Rhythmic Regularity beyond Meter and Isochrony

Jason Yust, Boston University

Rhythm in Music since 1900, McGill, Sept. 22–24

Keynote Presentation

What is rhythm?

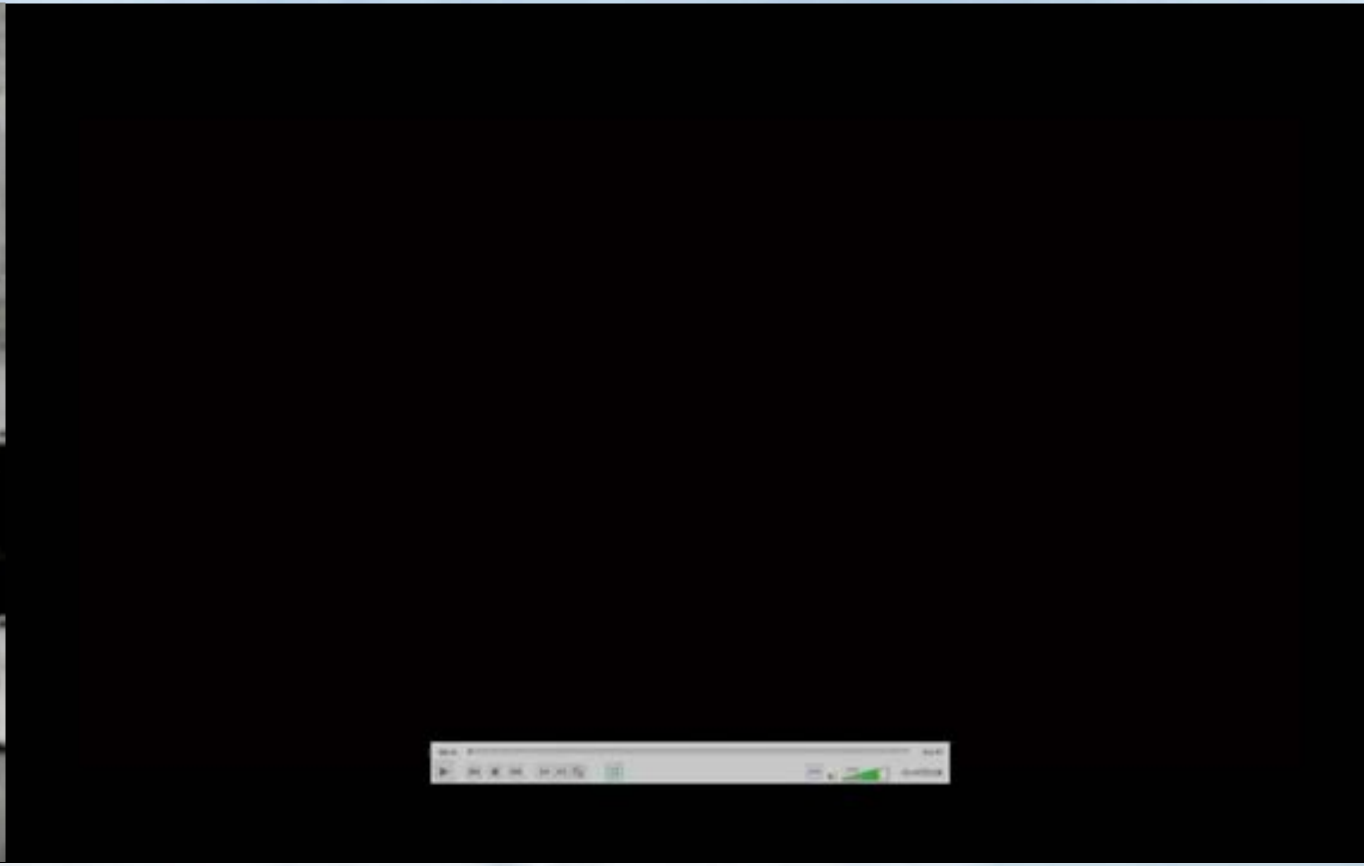
- Abstracted from timbre, pitch, loudness?
- Separate from meter or is meter a feature?
- Sound vs. notation
- On an isochronous grid?
- Discrete vs. continuous time

These questions all impinge on how we conceptually model rhythm!

What is rhythm?

Example: Bo Diddley rhythm

Bo Diddley on Ed Sullivan, 1955

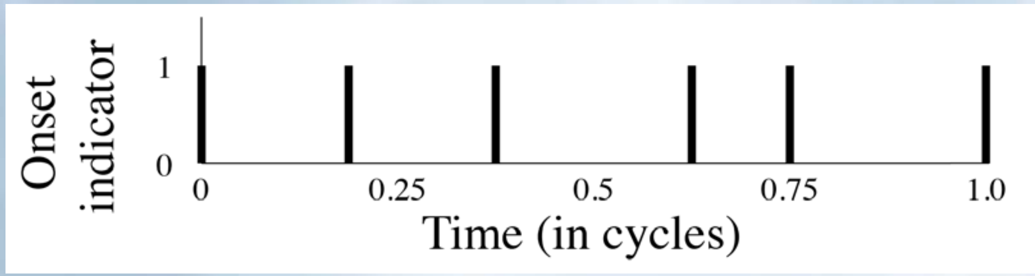


Bo Diddley in “All You Need is Love: The Story of Popular Music,” 1977

What is rhythm?: Bo Diddley

Is this an adequate representation?:

As a function over time:



Notation can be:

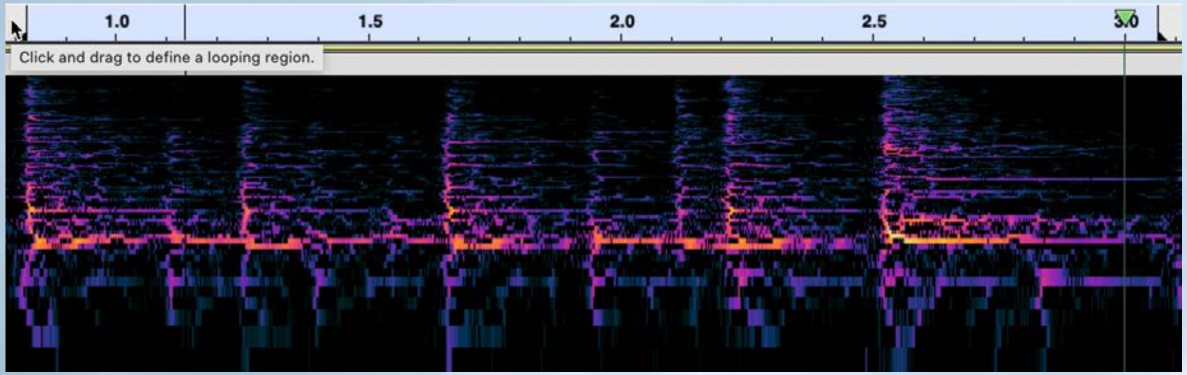
- Instruction for performance
- Descriptive: A summary of what musicians play (generalize over performances, different players)

What’s missing:

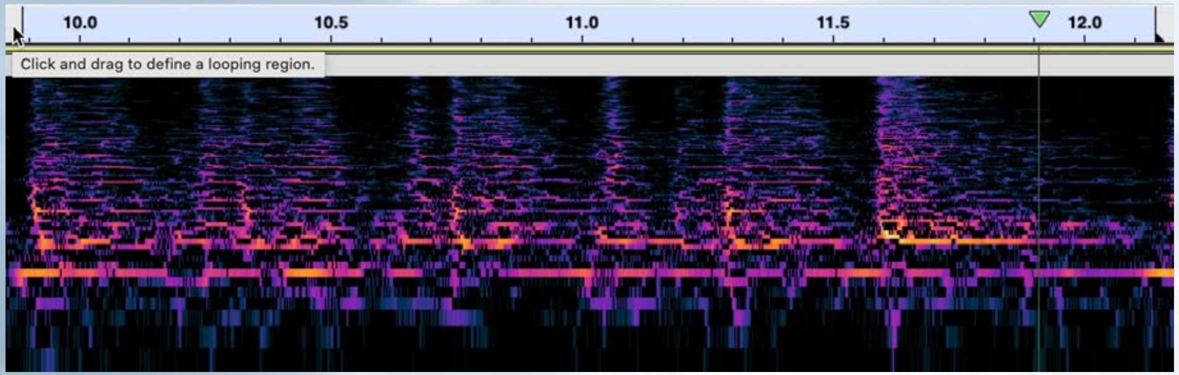
- Other things the musicians play
- Patterns of emphasis
- Meter
- Microtiming

What is rhythm?: Bo Diddley

Chess records, 1955, alt. take

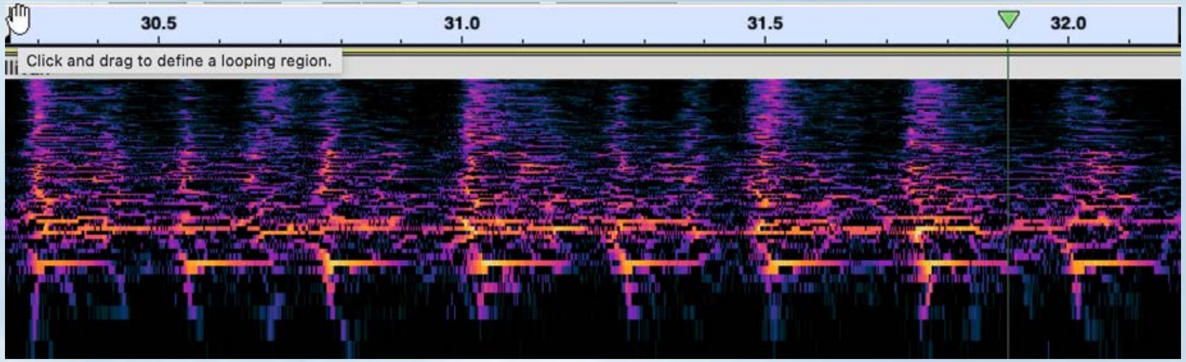


Drums alone



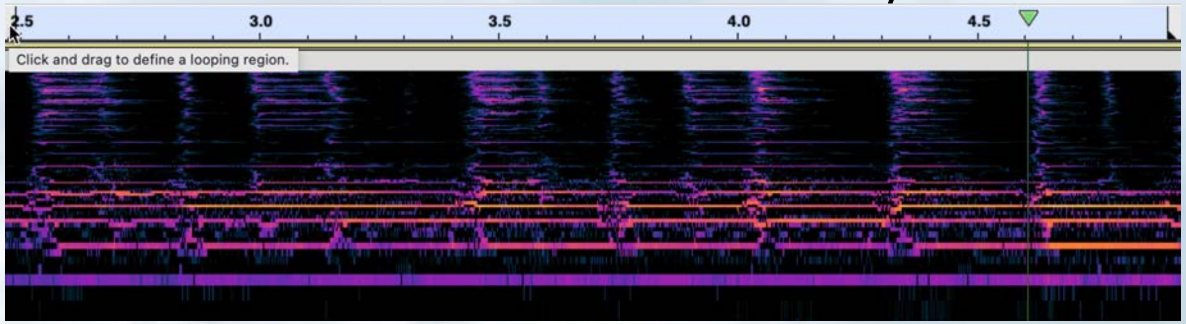
Full band (drums, bass, guitar, shaker)

Ed Sullivan Show 1955



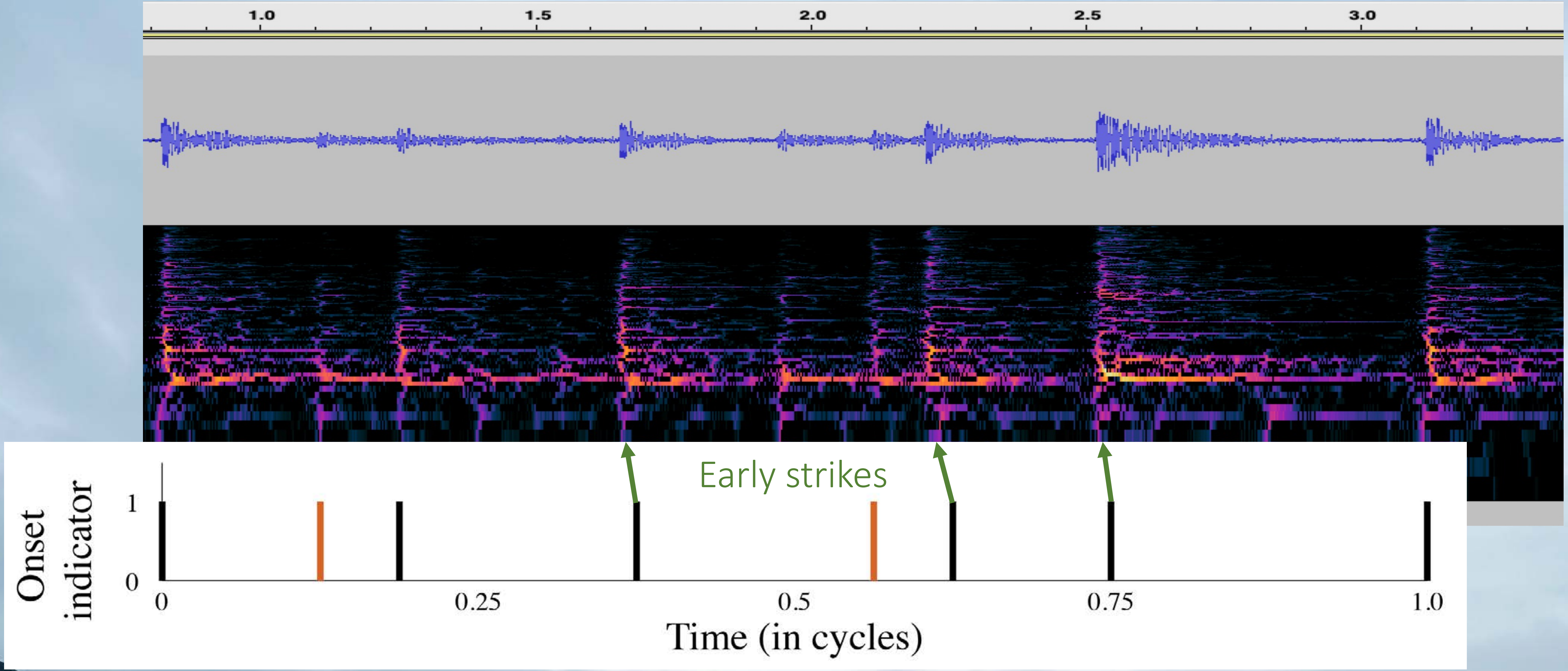
Full band + audience clapping

1977 Documentary



Guitar and bass only

What is Rhythm?: Bo Diddley



What is rhythm?: Bo Diddley

Other possible notated representations:

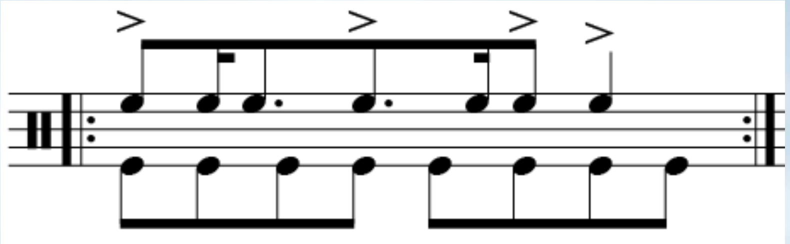
Emphasized onsets:

All onsets:

Emphasis as accents:

Showing microrhythm?:

With bass:

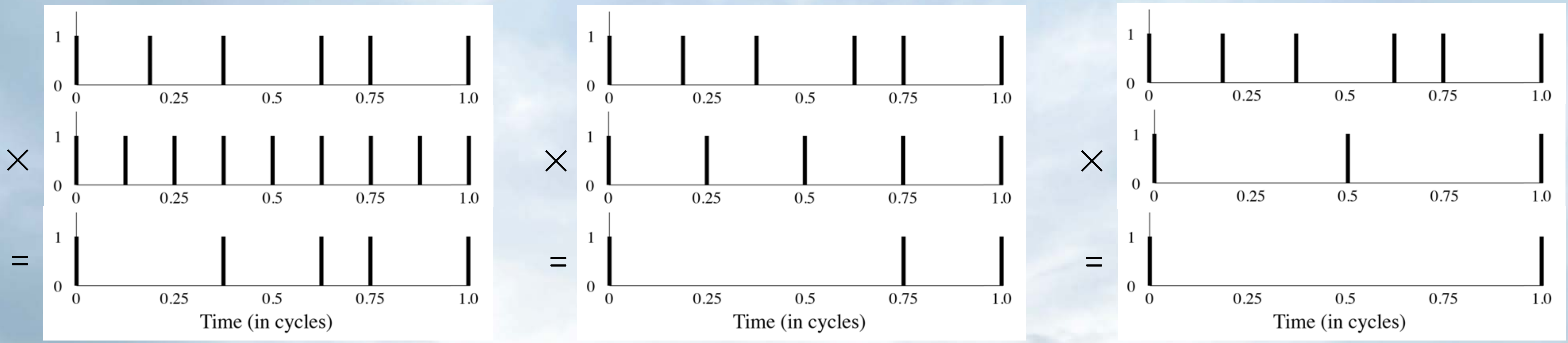


Or just a meter indication:

What is meter?

Meter is a kind of *interpretation* of a rhythm, which we can represent with a *filtering function.*

The Euro-classical concept of meter is notation-oriented, and based on the principle of **isochronous layers** (metronome model).



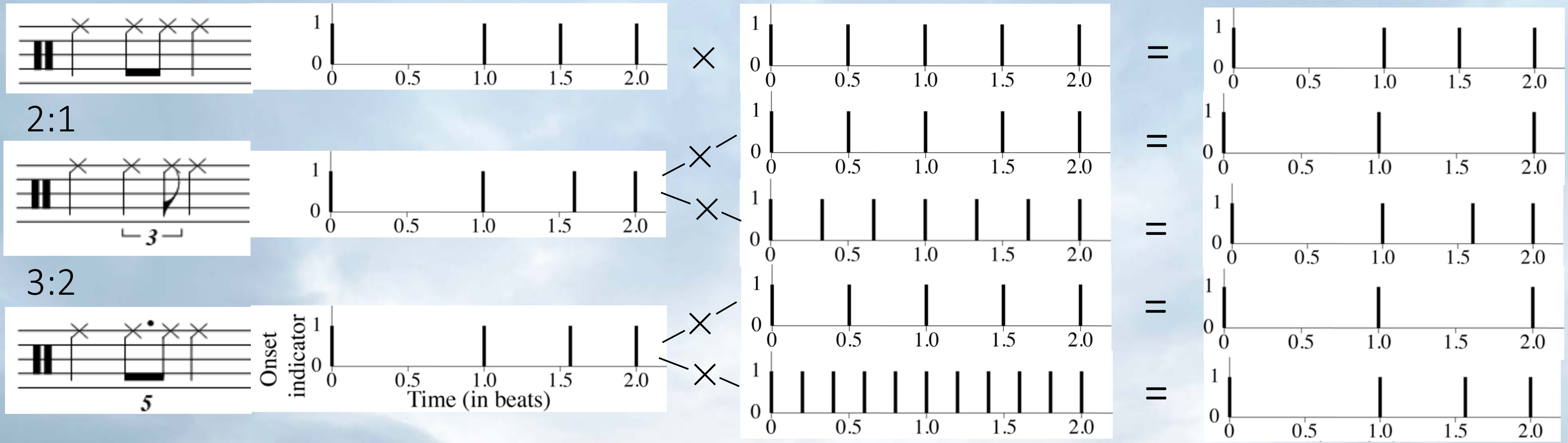
The rhythms and filters are both **discrete indicator functions.**

Continuity of time

Discrete filters on discrete signals do not reflect time as a continuous space.

Example: Swing ratios (See Benadon Forthcoming!)

1:1



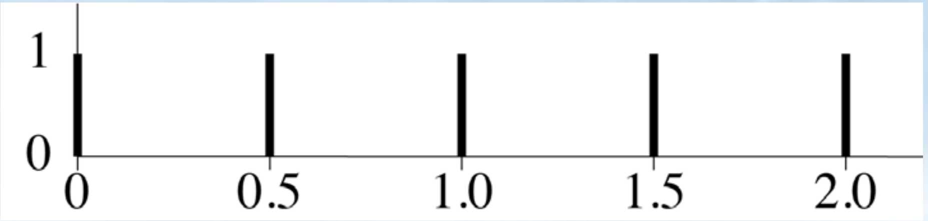
In the method of metrical notation, greater precision requires greater complexity.

European notation is optimized as a prescriptive, rather than descriptive, system.

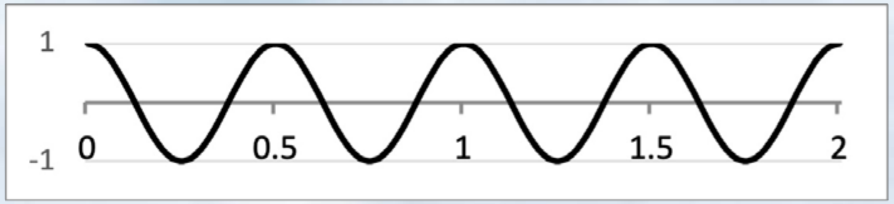
Isochrony vs. Periodicity

The traditional European concept of meter is based on **Isochrony** (discrete filters). An alternative (more general) concept is **periodicity.**

Instead of a **discrete filter:**

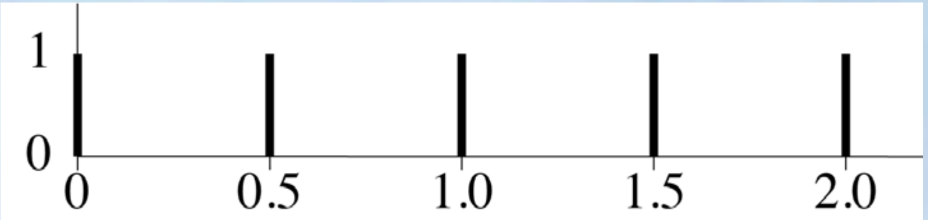


Consider a **continuous filter:**
(simple periodicity, AKA simple harmonic oscillator)



Isochrony vs. Periodicity

We can recover discrete isochronous functions from simple periodic ones.



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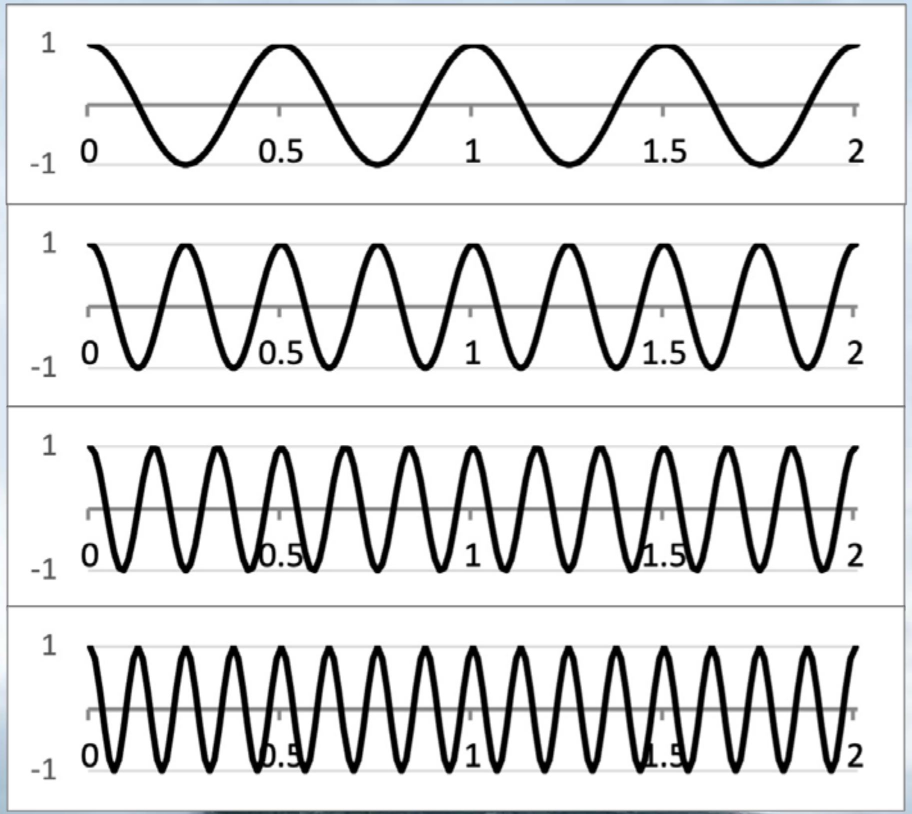

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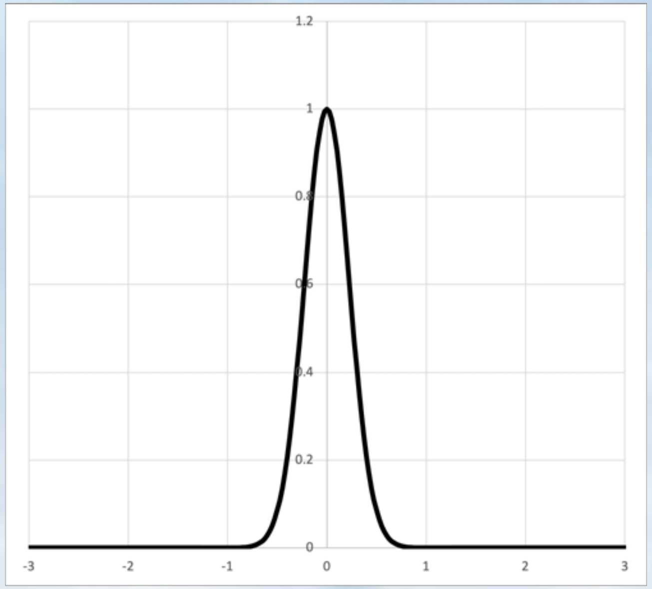
\+ . . .

Temporal **precision** can be modeled by high-frequency cutoff (low-pass filter) or sample rate.

All periodic functions are sums of simple periodic (sinusoidal) functions.

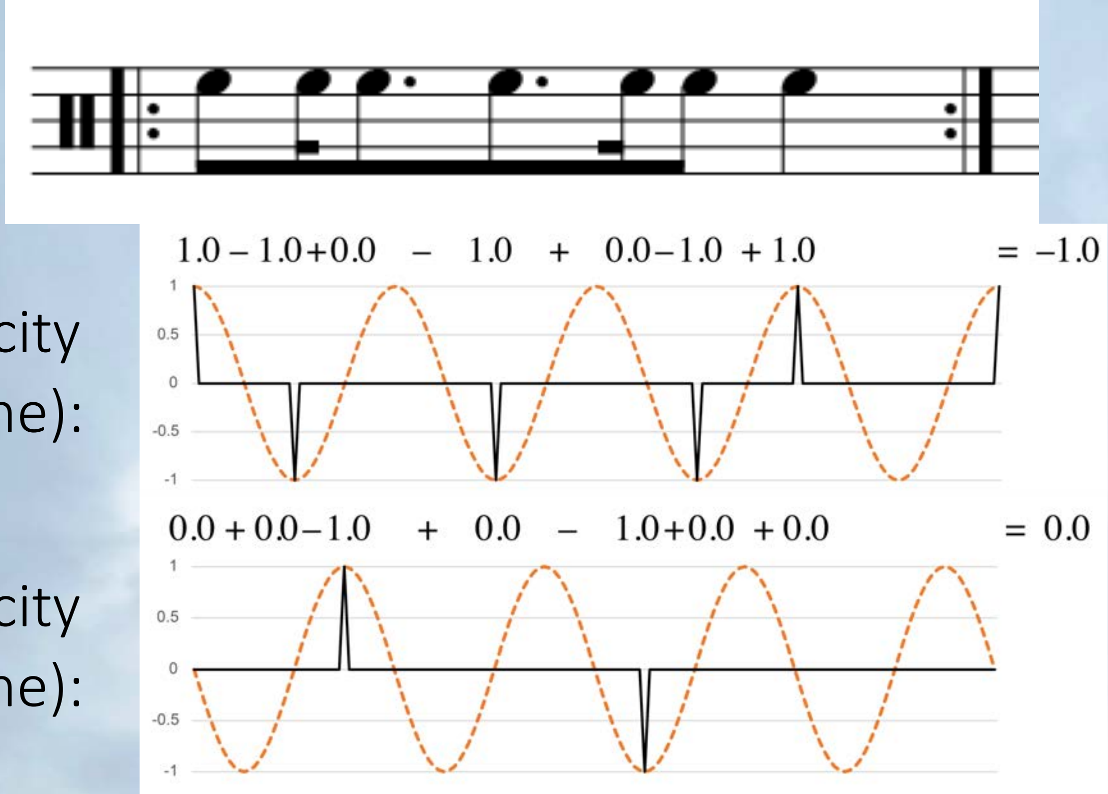
Beat bins

Anne Danielsen's concept of *beat bin* or Chris Stover's *beat span* can be represented as a **Gaussian filter:**



Sine and Cosine functions

Periodicities are two-dimensional.

