Rhythmic Qualities, Meter, and Reich's Cyclic Canons

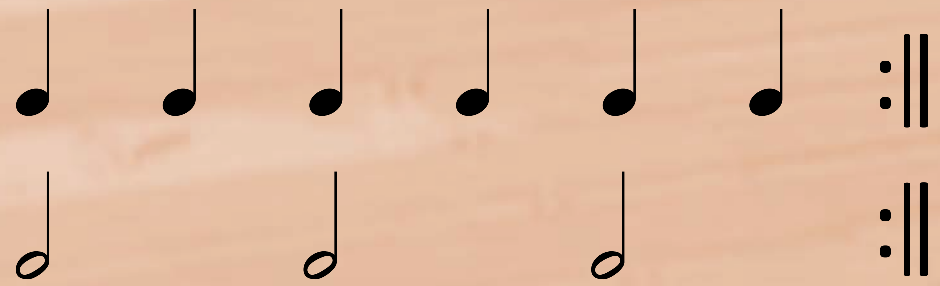
Presentation to SMT 2019, Columbus, Ohio

Jason Yust, Boston University

Goals

- Introduce a concept of ***rhythmic qualities*** analogous to Quinn's (2006) harmonic qualities.
- Relate rhythmic qualities to meter.
- Describe Reich's cyclic rhythms using rhythmic ***spectra.***
- Demonstrate mathematical properties of the DFT and their significance to understanding cyclic rhythms.

Pure metrical rhythms as periodic functions



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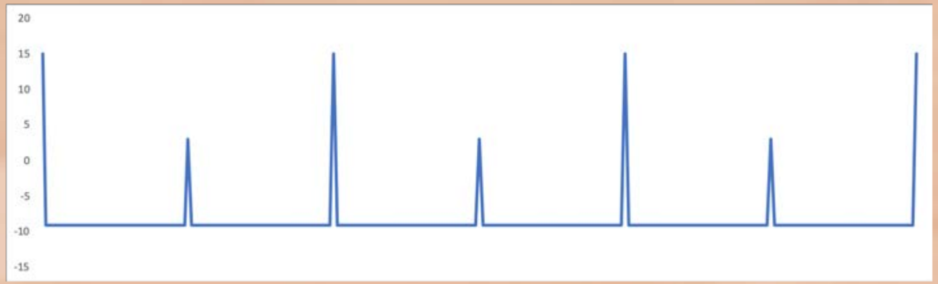

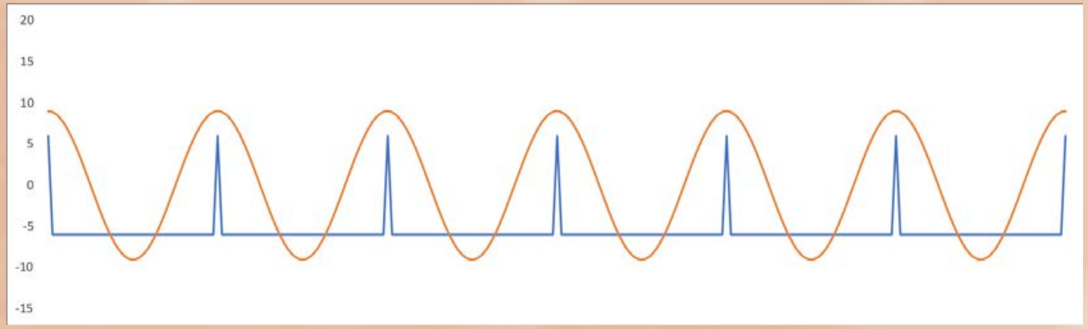

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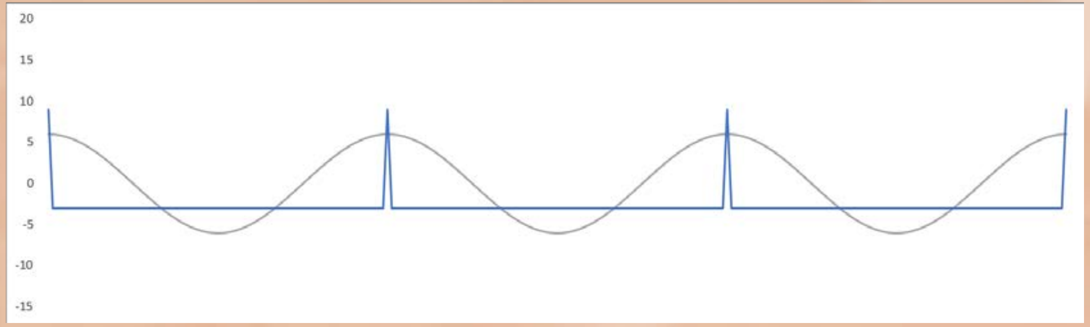

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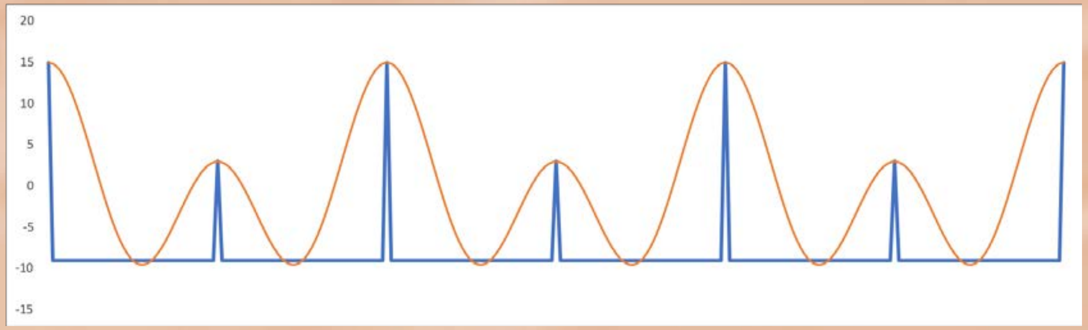

Pure metrical rhythms as periodic functions

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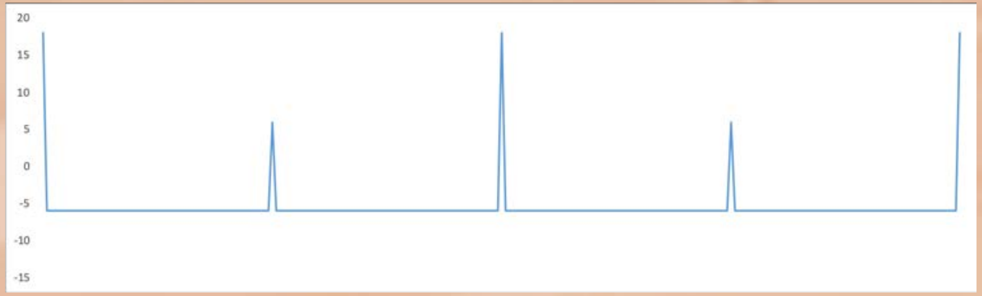

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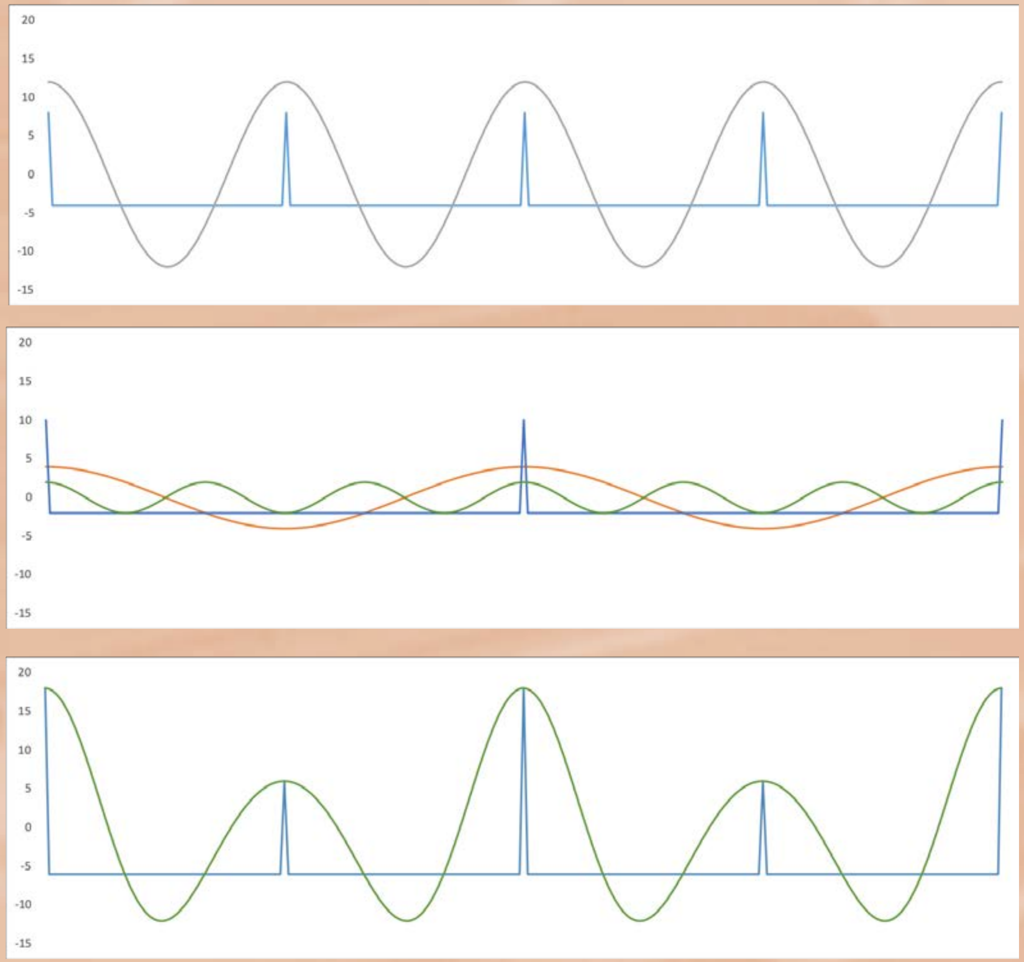


