs. Defense & Dynamic Games

Important: Check the course website for announcements frequently.

Do the evaluations for this course.
See the announcement on the course website.

Final Exam Schedule
Wednesday 15-Dec in the Law Auditorium,
EC101 DD, 12:00 – 2:00
EC101 EE, 3:00 – 5:00

Clicker Question
Using Game Theory to Analyze *Offense versus Defense*

- In many competitive situations the *offense* of one competitor battles the *defense* of the other.
  - True of many sports like soccer, basketball and American football.
- If the defense matches the offense, then the defense wins.
- If not, the offense wins.

**Example:** Military Strategies

- Attack from the *Left* or from the *Right*
- If the enemy correctly anticipates your action, you lose the battle.

**Example:** Business Strategies

- Master Card is better off if it guesses Visa’s market-penetration strategy, but…
- Visa would be better off if Master Card guessed wrong.
Matching Pennies

- “Matching pennies” is a game-theory model of offense-versus-defense.
- In this example, Eva plays offense; Esther plays defense.
- Eva and Esther each puts a penny on the table at the same time (they DON’T flip the coin).
- If Esther matches Eva (both heads or both tails), then Eva pays Esther $1.
- But if Esther fails to match Eva (one heads, one tails) Esther pays Eva $1.
- This is called a “zero-sum game,” because whatever amount one player wins, the other must lose.
- The game has no Nash equilibria with pure strategies (nonrandomized actions).

**Clicker Question**

In Matching Pennies,

a. both players have the same objectives.

b. both players can increase their payoffs by coordinating.

c. the sum of their payoffs is 0.

d. there are no winners.
Dynamic Games

- So far, we've analyzed static games, in which all players move at the same time.

- Now we will examine dynamic games, in which players move at different times, possibly with different information.

- **Dynamic Game Example:** Airline fares
  - British Airways (BA) sets its Boston-London fares.
  - Then, Delta sees what BA did and sets its own Boston-London fares.

The Battle of the Sexes: Simultaneous Moves

- Remember the **Battle of the Sexes**?

- Vanesa wants to go to a football match $F$, but Miguel wants to go to the opera $R$.

- If they both do $F$, then Vanesa gets utility 2, and Miguel gets 1,

- and if they both do $R$, then Vanesa gets 1 and Miguel gets 2.

- But if they do different things, then both get 0.

- Both must choose their strategies at the same time, without knowing what the other has done.

- There are two Nash equilibria: $\langle F, F \rangle$ and $\langle R, R \rangle$. 
The Battle of the Sexes: Sequential Moves

Now suppose that the players move at different times, first one, then the other.

For example, suppose that Vanessa moves first: she buys a ticket for either the football match or the opera.

She shows Miguel her ticket, so he knows what she has done.

Then Miguel moves: he buys his ticket for either the football match or the opera.

This is a dynamic game, because different actions are happening at different times.

What would happen in this game?

The answer is clear!

Vanesa (the selfish beast 😊) will choose football $F$…

and “force” Miguel to choose football $F$ as well.

$\langle F, F \rangle$ still looks like a Nash equilibrium.

We know they won’t choose $\langle R, R \rangle$, but is $\langle R, R \rangle$ still an equilibrium?

We can’t answer until we model strategies properly.

- If Vanessa moves first, and Miguel sees the result before he moves,…
- …then the matrix above does not correctly represent the game.
Dynamic-Game Strategies

- A **strategy** is a **complete plan of action** that specifies what a player will do **in every circumstance** that she can observe.

- From what strategies does Vanesa choose?
  - *F* and *R* (as before).

- What about Miguel? What are his strategy choices?
  - *F* and *R* are **NOT** strategies for Miguel.

  - A strategy is a **complete plan** that might tell you to do different things in **each circumstance you know about**.
  
  - Miguel knows whether Vanesa chose *F* or chose *R*.

  - So his strategies must reflect his knowledge of her action.

- Miguel's possible strategy choices are the following (with my own nicknames):

  - **Always F**: If Vanesa chose *F*, I will choose *F*. If Vanesa chose *R*, I will choose *F*.

  - **Copy**: If Vanesa chose *F*, I will choose *F*. If Vanesa chose *R*, I will choose *R*.

  - **Opposite**: If Vanesa chose *F*, I will choose *R*. If Vanesa chose *R*, I will choose *F*.

  - **Always R**: If Vanesa chose *F*, I will choose *R*. If Vanesa chose *R*, I will choose *R*.

- These four strategies form Miguel's strategy space.