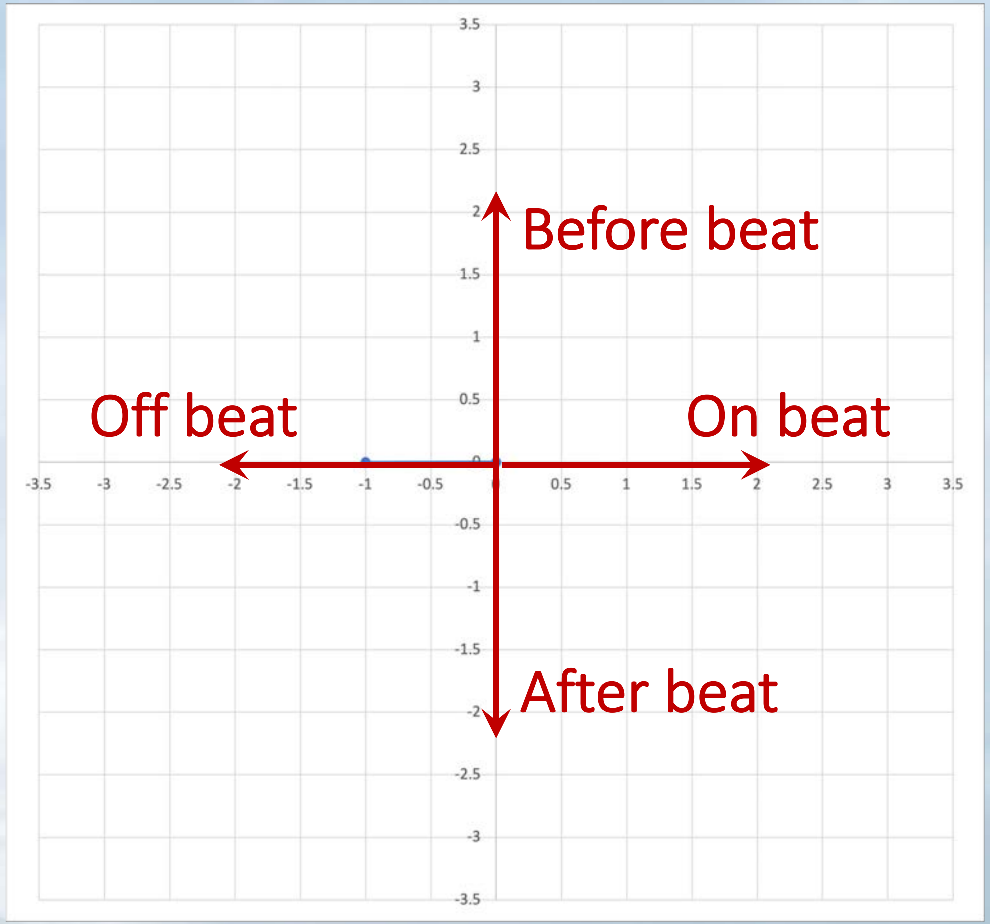


♩ periodicity (cosine):

♩ periodicity (–sine):

Frequency 4

On xy plane:

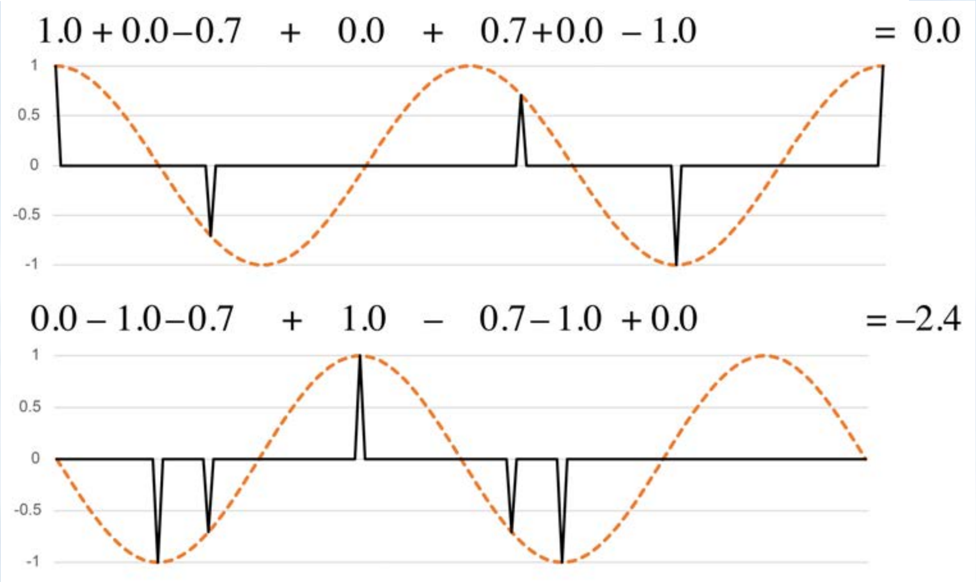


Sine and Cosine functions

Periodicities are two-dimensional.

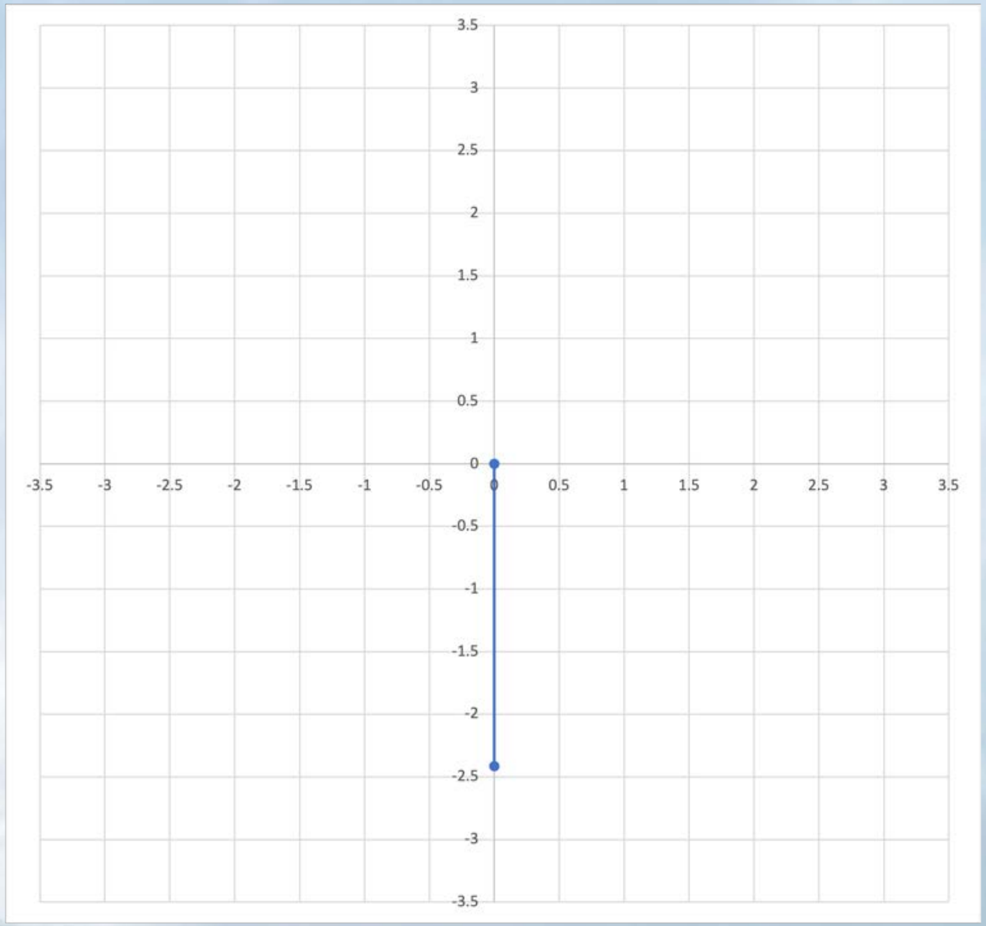
𝅗𝅥 periodicity (cosine):

𝅗𝅥 periodicity (–sine):



Frequency 2

On xy plane:

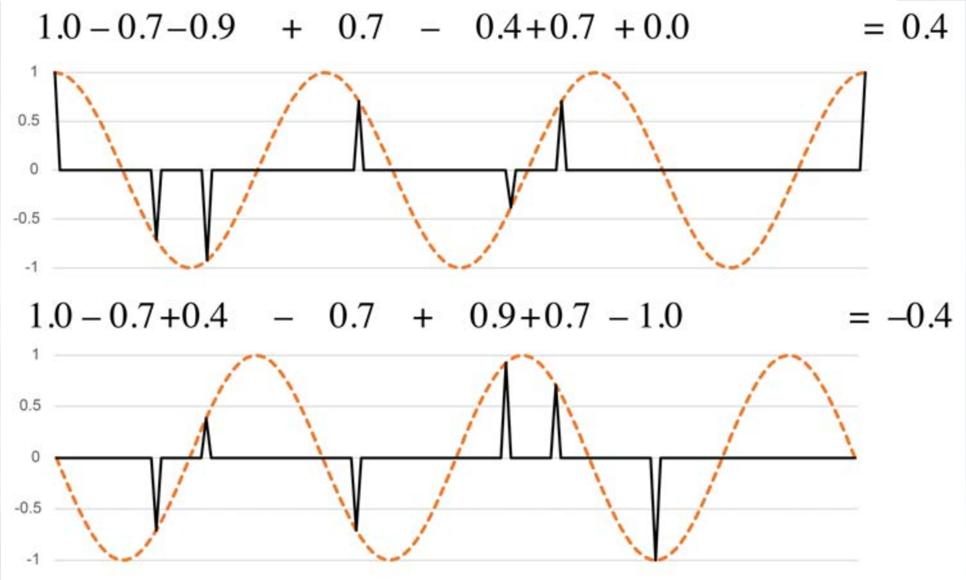


Sine and Cosine functions

Periodicities need not be integer multiples of the minimal duration.

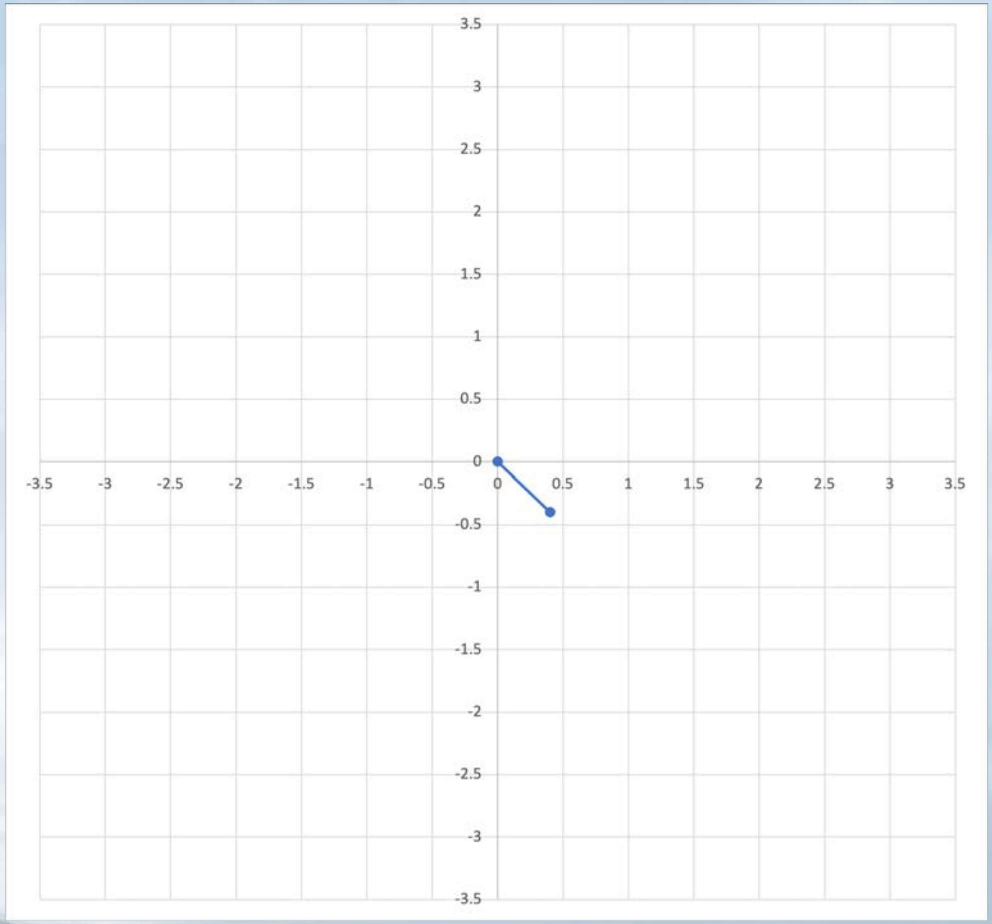
On xy plane:

1⅓♩ periodicity (cosine):



1⅓♩ periodicity (–sine):

Frequency 3

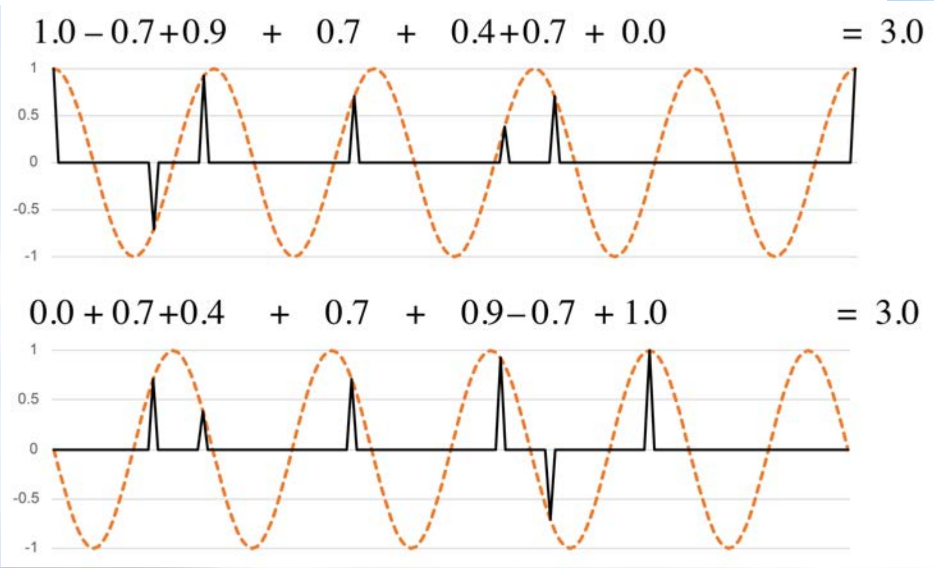


Sine and Cosine functions

Periodicities need not be integer multiples of the minimal duration.

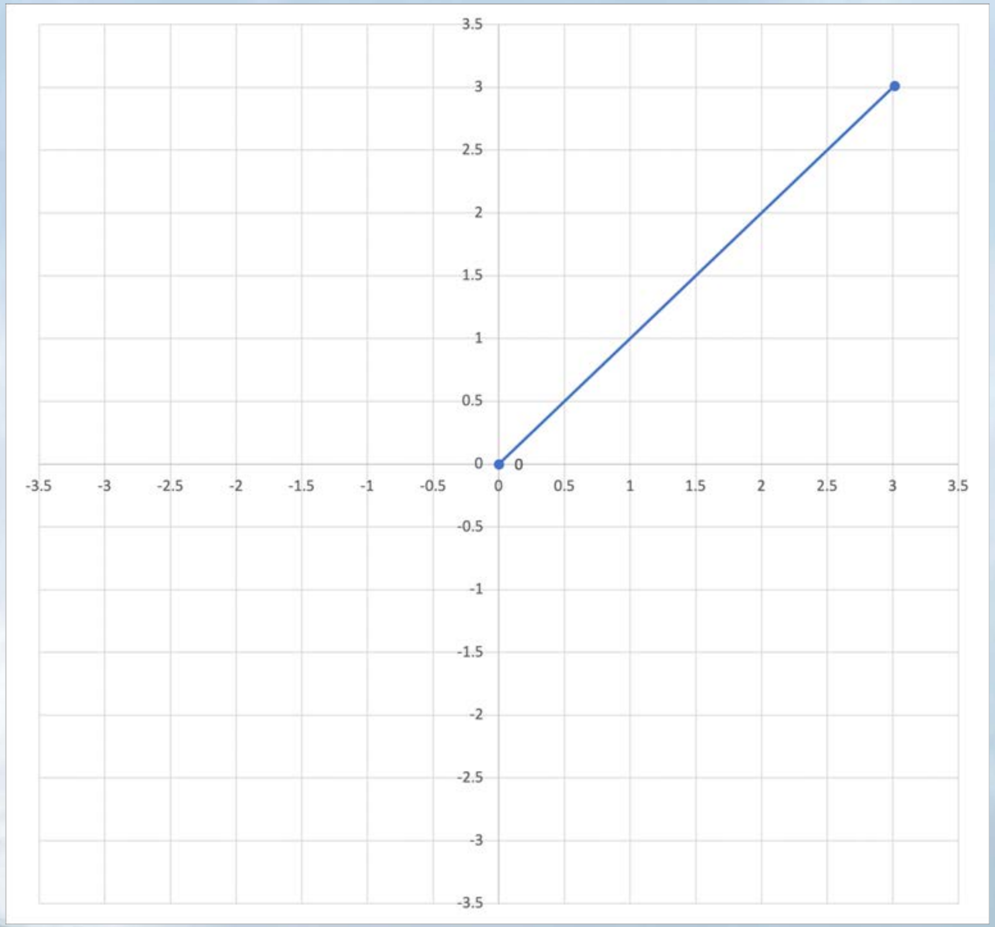
⅘♩ periodicity
(cosine):

⅘♩ periodicity
(–sine):



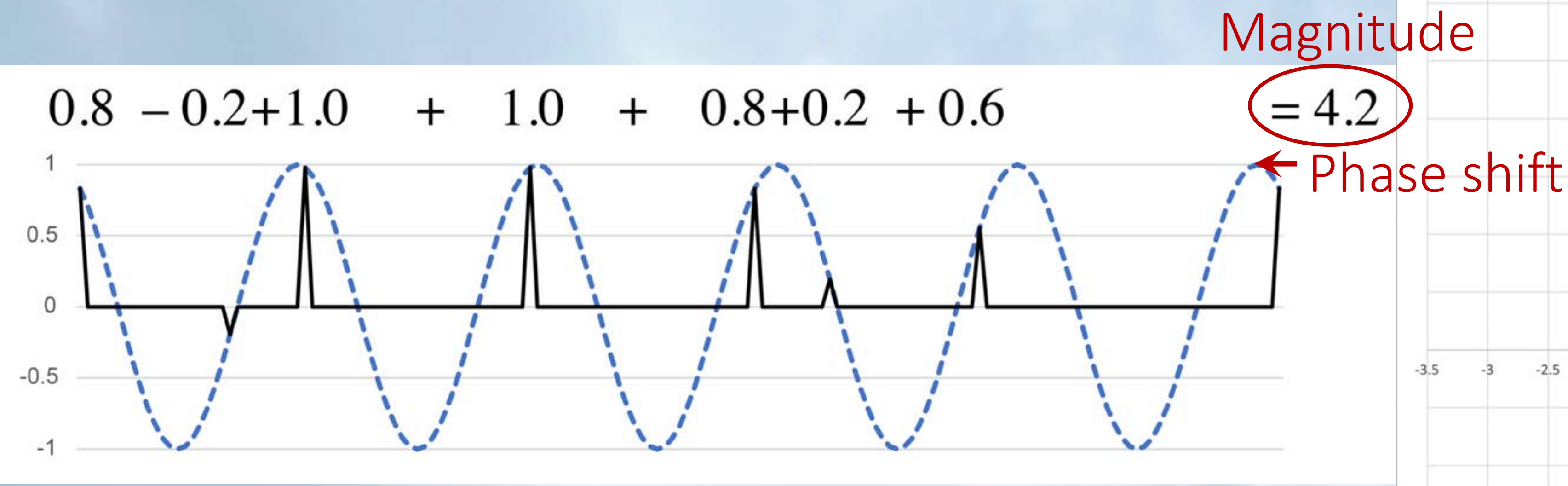
Frequency 5

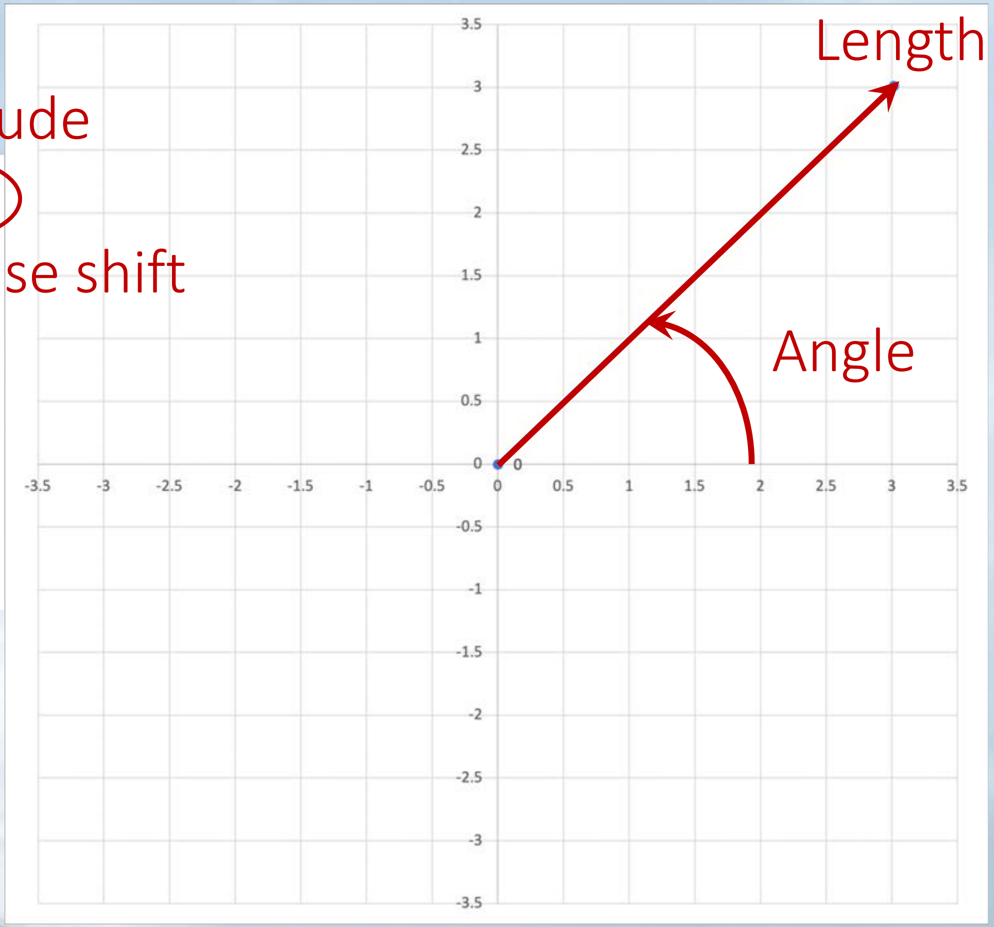
On xy plane:



Phase and magnitude

Angle in 2-d space corresponds to the **phase** of the best-matching wave.





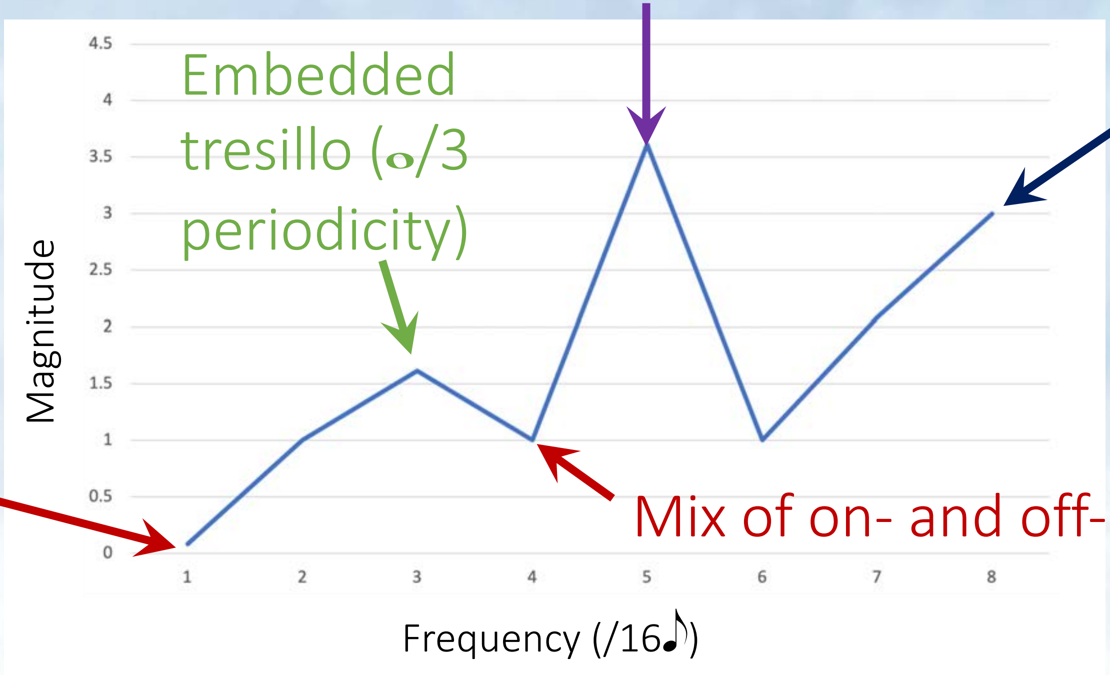
Rhythmic spectrum

The rhythmic spectrum shows the **magnitudes** at all possible frequencies

Rhythmic spectrum for the clave rhythm:

♪. ♪. ♩ ♪ ♩

The strongest frequency (periodicity 𝅝/5) shows that clave is close to 5 equally spaced onsets



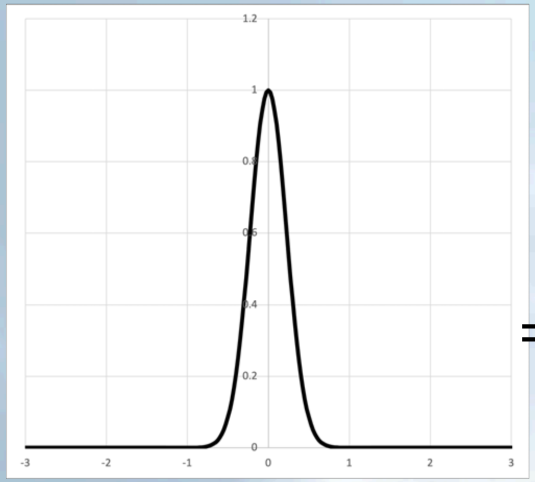
Mostly on-beat at ♪ level

Low 𝅝 periodicity: Relatively even distribution of onsets

N.B.: **Periodicities** are the reciprocals of **frequencies**

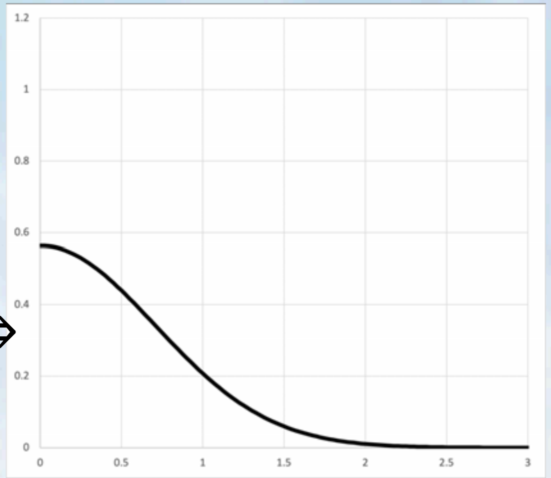
Beat bins

The spectrum of a Gaussian function is another Gaussian function with **inverse width**



⟵ Time ⟶

⇒

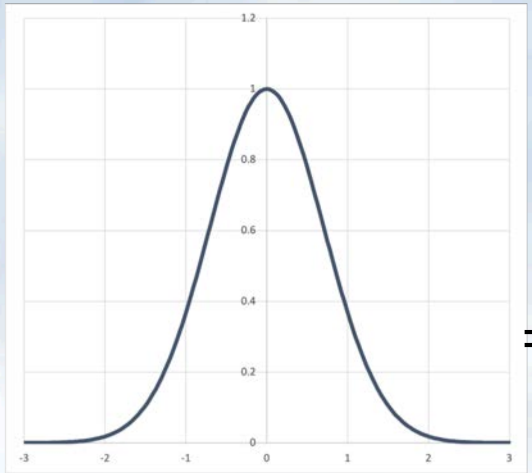


Frequency ⟶

This is **Heisenberg uncertainty** for rhythm:

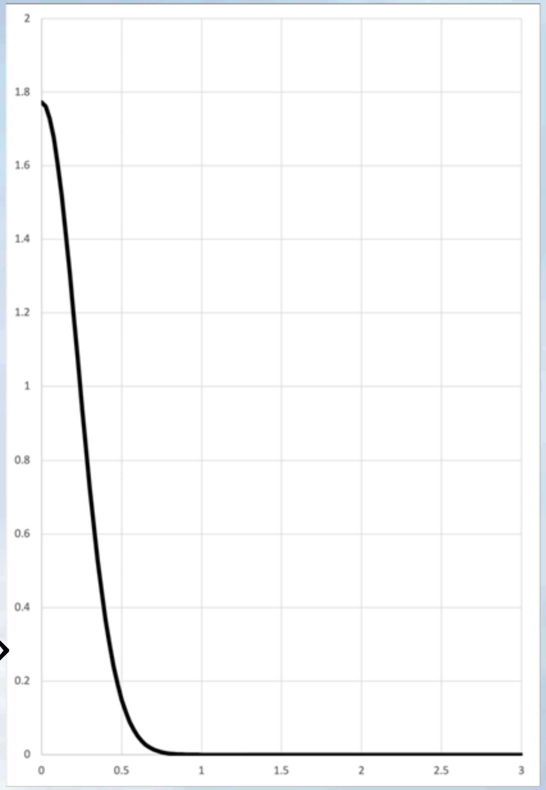
More precision in time ⇔ Less precision in frequency

More precision in frequency ⇔ Less precision in time



⟵ Time ⟶

⇒



Frequency ⟶

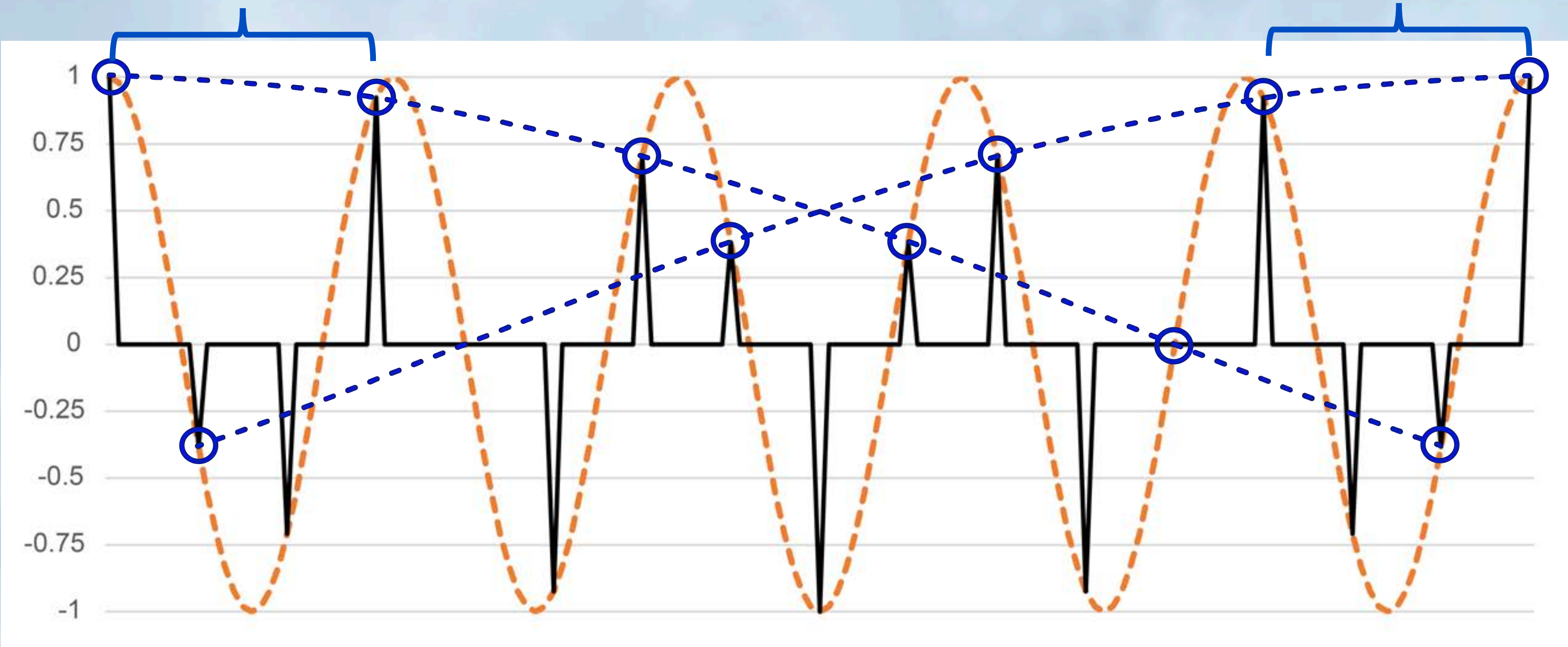
Maximal Evenness

Discretizing a periodic function

What happens when we sample a sine function along a discrete grid with a different period?