Rhythmic Qualities

Cycle of 12

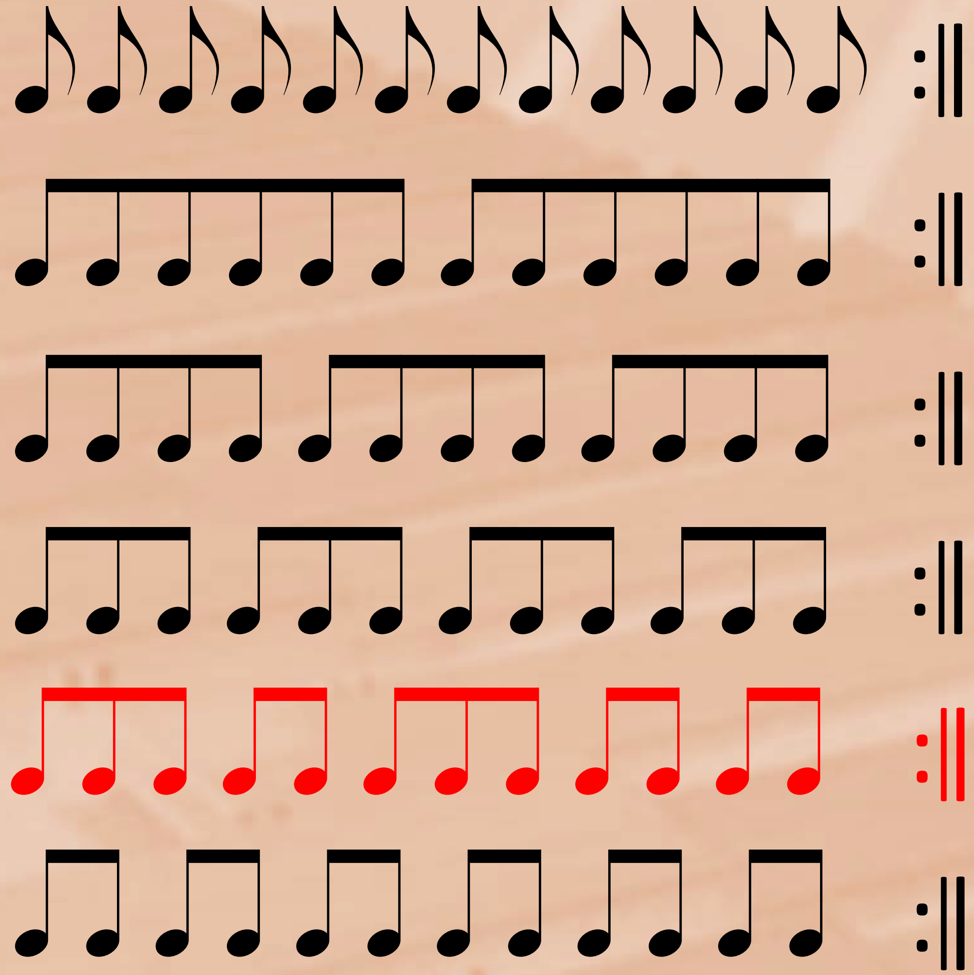
2 Subcycles

3 Subcycles

4 Subcycles

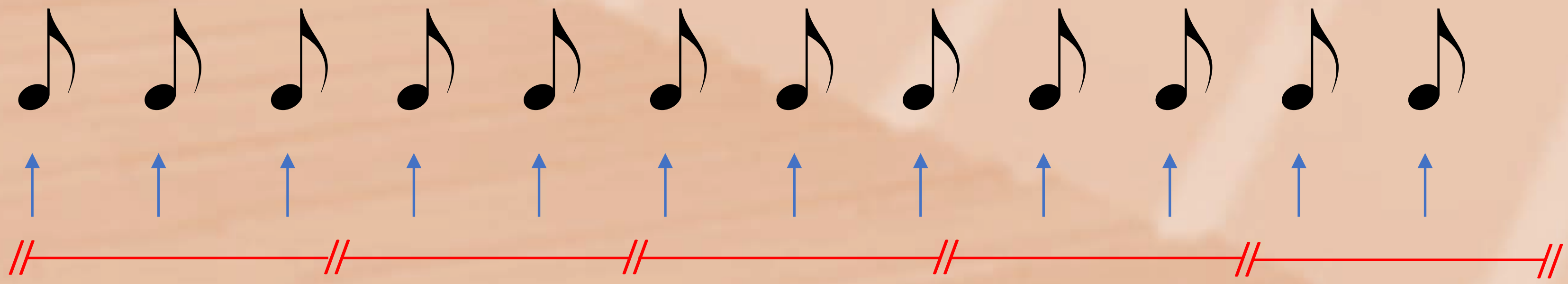
5 Subcycles?

6 Subcycles



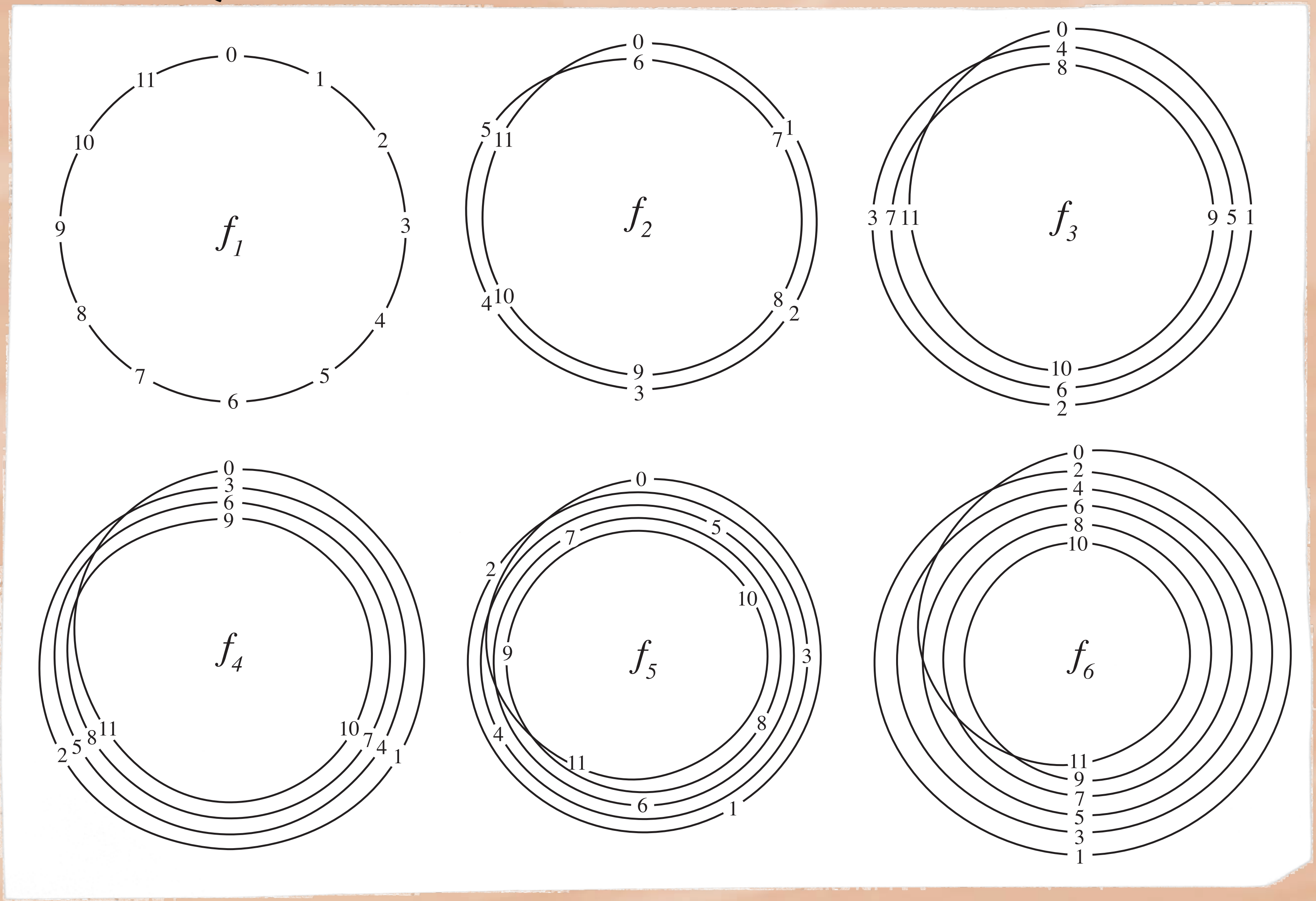
Rhythmic Qualities

Full cycle divided into 5 equal subcycles:



All 12 eighth-note timepoints fall in different positions within the subcycle.

