Tuesday, Nov 30, Lecture 21
Strategic Interaction and Game Theory

Strategic Interaction

- In perfectly competitive markets (like the market for corn), firms do not compete with other firms on an individual basis.
  - If Farmer Jane grows corn, she couldn't care less about what Farmer Jones is doing.
  - Farmer Jane looks up the market price of corn in the newspaper or online,…
  - …and she bases her business strategy on the price.
  - Farmer Jane does ***NOT interact strategically*** with her competitors.

- Monopolies, too, have **NO** ***strategic interaction*** with competitors (unless there are potential entrants, they have no competitors ☺).

- But suppose two fancy hotels are located across the street from one another (a duopoly).
  - The owner of each hotel will be interested in the ***business strategy*** of the other owner,
  - because potential customers are likely to choose between the two hotels.
  - Each owner will base her own business strategy on her beliefs about the strategy of her competitor.
  - This is an example of ***strategic interaction***.

- Strategic interaction is very important when a small number of people or firms engage in ***bargaining***, ***conflict*** or ***competition***.
  - Duopoly (two competing firms)
  - Oligopoly (several competing firms)
  - Contracts
  - Legal Disputes
  - Political campaigns

Clicker Question

In which of the following cases is strategic interaction ***least*** important?

a. Duopoly
b. Oligopoly
c. Perfect Competition
d. Political Campaigns

Game Theory

- ***Game Theory*** refers to a set of mathematical tools used to analyze strategic interaction.
  - Game theory is often applied in economics, political science, and military science,…
  - but game theory is ***not*** commonly applied to ordinary games like chess or tennis (at least not yet ☺).
- In game theory,
  - ***players*** (decision makers)…
  - adopt ***strategies*** (complete plans of action)…
  - and receive ***payoffs*** (rewards or punishments), which depend on the strategies of all the players.
- There must be at least two players in a game, but games with any number of players can be analyzed.

Strategies

- A ***strategy*** is a **complete plan** that describes the ***action*** a player will take ***in every circumstance*** that she can observe.
  - Sometimes, a strategy will involve only one action:

    ("I'll ask my boss for a raise [salary increase].")

  - But some strategies are complex plans that involve the choice of many possible actions depending on the observed situation *(e.g. military strategies)*.

Coordination in Business

- Sometimes firms can increase profits by coordinating their strategies.

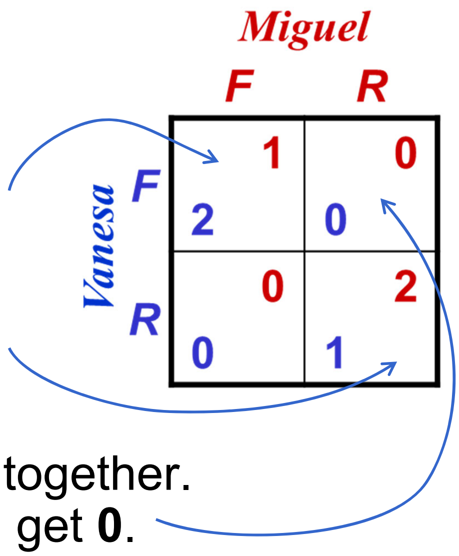
- **Example:** If a men's clothing shop and a women's clothing shop locate in the same shopping mall, both may attract more customers.

- **Example:** If two similar hardware stores locate further apart, they can charge higher prices, because they won't have to compete with each other.

- There are many other examples where firms can increase profits by coordinating.
  - One firm supplies inputs to another firm ***precisely when they are needed***.
  - All firms in a shopping center stay open during the ***same hours***. [Why?]
  - All car thieves steal cars on the ***same day***, so that police are spread thin.
  - Different firms put their trucks on the road at ***different times***, in order to avoid congestion.