3♪ = closest ♪-grid approximation to freq.-5 peak

Intervals are ordered by multiples of 3♪



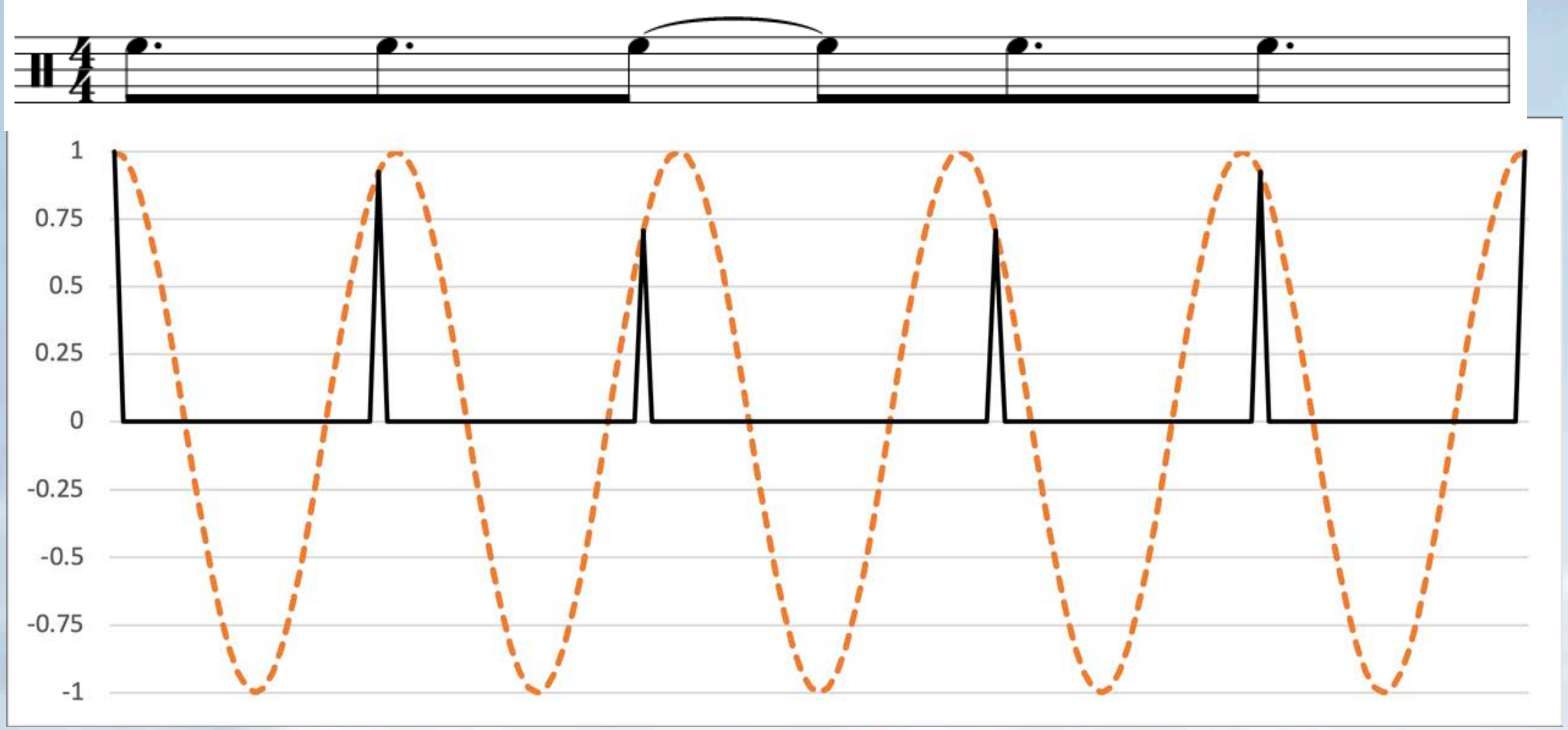
Ascending or descending

Freq.-5 periodic function sampled on 16^{th}-note grid

Generated and maximally even rhythms

Generated rhythms maximize the corresponding frequency.

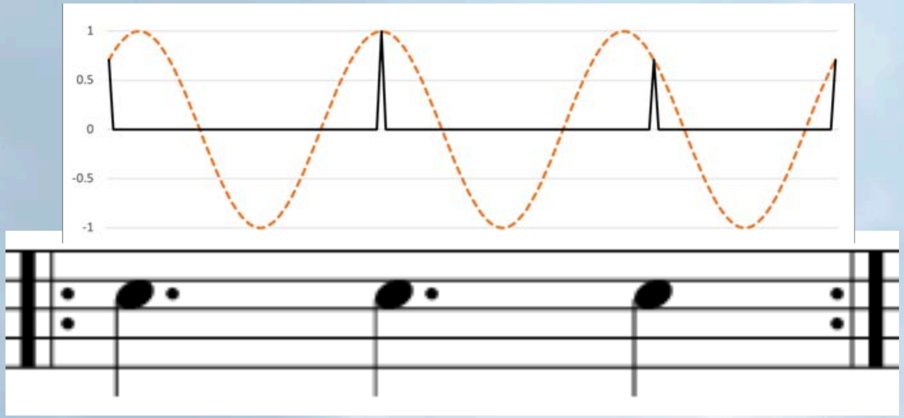
They are ***prototypes*** of the corresponding ***rhythmic quality***.



Maximally Even rhythms are a special case of generated rhythms with exactly one onset for each peak of the periodic function.

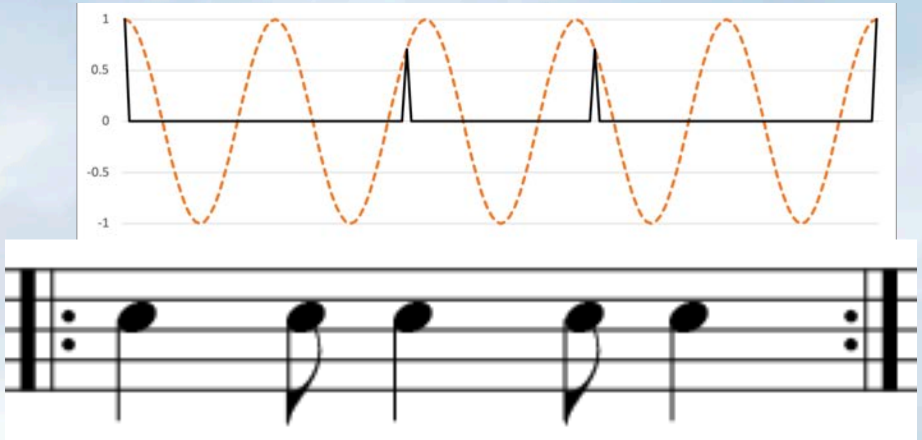
Generated and maximally even rhythms

Example: *Tresillo* (3-in-8) and *Cinquillo* (5-in-8)



Complement

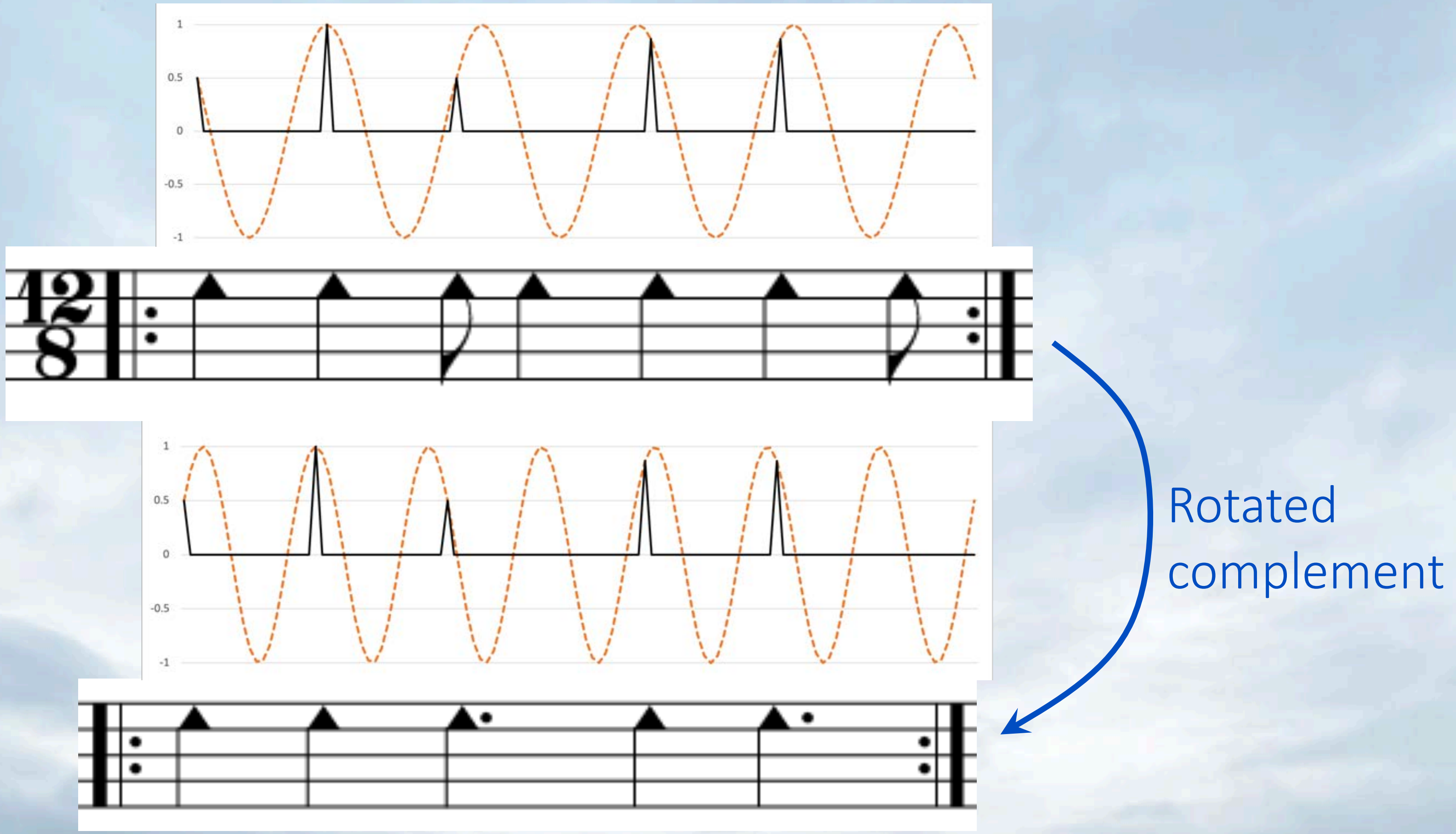
Rotate



Complementation and rotation affect phases only

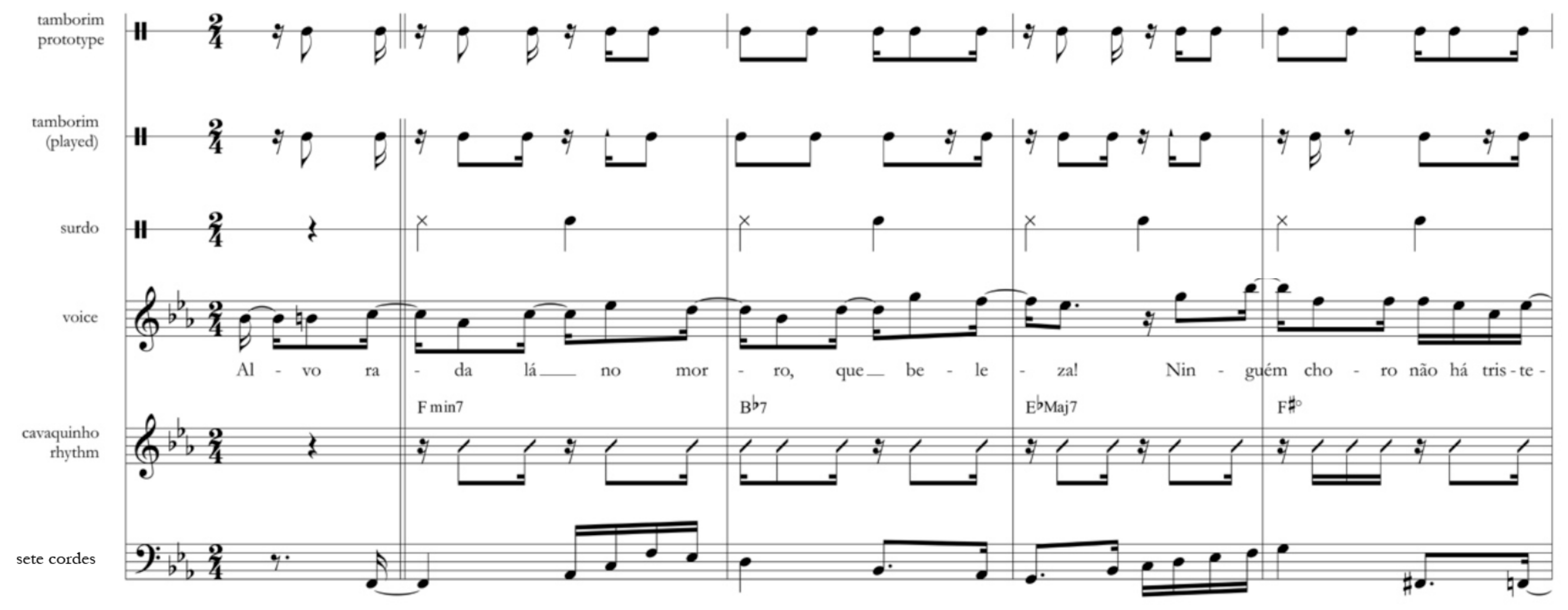
Generated and maximally even rhythms

Example: *Standard Pattern* (7-in-12 and 5-in-12)



Generated and maximally even rhythms

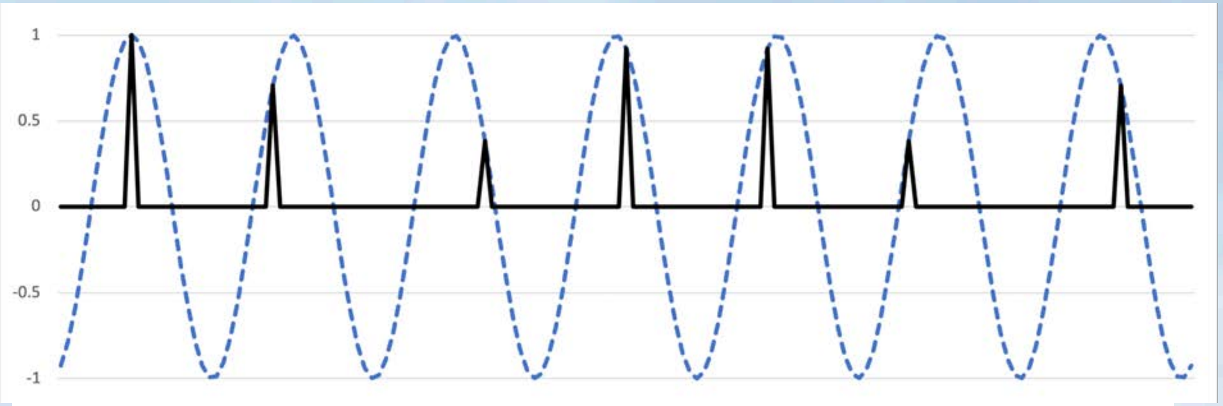
Example: *Samba Timeline* **(9-in-16)** (from Stover forthcoming)



"Alvorada" by Cartola

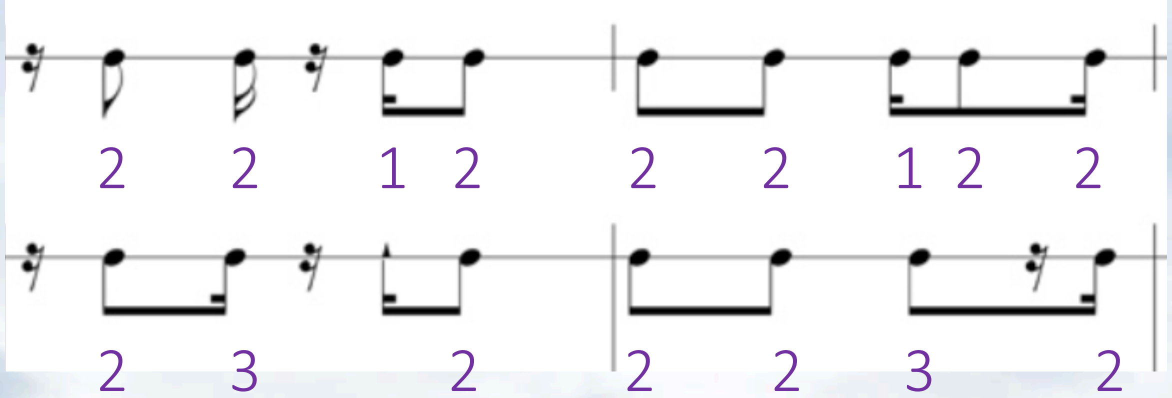
Generated and maximally even rhythms

Example: ***Samba Timeline* (9-in-16)** (from Stover forthcoming)



Stover's prototype:

Performed rhythm:



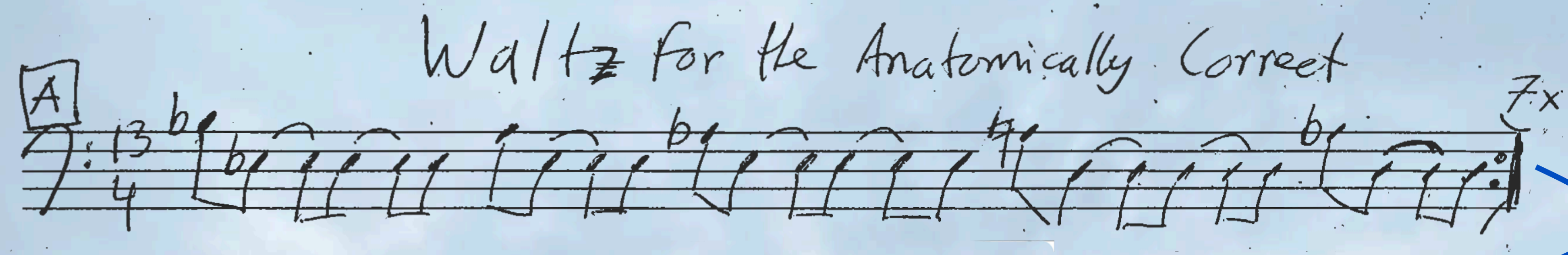
9-in-16 ME

Complements

7-in-16 ME

Generated and maximally even rhythms

Example: Rudresh Mahanthappa, *Waltz for the Anatomically Correct* (5-in-13)



Mod 26 complement

8-in-13 ME

Mod 13 complement

5-in-13 ME

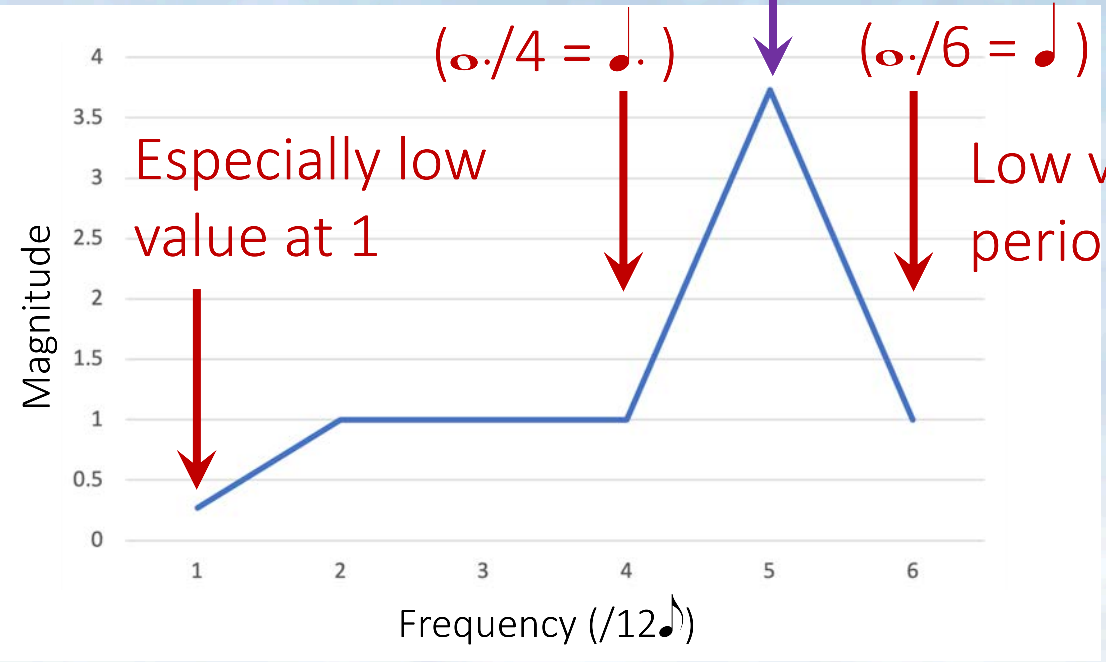
Spectrum of a maximally even rhythm

Maximally even rhythms emphasize a single frequency

High value at principal periodicity (𝅝./5)

Rhythmic spectrum for the standard pattern:
♩ ♩ ♪♩ ♩ ♩ ♪

Or its complement:
♩ ♩ ♩. ♩ ♩.