Rhythmic Qualities



Rhythmic Qualities

The *discrete Fourier transform* (DFT) converts any cyclic rhythm on a grid of *n* time points to a set of *n* rhythmic qualities by

(1) Wrapping the complete cycle into *k* subcycles (for all *k* s.t. $0 < k < n$).

(2) Summing the rhythm in the vector space (f_k) for each of these subcycles.

The results for complementary f_k and f_{n-k} are equivalent, so we consider only f_1 to $f_{u/2}$.

Each f_k is represented by a *magnitude* and a *phase*, representing the size of quality *k* and its orientation in the cycle.

DFT on pitch-class sets

David Lewin (1959), "Re: Intervallic Relations between Two Collections of Notes." *Journal of Music Theory* 3/2.

Ian Quinn (2006), "General Equal-Tempered Harmony." *Perspectives of New Music* 44/2–45/1

Emmanuel Amiot (2007), "David Lewin and Maximally Even Sets." *Journal of Mathematics and Music* 1/3

Jason Yust (2015), "Applications of DFT to the Theory of Twentieth-Century Harmony." *Mathematics and Computation in Music, Fifth International Conference (MCM 2015),* ed. T. Collins, D. Meredith, A. Volk (Springer)

——— (2016), "Special Collections: Renewing Set Theory." *Journal of Music Theory* 60/2

DFT on pitch-class sets and rhythms

Emmanuel Amiot (2009), "New Perspectives on Rhythmic Canons and the Spectral Conjecture" *JMM* 3/2.

——— (2011), "Structures, Algorithms, and Algebraic Tools for Rhythmic Canons." *PNM* 49/2

Emmanuel Amiot and William Sethares (2011), "An Algebra for Periodic Rhythms and Scales" *JMM* 5/3.

Emmanuel Amiot (2016), *Music through Fourier Space* (Springer).

Andrew Milne, David Bulger, Steffen Herff, and William Sethares (2015), "Perfect Balance: A Novel Organizational Principle for Musical Scales and Meters." *MCM 2015*.

Andrew Milne, David Bulger, and Steffen Herff (2017), "Exploring the Space of Perfectly Balanced Rhythms and Scales." *JMM* 11/2.

Matthew Chiu (2018), "Form as Meter: Metric forms through Fourier Space," Master's Thesis, Boston University.

On the Beat-Class–Pitch-Class Analogy

Milton Babbitt (1962), "Twelve-Tone Rhythmic Structure and the Electronic Medium." *PNM* 1/1.

Jeff Pressing (1983), "Cognitive Isomorphisms in Pitch and Rhythm in World Musics: West Africa, the Balkans, and Western Tonality." *Studies in Music* 17.

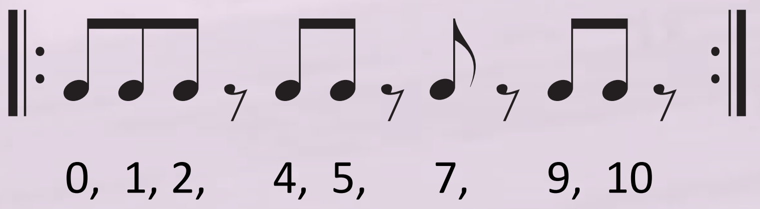
Richard Cohn (1992), "Transpositional Combination of Beat-Class Sets in Steve Reich's Phase-Shifting Music." *PNM* 30/2.

——— (2016), "A Platonic Model for Funky Rhythms." *Music Theory Online* 22/2.

Jay Rahn (1996), "Turning the Analysis Around: Africa-Derived Rhythms and Europe-Derived Music Theory." *Black Music Research Journal* 16/1.

John Roeder (2003), "Beat-Class Modulation in Steve Reich's Music." *Music Theory Spectrum* 25/2.

Example: Reich's signature rhythm (*Clapping Music*)



Used in . . .
Clapping Music (1972), *Music for Pieces of Wood* (1973), *Music for 18 Musicians* (1974–6), *Desert Music* (1984), *Sextet* (1985), *Three Movements* (1986), *Electric Counterpoint* (1987), . . .

