Dimensions of Atonality: Commentary on Hippel and Huron 2020

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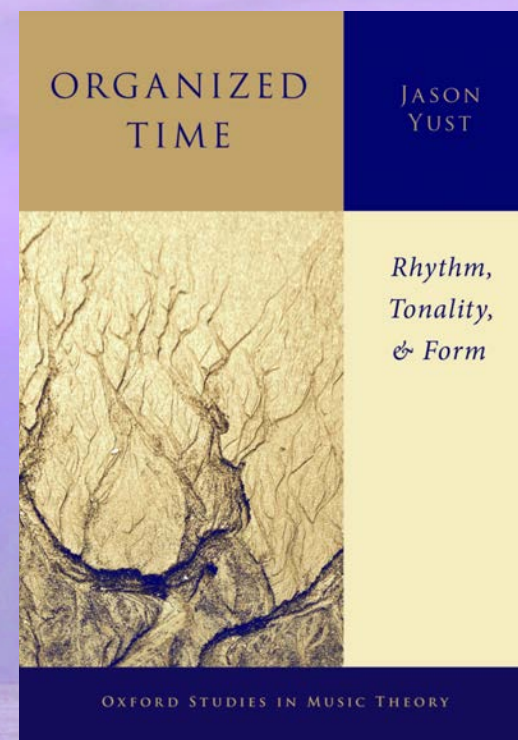
Music Cognition Seminar
11/15/2020



Fourier Transform on Pitch-Class Sets and Distributions

Using discrete Fourier transform, we can

- Identify harmonic qualities, through **spectra**
- Relate harmonies of similar quality through **phase spaces**

Krumhansl's tonal space is a phase space.

Fourier Transform on Pitch-Class Vectors: A brief history

Lewin, David (1959). "Re: Intervallic Relations between Two Collections of Notes," *JMT* 3/2.

——— (2001). "Special Cases of the Interval Function between Pitch Class Sets X and Y." *JMT* 45/1.

Quinn, Ian (2006–2007). "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1.

Callender, Cliff (2007). "Continuous Harmonic Spaces," *JMT* 51/2

Amiot, Emmanuel (2007). "David Lewin and Maximally Even Sets." *Journal of Mathematics and Music* 1/3.

——— (2016). *Music Through Fourier Space: Discrete Fourier Transform in Music Theory*. (Springer)

Yust, Jason (2015). "Schubert's Harmonic Language and Fourier Phase Spaces." *JMT* 59/1.

——— (2016). "Special Collections: Renewing Forte's Set Theory." *JMT* 60/2.

Pitch-Class Vectors

Key profiles are *pitch-class vectors*

Example:

	C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
Krumhansl-Kessler C major:	(6.4,	2.2,	3.5,	2.3,	4.4,	4.1,	2.5,	5.2,	2.4,	3.7,	2.3,	2.9)
Krumhansl-Kessler C minor:	(6.3,	2.7,	3.5,	5.4,	2.6,	3.5,	2.5,	4.8,	4.0,	2.7,	3.3,	3.2)

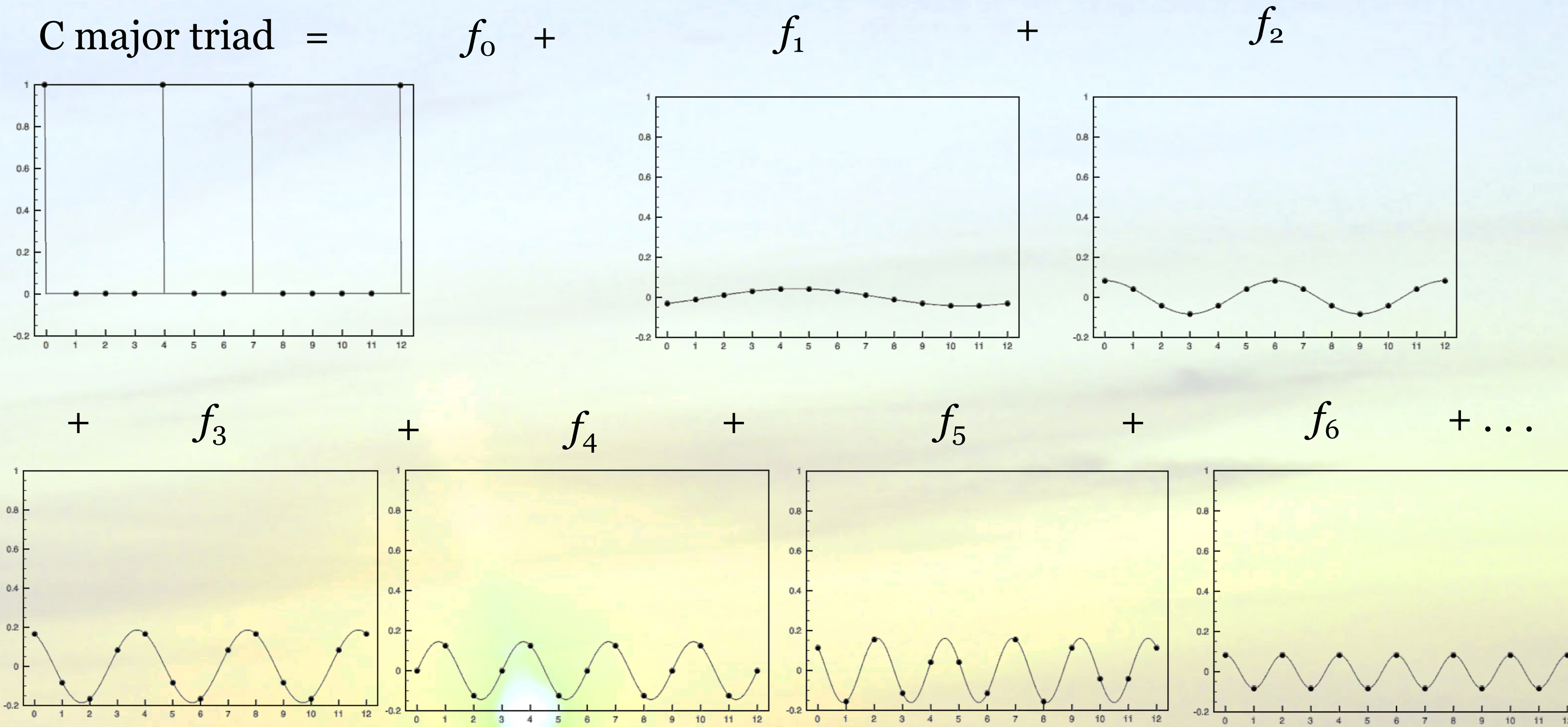
Pitch-class sets and multi-sets are also pitch-class vectors

Example:

	C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
C major triad:	(1,	0,	0,	0,	1,	0,	0,	1,	0,	0,	0,	0)
C major scale + tonic triad:	(2,	0,	1,	0,	2,	1,	0,	2,	0,	1,	0,	1)

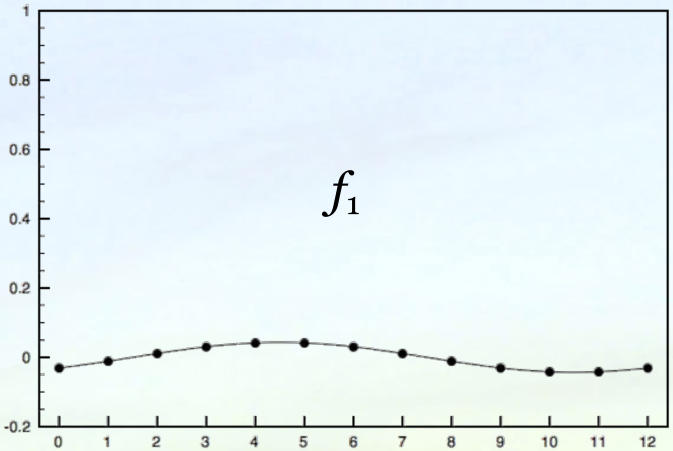
Other names: “Characteristic function,” “pitch-class distribution”

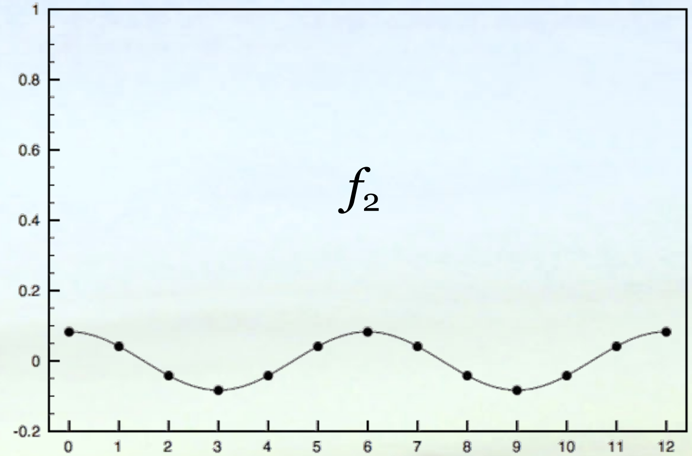
Fourier Transform of a Pitch-Class Vector



The DFT is a **lossless transformation** from a pitch-class vector to a sum of **periodic functions** dividing the octave into 1–6 parts.

Fourier Qualities

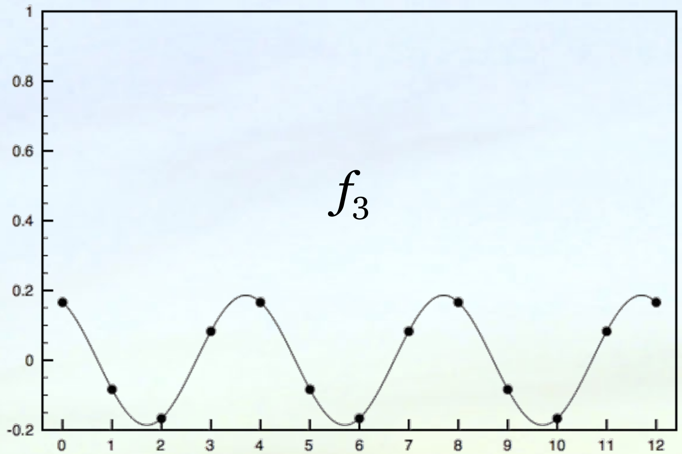


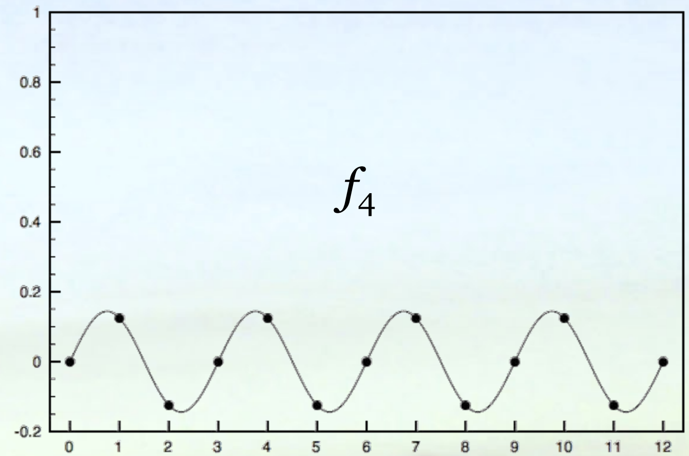


F_1 represents a concentration of pitch-class weight on the full pc circle.