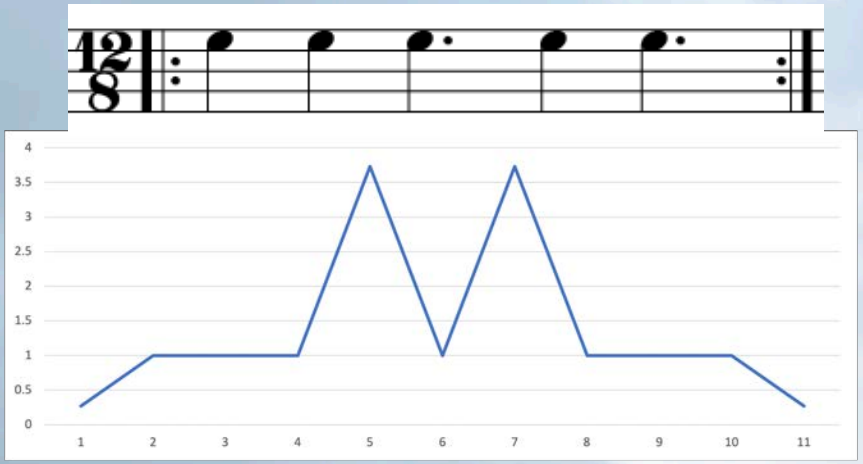


N.B.: **Periodicities** are the reciprocals of **frequencies**

Near-ME rhythms

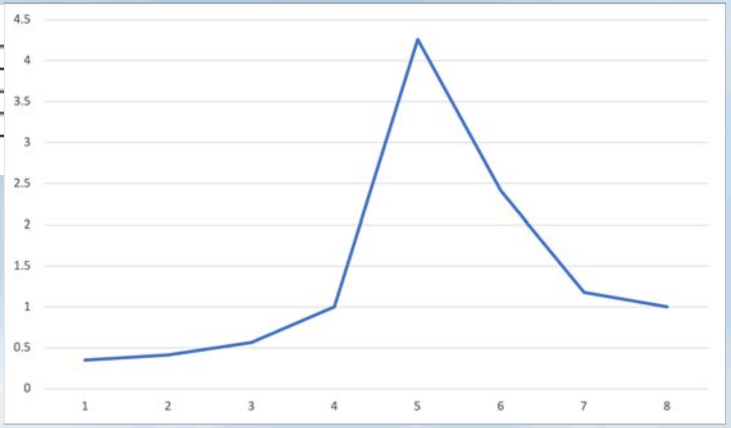
Rhythms close to ME also have large values at that frequency

12-cycle Son clave:



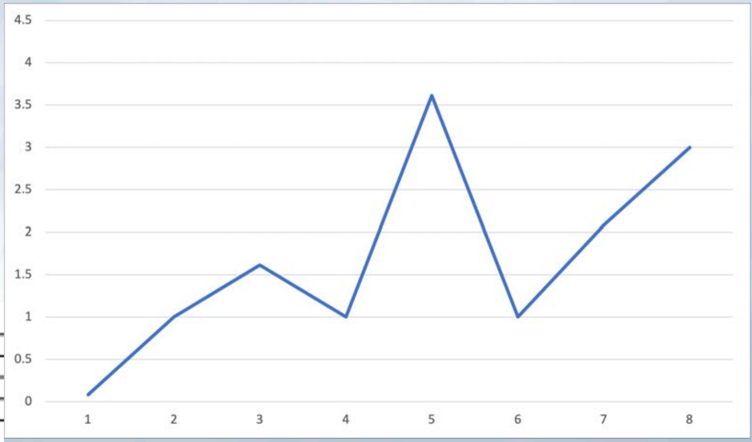
Full (2-sided) spectrum

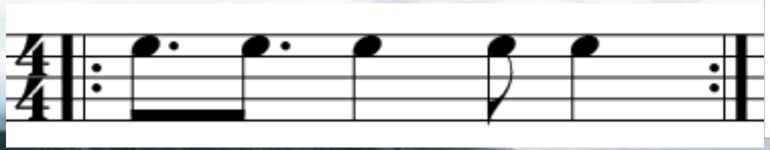
ME 5-in-16
(Bossa Nova)



(half spectra)

Son Clave,
near-ME

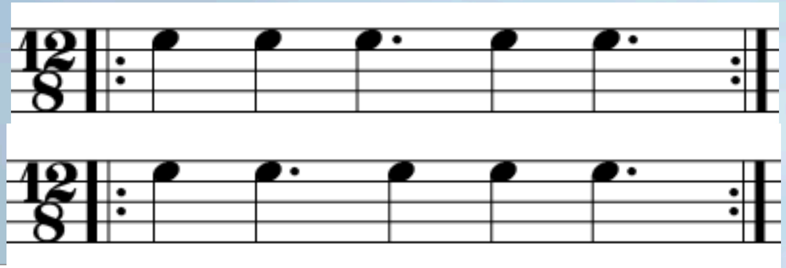


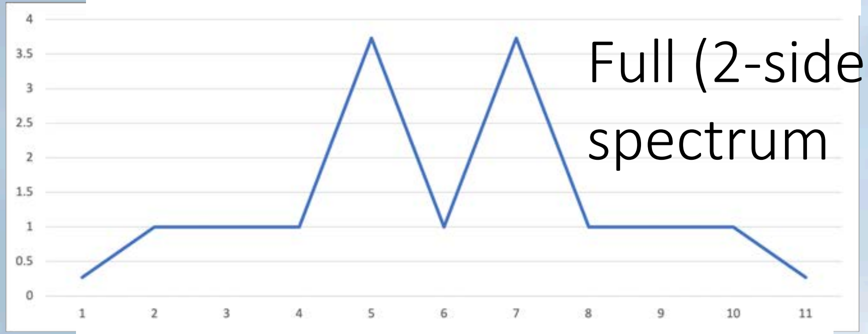


Near-ME rhythms

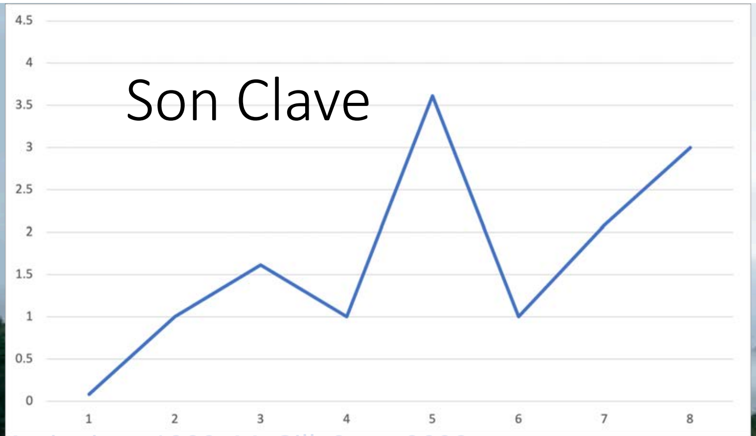
Subsets of ME rhythms also have large values at that frequency

12-cycle Son/ Rumba clave:





Full (2-sided) spectrum

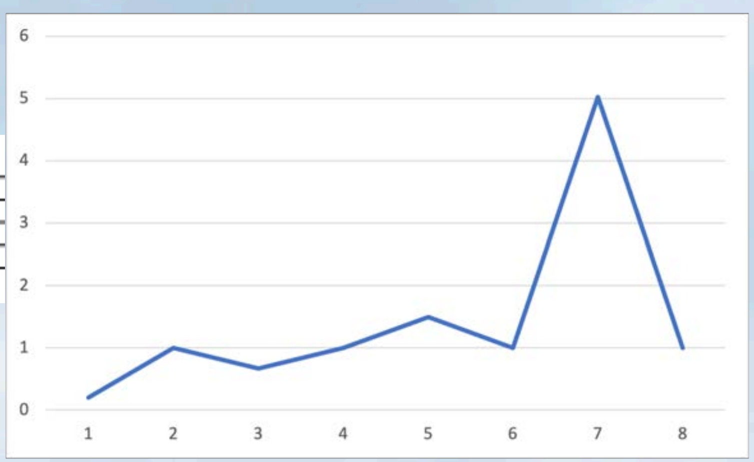


Son Clave

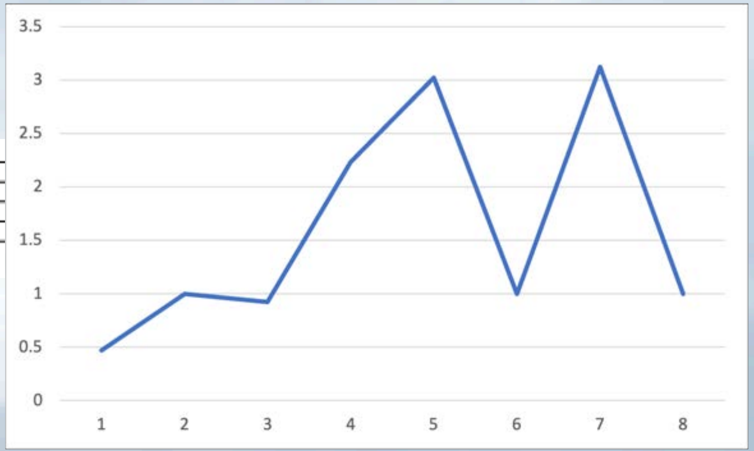
ME 7-in-16

Subset

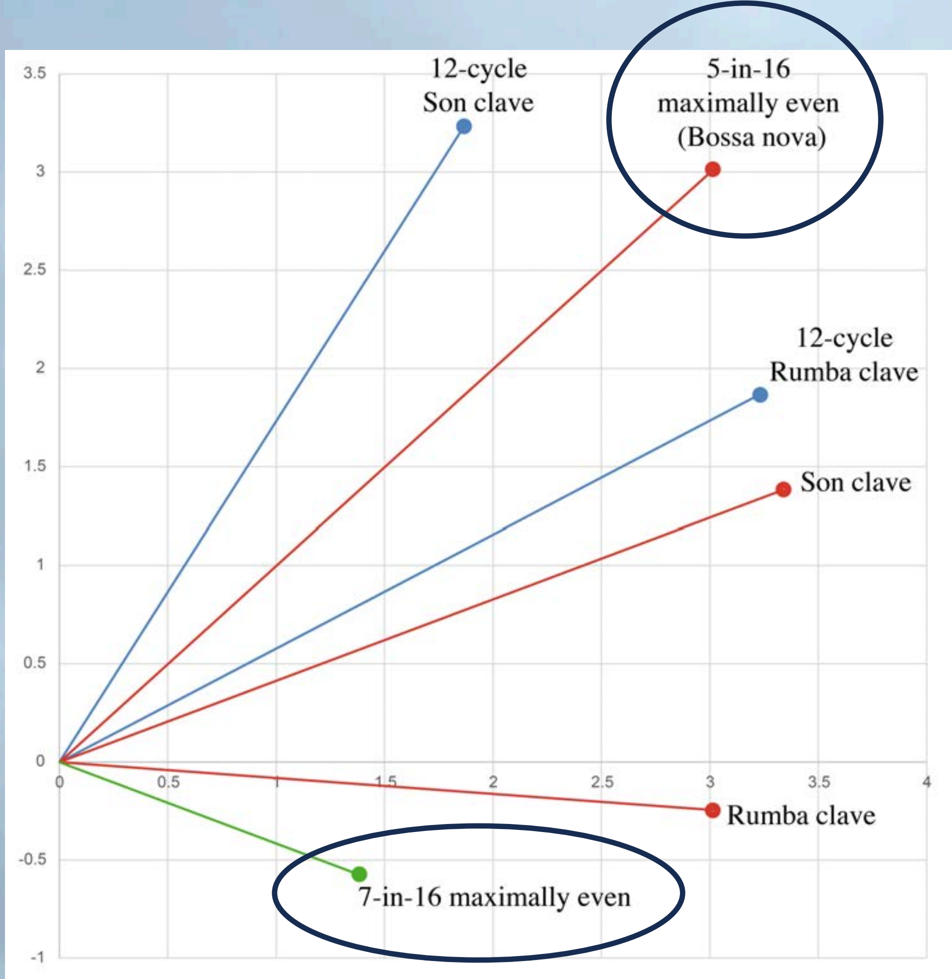
Rumba clave



(half spectra)

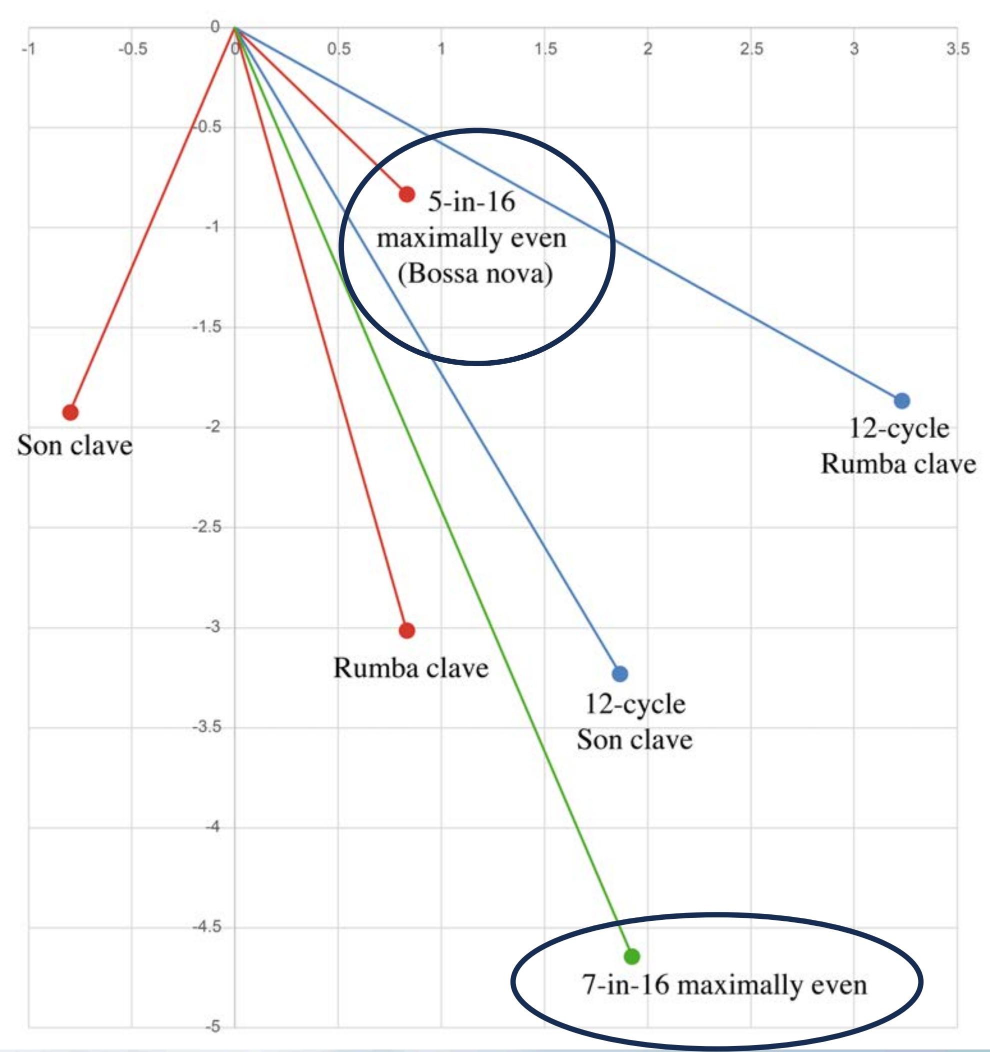


ME and near-ME rhythms on 2-d. planes



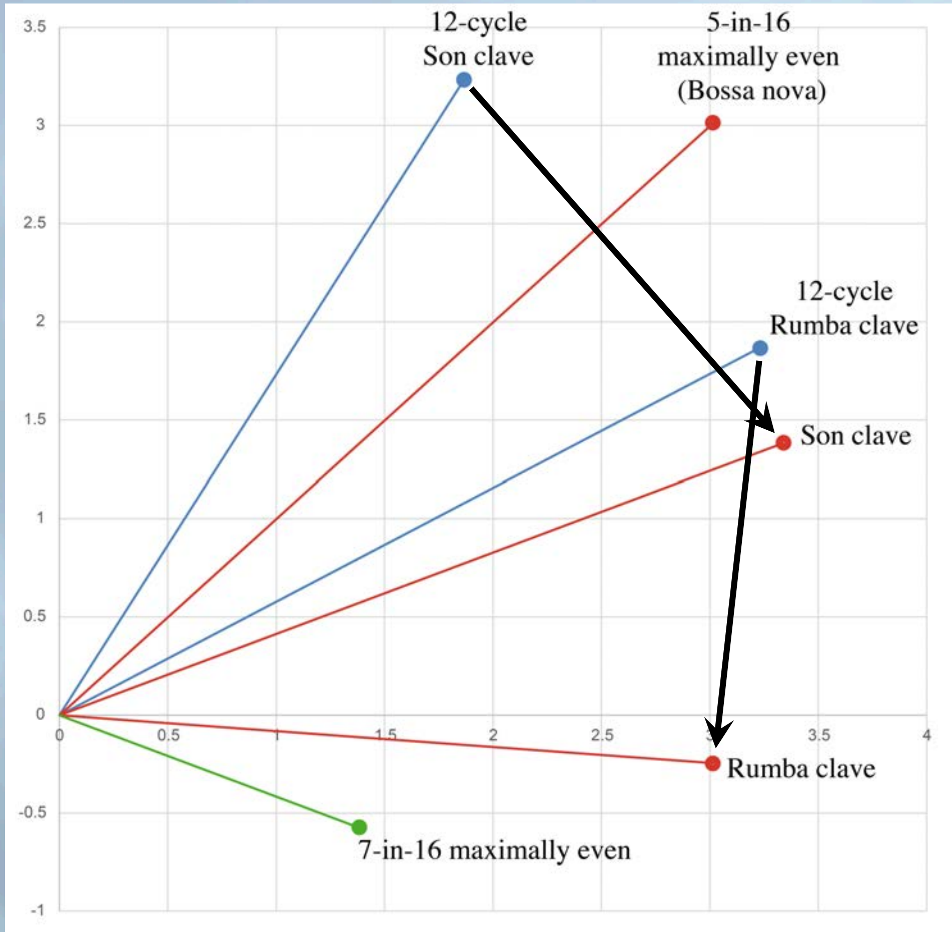
Freq.-5 space

The maximally even rhythms strongly favor one division or the other.



Freq.-7 space

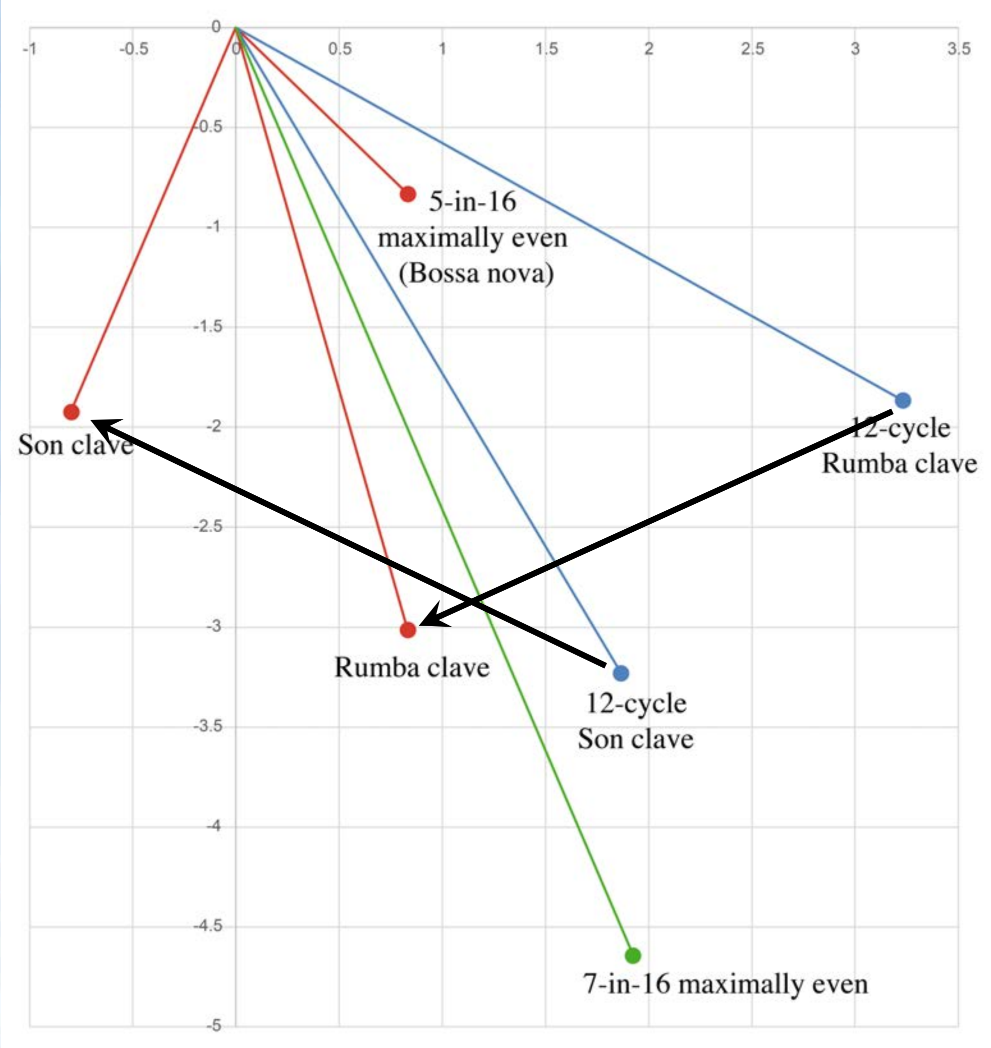
ME and near-ME rhythms on 2-d. planes



Freq.-5 space

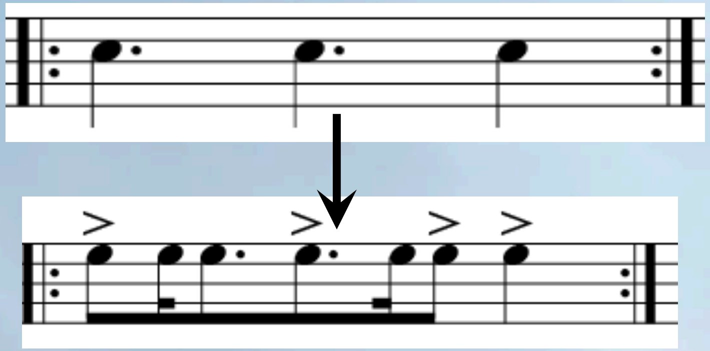
The shift from 12-cycle to 16-cycle moves onsets forward.

Clave rhythms that move between duple and triple feel occupy these intermediate spaces.



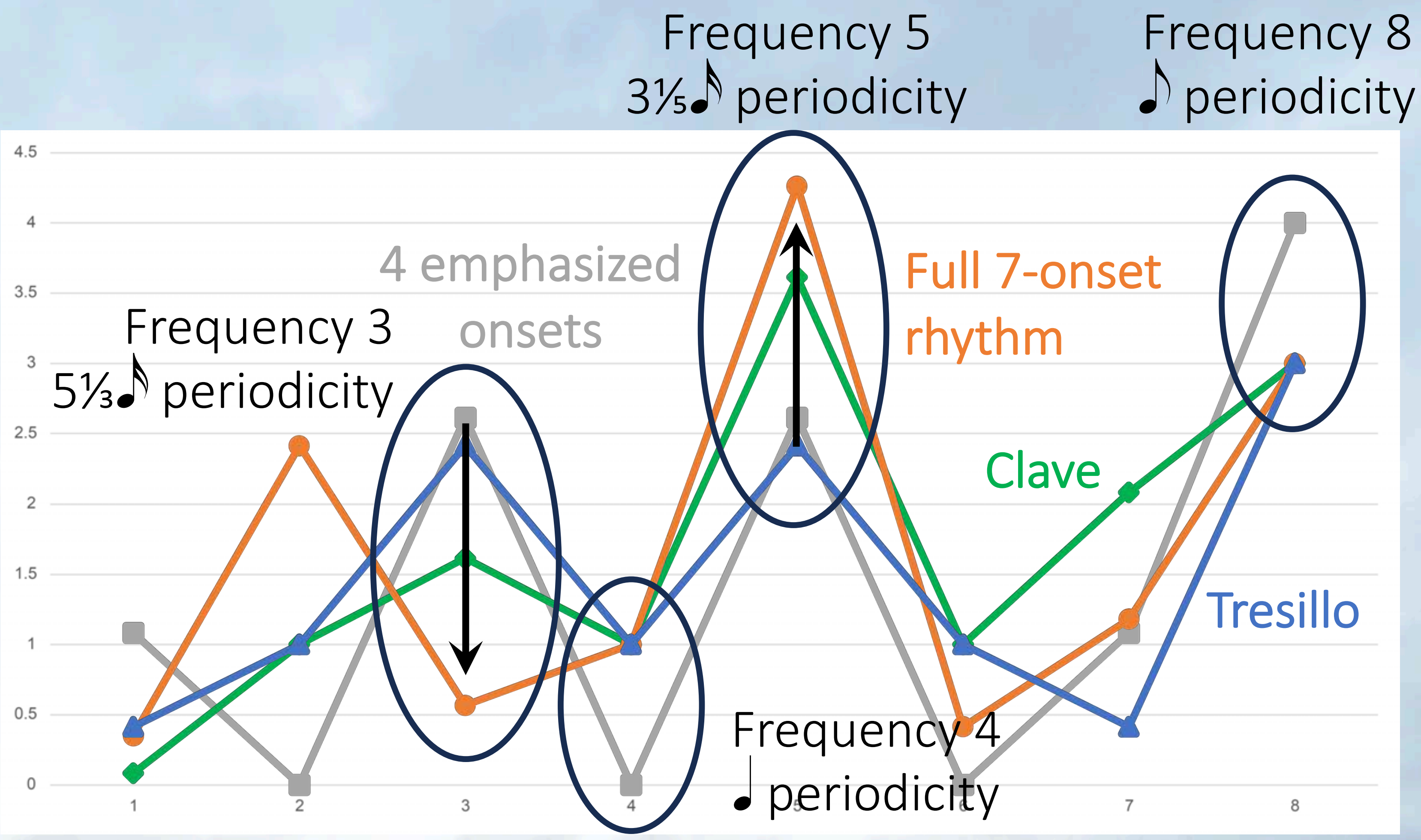
Freq.-7 space

Back to Bo Diddley . . .



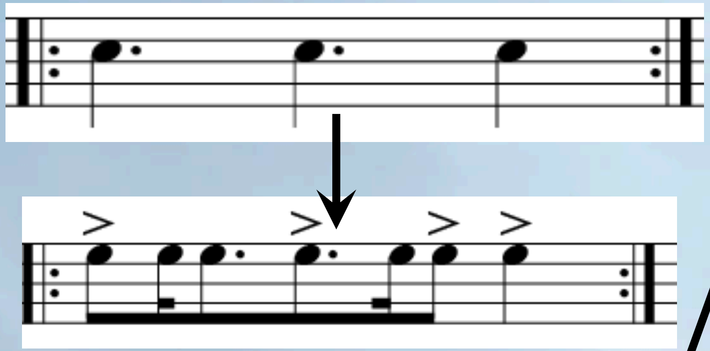
Added onsets:

- Reduce frequency 3
- Enhance frequency 5
- Preserve balance at ♩ level
- Preserve weighting at ♪ grid



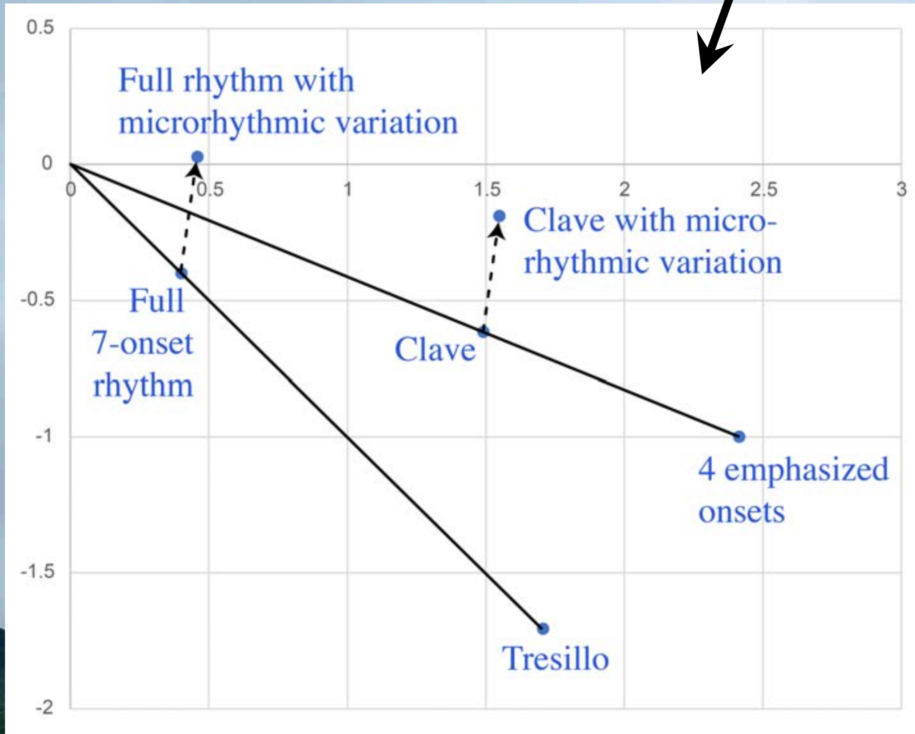
Rhythmic spectra

Back to Bo Diddley . . .

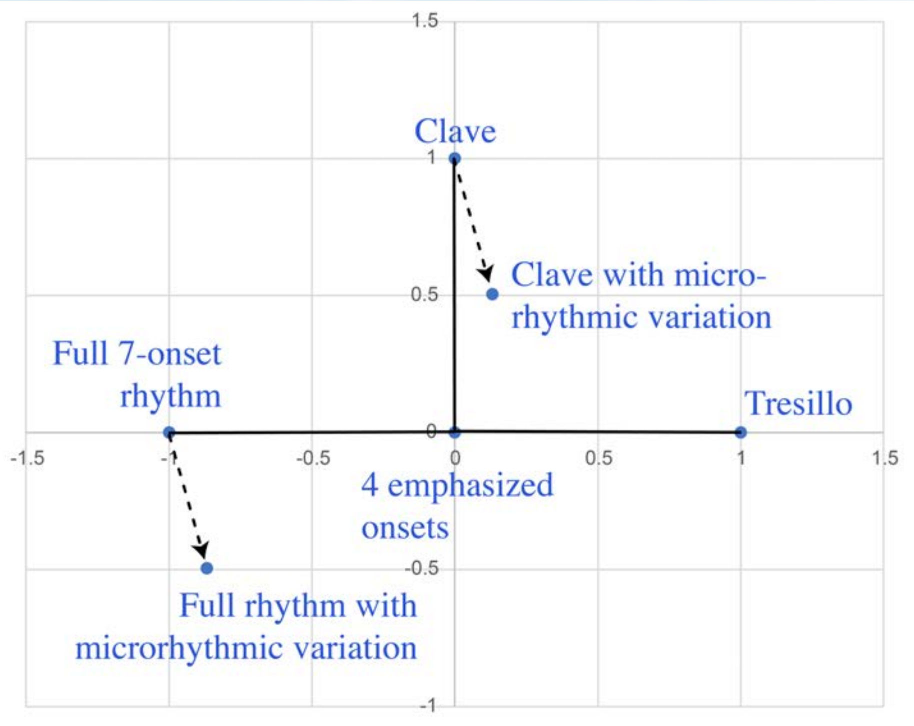


Full 7-onset rhythm agrees with tresillo in phase these spaces (unlike clave). Microrhythmic variation induces additional phase shift.

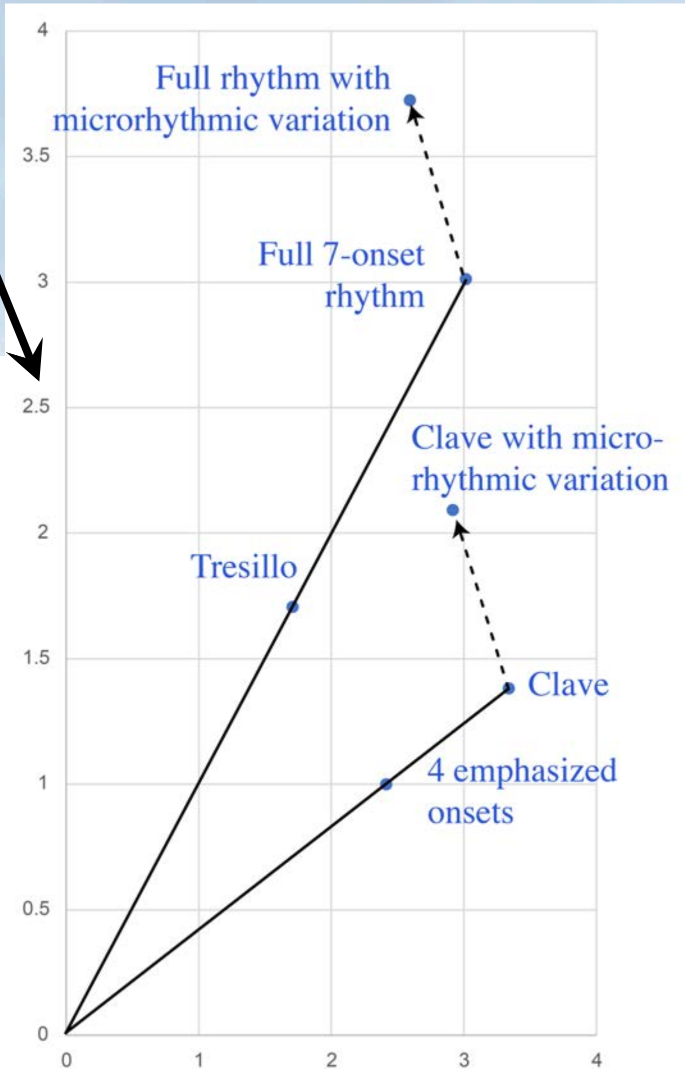
Frequency 3:



Frequency 4:



Frequency 5:



Flow in Jazz, Old and New:

Donald Byrd and Marquis Hill, “Fly Little Bird Fly”

Flow in jazz: Byrd/Hill “Fly Little Bird Fly”

Donald Byrd’s “Fly Little Bird Fly” (1967) has a distinctive melodic rhythm in the head.