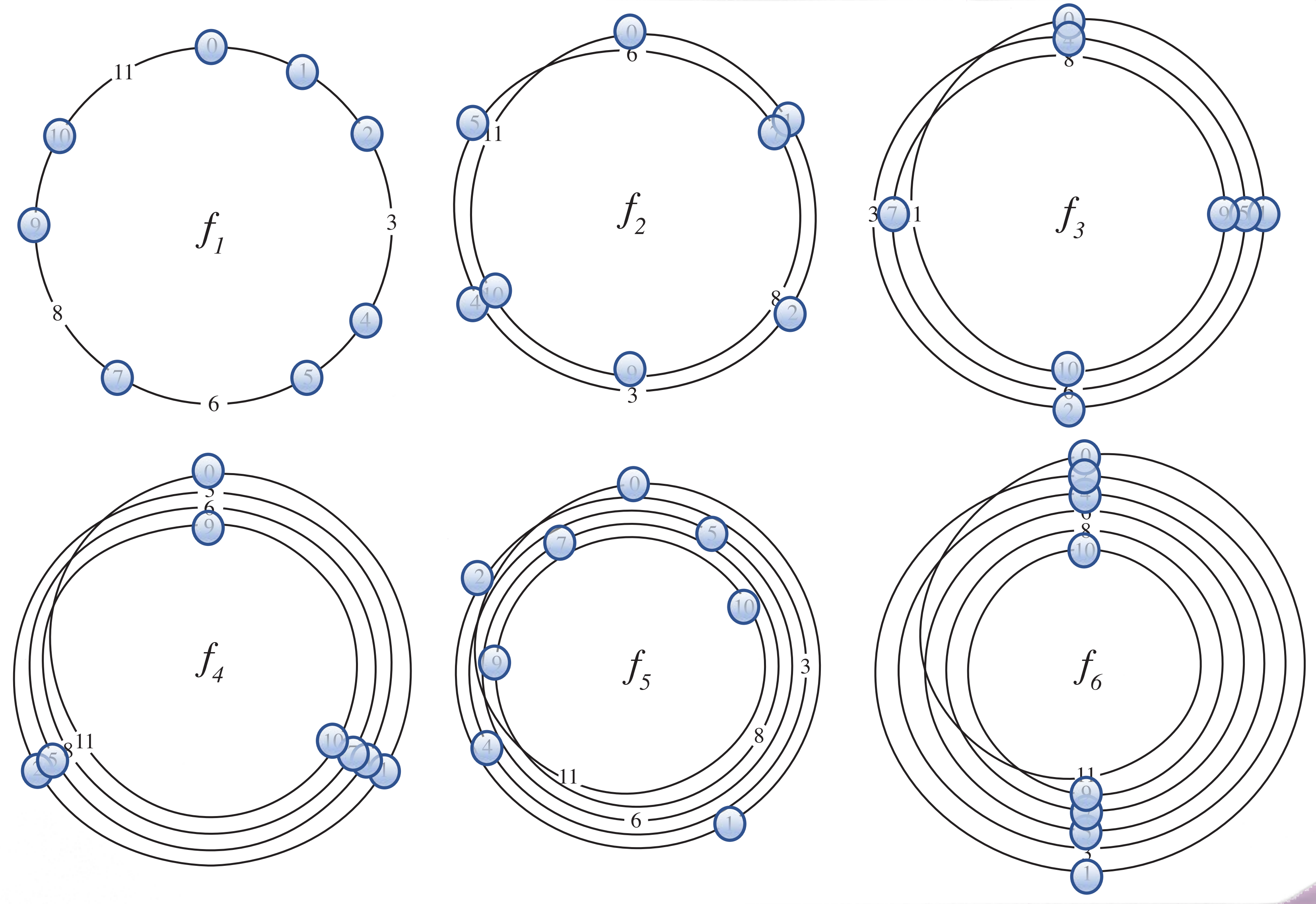
Clapping Music

Music for Pieces of Wood

Sextet I

Example: Reich's signature rhythm

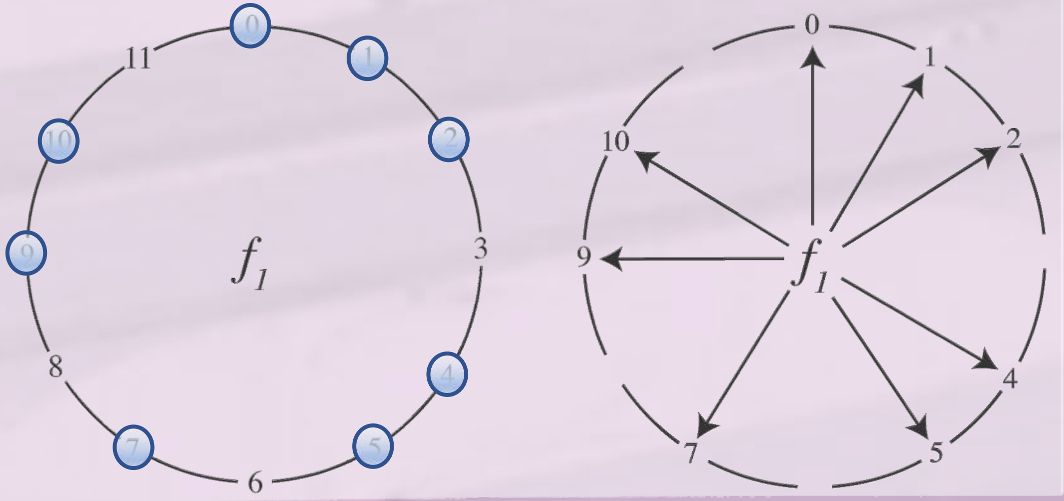
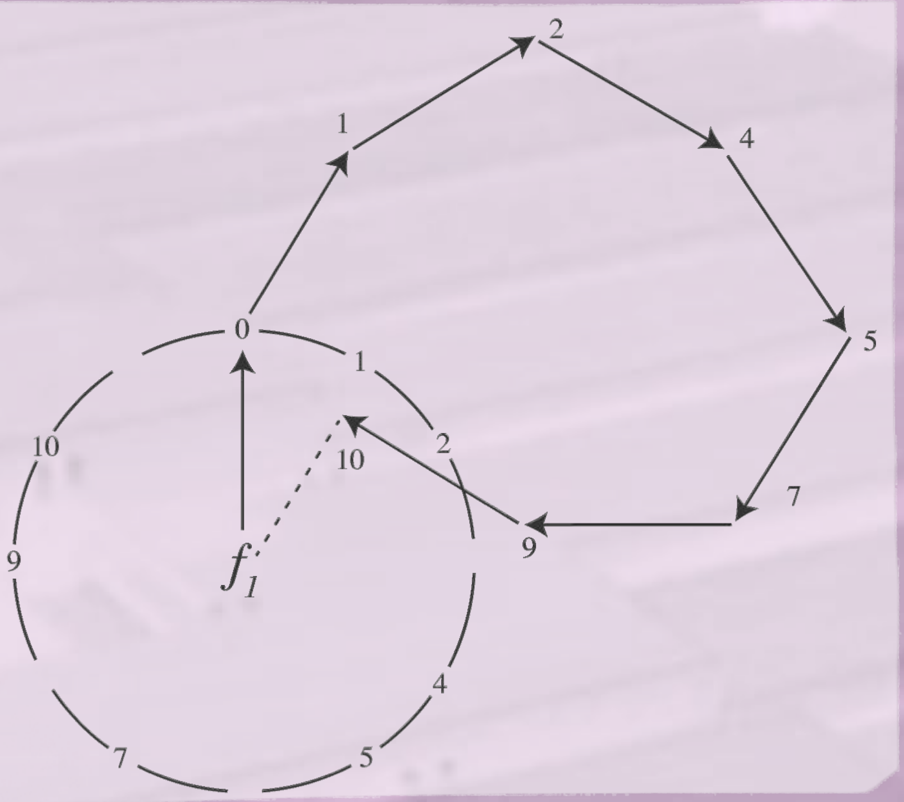
0, 1, 2, 4, 5, 7, 9, 10