Battle of the Sexes

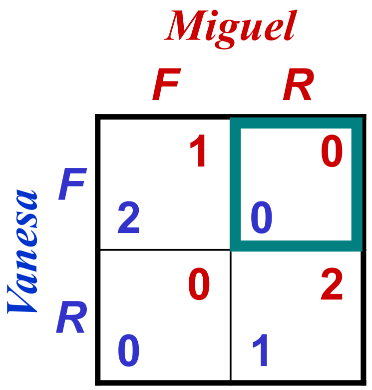
- The ***Battle of the Sexes*** is a game-theory model of coordination in business (or in personal relationships).
- Vanesa wants to go to a football match ***F***, but Miguel wants to go to the opera ***R*****.**
- If they both do ***F***, then Vanesa gets payoff ***2*** , and Miguel gets ***1***,
- and if they both do ***R***, then Vanesa gets ***1*** and Miguel gets ***2***.
- But they are in love and want to be together. If they do different things, then both get **0**.
- Each must buy his/her ticket without knowing what the other is doing. *[Miguel forgot to charge his cell phone.]*



		Miguel	
		F	***R***
Vanesa	***F***	1 2	0 0
	R	0 0	2 1

Game-Theory Terminology

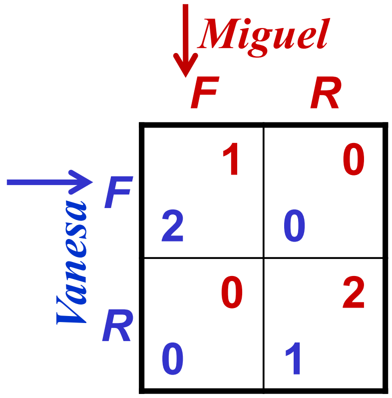
- Vanesa and Miguel are ***players.***
- ***F*** and ***R*** are ***strategies.*** (In this case each strategy involves only one action.)
- ***{F, R}*** is the ***strategy space*** (the set of allowable strategies).
- ***2, 1*** and ***0*** are payoffs.



Vanesa \ Miguel	***F***	***R***
F	1 2	0 0
R	0 0	2 1

- Each cell in the table corresponds to a ***strategy profile*** (one strategy for each player), and the contents of the cell are the payoffs corresponding to that profile.
  - For example, the top-right cell represents the strategy profile ⟨***F, R*** ⟩ (Vanesa chooses ***F***; Miguel chooses ***R***).
  - ***0*** for Vanesa and ***0*** for Miguel are the corresponding payoffs.

- The Battle of the Sexes is modeled as a ***normal-form game***.
  - Each row represents a strategy for one player (Vanesa),…
  - Each column represents a strategy for the other player (Miguel).
  - The row player chooses a strategy (up or down);
  - the column player chooses a strategy (left or right).



Vanesa \ Miguel	***F***	***R***
F	1 2	0 0
R	0 0	2 1

Vanesa \ Miguel	***F***	***R***
F	2, 1	0, 0
R	0, 0	1, 2

- In books and exams, the game is illustrated in black and white,…
  - with the first number inside each cell representing the payoff to the row player (Vanesa, in this case),
  - and the second number to the column player (Miguel).

Applying Game Theory

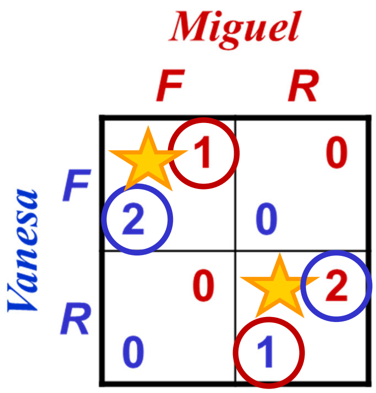
- Can we use game theory to predict the outcomes of strategic interaction?
- What strategies should we expect Vanesa and Miguel to adopt in their "Battle of the Sexes"?
- Unfortunately, game theory has a number of different ***"solution concepts"*** that sometimes predict different outcomes.
- The most commonly used solution concept is the ***Nash equilibrium***, named after the mathematician *John Nash* *[Nobel Prize, 1994]*. ***Movie: A Beautiful Mind***
  - Sometimes we call it simply "an equilibrium."

Nash Equilibrium

- A player's ***best response*** to the strategies of the other players…
- …is a strategy that provides the player with the highest payoff when other players use their strategies.
- A [***Nash***] ***equilibrium*** is a strategy profile in which each player's strategy is a best response to the strategies of the other players.

- Equivalently, in a ***Nash equilibrium***, if after choosing her own strategy, each player finds out what the other players had chosen…
- …***no*** player would want to ***deviate*** *[change]* from her previously chosen strategy.
- Does the word "equilibrium" make sense for this this situation? Why?