Conventional notation:

With triplet swung eighths:

Alternate triplet rhythm:

Approximately as measured from the studio recording:

Flow in jazz: Byrd/Hill “Fly Little Bird Fly”

Marquis Hill’s version (2016) puts the whole tune in 7/4.

Byrd:

or

Hill:

Marquis Hill on Donald Byrd:

“Just talking about the similarities in the music, definitely the groove aspect. Even his bebop, straight-ahead stuff of that era still had that aspect of groove. That essence was from where this music comes. And I try to capture that in my music—even my more hip-hop or funky stuff to my more swinging jazz stuff. It’s all about capturing that feeling and being able to transfer it to people.

Jazz Times Oct. 24, 2017

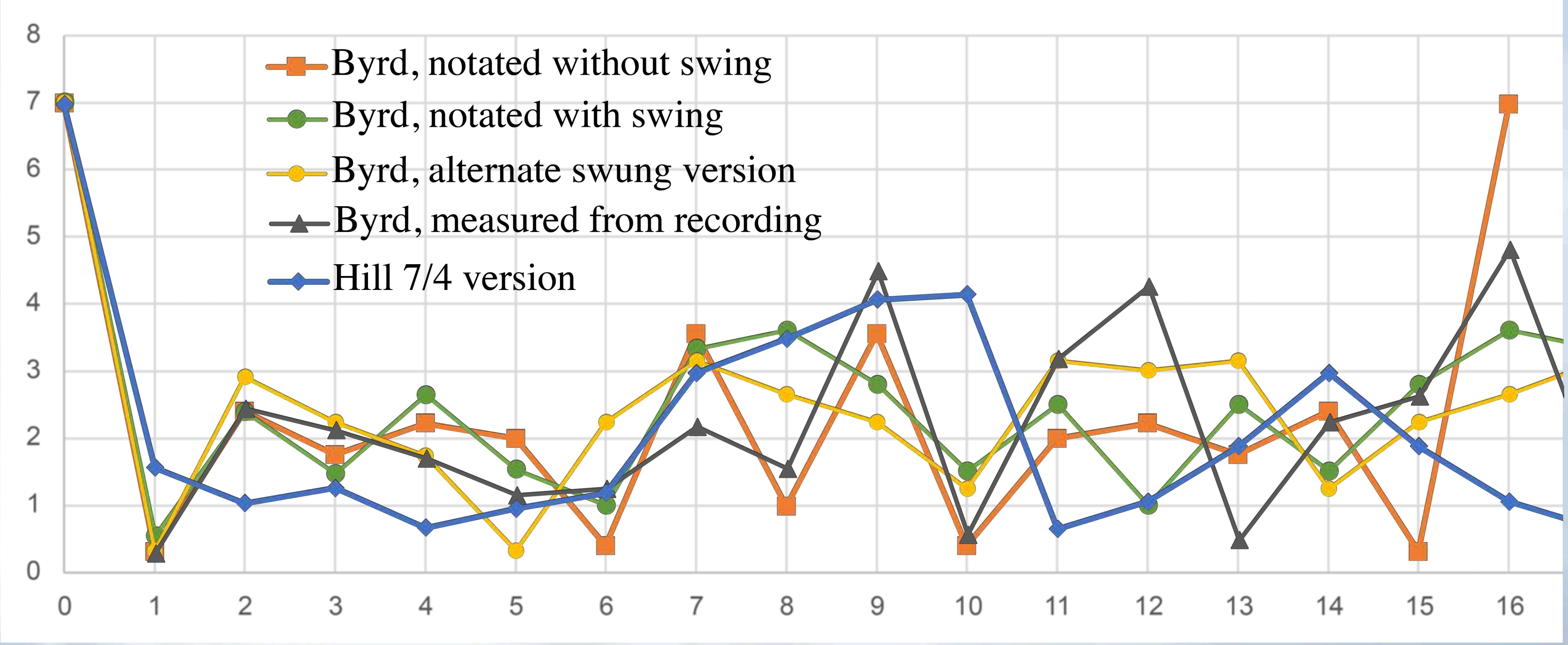
Flow in jazz: Byrd/Hill "Fly Little Bird Fly"

Frequencies 7 and 9 are prominent in all spectra

The version with standard swing is similar, except with a single broad peak across 7–8–9

The alternate swung version weakens frequency 9, favoring 7

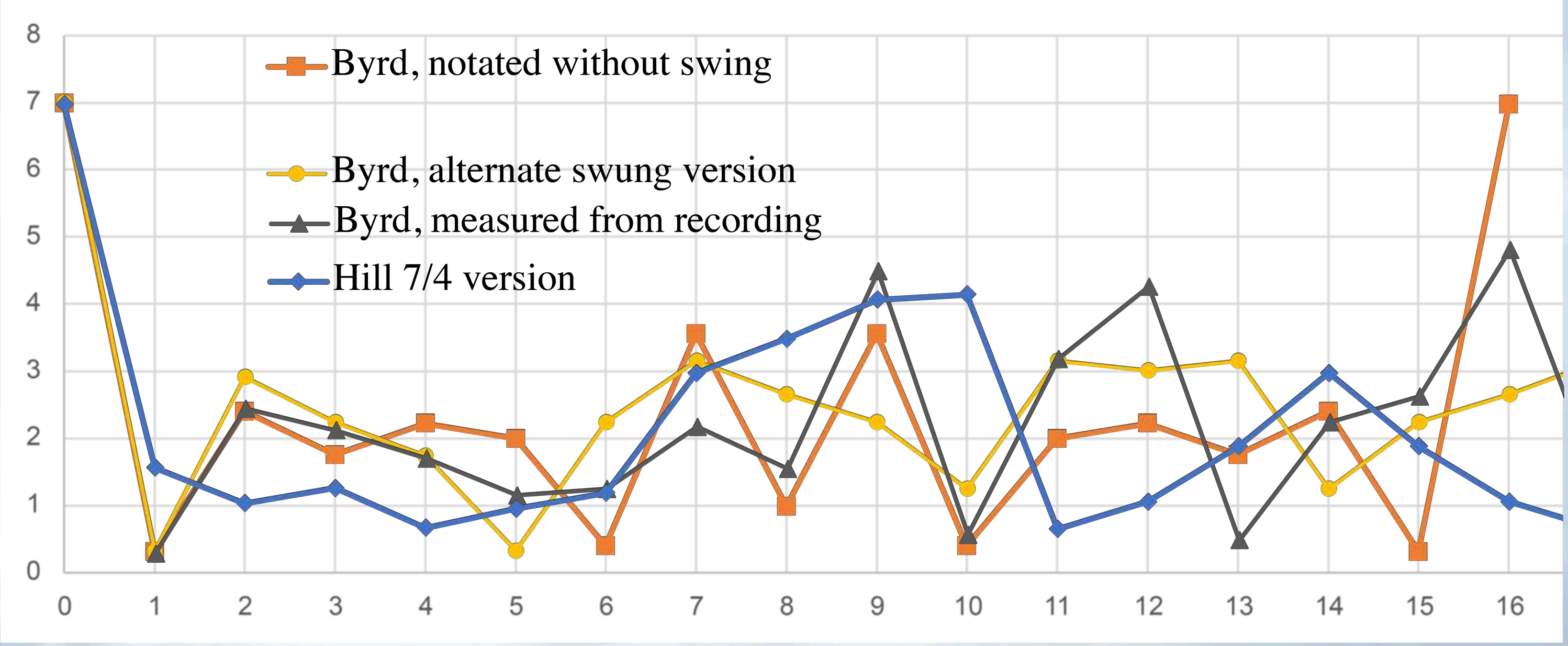
The onsets as measured from the recording favor frequency 9



Flow in jazz: Byrd/Hill "Fly Little Bird Fly"

Frequencies 7 and 9 are prominent in all spectra

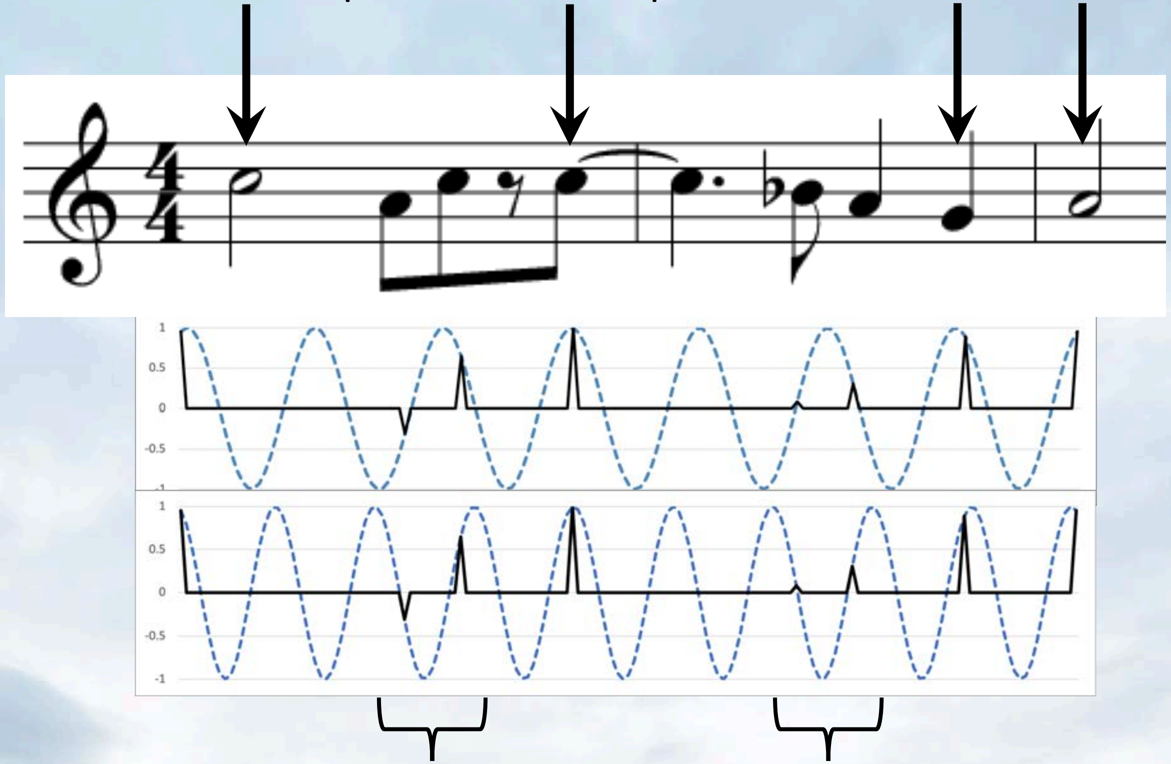
Hill's recomposed rhythm favors frequency 9 like the Byrd's recorded rhythm, but also includes frequency 7 as the underlying beat



Flow in jazz: Byrd/Hill “Fly Little Bird Fly”

Frequencies 7 and 9 as interpretations of the rhythm

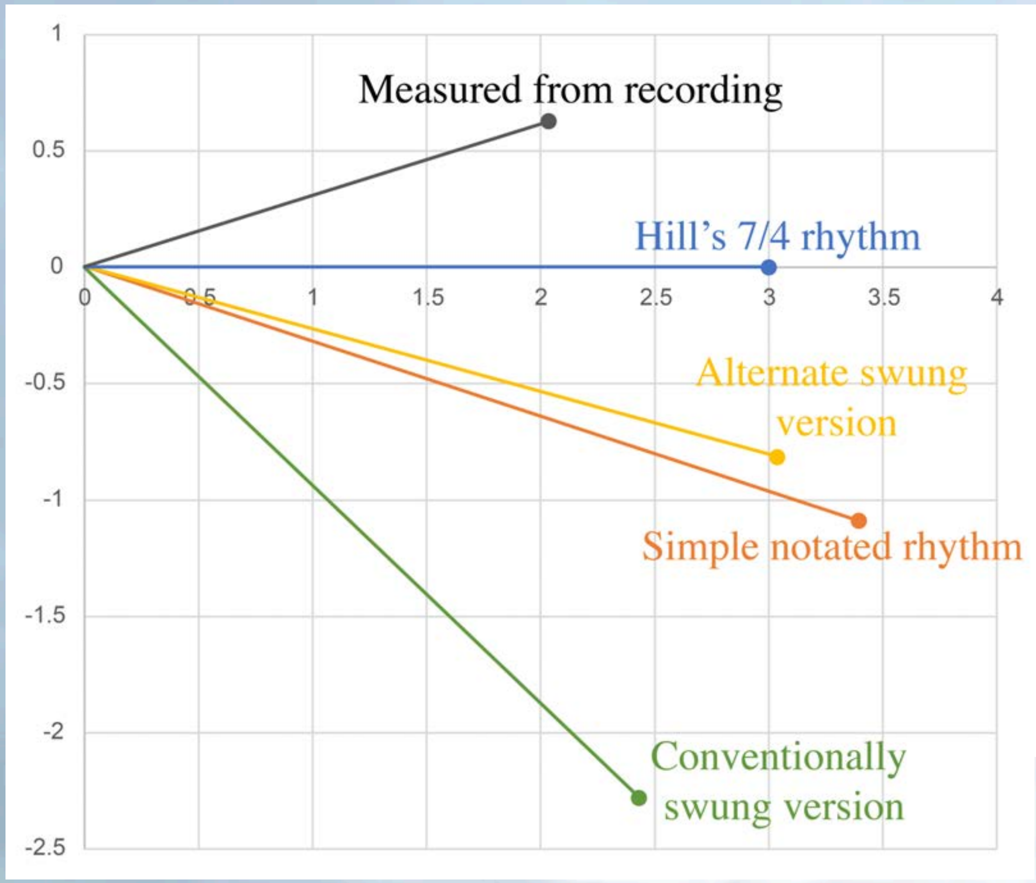
Both frequencies emphasize these notes.



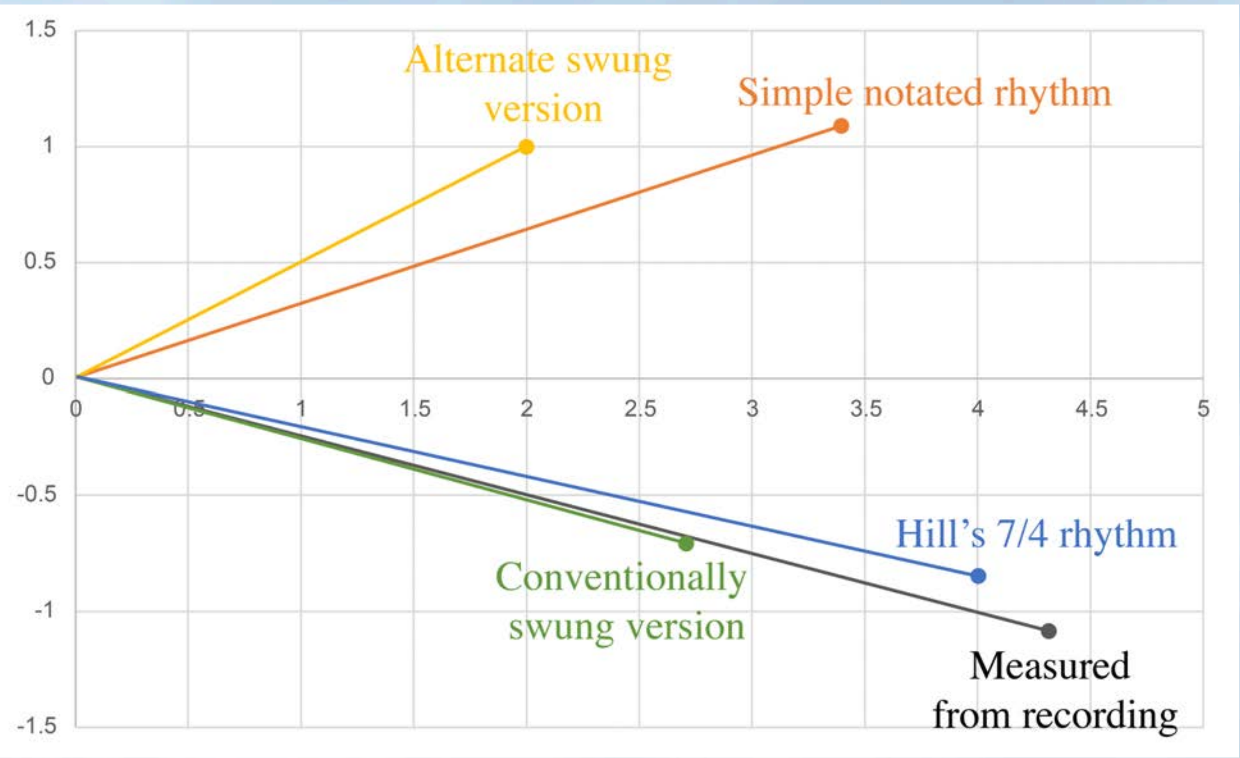
Frequency 7 groups the other two pairs of onsets.
Frequency 9 splits them up.

Flow in jazz: Byrd/Hill “Fly Little Bird Fly”

Frequency-7 space



Frequency-9 space



In both spaces, Hill’s rhythm is closer to the measured one than other notatable versions.

Interaction of Frequencies

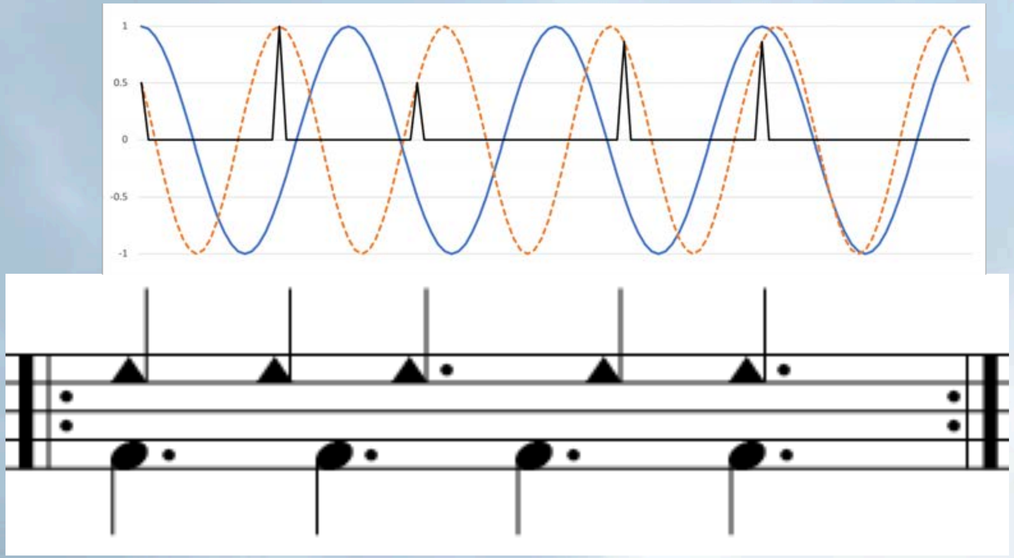
Hidden polyrhythm in Timeline Music, Ligeti, Dave King, Miles Okazaki

Interaction of frequencies

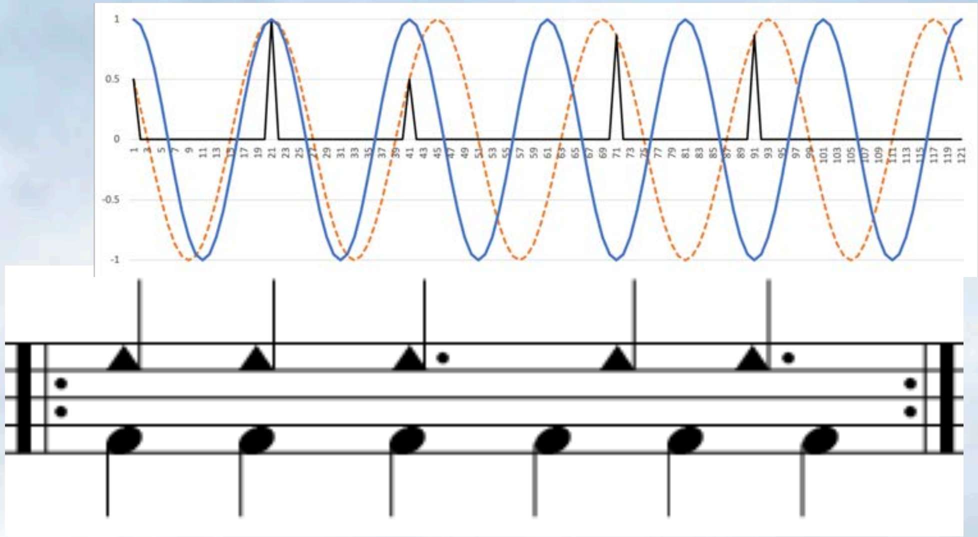
When two frequencies are close together, we get a **slow phase shift.**

Example: Standard pattern against 4 main beat, 6 secondary beats

→ Out of phase → In phase →



→ In phase → Out of phase →



See Ladzekpo 1995, Peñalosa 2009, Stover Forthcoming

Interaction of frequencies

C.K. Ladzekpo (1995) on cross-rhythm in Anlo-Ewe thought:

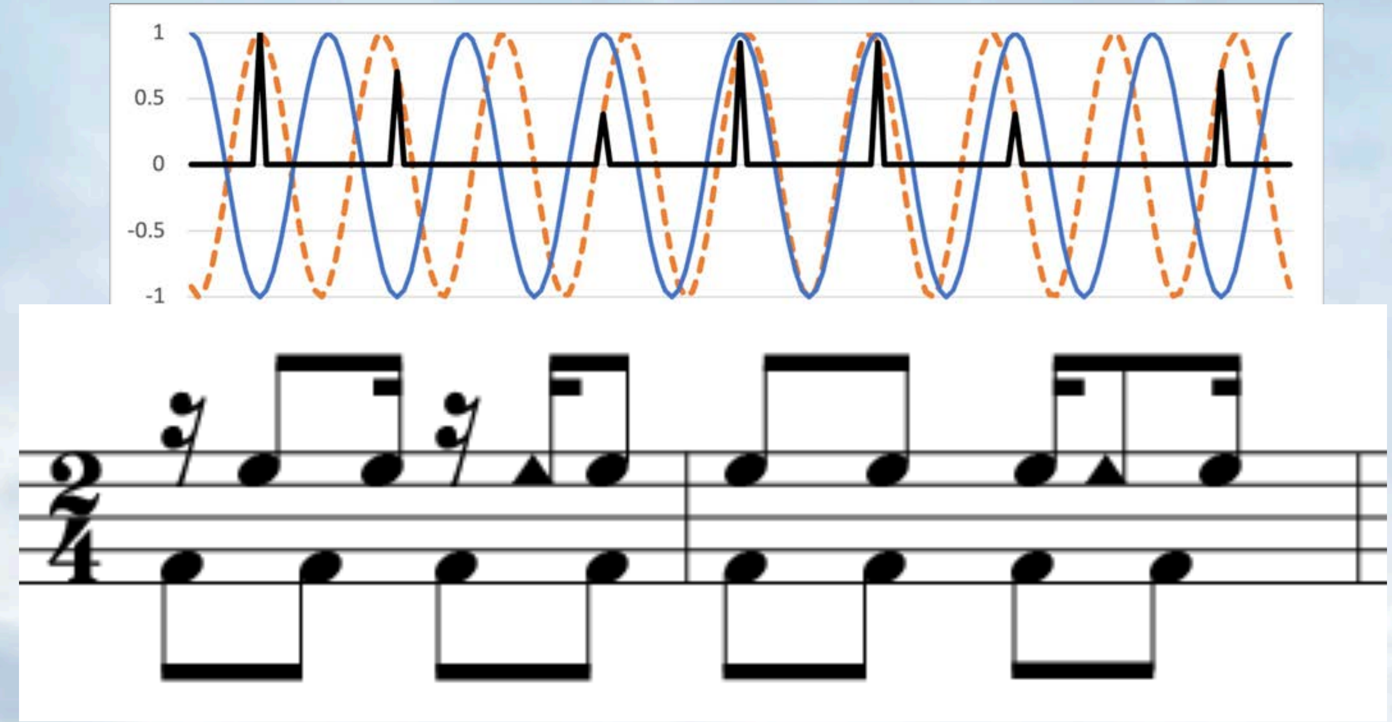
“In aesthetic expression, a moment of resolution or peace occurs when the beat schemes coincide and a moment of conflict occurs when the beat schemes are in alternate motion. These moments are customarily conceived and expressed as physical phenomena familiar to a human being. A moment of resolution is expressed as a human being standing firm or exerting force by reason of weight alone without motion while moment of conflict is expressed as a human being travelling forward alternating the legs.

“In the cultural understanding, the *technique of composite* rhythm embodies the lessons of establishing contact between two dissimilar states of being, or in particular, the right way to look at despair. . . . Those in despair recognize the facts of their existence, rather like a drowning swimmer admitting the water is there. If you block off the despair, you block off the joy. More simply, an avoidance of contrasting obstacles is an avoidance of the real challenges of life. It will only stifle progress.”