F_2 represents a concentration of pitch-class weight on a half-octave (tritone) cycle.

Fourier Qualities

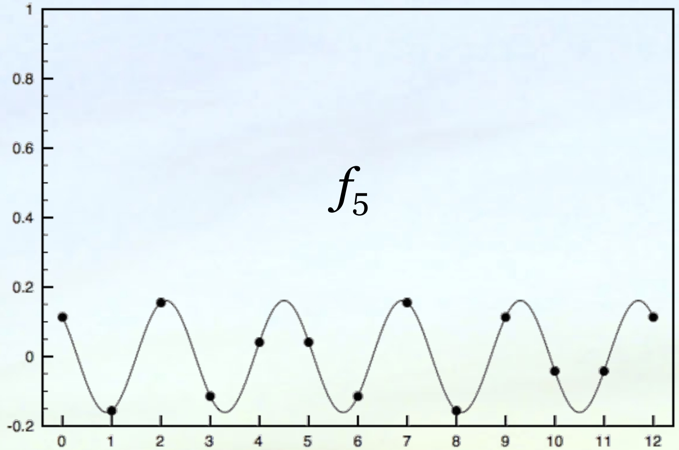




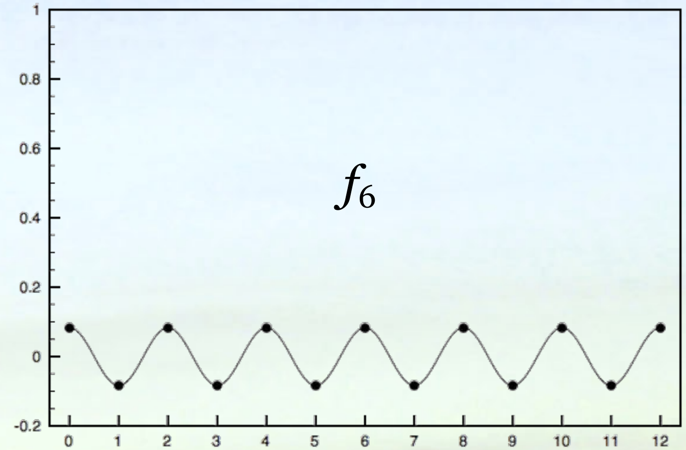
f_3 gives the weighting on the nearest *augmented triad* or ***hexatonic*** *scale*.

f_4 gives the weighting on the nearest *diminished seventh* or ***octatonic*** *scale*.

Fourier Qualities

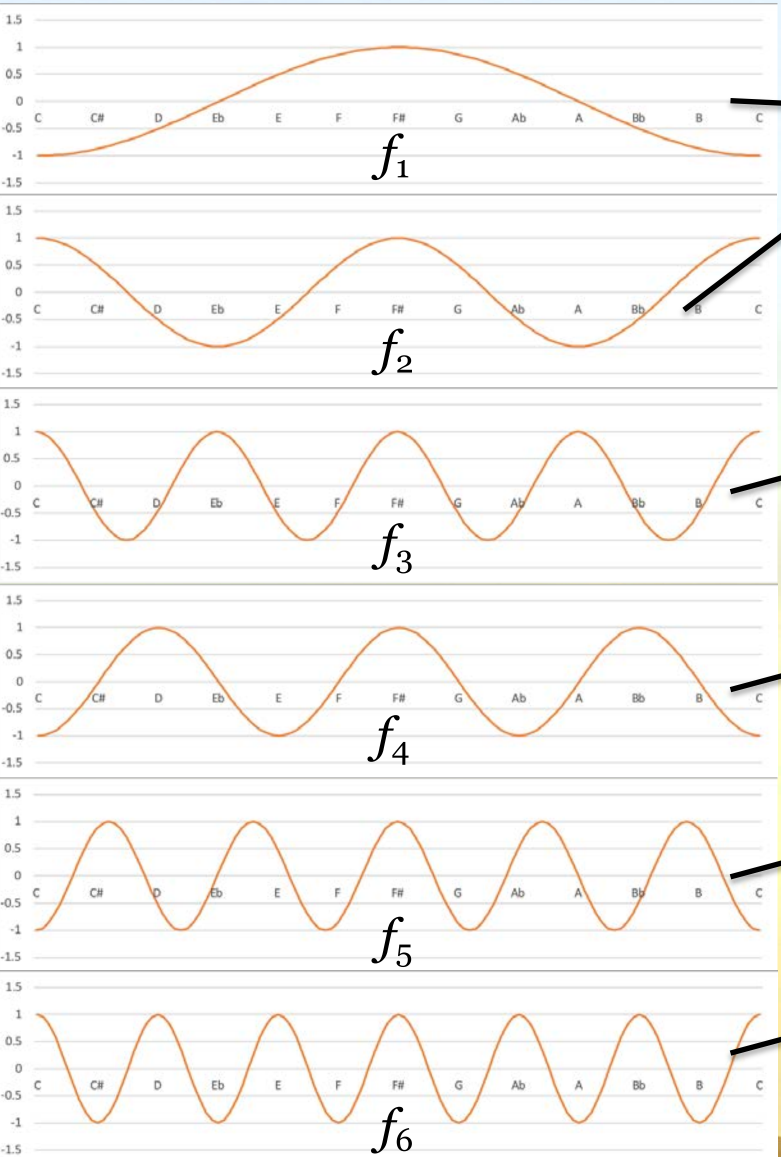


f_5 give the balance on the ***circle of fifths***



f_6 gives the weighting on one of the two ***whole-tone*** collections.

The Six Harmonic Qualities



Use of f_1 and f_2 requires dissonant harmony and abandonment of traditional voice-leading principles

f_3 represents the **triadic/ functional** element of tonality.

f_4 represents **octatonicism**, a potential extension of tonality.

f_5 represents the **diatonic** element of tonality.

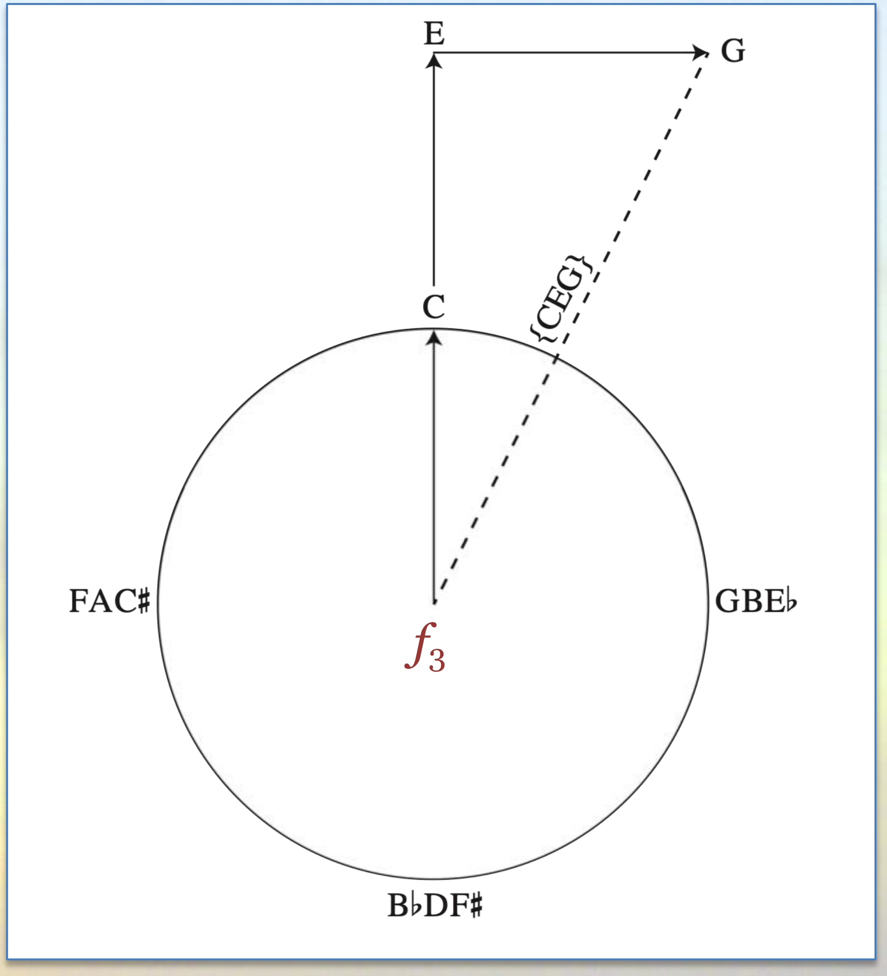
f_6 represents the balance between **whole tone scales.**

Fourier Transform as Vector Sums

Fourier component f_k can be derived as a vector sum with each pitch class as a unit vector, where the unit circle is the 8ve/k.