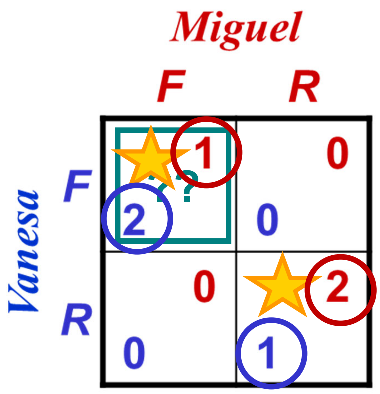
Equilibrium in the Battle of the Sexes



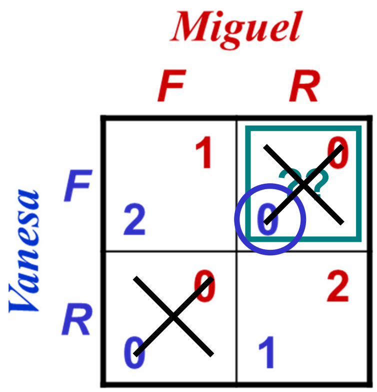
- What is Vanesa's best response
  - if Miguel chooses F ?
    - Answer: Vanesa chooses F and gets 2 instead of 0.
  - if Miguel chooses R ?
    - Answer: Vanesa chooses R and gets 1 instead of 0.
- What is Miguel's best response
  - if Vanesa chooses ***F*** ?
    - Answer: Miguel chooses ***F*** and gets ***1*** instead of ***0***.
  - if Vanesa chooses ***R***
    - Answer: Miguel chooses ***R*** and gets **2** instead of ***0***.
- Result: if a cell has two circles, then each strategy in the profile is a best response to the other strategy in the profile.
- Therefore, ⟨***F, F*** ⟩ and ⟨***R, R*** ⟩ are both Nash equilibria.

Finding Equilibria by Checking for Deviations

- Suppose both Vanesa and Miguel decide to go to the football match.
  - Is that an equilibrium?
  - Given that Miguel has chosen ***F***, what happens to Vanesa if she deviates from ***F*** to ***R*** ?
    - Answer: she would get ***0*** instead of **2**.
    - So ***F*** is Vanesa's best response to Miguel's ***F***, and ***she would not deviate***.
  - Given that Vanesa has chosen ***F***, what happens to Miguel if he deviates from ***F*** to ***R*** ?
    - Answer: he would get ***0*** instead of ***1***.
    - So ***F*** is Miguel's best response to Vanesa's ***F,*** and ***he would not deviate***.
  - Result: the strategy profile ⟨***F, F*** ⟩ **IS** an equilibrium!
  - Likewise, ⟨***R, R*** ⟩ is an equilibrium.

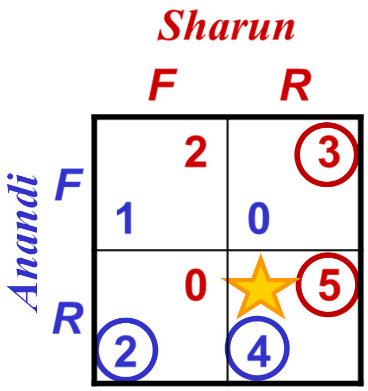


- Suppose Vanesa goes to football and Miguel goes to the opera ⟨***F, R***⟩.
  - Is ⟨***F, R***⟩ an equilibrium?
  - Given that Miguel has chosen ***R***, what happens to Vanesa if she deviates from ***F*** to ***R*** ?
    - Answer: she would get ***1*** instead of ***0***, so ***she would deviate***.
    - ***F*** is ***not*** Vanesa's best response to Miguel's ***R***.
  - Therefore ⟨***F, R*** ⟩ is ***not*** an equilibrium!
  - We do not have to ask if Miguel would also deviate.
- Likewise, ⟨***R, F*** ⟩ is not an equilibrium.



- In the “Battle of the Sexes” coordination failure is not an equilibrium!
- In equilibrium, Miguel would have to do what Vanesa wants, or *vice versa*.
- Both of these equilibria are called ***pure-strategy*** equilibria, because neither player’s strategy contains actions chosen at random.
- There is a ***mixed-strategy*** equilibrium also:
  Vanesa goes to football with probability ***2/3*** and to the opera with probability ***1/3***. Miguel does the opposite. *[You are not required to know this.]*
  - Extra credit: prove that this is an equilibrium ☺ **!**

The Compatible Couple



- Anandi and Sharun like to be together, but they both like ***R*** more than ***F***.
- Anandi gets **2** if she is alone at ***R*** and **4** if they are together at ***R***. Sharun gets **3** and **5**.
- For Anandi, we say that ***R*** is a ***strictly dominant strategy***, because she always prefers ***R*** to ***F*** no matter what Sharun does. Anandi’s other strategies are ***strictly dominated***.
- For Sharun, ***R*** is also strictly dominant.
- ⟨***R***, ***R*** ⟩ is a Nash equilibrium, because a strictly dominant strategy is always a best response to another player’s strictly dominant strategy (and to any other strategy).
- Also, ⟨***R***, ***R*** ⟩ is the only Nash equilibrium, because either player would always deviate to his strictly dominant strategy.