Interaction of frequencies

Example: **Samba Timeline (9-in-16)** (from Stover forthcoming)

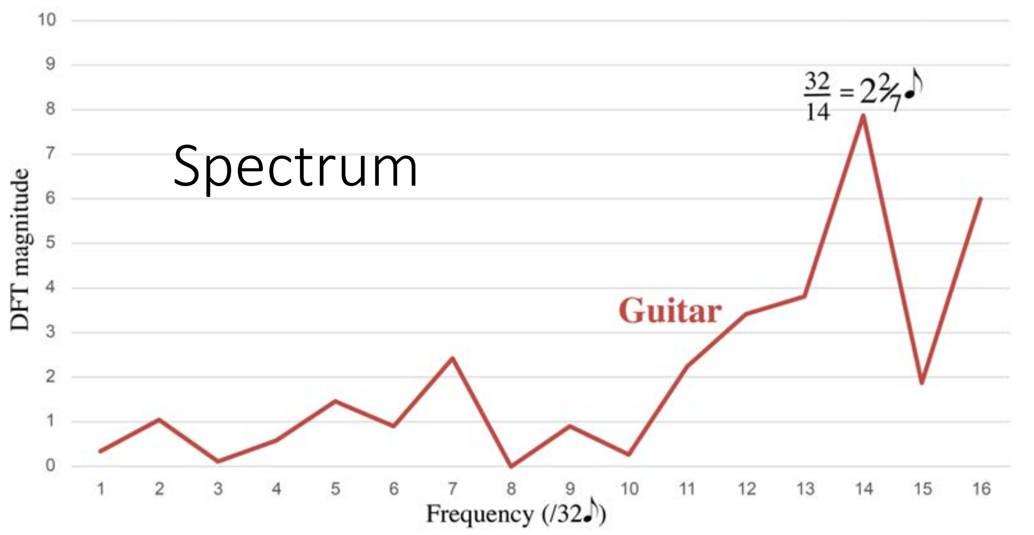
Out of phase ⟶ In phase ⟶



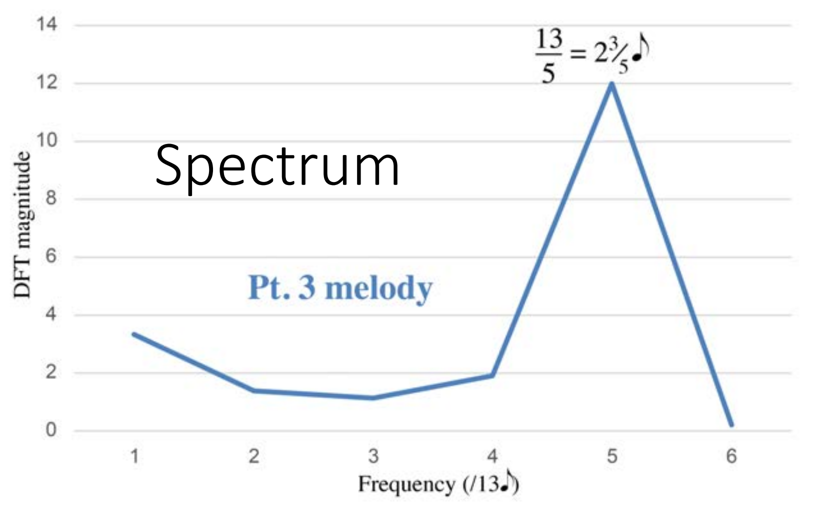
Interaction of frequencies

Ex.: “You Can’t Say Poem in Concrete” by Dave King Trucking Company

Pt. 2 guitar ostinato



Pt. 3 melody



Approximates 14-in-32 maximally even rhythm:

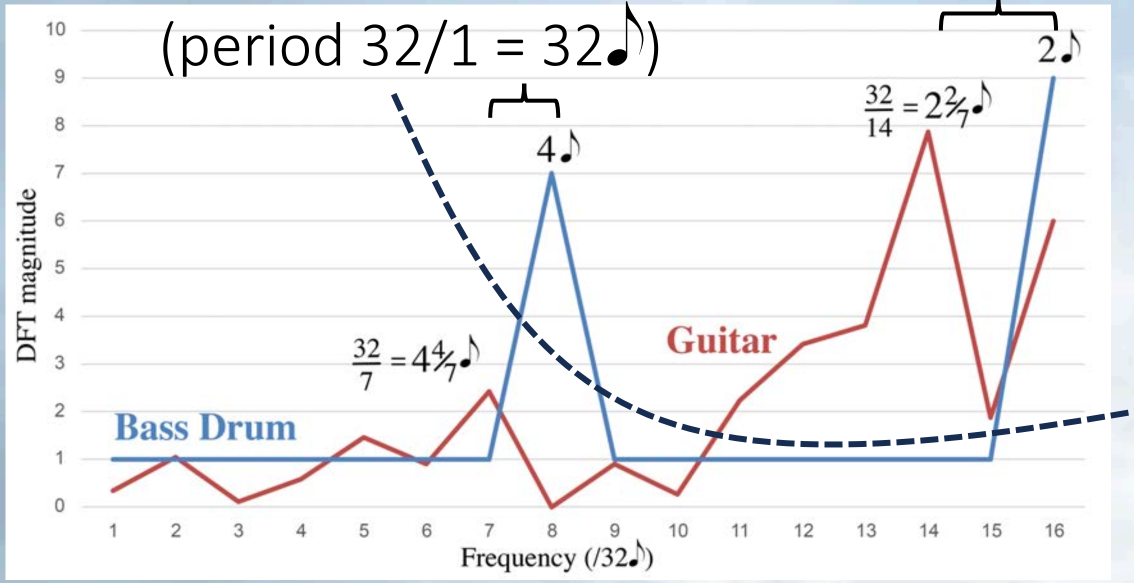
Approximates 5♪-generated rhythm:

Interaction of frequencies

Ex.: "You Can't Say Poem in Concrete" by Dave King Trucking Company

Frequency difference of 1 (period 32/1 = 32♪)

Frequency difference of 2 (period 32/2 = 16♪)



Spectrum

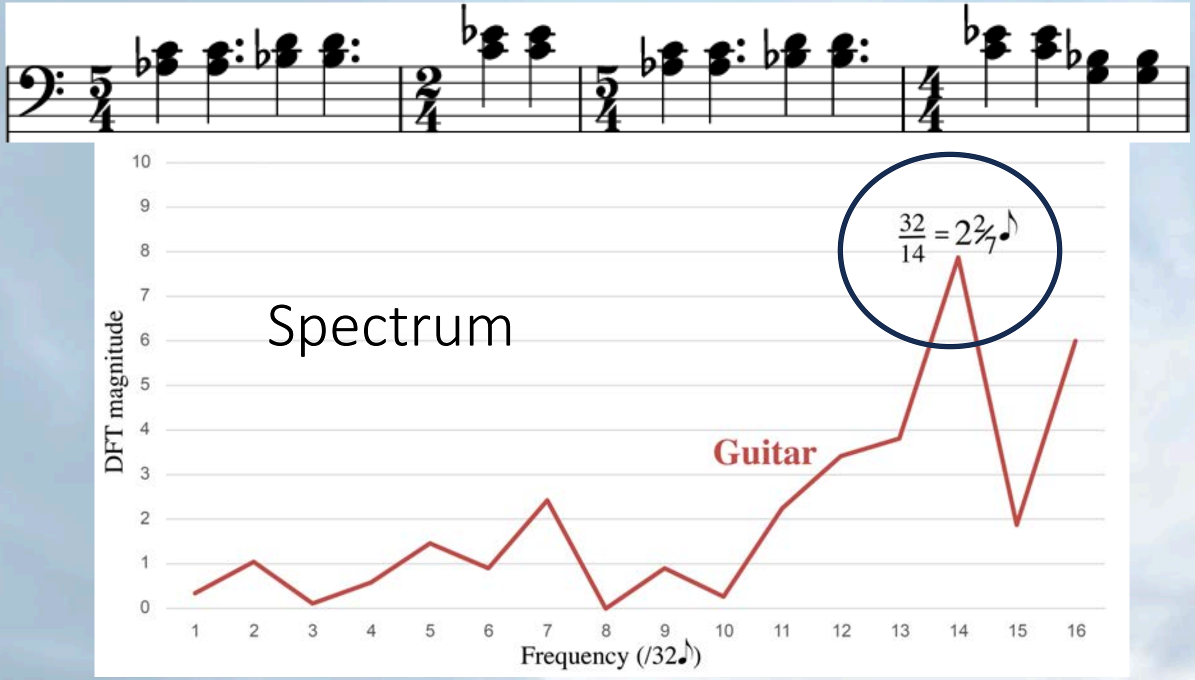
In phase → Out of phase → In phase → Out of phase → In phase

In phase → Out of phase → In phase

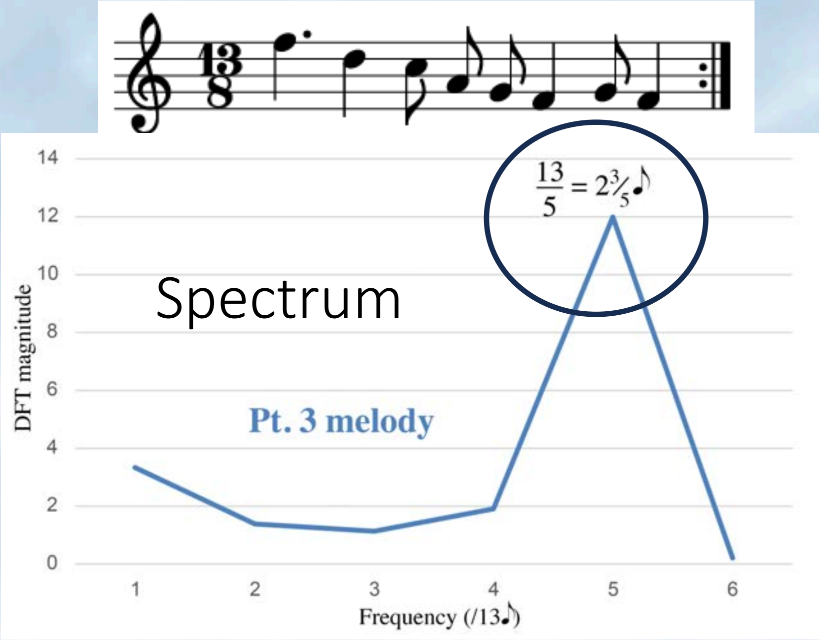
Interaction of frequencies

Ex.: “You Can’t Say Poem in Concrete” by Dave King Trucking Company

Pt. 2 guitar ostinato



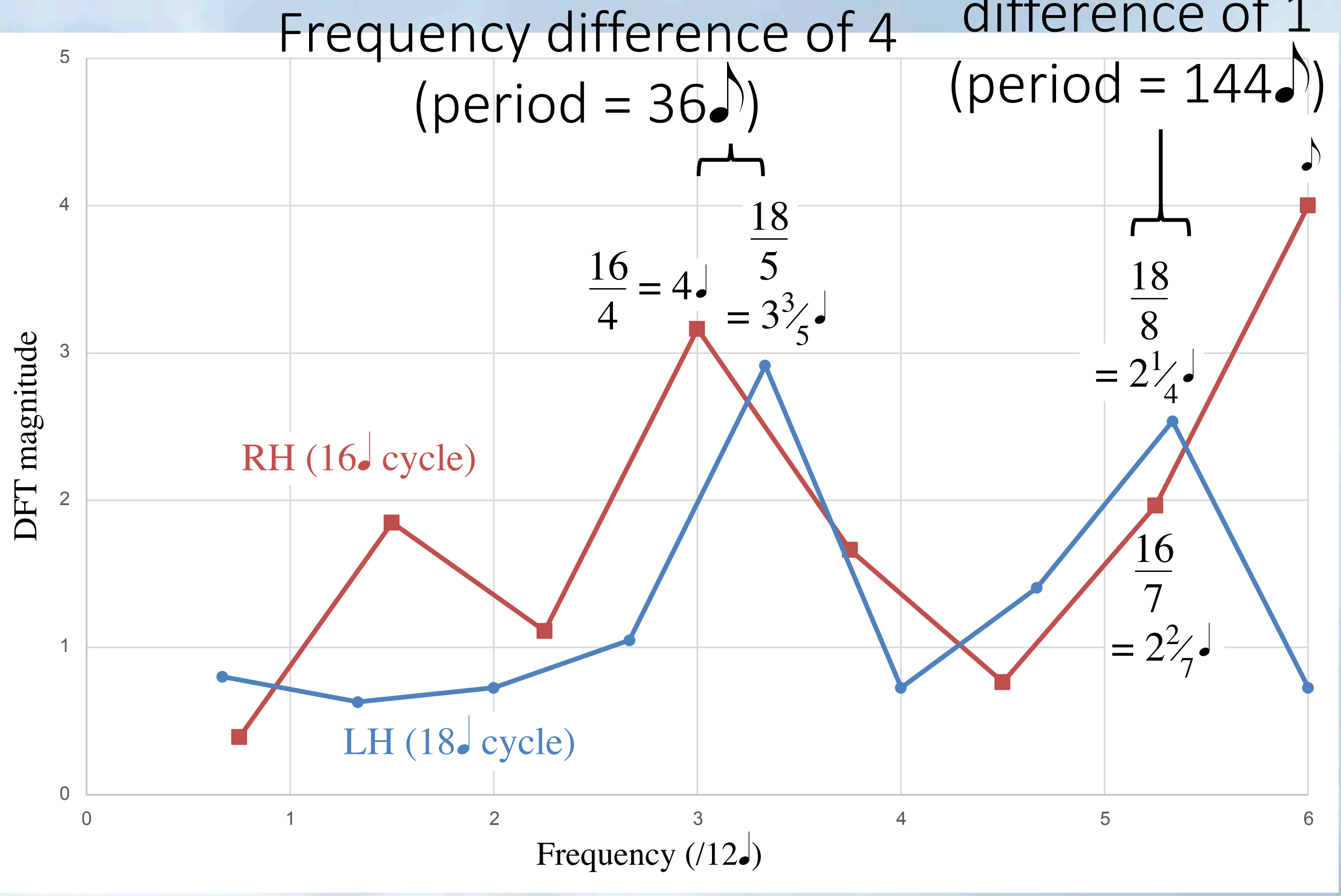
Pt. 3 melody



Despite having very different cycles (32♪ vs. 13♪), the principal frequency is very close.
The band also speeds up slightly from ♪ = 230 to ♪ = 236, making
$2^{2}/_{7}$ ♪ ≅ MM101 and $2^{3}/_{5}$ ♪ ≅ MM91

Interaction of frequencies

Ex.: Ligeti, Etude 8, “Fém”



Spectra

Interaction of frequencies

Ex.: Ligeti, Etude 8, “Fém”

36♪ wave: Interlocking → Together → Interlocking →

144♪ wave: Mostly interlocking - - - →

→ Together → Interlocking

- - - → More together - - - →

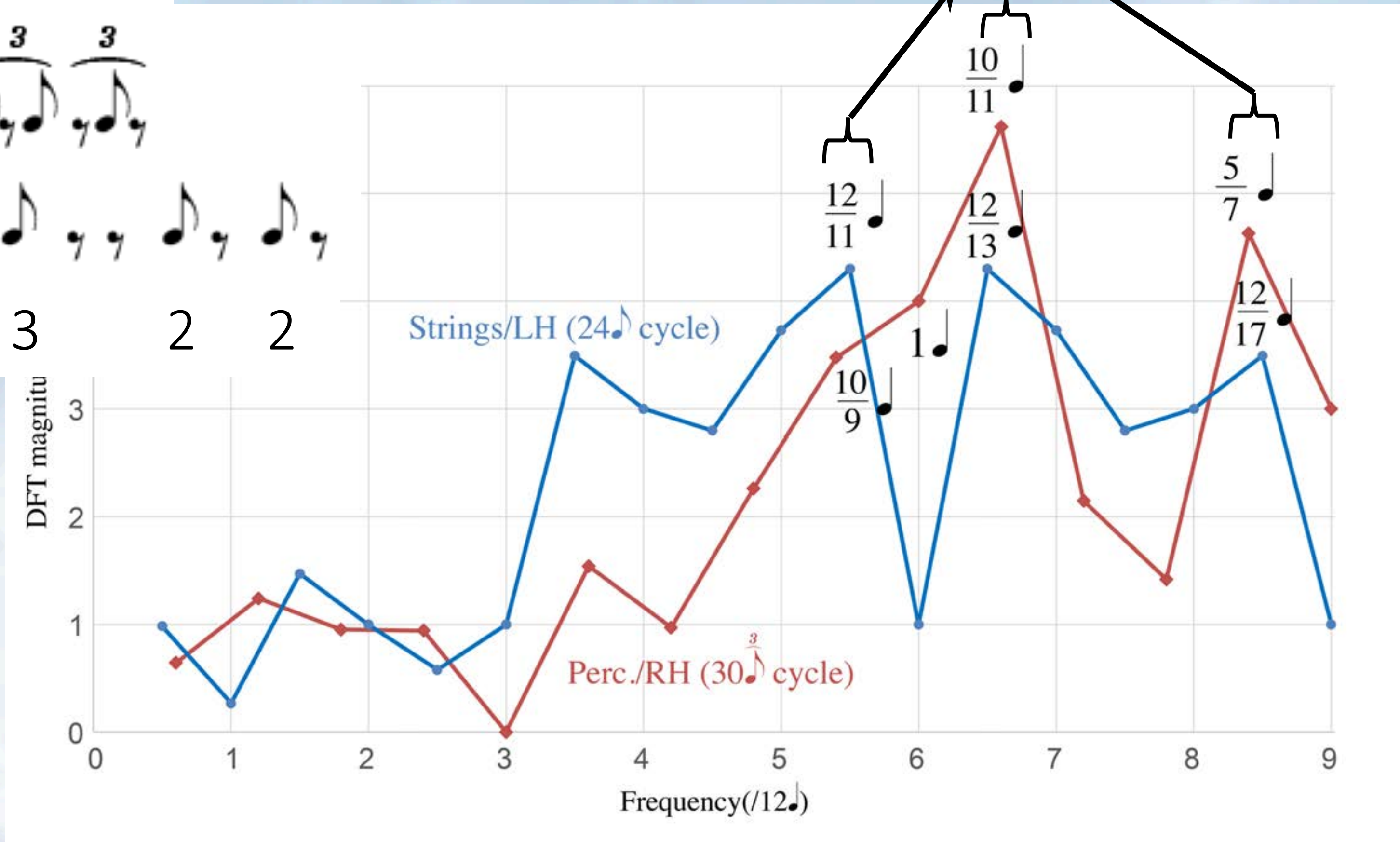
Interaction of frequencies

Ex.: Ligeti, *Piano Concerto* Mvt. 1

Frequency differences of $^{1}/_{60}$ (period = 60♩)

3 3 3 2 3 3 3 4 2 2 2

3 3 3 4 2 2 3 2 2



Interaction of frequencies

Ex.: Ligeti, *Piano Concerto* Mvt. 1

. . . 2 2 2 3 3

. . . 2 3 2 2

↓ ↓

Coinciding attacks

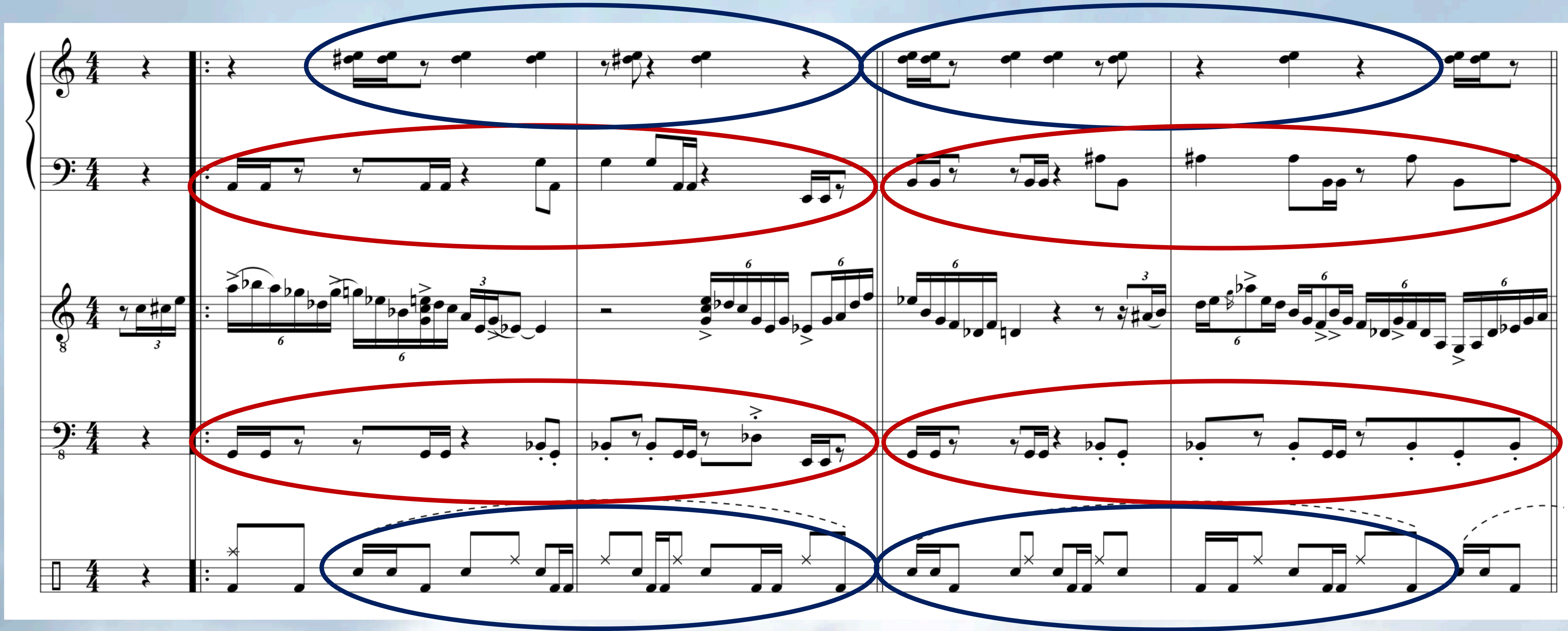
. . . 2 2 2 3 3 3 2 3 3 3 4 2 2 2

. . . 4 2 2 3 2 2 3 3 3

Interlocking

Interaction of frequencies

Ex.: Okazaki, “Box in a Box” (2017)

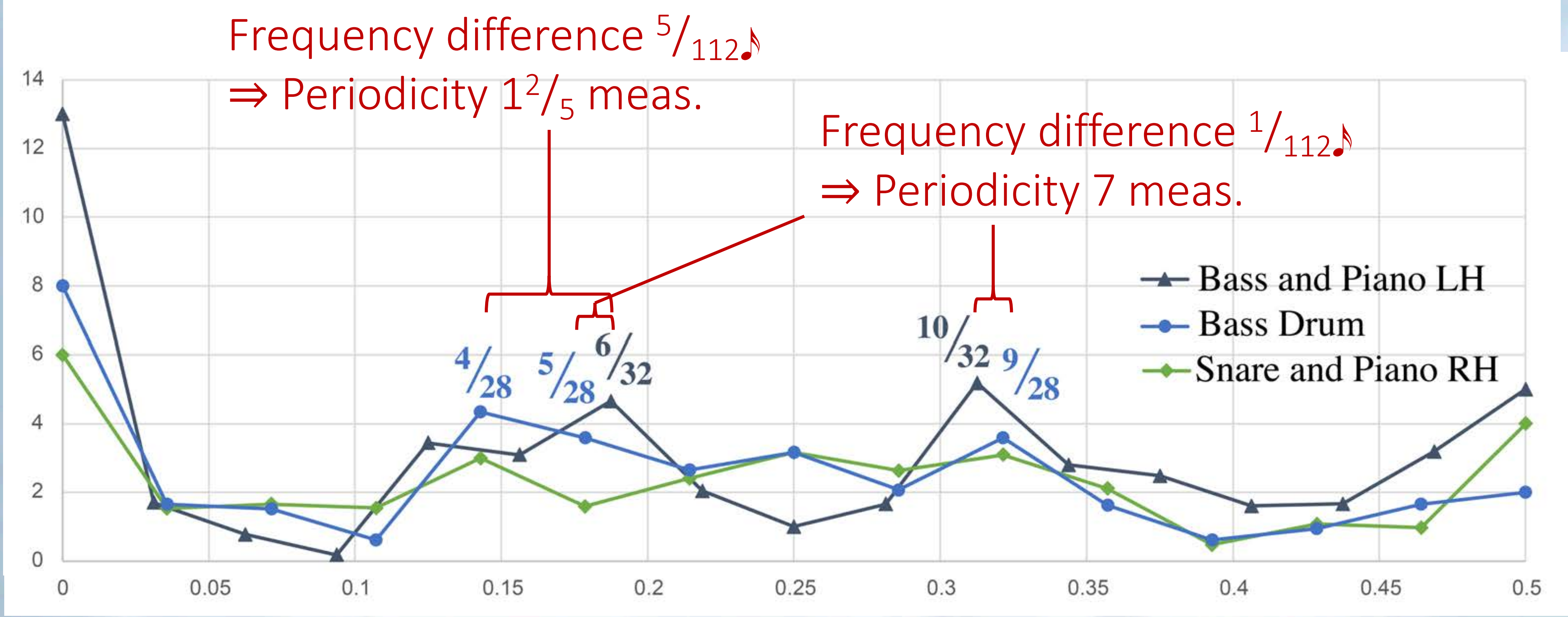


Bass and Piano LH: 8♩ ostinato

Drums and Piano RH: 7♩ ostinato

Interaction of frequencies

Ex.: Okazaki, “Box in a Box” (2017)



Rhythmic spectra

Interaction of frequencies: “Box in a Box”

Bass/
Piano LH

Bass Drum

$1\frac{2}{5}$ meas. periodicity

→ C → I → (C) → I → C → I

More interlocking ------------------------→ More coinciding

7 meas. periodicity

→ C → I → (C) → I → C → I → C

-------------→ More interlocking -----------

→ I → C → (I) → C → I → (C) → I

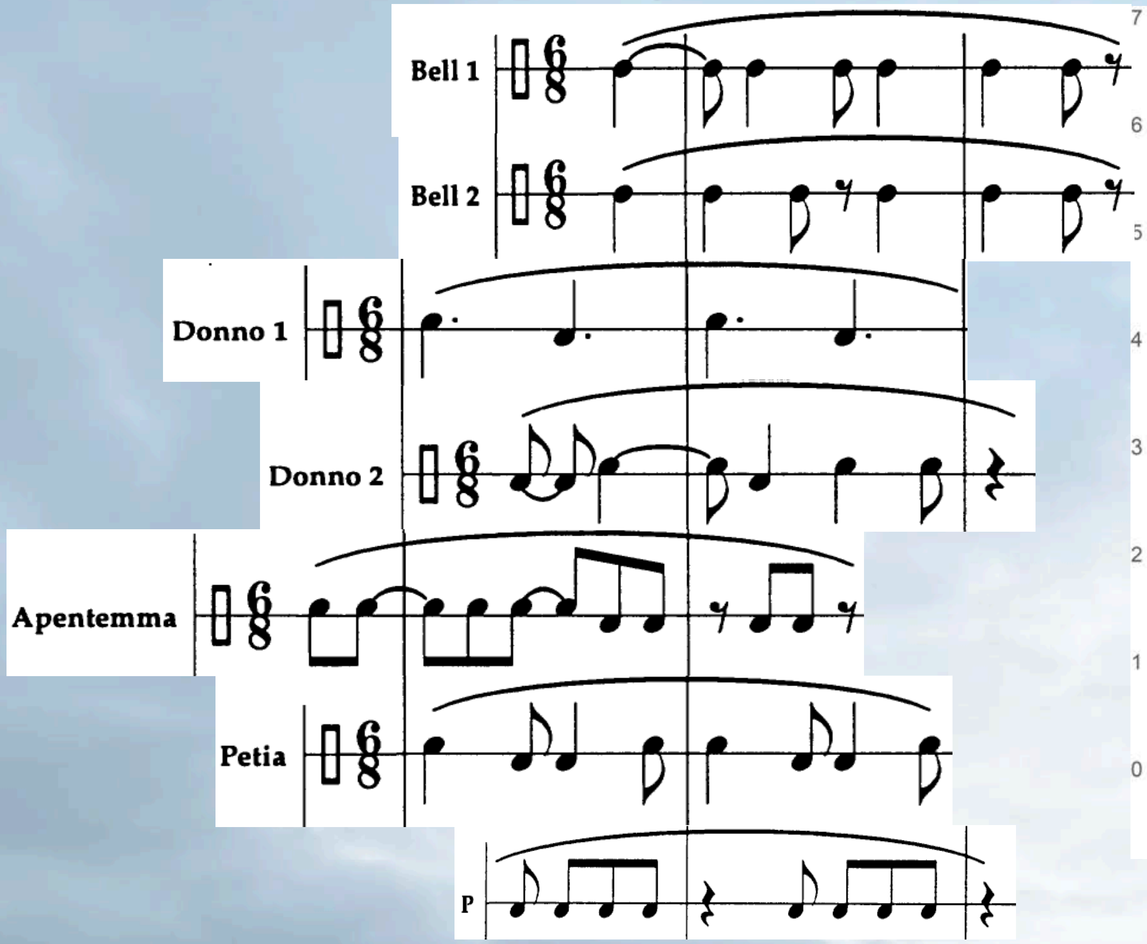
------------→ More coinciding ------------→

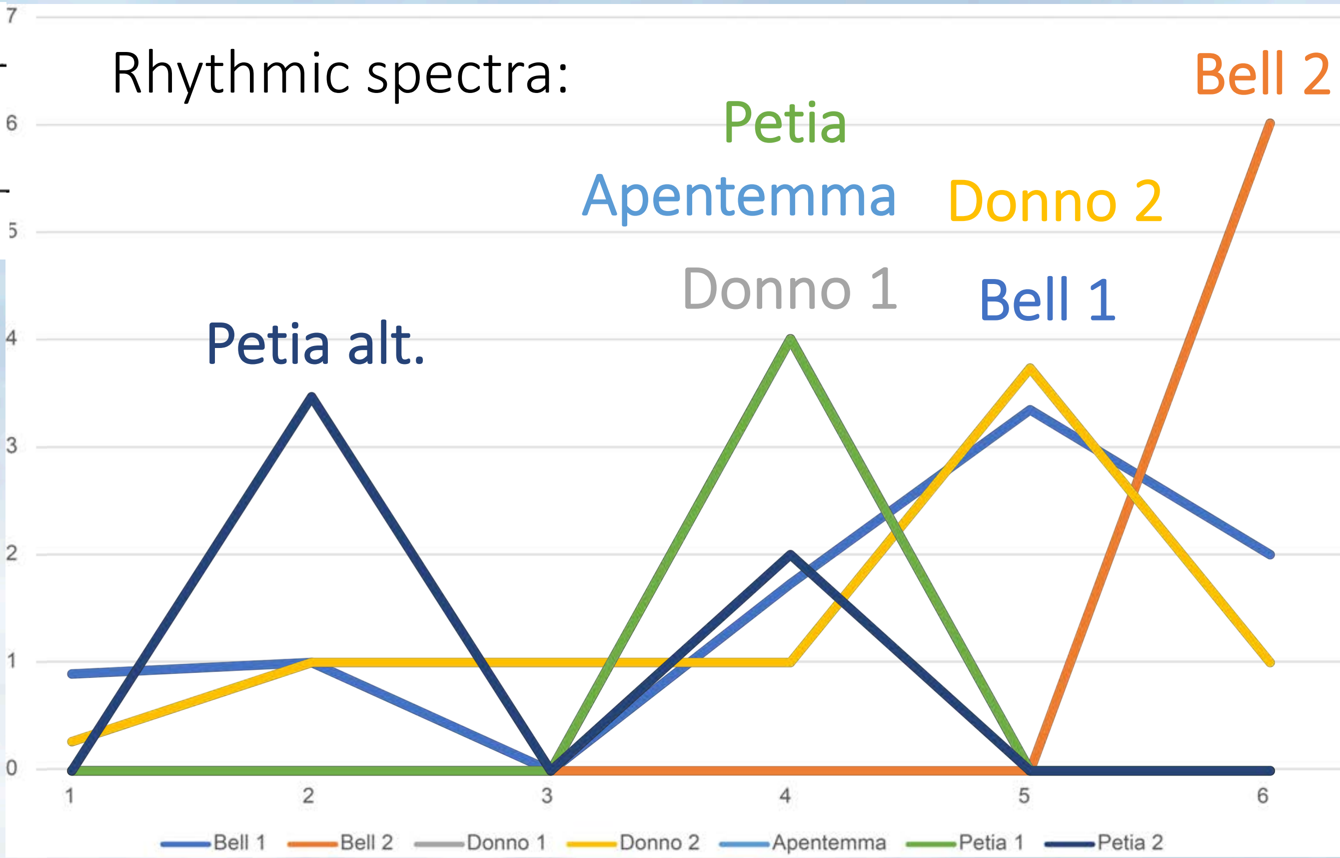
Composite Rhythm in Adowa

Analysis of a transcription by Willi Anku

Adowa

The Adowa ensemble (in Anku's transcription)