- Some of Miguel's strategies (e.g. **Opposite**) might be bad strategies, but they are still strategies.
Representing the Dynamic Game

- The dynamic Battle of the Sexes can be represented as follows:

```
<table>
<thead>
<tr>
<th></th>
<th>Always F</th>
<th>Copy</th>
<th>Opposite</th>
<th>Always R</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
```

- Notice that if Vanessa does F, then Miguel’s strategies Always F and Copy require the same actions and lead to the same payoffs.

But what are the Nash equilibria of this game?

- If we check each cell, we can see that there are exactly 3 pure-strategy equilibria:
  - \( \langle F, \text{Always F} \rangle \)
  - \( \langle F, \text{Copy} \rangle \)
  - \( \langle R, \text{Always R} \rangle \)

- In each equilibrium, rational players have no incentive to deviate from their chosen strategies.

- However, it turns out that only \( \langle F, \text{Copy} \rangle \) is formed from strategies (plans) that would actually be followed during the game.

- What’s wrong with the strategies in the other equilibria?

- **Answer:** Some of them aren’t time-consistent …
***Clicker Question***

What is true about \( \langle R, \text{Copy} \rangle \)?

- a. It is a Nash equilibrium.
- b. Vanesa would deviate.
- c. Miguel would deviate.
- d. *None* of the above

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**Time Consistency***

*The formal term in game theory is “sequential rationality.”*

- A **strategy** is a plan of action that specifies what a player will do *in every circumstance* that she can observe.

- A player’s strategy is **time-consistent** if the player is willing to follow it in every circumstance that might arise.

  - **Example:** Suppose your strategy is to study economics Saturday night, but if your friends have a party, you will go to the party for a half hour and then go back to your dorm and continue to study.

  - But after the party begins, you decide that one half-hour isn’t your best response to the party.

  - You decide to stay there all night and study some other time.

  - Your original strategy was not **time-consistent**.
In our Battle-Sexes example, Vanesa buys her ticket first.
But if Miguel says he will go to opera no matter what Vanesa does,
…wouldn’t Vanesa be “forced” to buy an opera ticket?

- It’s true that \( \{R, \text{Always } R\} \)
  is a Nash equilibrium!

But Vanesa would ignore Miguel’s statement!
- Vanesa knows that if she chooses \( F \), Miguel will change
  his mind about \( \text{Always } R \).
- She thinks: “\text{Miguel might choose the strategy } \text{Always } R \text{ at first,…}
  “…but if I have chosen } F, \text{ then, when it’s Miguel’s turn to buy a
  ticket, he won’t follow the } \text{Always-R } \text{ strategy. He will copy me.”}
- \( \text{Always } R \) is not a best response to \( F \), so Miguel will deviate
  after Vanesa chooses \( F \). \( \text{Always } R \) is not time-consistent
  [not sequentially rational].

**Clicker Question**

If Vanesa moves first, what is true about \( \{R, \text{Always } R\} \)?

a. It is a Nash equilibrium.

b. Miguel would deviate.

c. Vanesa would deviate.

d. Both players would
  deviate.
Yes, \( \{R, \text{Always } R\} \) is a Nash equilibrium, because no player wants to deviate from that strategy profile.

But it will not occur if both players are economically rational.

This is because Miguel’s strategy \textit{Always } R is not time-consistent [sequentially rational].

Miguel would not always follow \textit{Always } R during the game,…

…and Vanesa \textbf{knows} he won’t.

So, if Vanesa moves first, she will choose \( F \), even if Miguel \textit{says} he will follow \textit{Always } R .

We will make sense of this situation next time in the last lecture.

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**A New Kind of Equilibrium**

In general, the Nash equilibrium does not guarantee that the equilibrium strategies of dynamic games will be time consistent,…

…because the Nash-equilibrium concept doesn’t require a best response at every player’s turn to move during the game.

However, there’s a special kind of Nash equilibrium that does guarantee time-consistent equilibrium strategies…

…the \textit{subgame-perfect Nash equilibrium}.
Normal-Form and Extensive-Form Games

- So far, we’ve described games with a matrix in which each row or column represents a player’s strategy: the **normal-form game**.

- But to find a **subgame-perfect Nash equilibrium** we need a different game structure: the **extensive-form game**.

- We’ll explain the extensive-form game in the next (and last) lecture,…

- …and we’ll use it to find an equilibrium with time-consistent strategies.

Please complete your EC101 course evaluations in the remaining class time or after class.

Go to the Course Evaluations announcement in the course website.
End of Lecture 23