Clicker Question

A strategy profile is a Nash equilibrium of a game if given the strategies of the other players

a. all players want to deviate from their chosen strategies.

b. at least one player does not want to deviate.

c. no player wants to deviate.

d. ***None*** of the above

Cooperation versus Competition

- Sometimes cooperation is more profitable or productive than competition.
- But cooperation can be hard to maintain.
- If all other firms (or players) are cooperating, it may be profitable for an individual firm to “defect” or cheat.

- **Example:** Coke and Pepsi would increase profits if they both spent less on advertising.

- **Example:** The U.S. and Russia would both be better off if they could commit to keeping fewer nuclear weapons.

- The game-theory model of cooperation vs. competition is called the "***Prisoners' Dilemma***"

Prisoners' Dilemma

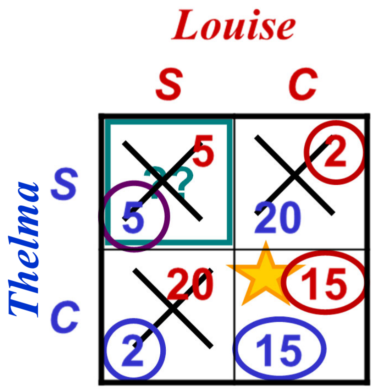
Years in Prison

		Louise	
		S	***C***
Thelma	***S***	**5** **5**	**2** **20**
	C	**20** **2**	**15** **15**

- Thelma and Louise have been caught by the police.
  - Police have evidence to put them behind bars for 5 years each,…
  - but with a confession, the police could get 20-year sentences.
  - So the police offer them the following terms:
    - If only one person confesses, she will get only 2 years in prison, but the other gets 20 years,
    - …but if both confess, each gets 15 year in prison.
- Thelma and Louise each has two possible strategies:
  - Silence (S) *[Try to cooperate with the other player.]*
  - Confession (C) *[Follow narrow self-interest.]*
- Each has to make her choice without knowing what the other will do.

Equilibrium in the Prisoners' Dilemma

- Suppose both Thelma and Louise decide to stay silent (**S**).
  - Is that an equilibrium?
  - Given that Louise has chosen ***S***, what happens to Thelma if she deviates from ***S*** to ***C*** ?
    - ♦She would get 2 years in prison instead of 5.
    - ♦So Thelma ***would deviate*** to ***C !***
  - Therefore, ⟨***S, S*** ⟩ **IS NOT** an equilibrium! In fact, for each player, confession ***C*** is a strictly dominant strategy—i.e. it is better to play ***C***, no matter what the other person does.



Years in Prison
small numbers are better

- ⟨***C, C***⟩ **is** an equilibrium—the ***only*** equilibrium, even though both would be better off ***if they could commit*** to silence ***S***! [⟨***S, S*** ⟩ Pareto improves ⟨***C, C***⟩.]

Cooperation and the Prisoners' Dilemma

- The prisoners' dilemma illustrates how difficult it is for competing firms to cooperate with each other, even when cooperating is Pareto efficient.
- Whatever they have agreed to, each player can do better by cheating (following narrow self-interest).
- That is why OPEC countries cheat and overproduce.
- That is why firms and political candidates employ negative advertising.
- Too bad (for them) that they cannot make a binding commitment.

Clicker Question

Which strategy profiles are not Pareto efficient?

Years in Prison

Louise

		S	C
Thelma	S	5 5	2 20
	C	20 2	15 15

a. ⟨*S*,*S*⟩

b. ⟨*S*,*C*⟩ and ⟨*C*,*S*⟩

c. ⟨*S*,*C*⟩, ⟨*C*,*S*⟩ and ⟨*C*,*C*⟩

d. ⟨*C*,*C*⟩

End of Lecture 21