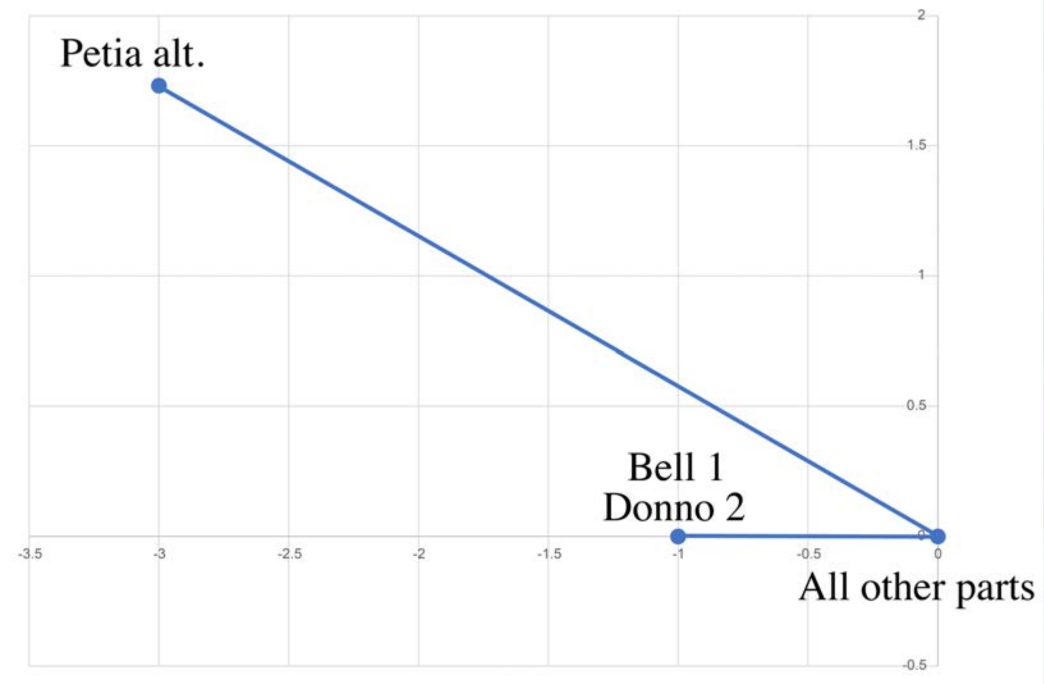




Frequencies 4 and 5 are prominent and represented by multiple parts

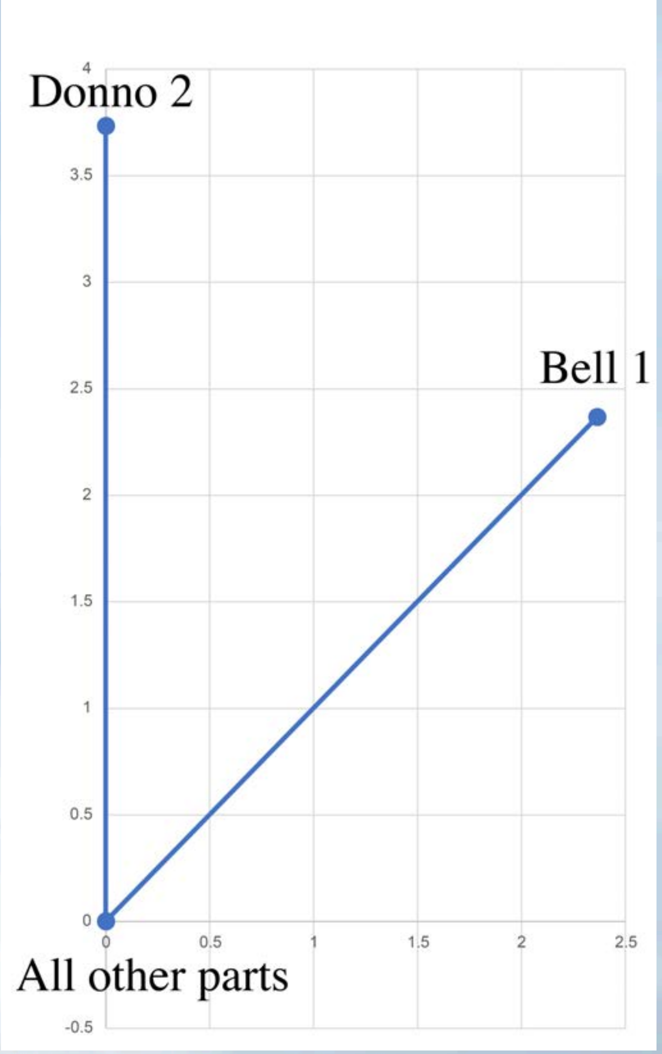
Adowa

The Adowa ensemble on 2-d. planes



Frequency-2 space

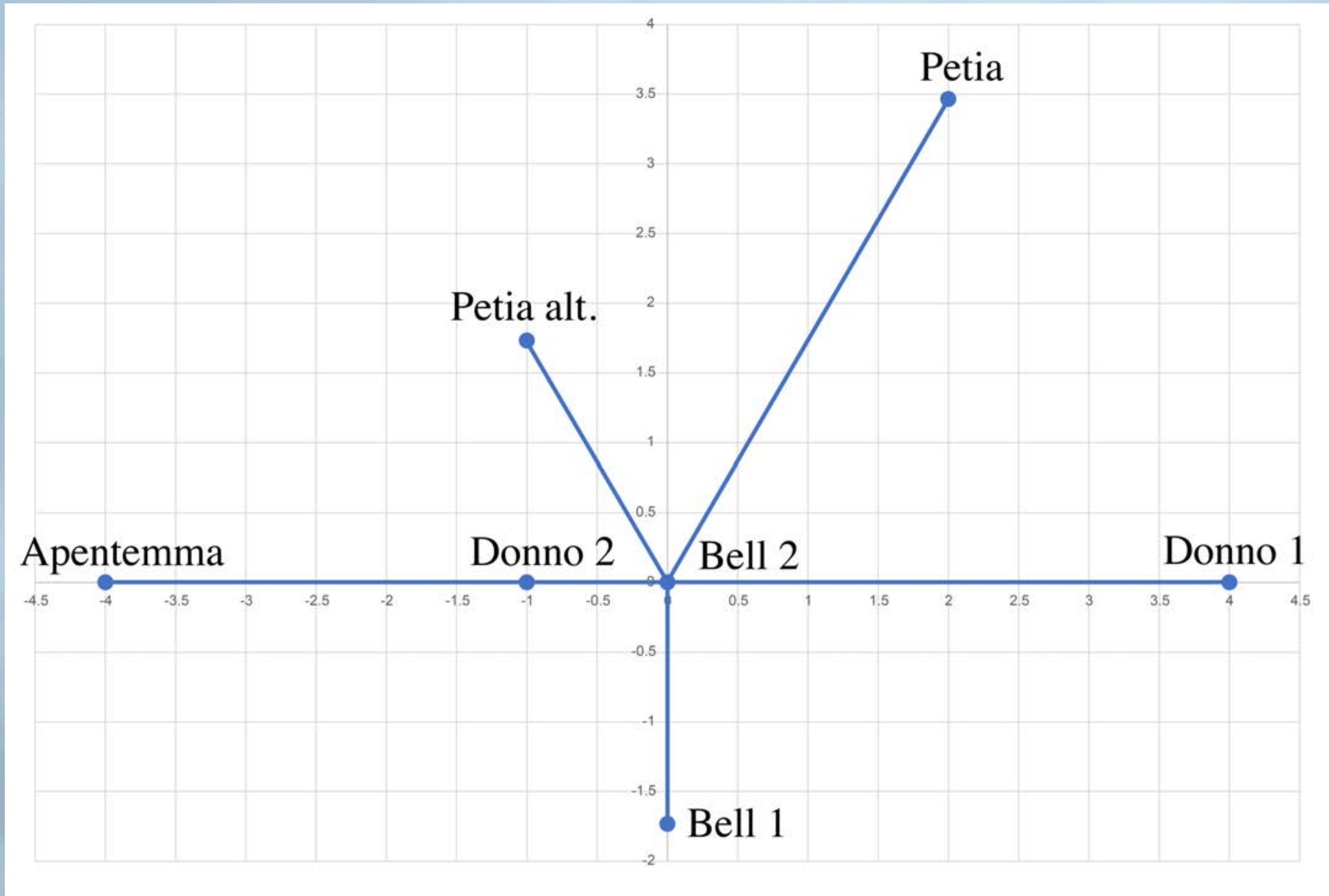
Frequencies 2 and 5 (periodicities 𝅗𝅥. and 4/5♩.) are both represented by just 2–3 parts, similar in phase.



Frequency-5 space

Adowa

The adowa ensemble on 2-d. planes



Frequency-4 space

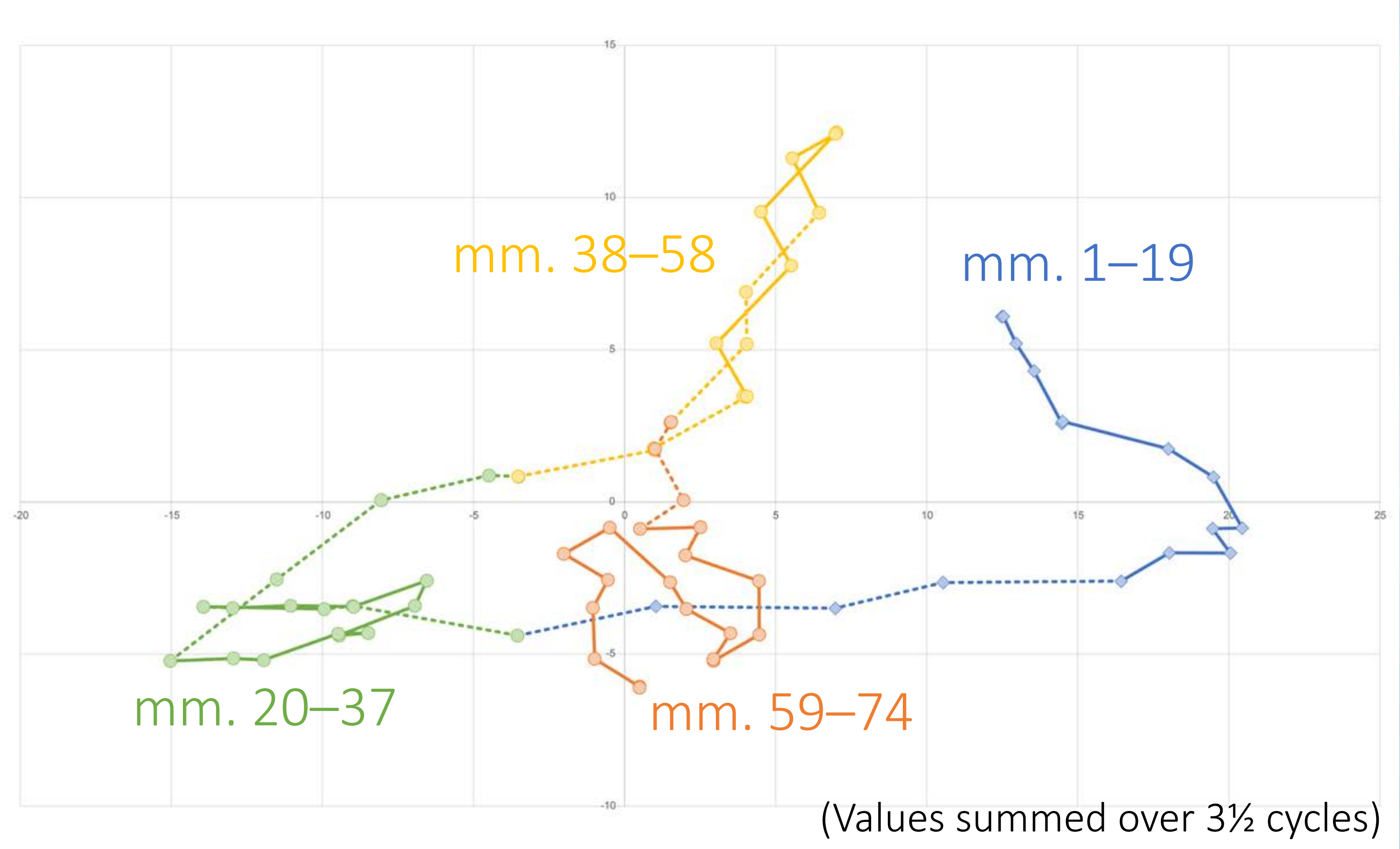
Frequency 4 represents the periodicity of the beat (♩.) and here multiple parts cover different regions of the space (on-beat, off-beat, ahead of beat, behind beat).

Adowa: Atumpan (lead drum)

Anku's transcription of a performance by Solomon Amonquandoh divided into 4 sections

Amonquandoh begins centered on the main two beats.

Then he explores each region of the space in turn: off-beats, ahead of the beat, and behind the beat.



Frequency-2 space

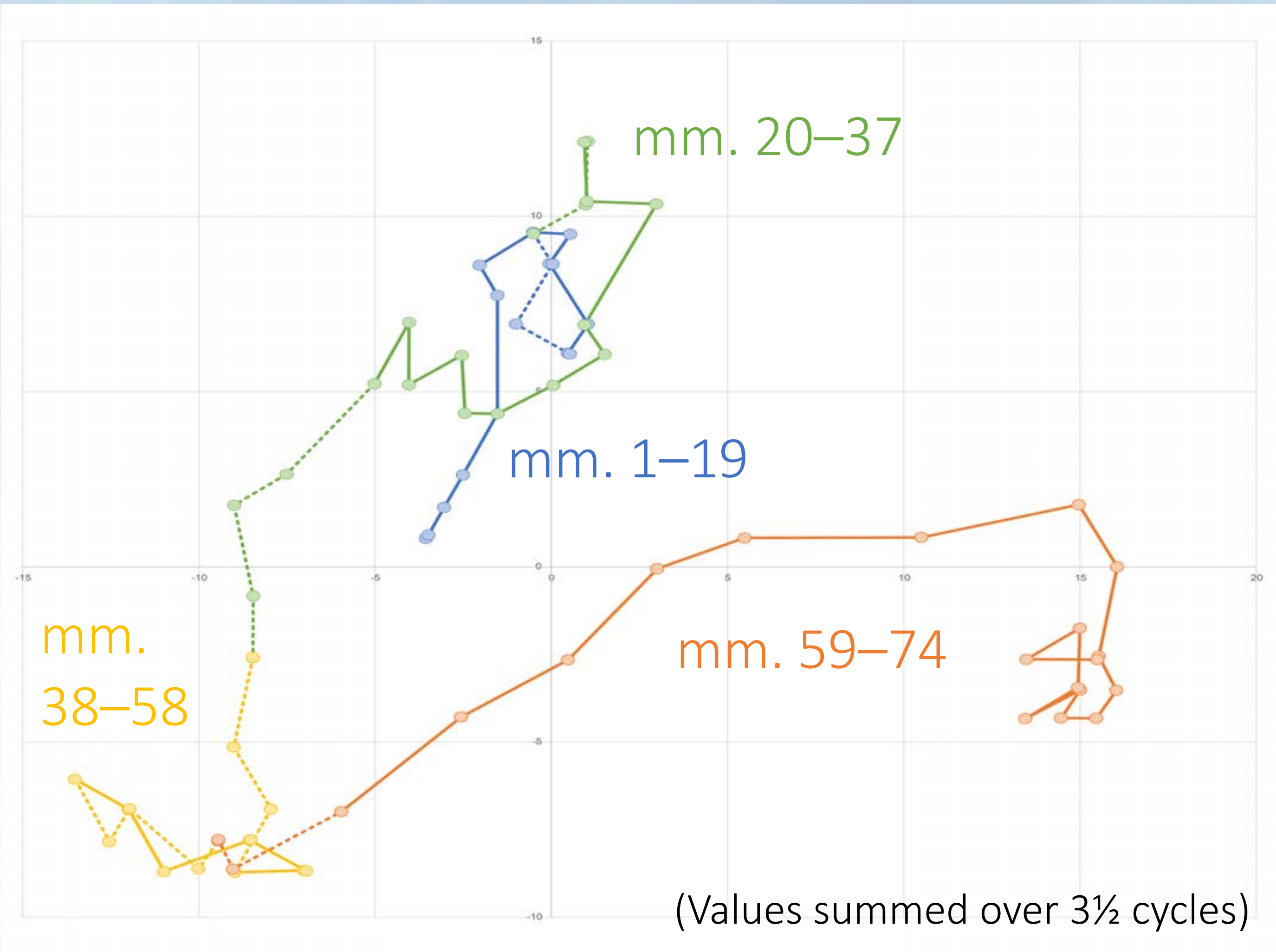
Adowa: Atumpan (lead drum)

Anku's transcription of a performance by Solomon Amonquandoh divided into 4 sections

Frequency 4 measures orientation with respect to the main 4 ♩. beats.

The performance also explores all the regions of this space in sequence.

The first two sections are consistently ahead of the beat, the third section behind, and the last section on the beat.

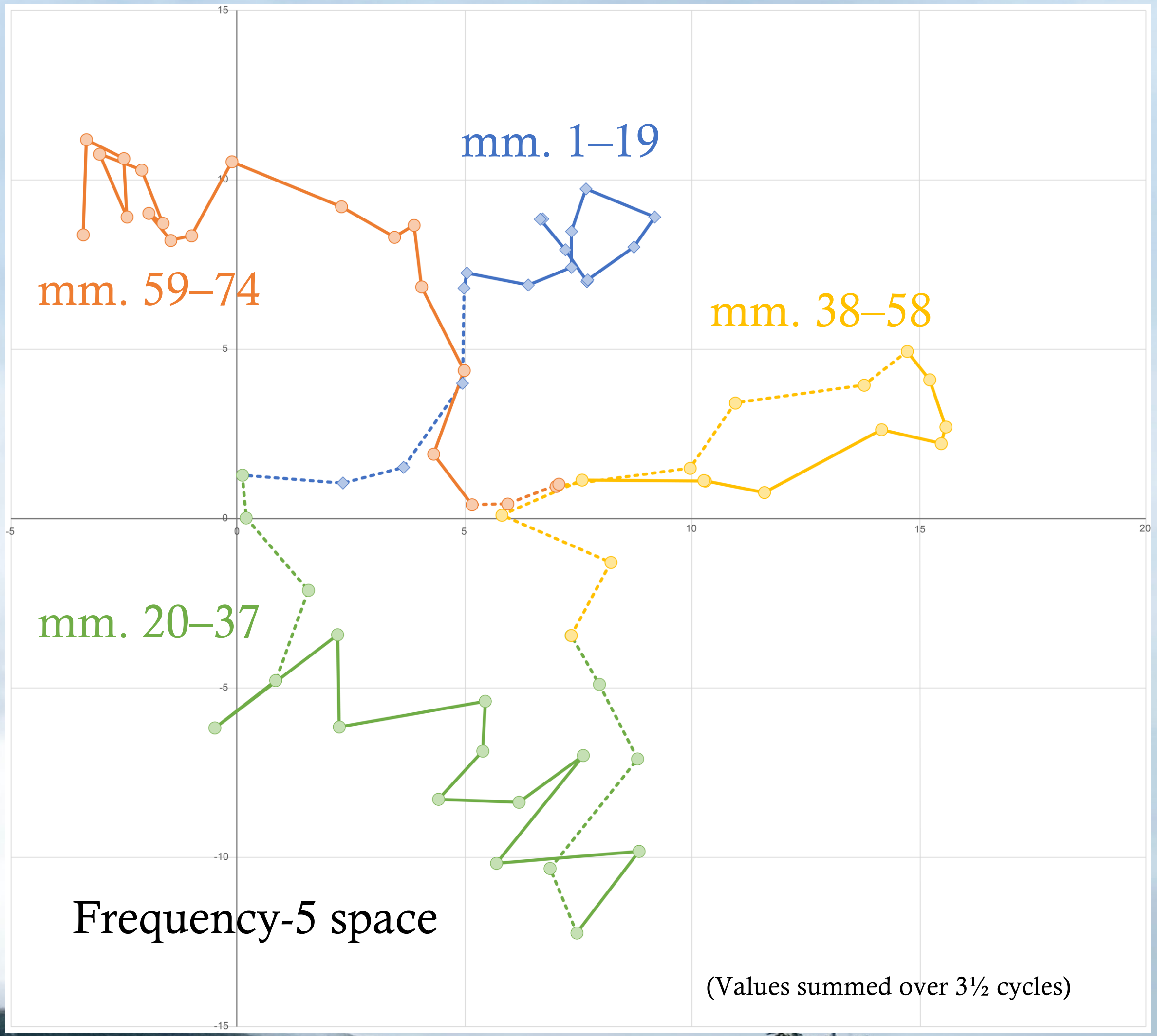


Frequency-4 space

Adowa: Atumpan (lead drum)

Anku's transcription of a performance by Solomon Amonquandoh divided into 4 sections

This is frequency articulated by the timeline rhythm of the bell, which is in the upper right quadrant of the space. Amonquandoh starts in the vicinity of the bell and explores all the adjacent regions, avoiding the half of the space across from the bell.



Iqa’at

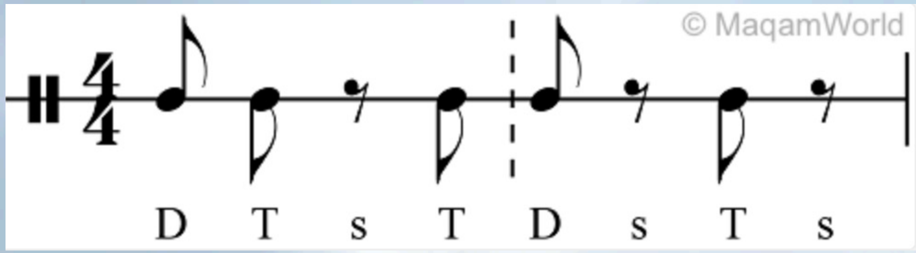
Arabic Iqa'at

Examples are all taken from Farraj and Shumays, *Inside Arabic Music,* and maqamworld.com (excellent resources!)

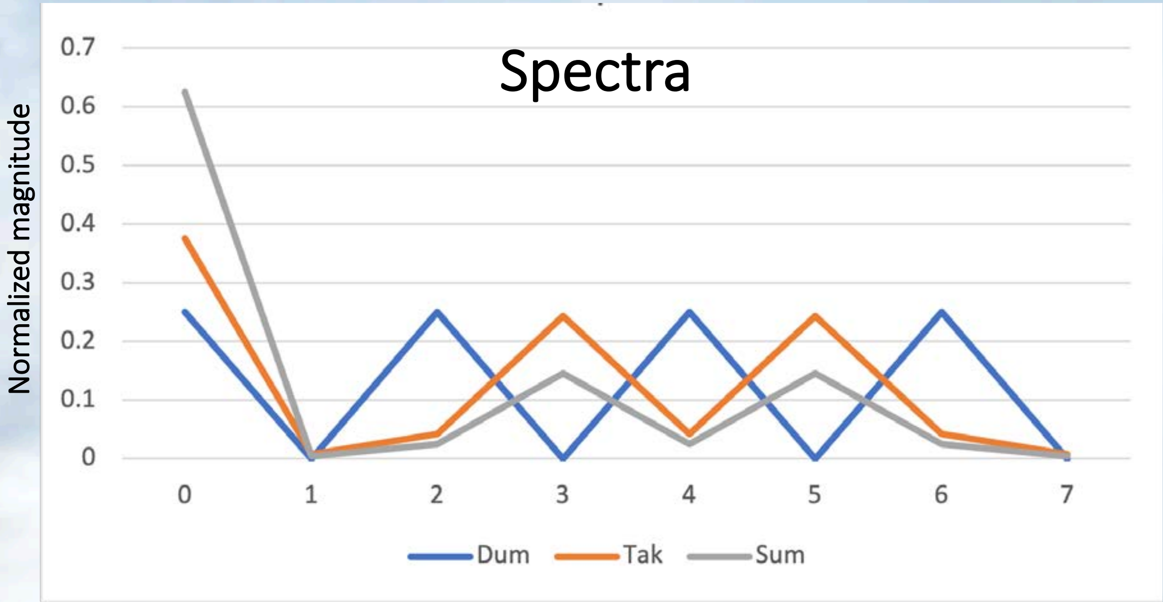
Musicians describe rhythmic types as a pattern of "Dums" (low resonant strokes), "Taks" (high-energy strikes) and rests.

Um Kulthum, Darit el-Ayyam:

Example: Maqsum



The Dum rhythm has only even frequencies. The Tak rhythm emphasizes the odd frequencies and this is preserved in the sum

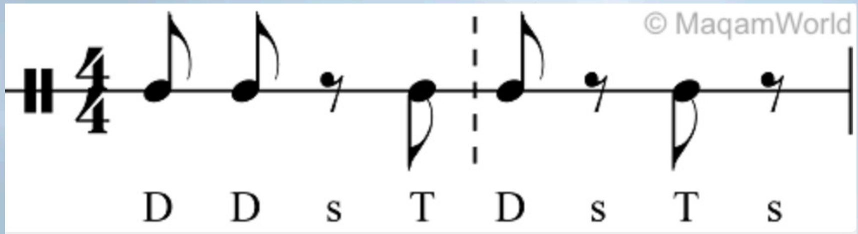


Arabic Iqa’at

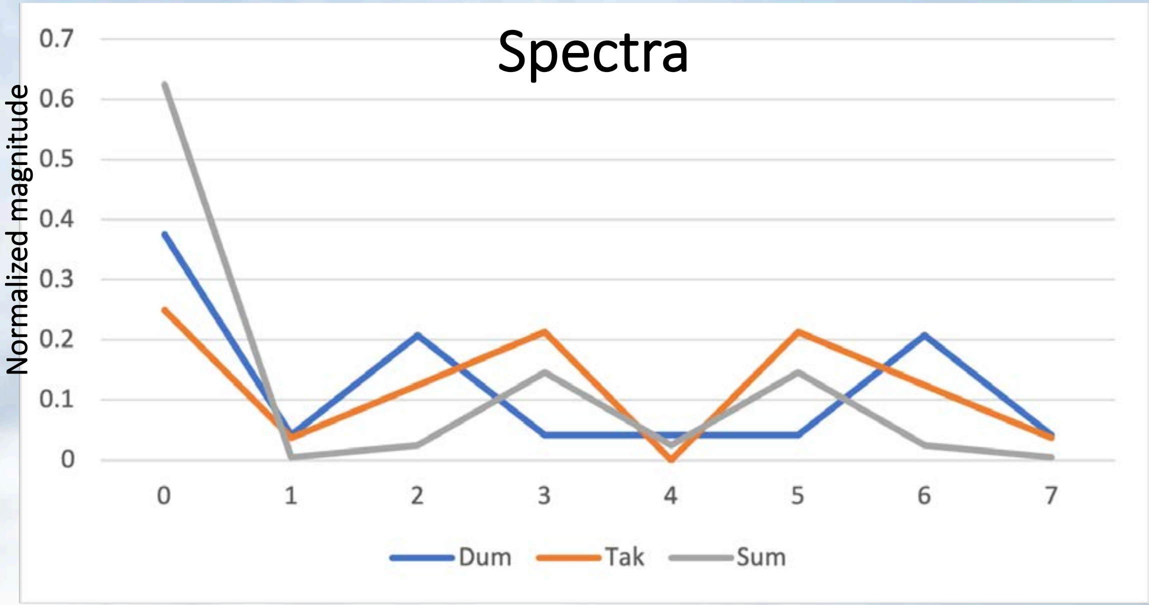
Examples are all taken from Farraj and Shumays, *Inside Arabic Music,* and maqamworld.com (excellent resources!)