The length of the resulting vector is the **magnitude** of the component, and the angle is its **phase.**

Example: C maj. triad, $k = 3$

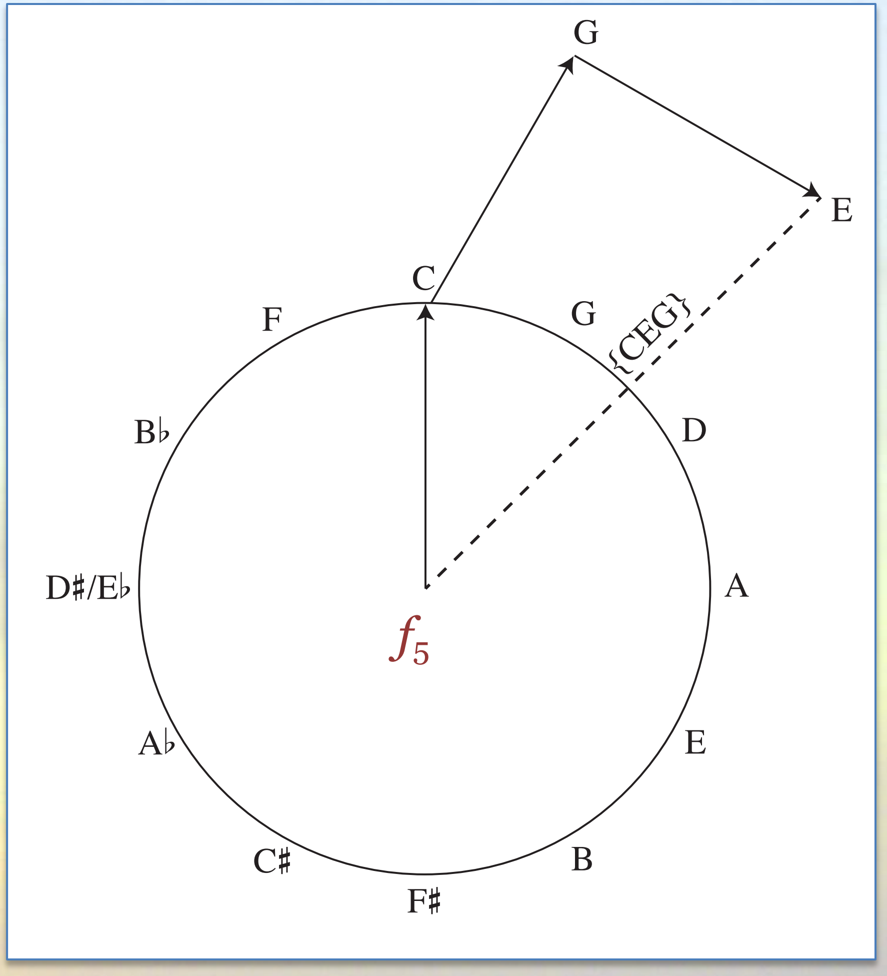


Fourier Transform as Vector Sums

Fourier component f_k can be derived as a vector sum with each pitch class as a unit vector, where the unit circle is the 8ve/k.

The length of the resulting vector is the **magnitude** of the component, and the angle is its **phase.**

Example: C maj. triad, $k = 5$

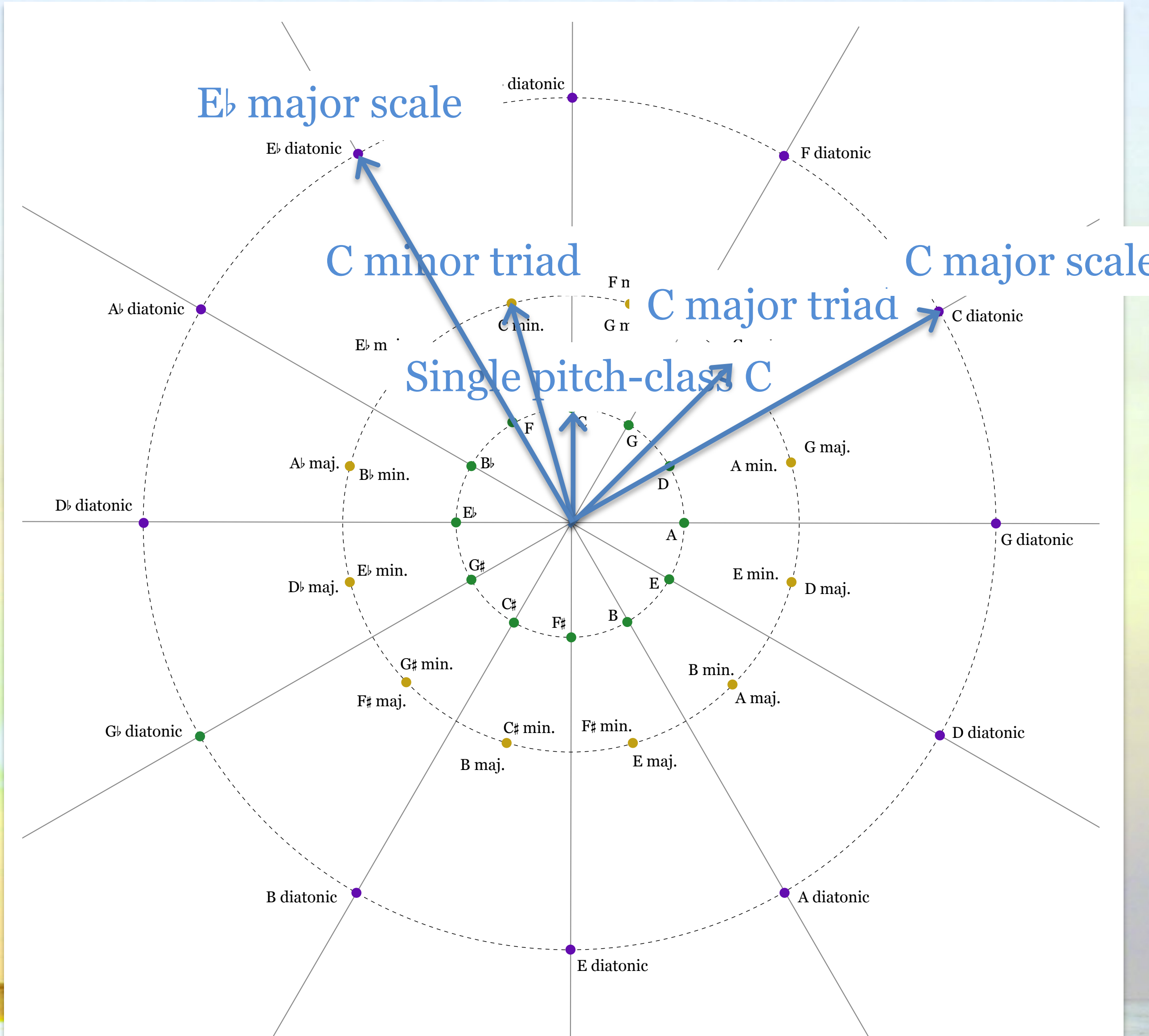


Single component spaces (complex plane)

Example: F_5 space

Distance from the center is the ***magnitude*** of f_5

Angle is the ***phase*** of f_5



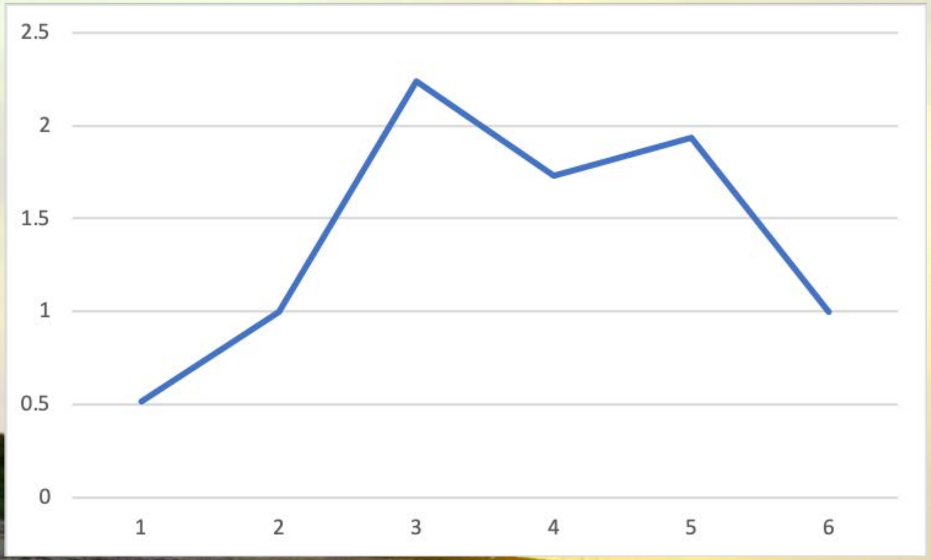
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

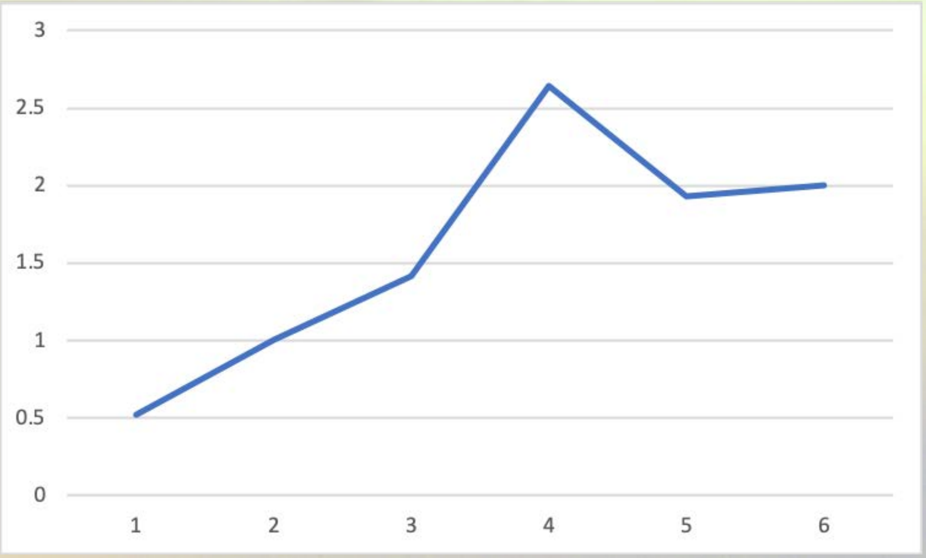
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Major/minor triad



Dominant 7th



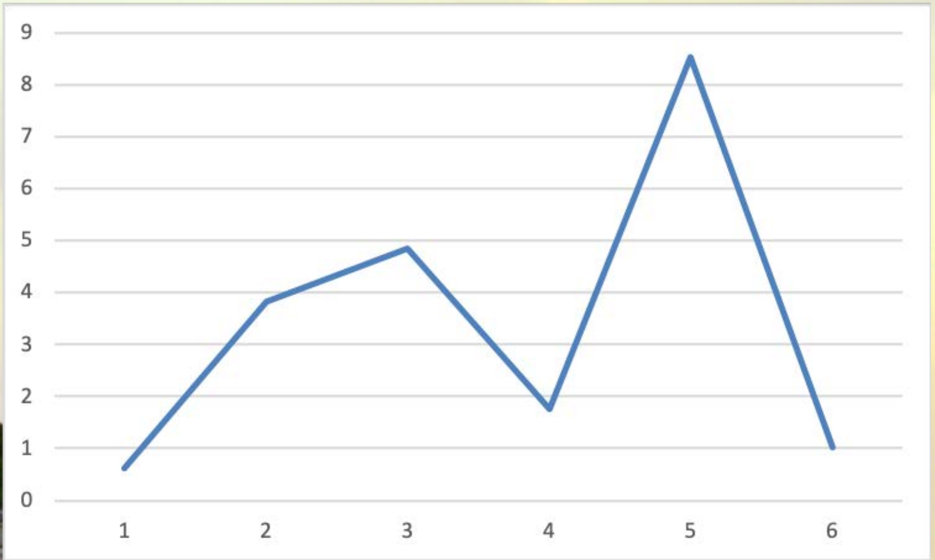
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