Example: Reich’s signature rhythm

0, 1, 2, 4, 5, 7, 9, 10

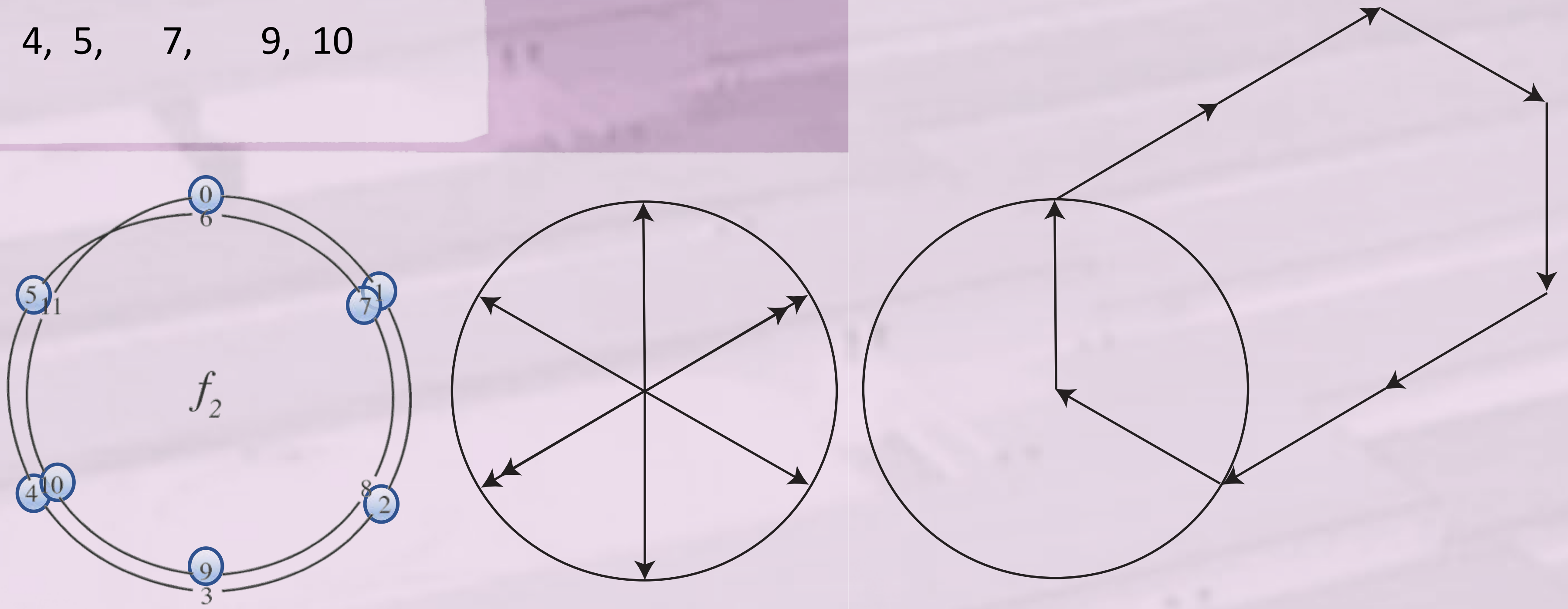
Vector addition for f_1

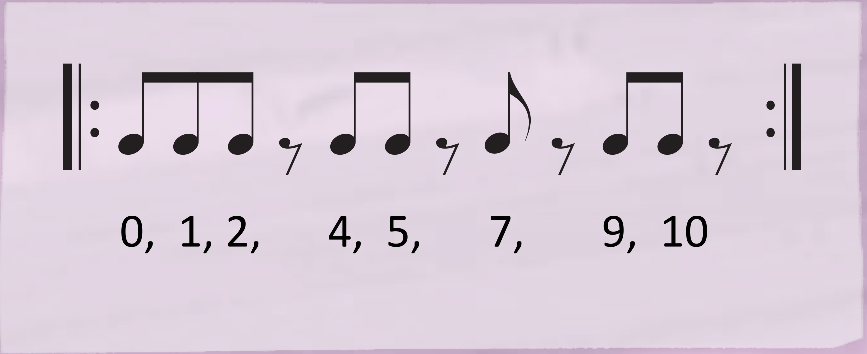
Example: Reich's signature rhythm

Vector addition for f_2

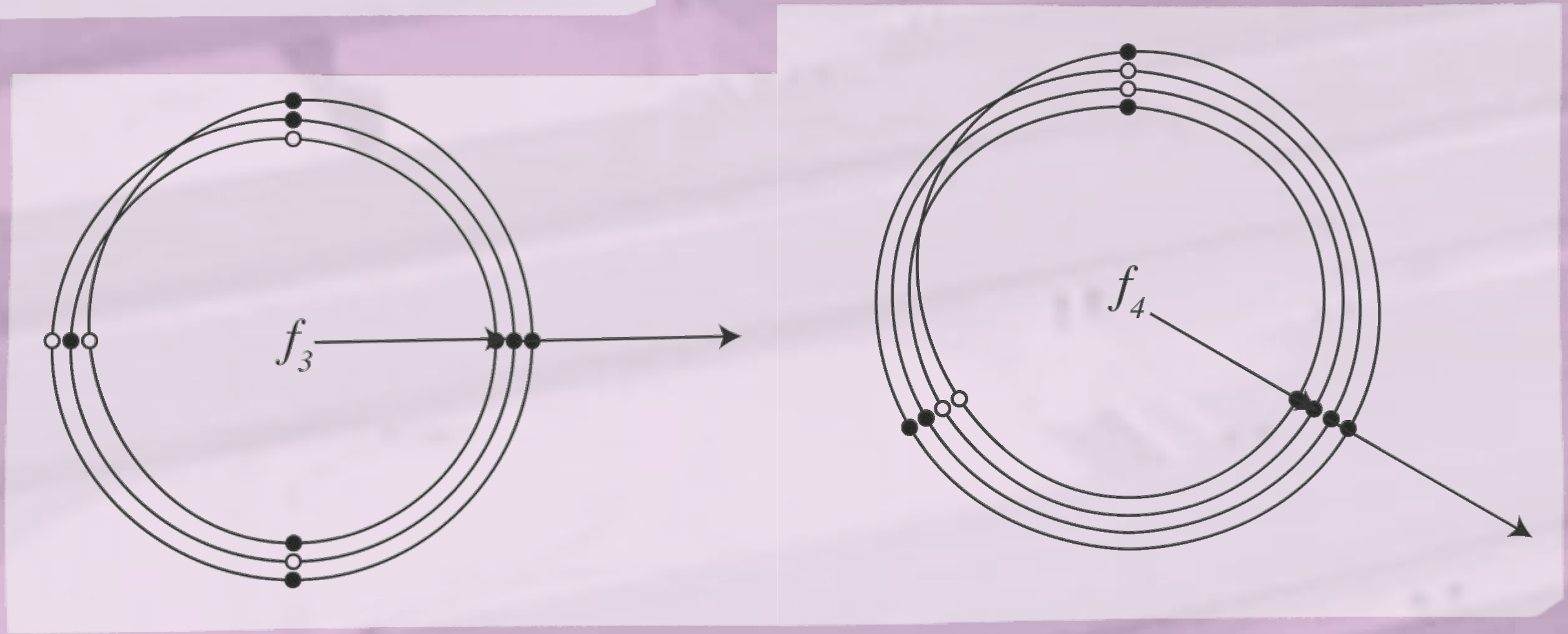
0, 1, 2, 4, 5, 7, 9, 10



Example: Reich's signature rhythm

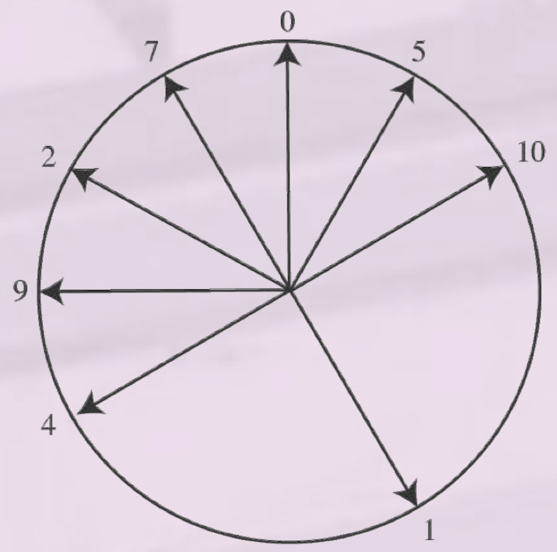


Vector sums for f_3 and f_4

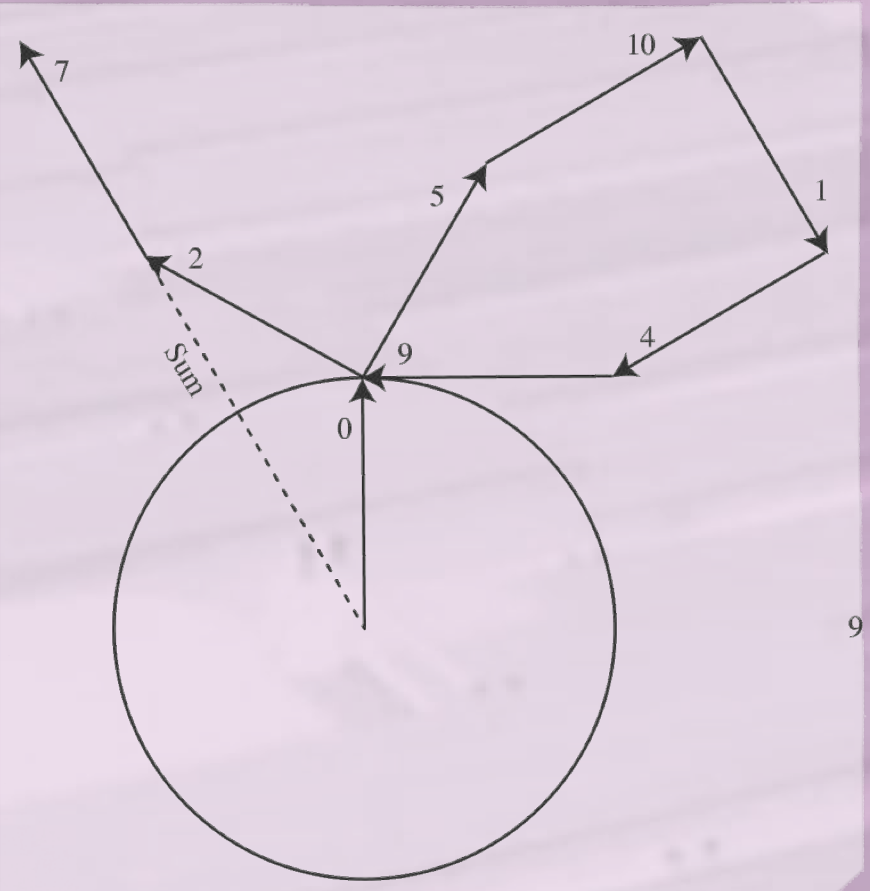


Example: Reich's signature rhythm

0, 1, 2, 4, 5, 7, 9, 10