Hassan al-Hafar,
Talumuni Wa Lam Tarthu Li Hali

Example: Baladi



The sum is a cinquillo rhythm

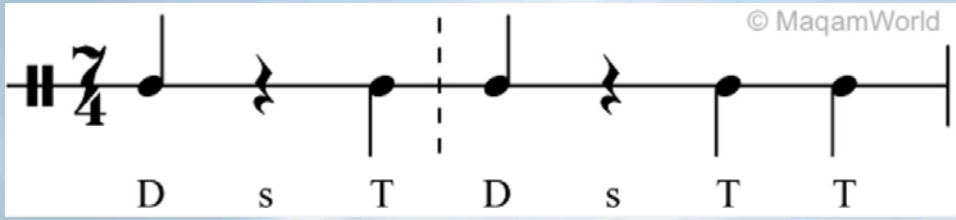


Arabic Iqa'at

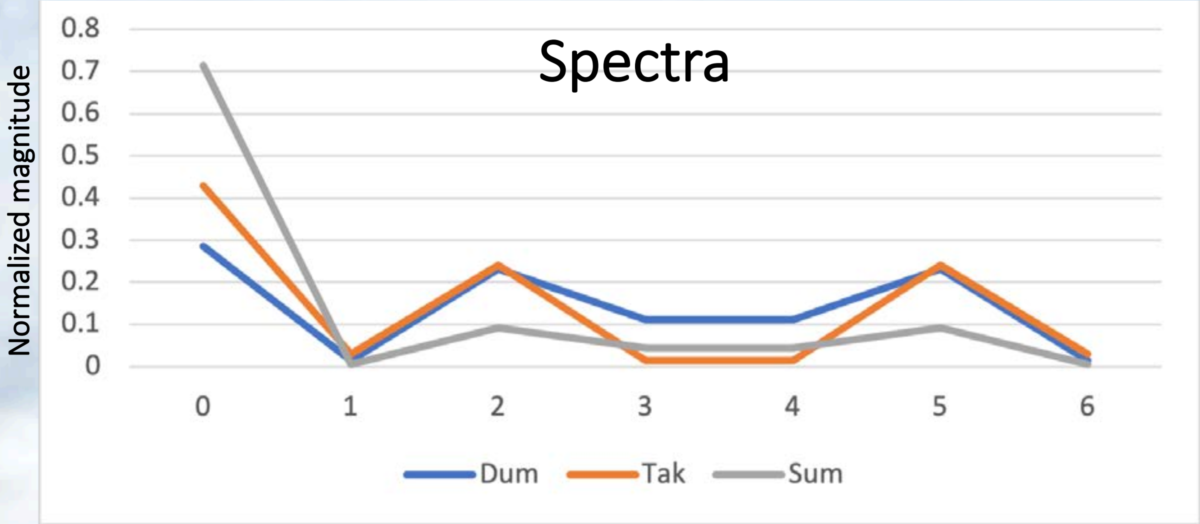
Examples are all taken from Farraj and Shumays, *Inside Arabic Music,* and maqamworld.com (excellent resources!)

Ensemble Markos
Muwashah Jalla Man Qad Sagha Badran

Example: Nahwakht



Frequency 2 is prominent in all components of the rhythm.



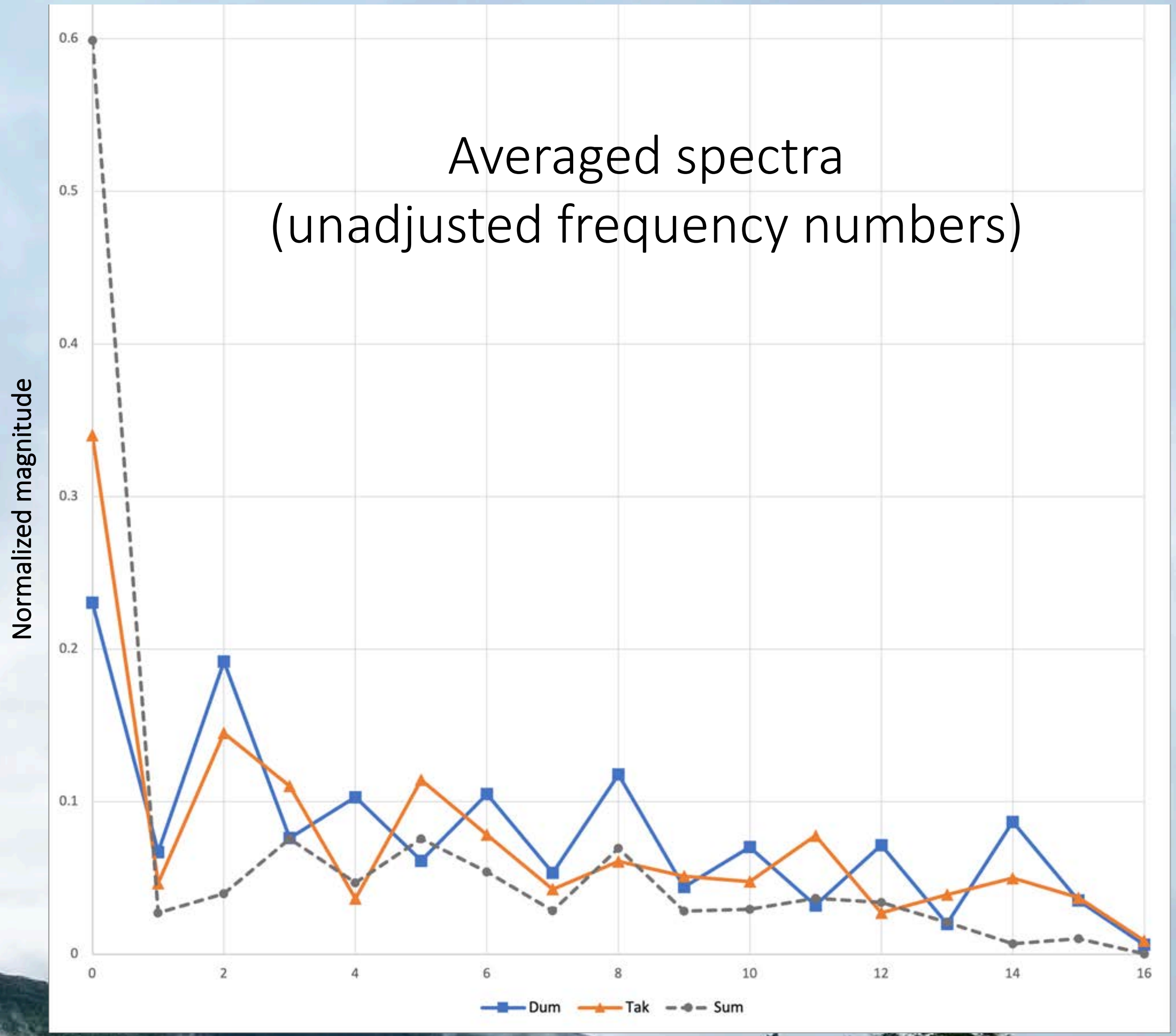
Arabic Iqa’at

Descriptive analysis of 37 iqa’at included in Farraj and Shumays *Inside Arabic Music*.

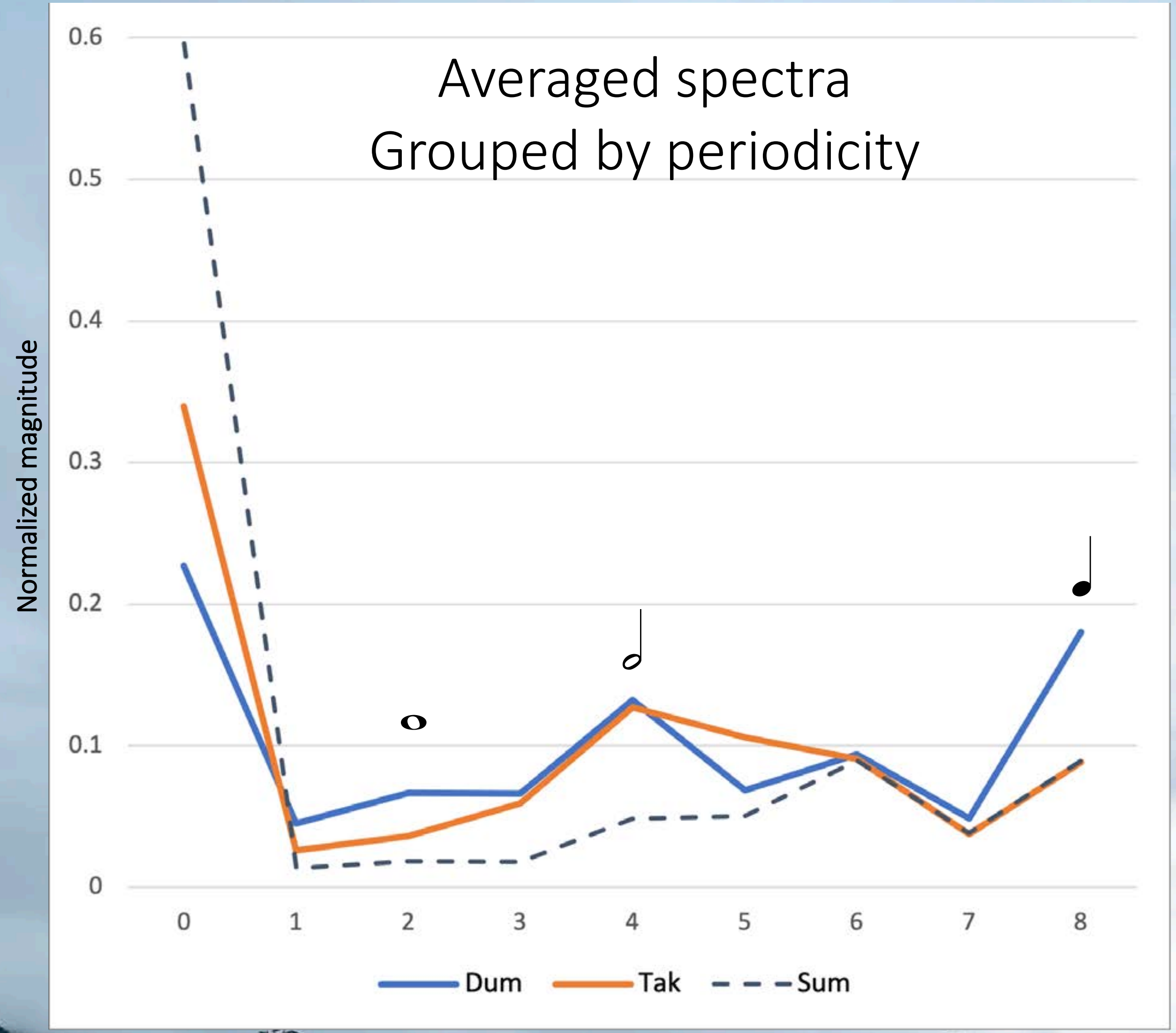
Here I average frequencies as divisions of the cycle.

The even-numbered frequencies clearly favor Dum while 3 and 5 favor Tak.

We see destructive interference at 2: Dum and Tak tend to be large but opposed in phase so they sum to a small value.



Arabic Iqa'at



Here I align spectra based on periodicity (multiples of ♪) rather than frequency number.

This requires grouping nearby periodicities into 8 bins, so periodicities are approximate.

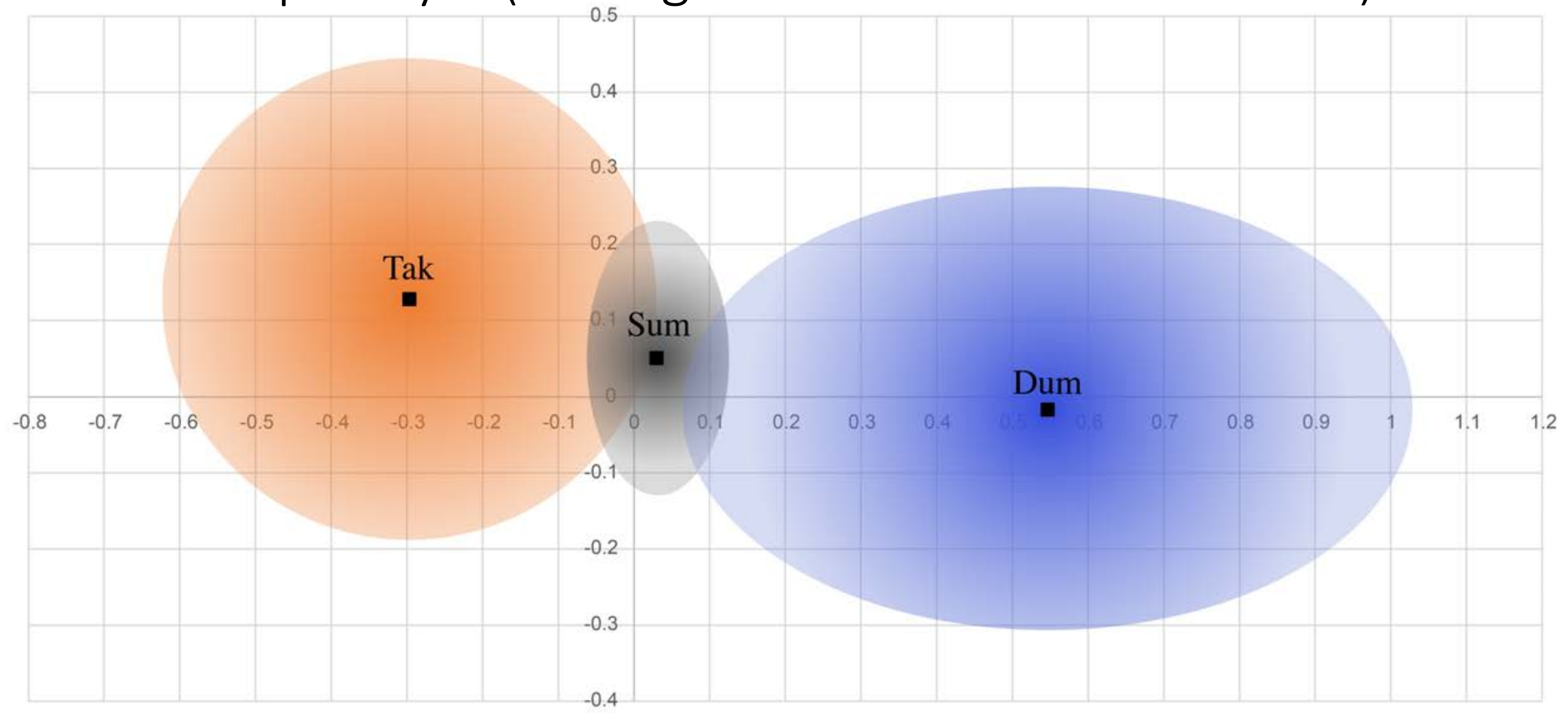
The lower frequencies tend to have destructive interference.

Both individual parts peak at 𝅗𝅥 and Dum peaks at ♩

Arabic Iqa’at

Averaging values in 2-d. space shows phase values.

Frequency 2 (Averages and standard deviations)

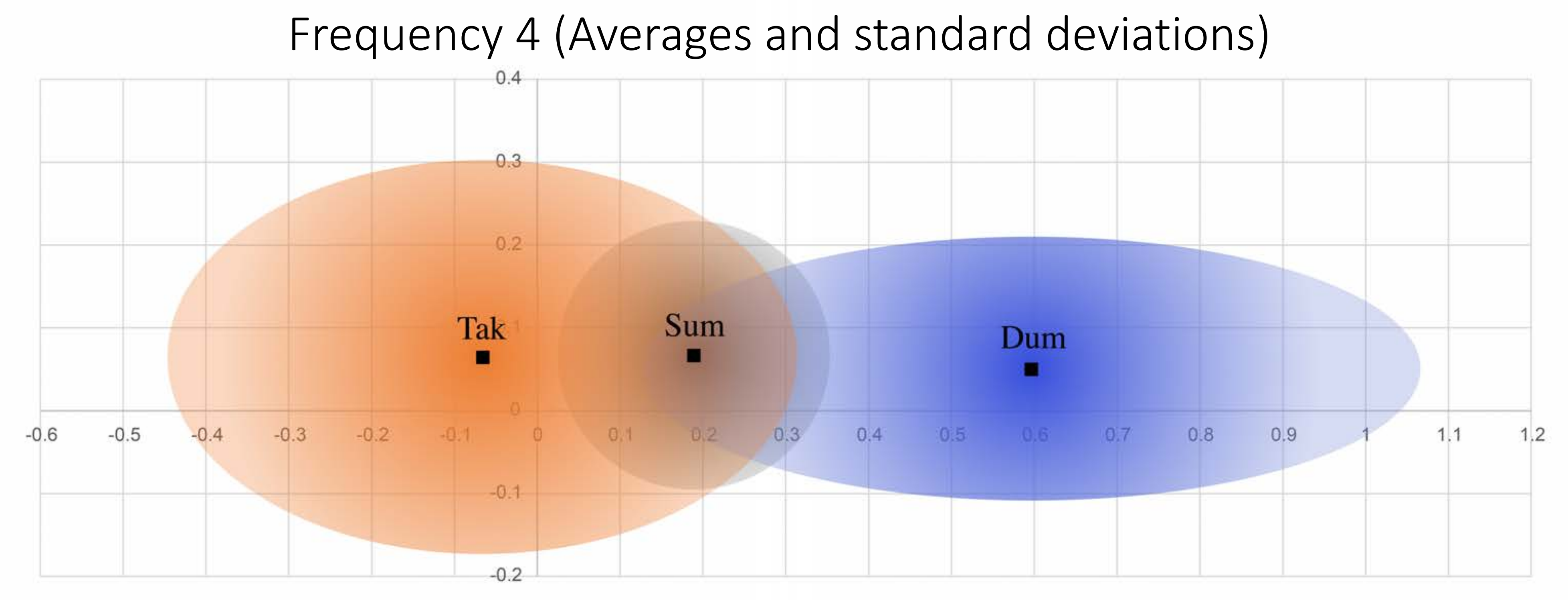


–A clear on-beat/off-beat division between Dum and Tak.

–The resulting sums are weakly on-beat.

Arabic Iqa’at

Averaging values in 2-d. space shows phase values.

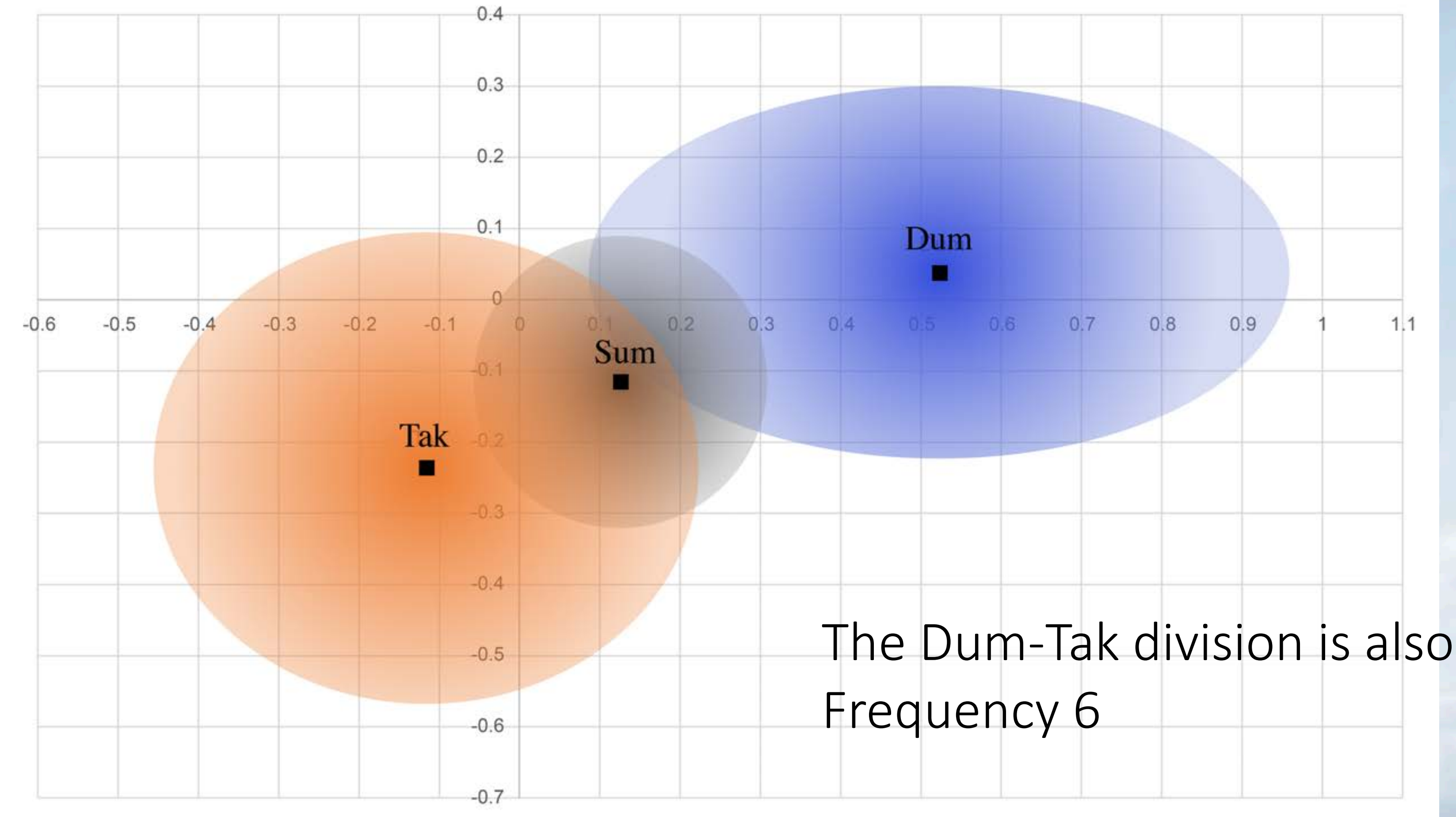


–Frequency 4 has a similar division between Dum and Tak, although Tak is not exclusively off-beat.

Arabic Iqa'at

Averaging values in 2-d. space shows phase values.

Frequency 6 (Averages and standard deviations)

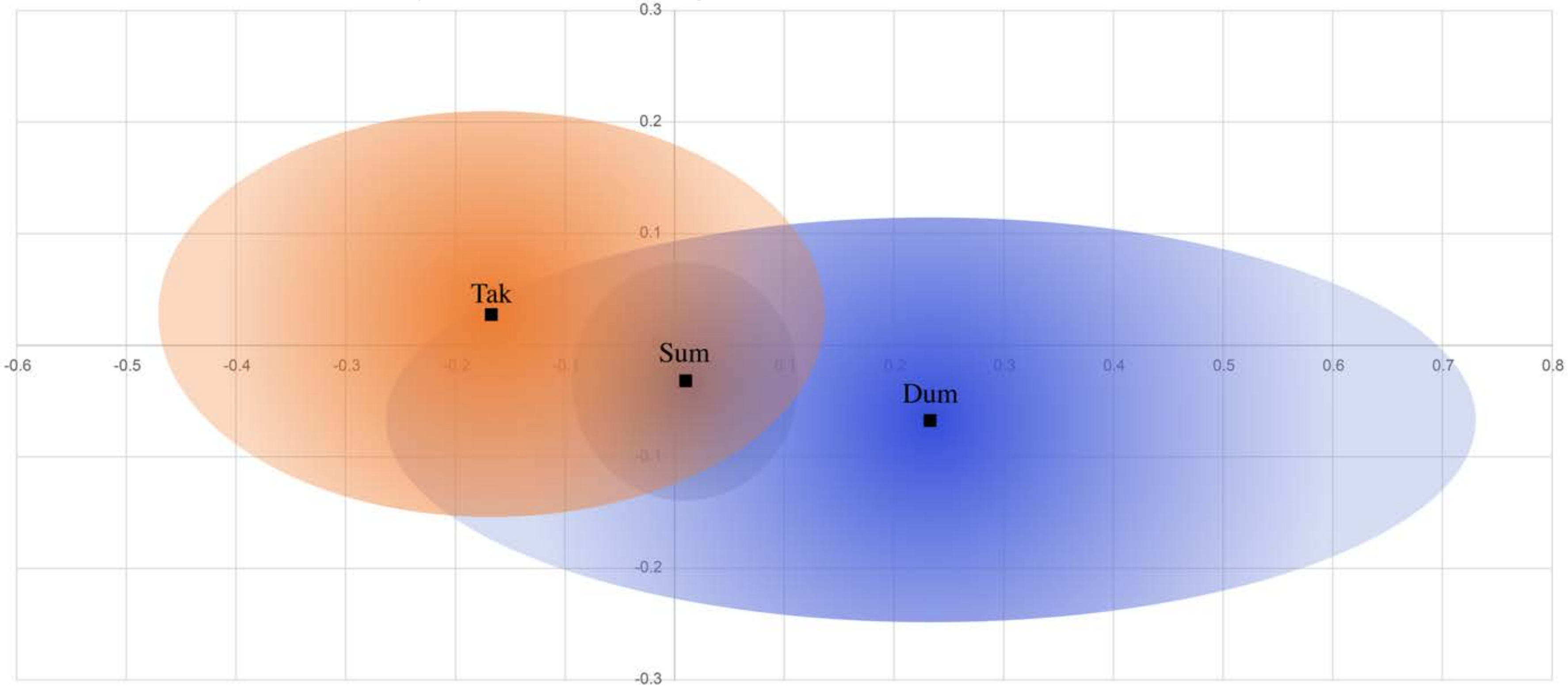


The Dum-Tak division is also very clear in Frequency 6

Arabic Iqa'at

Averaging values in 2-d. space shows phase values.

Periodicity ≅ 𝅝 (Averages and standard deviations)

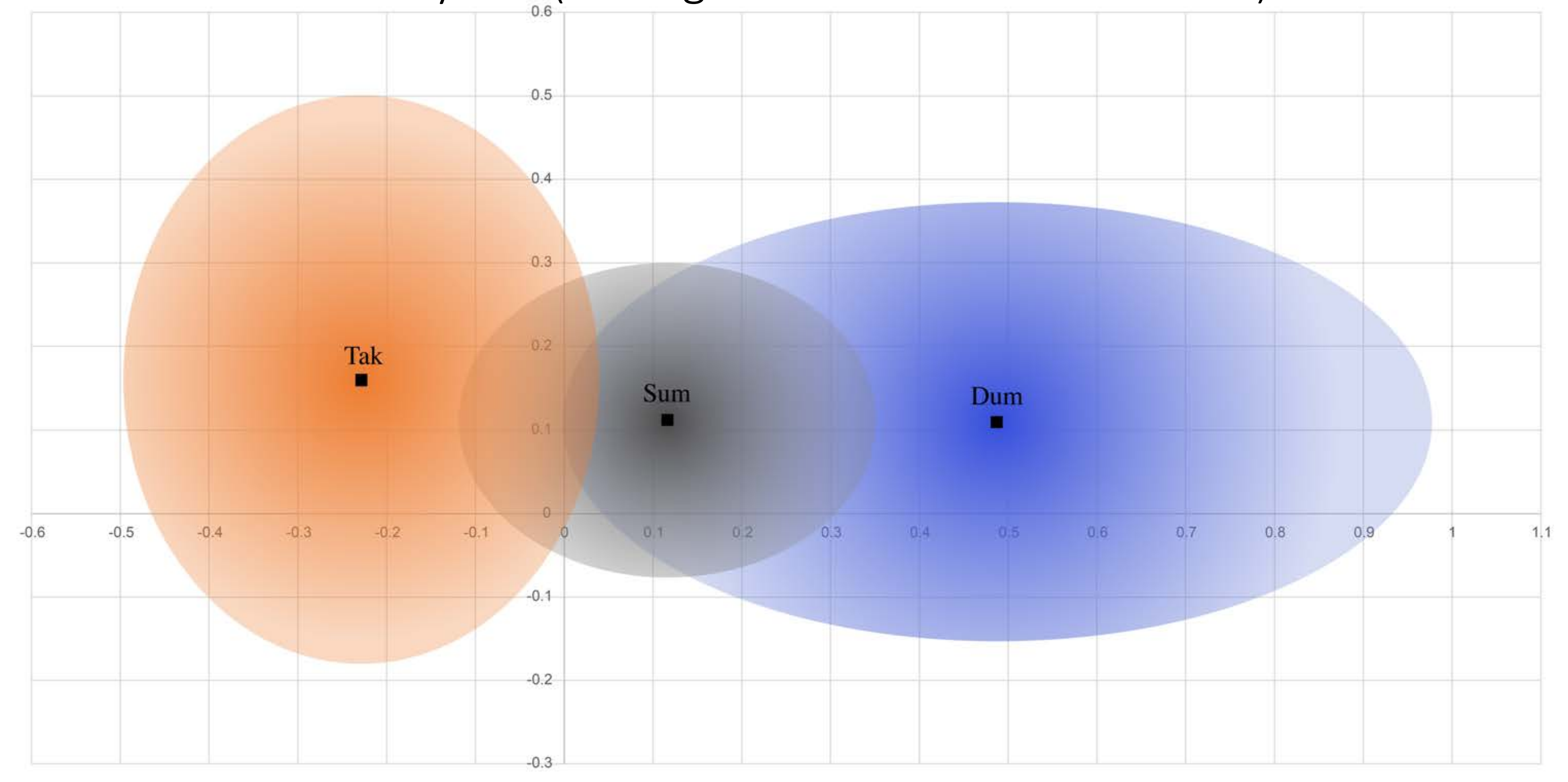


We get similar Dum-Tak divisions grouping by approximate periodicity.

Arabic Iqa'at

Averaging values in 2-d. space shows phase values.

Periodicity ≅ 𝅗𝅥 (Averages and standard deviations)



Of the periodicity-grouped data, the 𝅗𝅥 periodicity has the clearest Dum-Tak division.

Summary

- Traditional concepts of meter originate in the European notational system, which has a problematic time-discrete basis.
- Simple periodicity can both recast traditional concepts of meter and lead to new ones. It can deal with microtiming.
- Individual periodic functions relate in a two-dimensional space.
- A rhythmic spectrum summarizes all the periodicities of a rhythm.
- Maximally even and near-maximally-even rhythms are important in many kinds of music, because they articulate periodicities that are not simple multiples of a basic time unit.
- When music incorporates multiple nearby periodicities, they interact to produce slower periodic processes.
- Rhythms with different cycles can be compared as periodicities or frequencies.

Thanks!

Preprints and powerpoints here:

sites.bu.edu/jyust