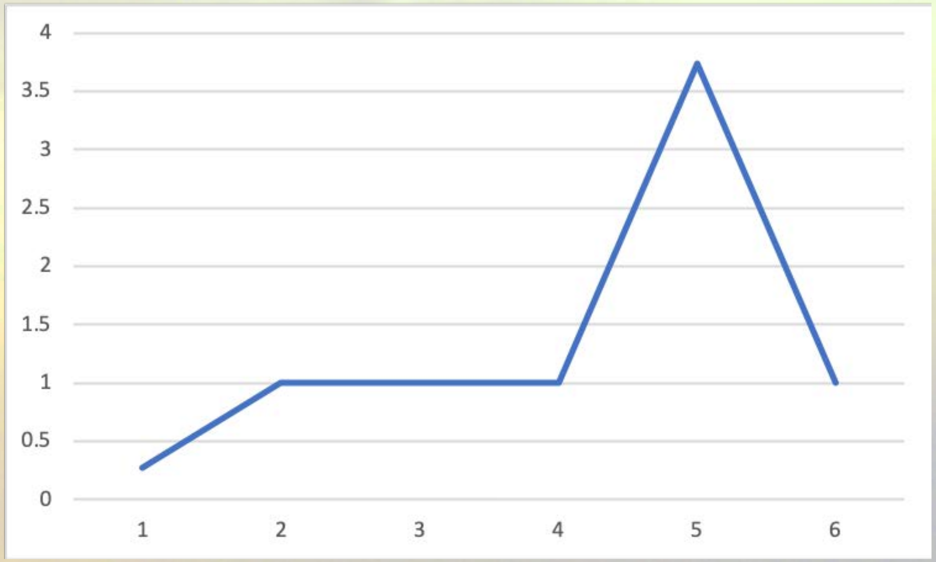
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Krumhansl-Kessler major key

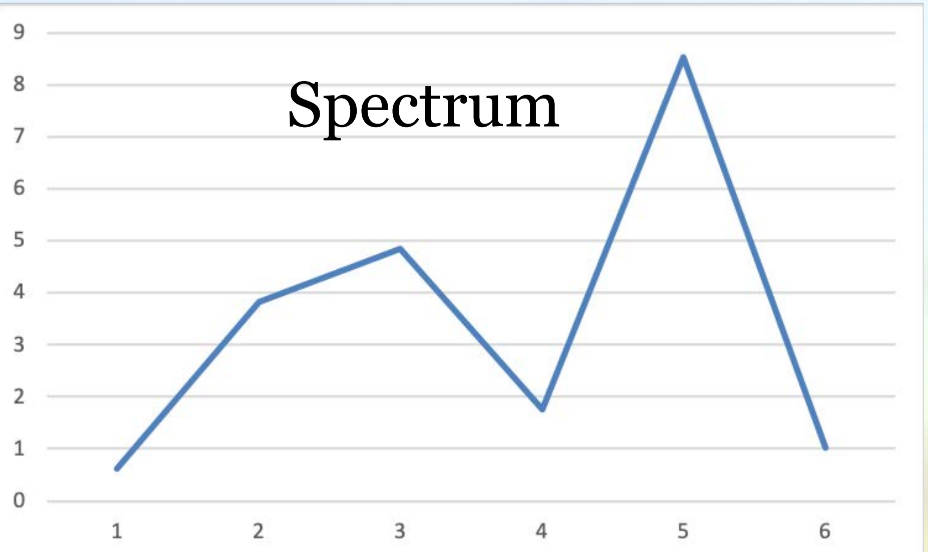


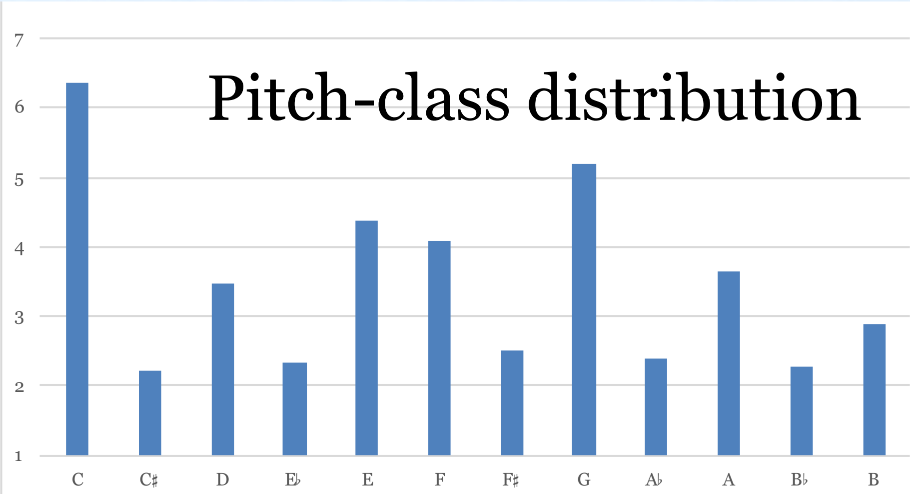
Diatonic scale



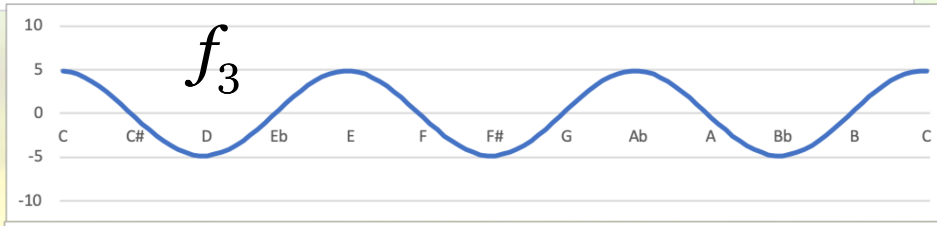
From Spectra to Phase Spaces

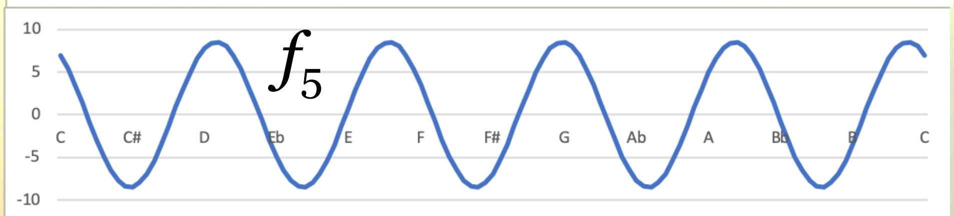
Krumhansl-Kessler major key



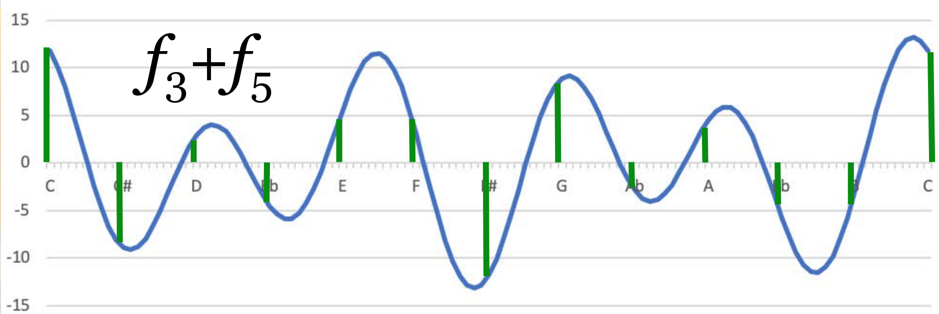


Largest coefficients:



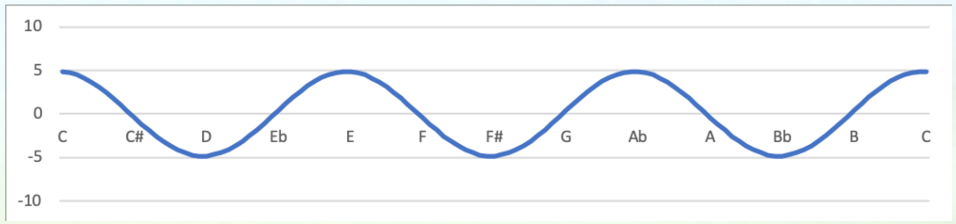


The sum of these gives a good approximation:

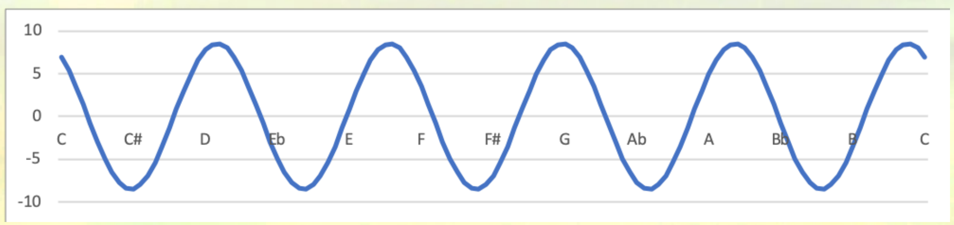


From Spectra to Phase Spaces

Transpositions of the key correspond to phase shifts of the components

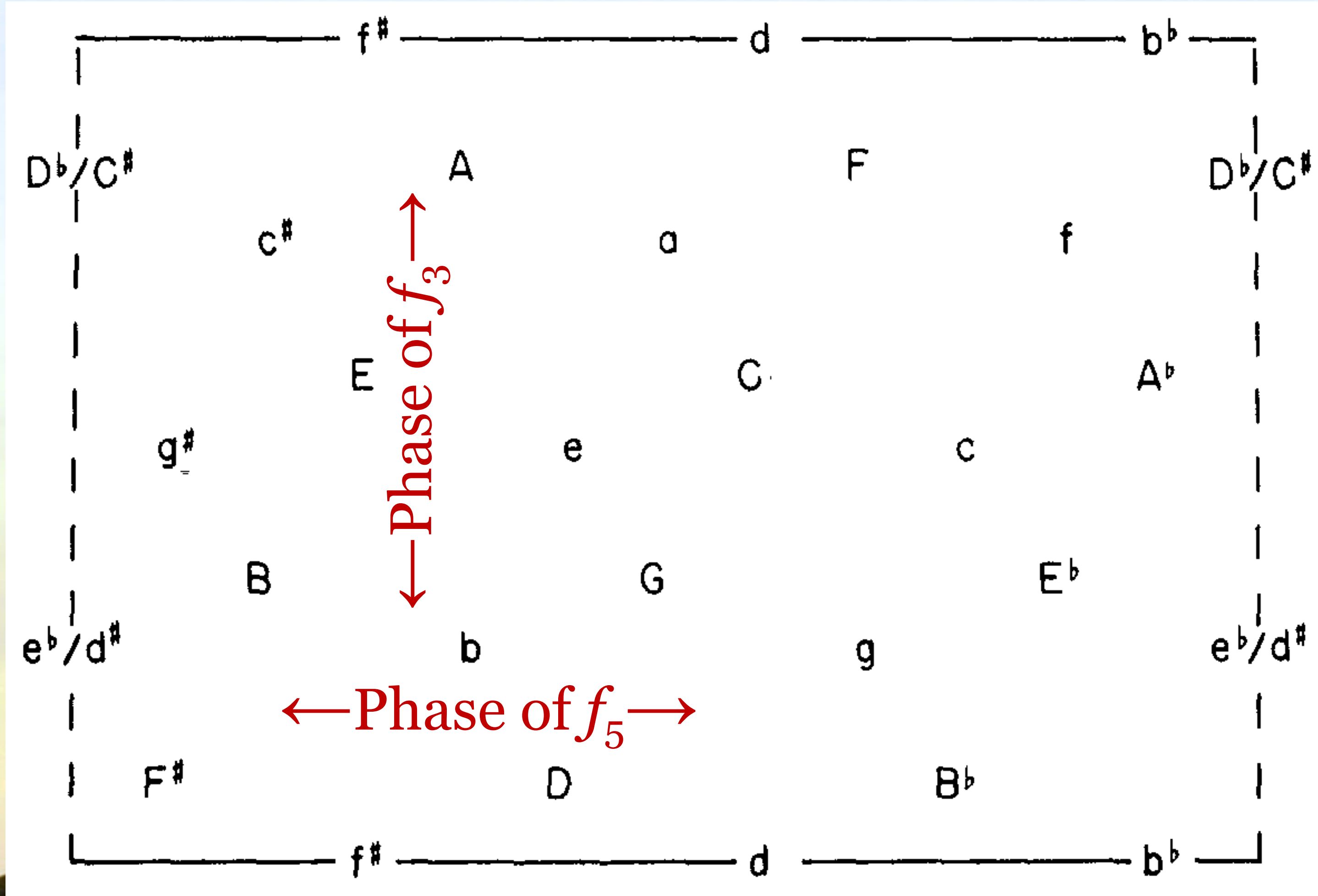


⟵Phase of f_3⟶



⟵Phase of f_5⟶

From Spectra to Phase Spaces



Krumhansl and Kessler Tonal Space

Application to 12-tone rows

Dimensions of atonality

Correlation and Convolution

Convolution theorem:

$$\mathrm{DFT}(\text{cross-correlation}(A, B)) = \overline{\mathrm{DFT}(A)} \times \mathrm{DFT}(B)$$

In mathematical notation (c_k is the cross-correlation and ^ indicates the DFT):

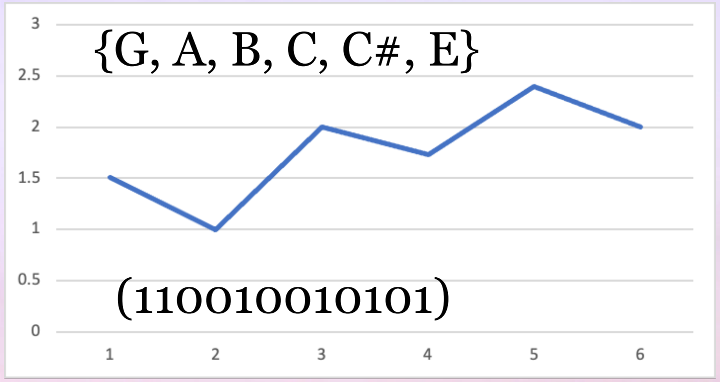
$$\hat{c}_k = \hat{a}_k \hat{b}_k = |\hat{a}_k||\hat{b}_k| e^{i(\varphi_{b_k} - \varphi_{a_k})}$$

i.e., the magnitudes are a *componentwise product* and the phases are a *componentwise difference*

Example

Schoenberg, Op. 28/1

Spectrum of hexachord



a (1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1)

b × (6.4, 2.2, 3.5, 2.3, 4.4, 4.1, 2.5, 5.2, 2.4, 3.7, 2.3, 2.9)

= 6.4+2.2 + 0 + 0 + 0 + 4.1 + 0 + 5.2 +0 + 3.7 + 0 +2.9 = 24.7

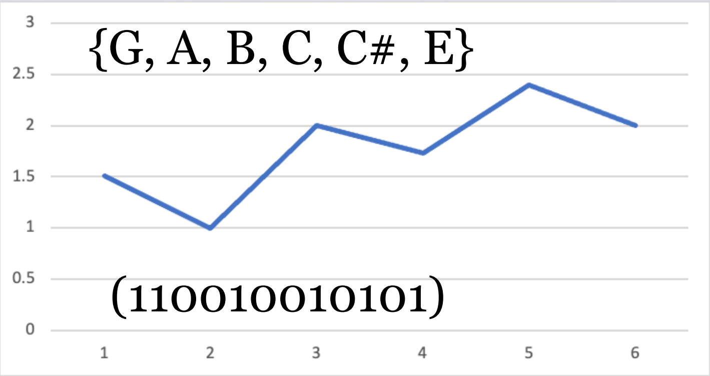
c

C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(24.7,	18.8,	21.6,	17.5,	24.0,	20.8,	18.7,	24.5,	18.3,	24.0,	17.1,	20.8)

Example

Schoenberg, Op. 28/1

Spectrum of hexachord



a (1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1)

b × (2.2, 3.5, 2.3, 4.4, 4.1, 2.5, 5.2, 2.4, 3.7, 2.3, 2.9, 6.4)

= 2.2+3.5 + 0 + 0 + 0 + 2.5 +0 +2.4 + 0 +2.3 +0 + 6.4 = 18.8

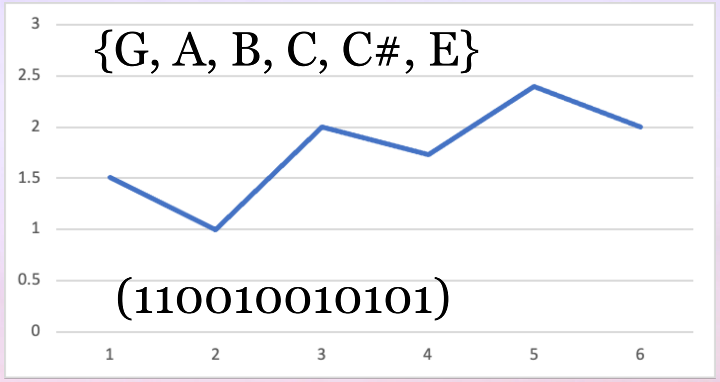
c

C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(24.7,	18.8,	21.6,	17.5,	24.0,	20.8,	18.7,	24.5,	18.3,	24.0,	17.1,	20.8)

Example

Schoenberg, Op. 28/1

Spectrum of hexachord



a (1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1)

b × (3.5, 2.3, 4.4, 4.1, 2.5, 5.2, 2.4, 3.7, 2.3, 2.9, 6.4, 2.2)

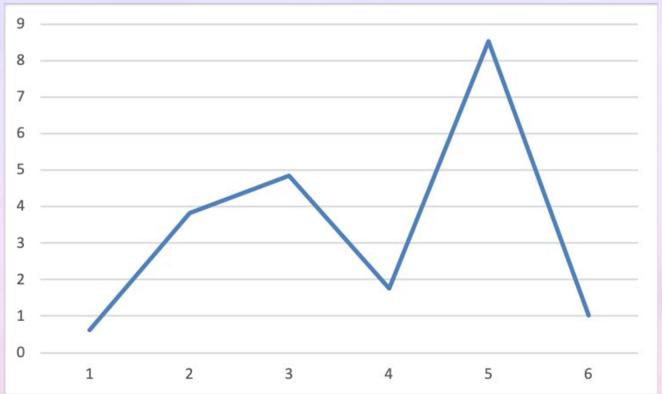
= 3.5+2.3 + 0 + 0 + 0 +5.2 + 0 +3.7 + 0 + 2.9 +0 +2.2 = 21.6

c

C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(24.7,	18.8,	21.6,	17.5,	24.0,	20.8,	18.7,	24.5,	18.3,	24.0,	17.1,	20.8)

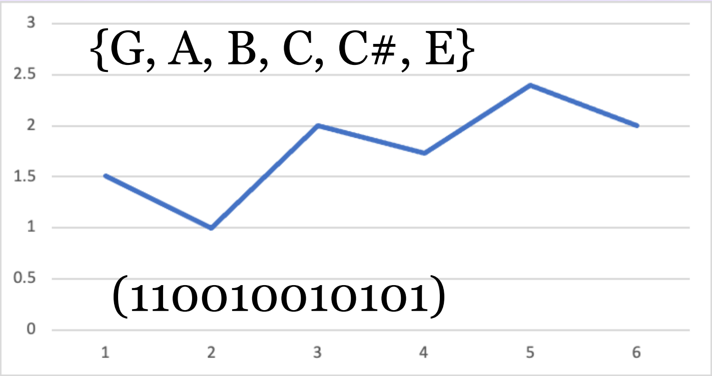
Simplification 1: Multiply Spectra

Krumhansl-Kessler major key



×

Spectrum of hexachord



High values in the cross-correlation will occur when spectra are similar

Simplification 1: Multiply Spectra (Covariance)

Hippel and Huron procedure
Top 20 most tonal

1	Berg	*Lulu* Principal 3
2	Schoenberg	Op.27 no.4
3	Berg	*Lulu* Principal 4
4	Berg	*Lulu* Principal 1
5	Berg	*Lyric Suite* 1
6	Berg	*Lulu* Schoen
7	Berg	“Der Wein”
8	Schoenberg	*Phantasia*
9	Berg	*Lulu* Lulu
10	Schoenberg	Op.50a
11	Berg	*Lulu* Alwa
12	Berg	Violin Concerto
13	Berg	*Lulu* Countess
14	Schoenberg	Op.28-1
15	Schoenberg	Op.29
16	Berg	Chamber Conc. 1-3
17	Berg	*Lulu* Acrobat
18	Schoenberg	Op.35 no.2
19	Schoenberg	Op.42
20	Berg	*Lulu* Principal 2