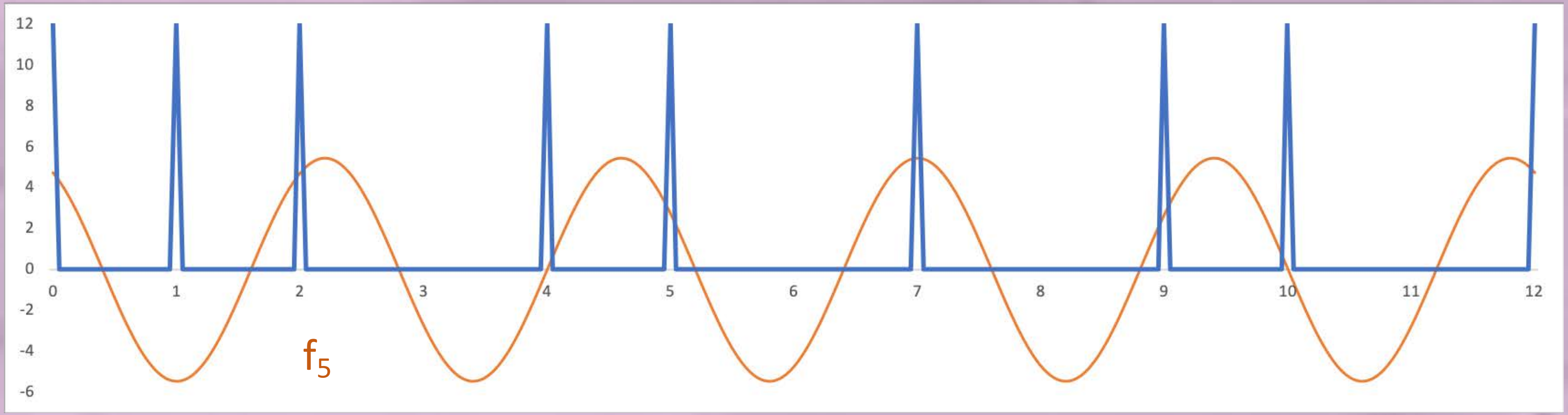
Vector sum for f_5



Example: Reich's signature rhythm

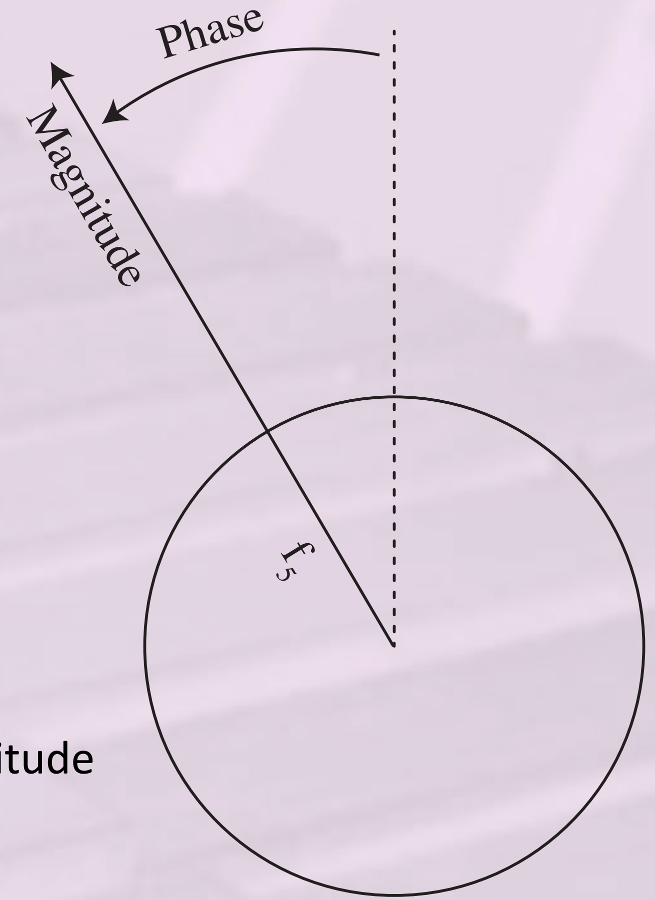
0, 1, 2, 4, 5, 7, 9, 10

Rhythm



Phase ←

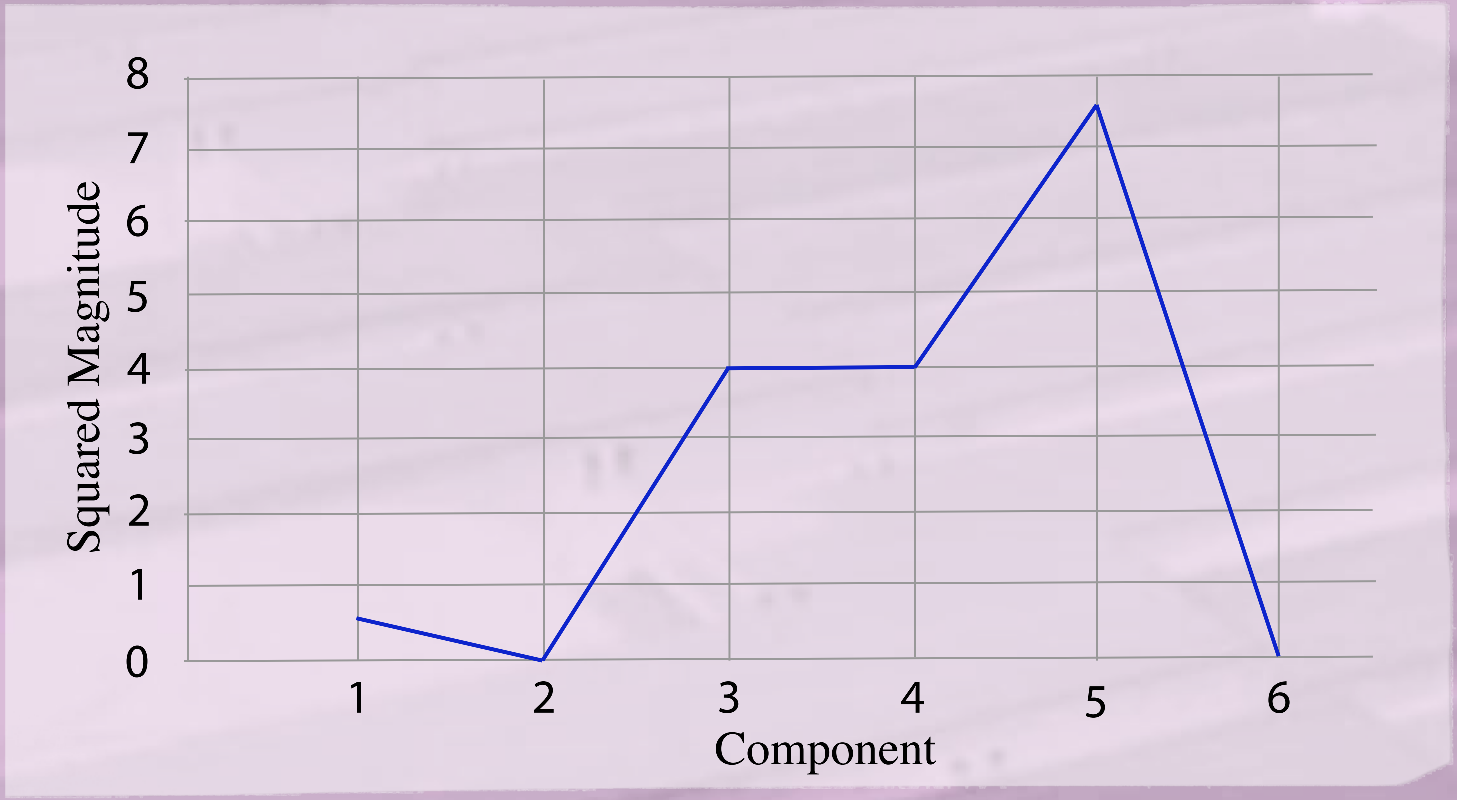
↑ Magnitude



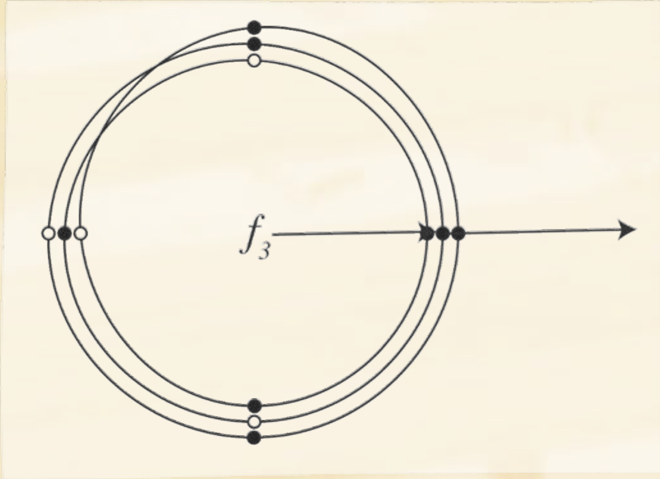
Example: Reich's signature rhythm

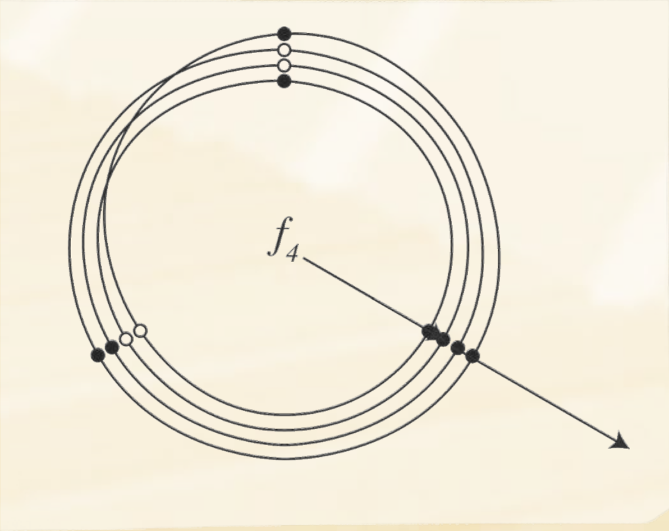
The ***spectrum*** of a rhythm shows the sizes of each component as squared magnitudes

Spectrum of the *Clapping Music* rhythm:



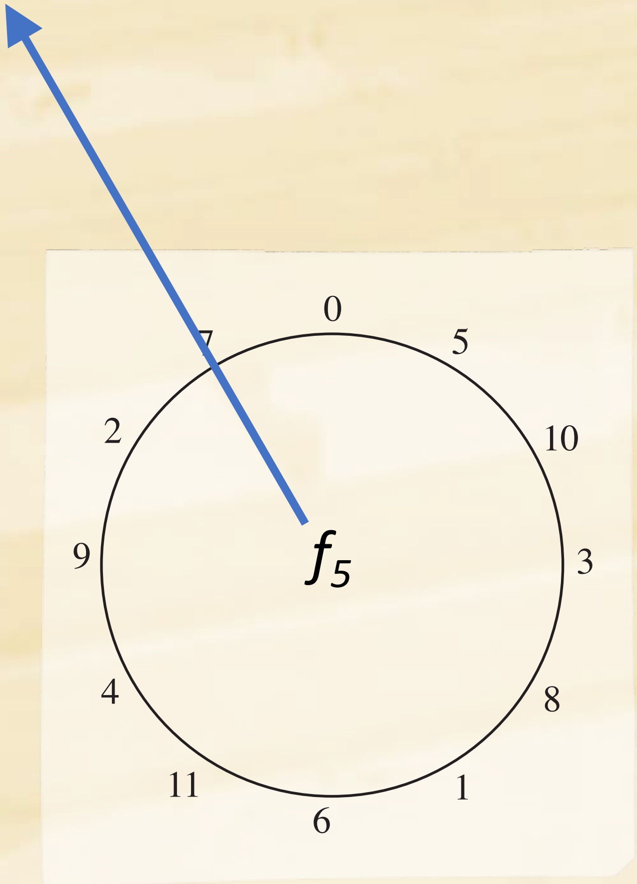
Meter and Rhythmic Qualities





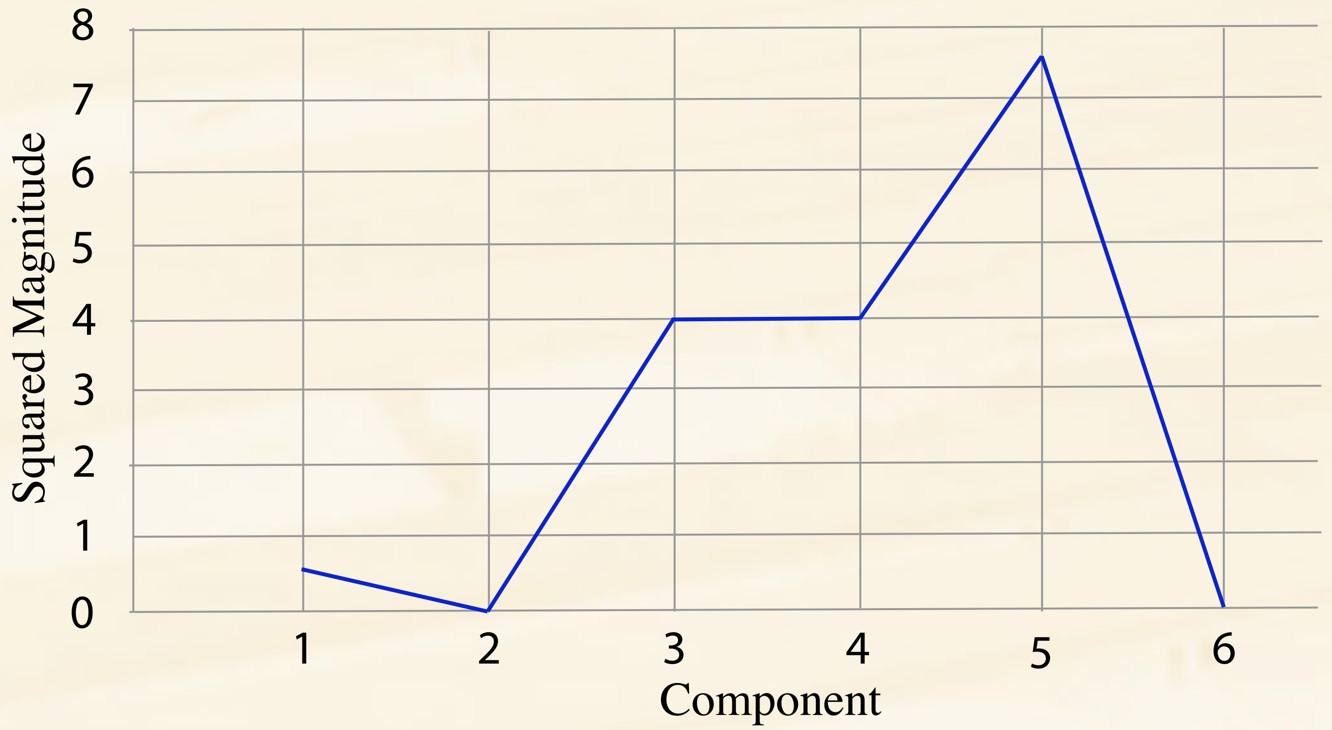
Heard in 3/2 (division of the measure in 3) or 12/8 (division of the measure in 4) the *Clapping Music* rhythm has a feel that is *metrical* but *syncopated.*