Spectrum Covariance
Top 20 most tonal

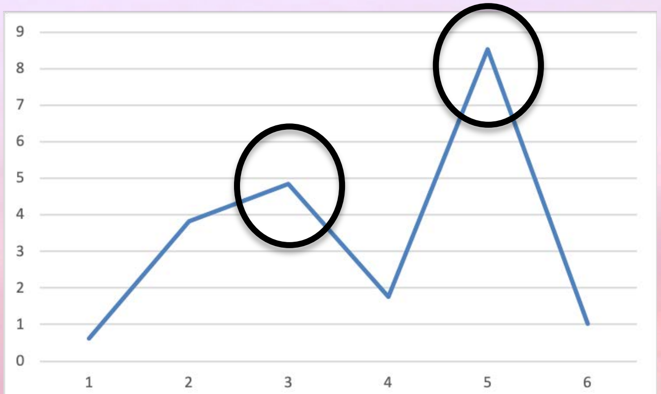
1	Berg	*Lulu* Principal 4
2	Schoenberg	Op.27 no.4
3	Berg	*Lulu* Principal 3
4	Berg	*Lulu* Principal 1
5	Berg	*Lyric Suite* 1
6	Berg	*Lulu* Lulu
7	Berg	*Lulu* Schoen
8	Berg	*Lulu* Countess
9	Schoenberg	Op.50a
10	Berg	*Lulu* Acrobat
11	Berg	*Lulu* Alwa
12	Berg	“Der Wein”
13	Berg	Violin Concerto
14	Schoenberg	*Phantasia*
15	Schoenberg	Op. 45
16	Schoenberg	Op.29
17	Schoenberg	Op.42
18	Schoenberg	Op.23
19	Berg	*Lulu* Principal 2
20	Berg	*Lulu* Chorale

Spearman rank r = .95

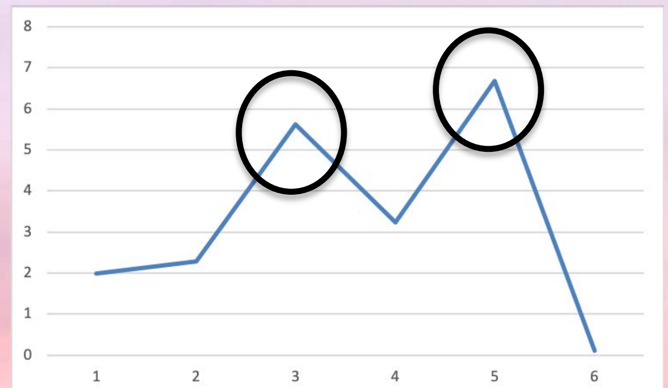
Simplification 2: Maximum $2f_3+3f_5$

Covariance with the KK profiles is determined primarily by $|f_5|$ and $|f_3|$

Krumhansl-Kessler major key



Krumhansl-Kessler minor key



A further simplified measure of tonalness is therefore given by $3|f_5| + 2|f_3|$

Simplification 2: Maximum $2f_3+3f_5$

Hippel and Huron Top 20 most tonal

1	Berg	*Lulu* Principal 3
2	Schoenberg	Op.27 no.4
3	Berg	*Lulu* Principal 4
4	Berg	*Lulu* Principal 1
5	Berg	*Lyric Suite* 1
6	Berg	*Lulu* Schoen
7	Berg	"Der Wein"
8	Schoenberg	*Phantasia*
9	Berg	*Lulu* Lulu
10	Schoenberg	Op.50a
11	Berg	*Lulu* Alwa
12	Berg	Violin Concerto
13	Berg	*Lulu* Countess
14	Schoenberg	Op.28-1
15	Schoenberg	Op.29
16	Berg	Chamber Conc. 1-3
17	Berg	*Lulu* Acrobat
18	Schoenberg	Op. 35 no.2
19	Schoenberg	Op. 42
20	Berg	*Lulu* Principal 2

Spectrum Covariance Top 20 most tonal

1	Berg	*Lulu* Principal 4
2	Schoenberg	Op.27 no.4
3	Berg	*Lulu* Principal 3
4	Berg	*Lulu* Principal 1
5	Berg	*Lyric Suite* 1
6	Berg	*Lulu* Lulu
7	Berg	*Lulu* Schoen
8	Berg	*Lulu* Countess
9	Schoenberg	Op.50a
10	Berg	*Lulu* Acrobat
11	Berg	*Lulu* Alwa
12	Berg	"Der Wein"
13	Berg	Violin Concerto
14	Schoenberg	*Phantasia*
15	Schoenberg	Op. 45
16	Schoenberg	Op. 29
17	Schoenberg	Op. 42
18	Schoenberg	Op. 23
19	Berg	*Lulu* Principal 2
20	Berg	*Lulu* Chorale

Spearman rank r = .95

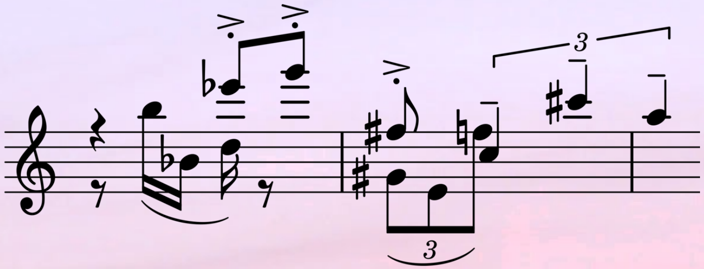
$2f_3 + 3f_5$ Top 20

1	Schoenberg	Op.27 no.4
2	Berg	*Lulu* Principal 3
3	Berg	*Lulu* Principal 4
4	Berg	*Lulu* Principal 1
5	Berg	*Lyric Suite* 1
6	Schoenberg	Op. 29
7	Berg	*Lulu* Lulu
8	Berg	*Lulu* Scoen
9	Berg	Violin Concerto
10	Webern	Op. 24
11	Berg	"Der Wein"
12	Berg	*Lulu* Countess
13	Schoenberg	*Phantasia*
14	Schoenberg	Op. 50a
15	Schoenberg	Op. 50c
16	Schoenberg	Op. 37
17	Schoenberg	Op. 28 no. 1
18	Berg	*Lulu* Alwa
19	Berg	*Lulu* Chorale
20	Schoenberg	Op. 42

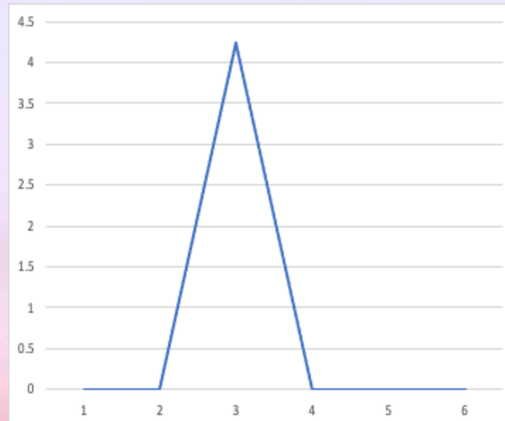
Spearman rank r = .91

Hexatonic Rows

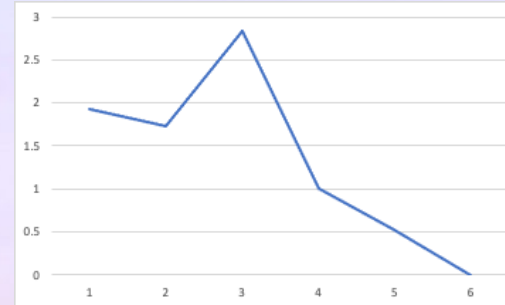
Webern Op. 24
Concerto for Nine Instruments



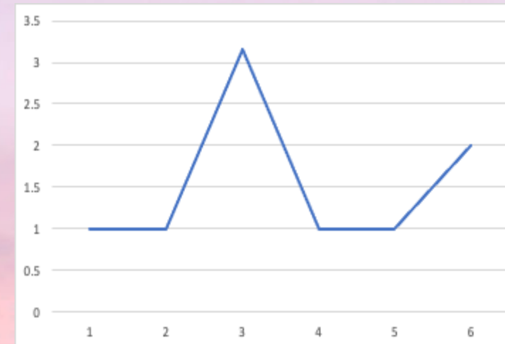
Hexachord:



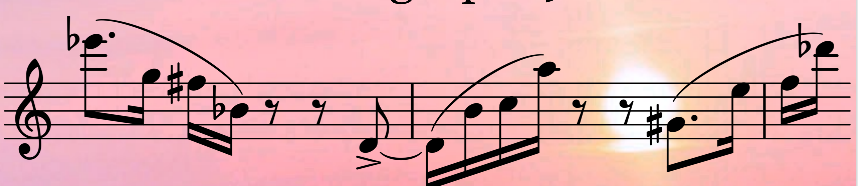
Initial tetrachord:



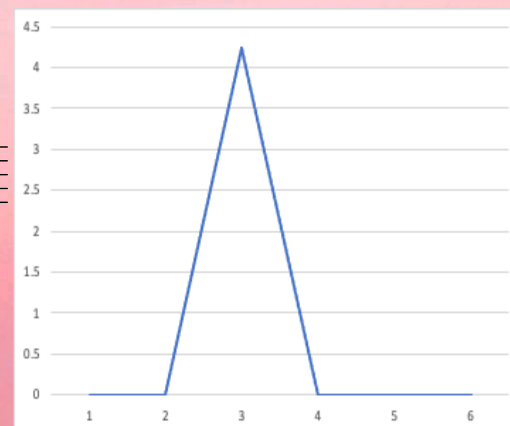
Final tetrachord:



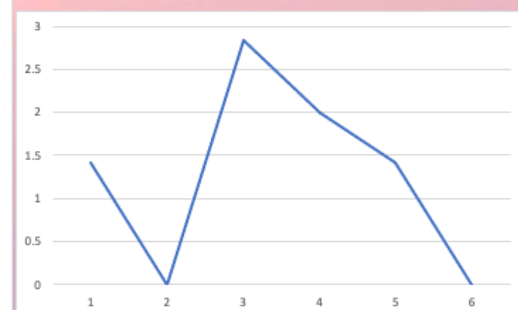
Schoenberg Op. 29 *Suite*



Hexachord:

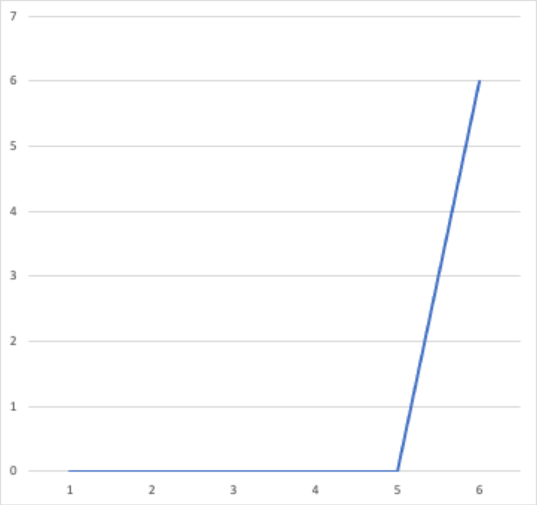


Both tetrachords:



Highly atonal rows

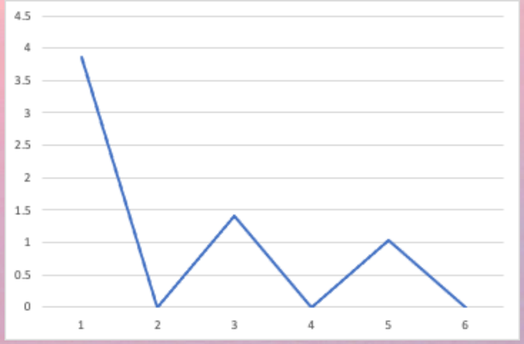
Hexachord spectrum (Whole tone):



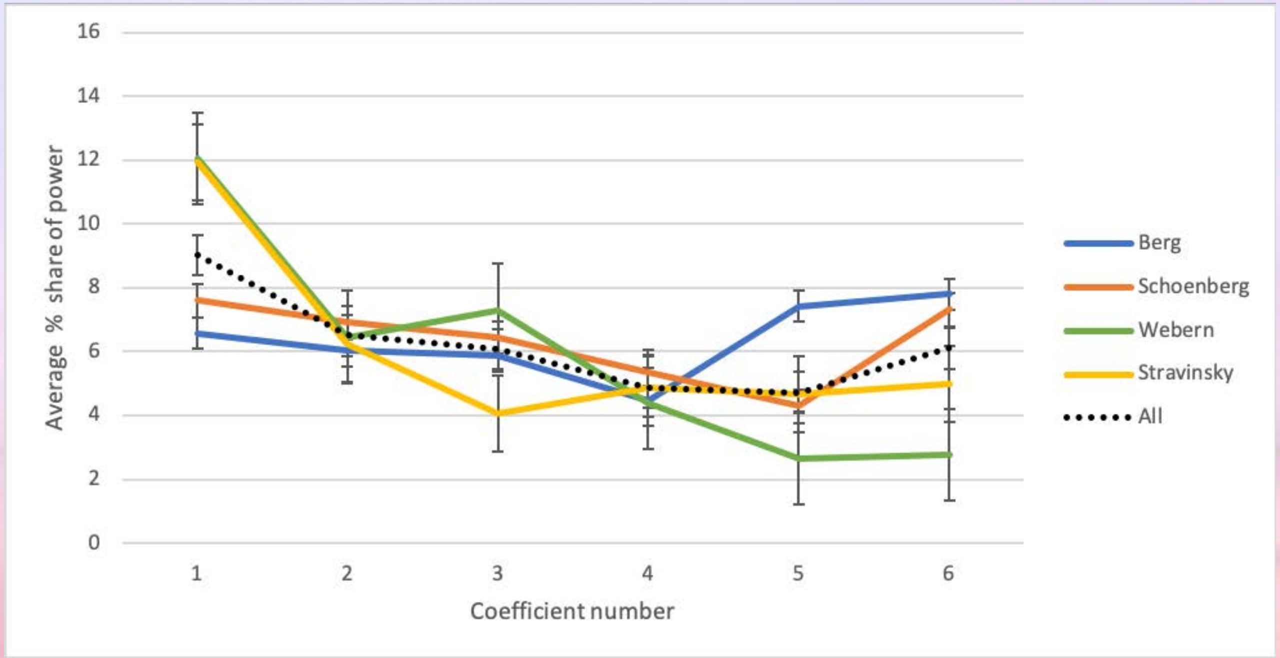
Schoenberg Op. 48 no. 3

Webern Op. 21 Symphony

Hexachord spectrum (Chromatic):



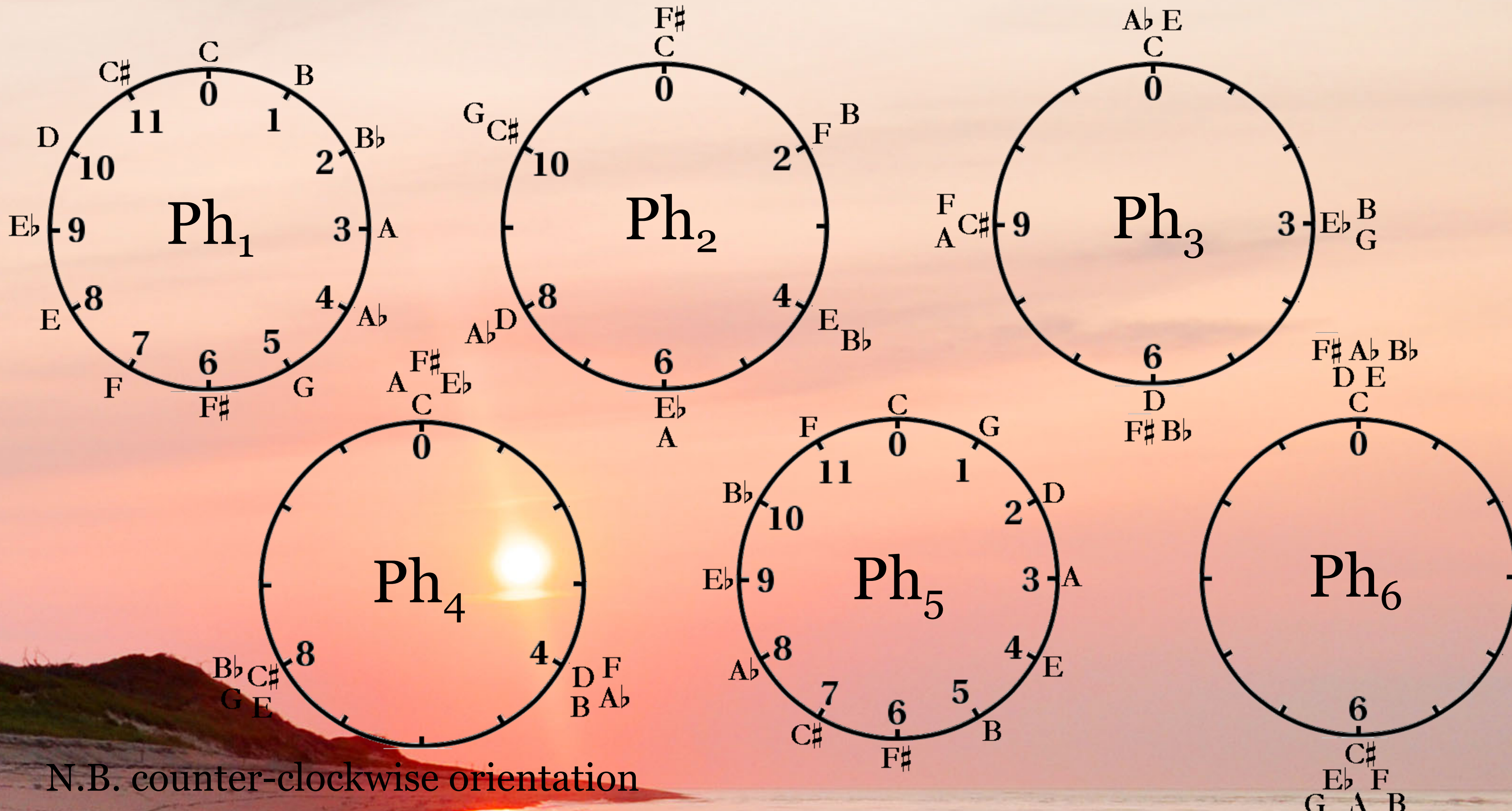
Average spectrum by composer



- **Berg's** rows are more tonal because they are **more diatonic**
- There are two strategies to construct more atonal rows:
  - —High f_1 (chromatic concentration), typical of **Webern** and **Stravinsky**
  - —High f_6 (whole tone quality), typical of **Schoenberg**

Phase Spaces

One-dimensional phase spaces are Quinn's *Fourier balances*, superimposed n-cycles created by multiplying the pc-circle by n.



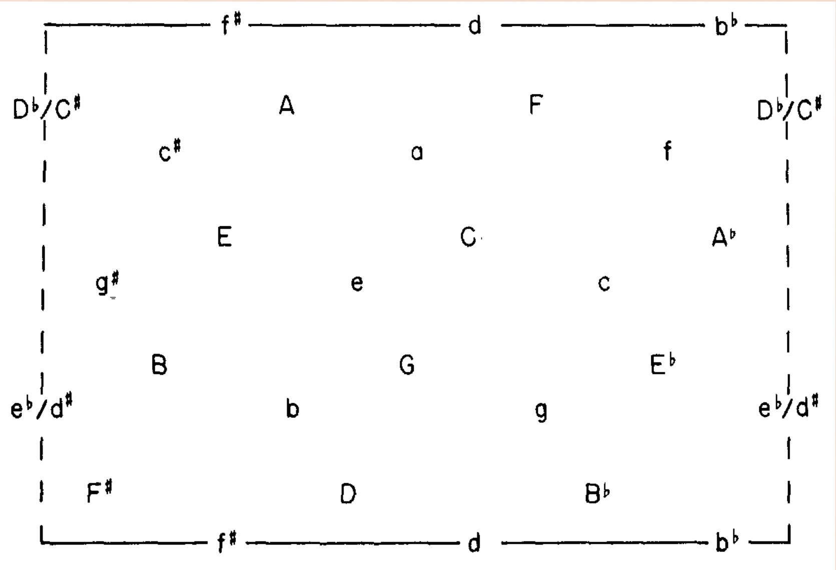
N.B. counter-clockwise orientation

Krumhansl's Tonal Space

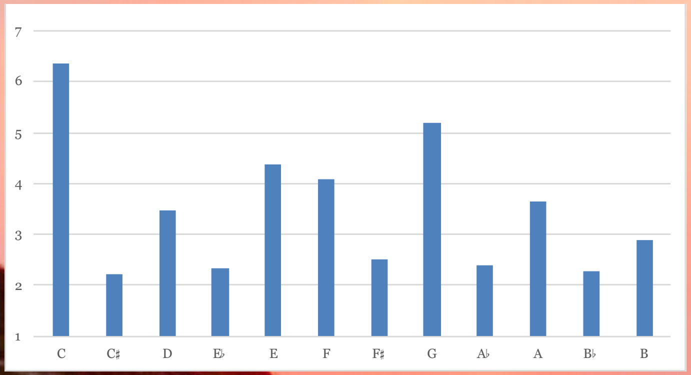
Krumhansl's space is based on **correlations** between ***key profiles.***

Key profiles may be derived from

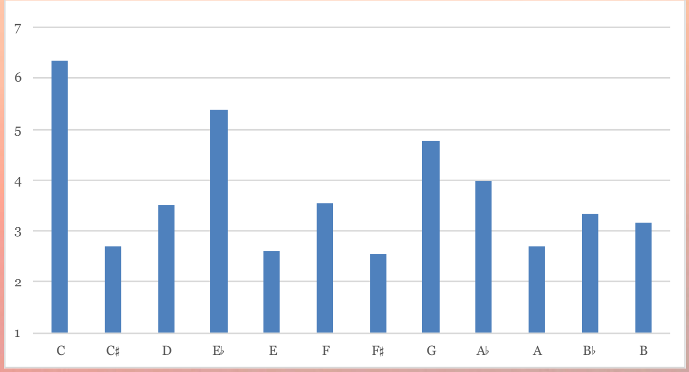
- Listener rankings of stability of tones in a given context,
- Frequency of occurrence of tones in the given key,
- etc.



Krumhansl and Kessler's key profiles based on listener ratings

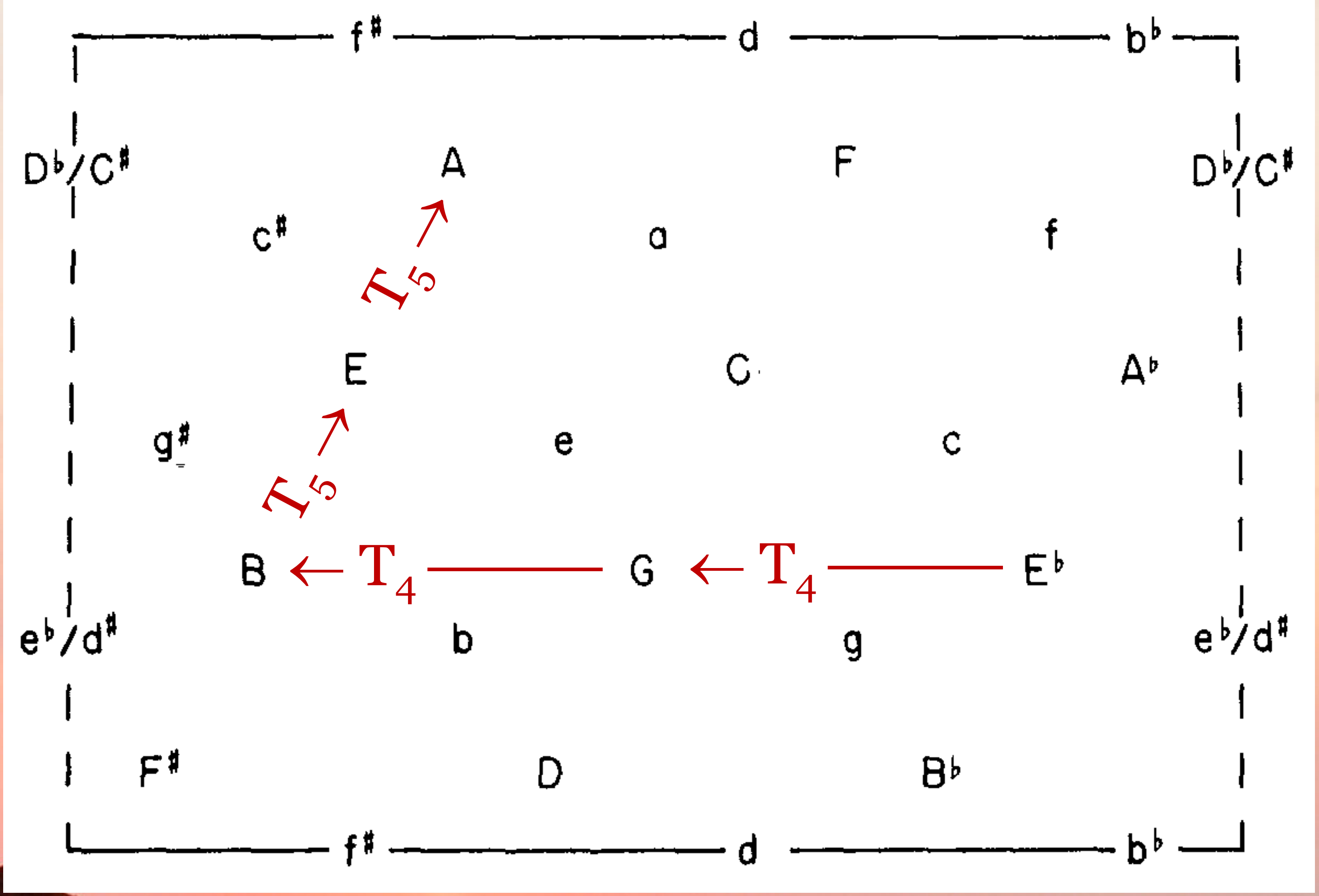


C major



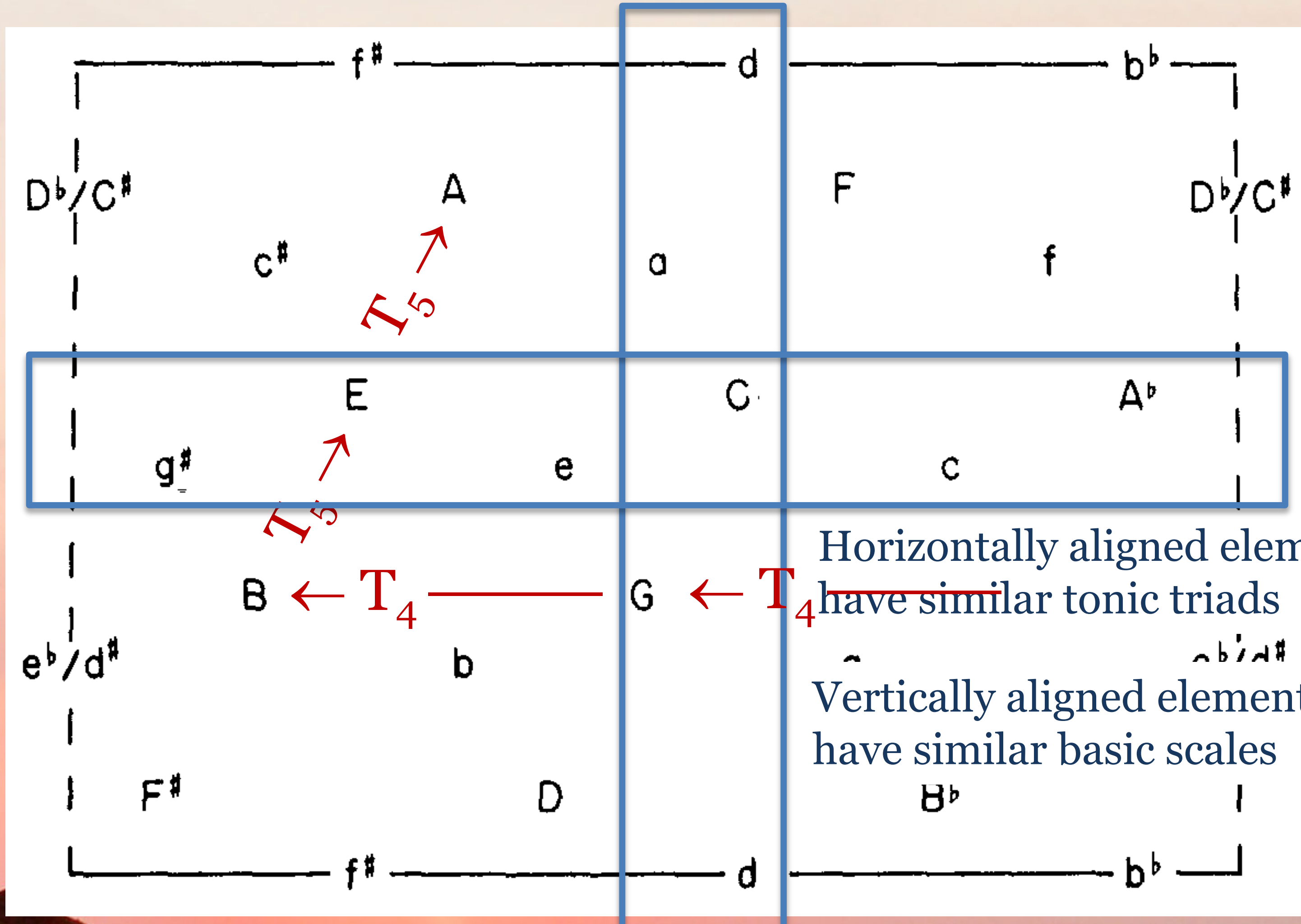
C minor

Krumhansl's Tonal Space



Toroidal map of key profiles from Krumhansl and Kessler 1982

Krumhansl's Tonal Space



Horizontally aligned elements have similar tonic triads

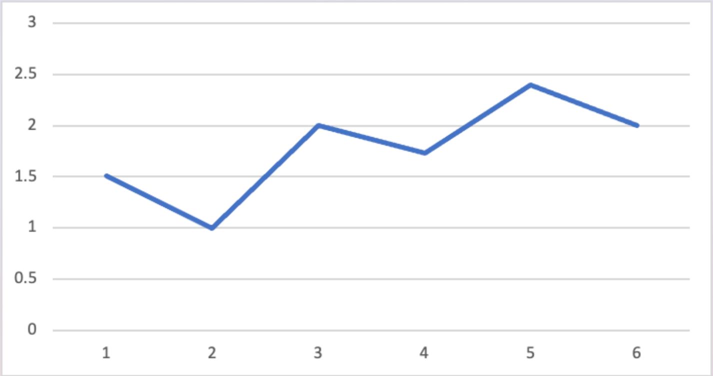
Vertically aligned elements have similar basic scales

Toroidal map of key profiles from Krumhansl and Kessler 1982

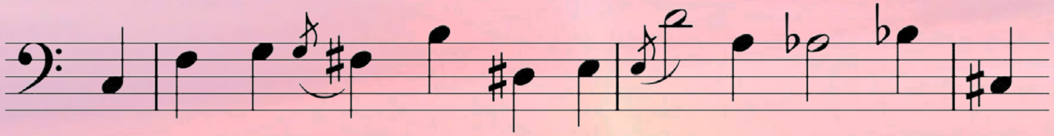
Examples

Schoenberg, Op. 28/1

Spectrum of hexachord



Berg, “Akrobat” row



Spectrum of hexachord