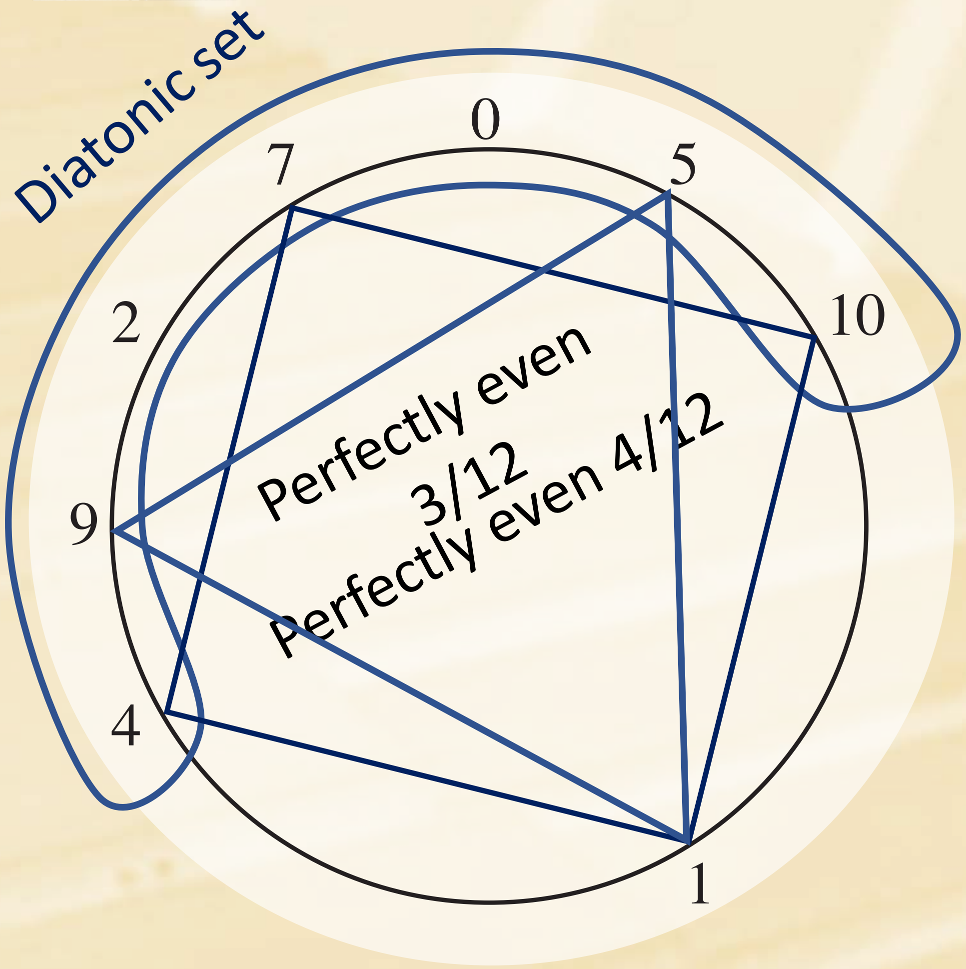
Meter and Rhythmic Qualities



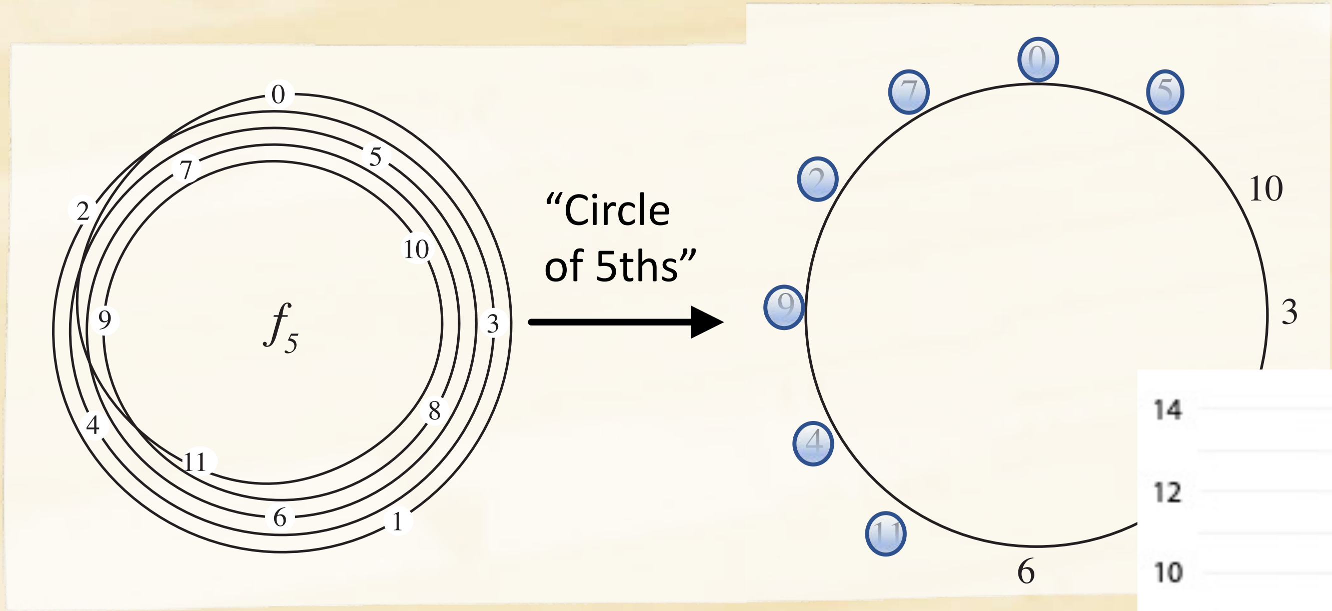
The largest rhythmic quality, f_5, is roughly *in phase* with the downbeat, but is not metrical in any regular meter.

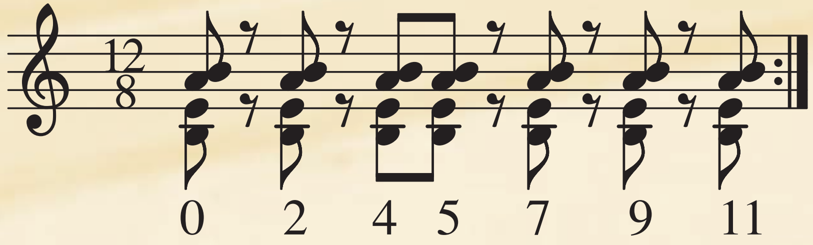
The signature rhythm



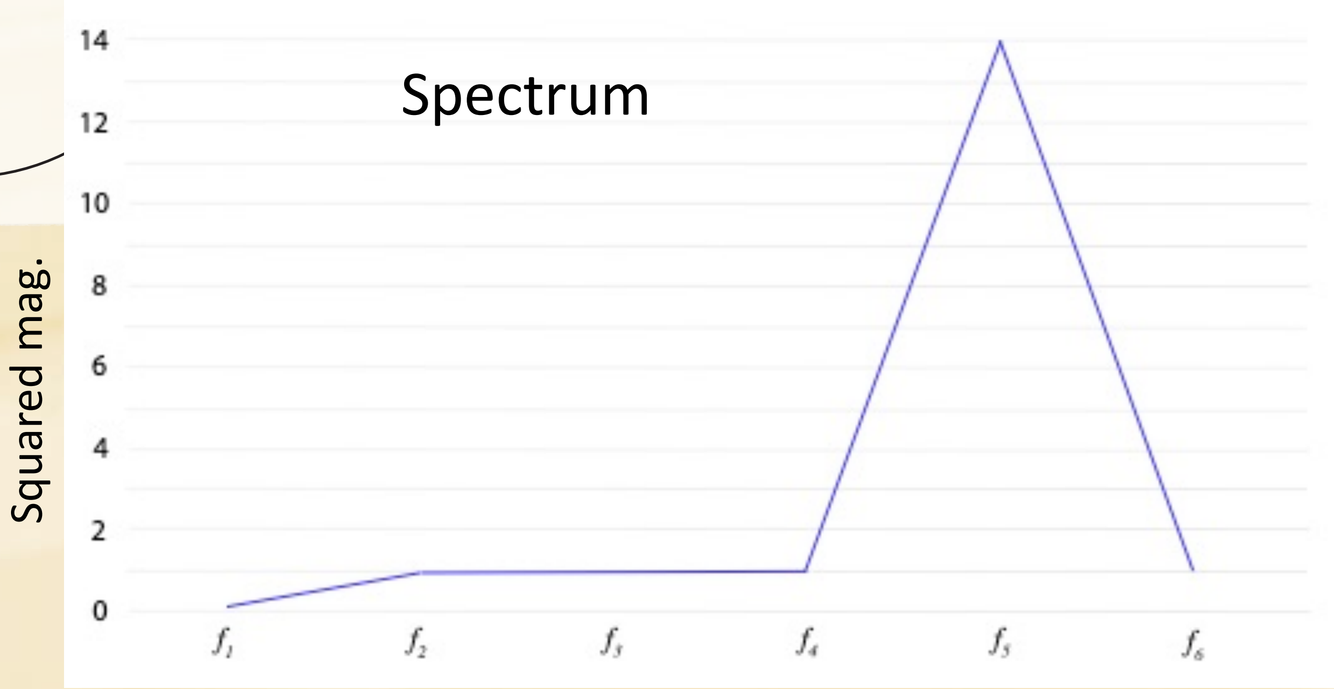


Maximizing f_5





“Diatonic rhythm” from *Sextet* mvt. 5



Mathematical properties of the DFT

Invertibility:
{Weighted collections of onsets} ←One-to-one→ {Qualities}

Conservation of power:
Sum of squared onset weights = Sum of squared qualities

Magnitudes are invariant over rotation and retrograde (transposition and inversion)

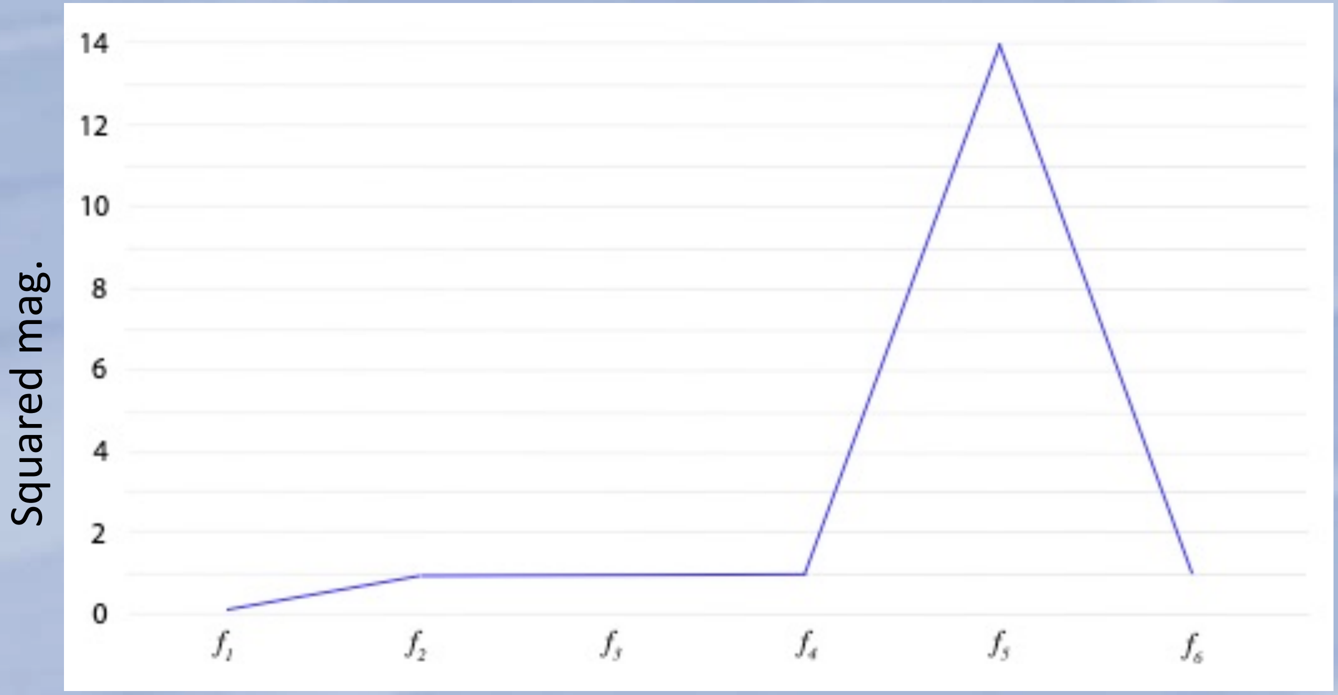
The *maximally even* rhythm of cardinality n maximizes f_n

Mathematical properties of the DFT

Conservation of power:

Maximizing f_5 entails minimizing all other qualities

“Diatonic rhythm” from *Sextet* mvt. 5



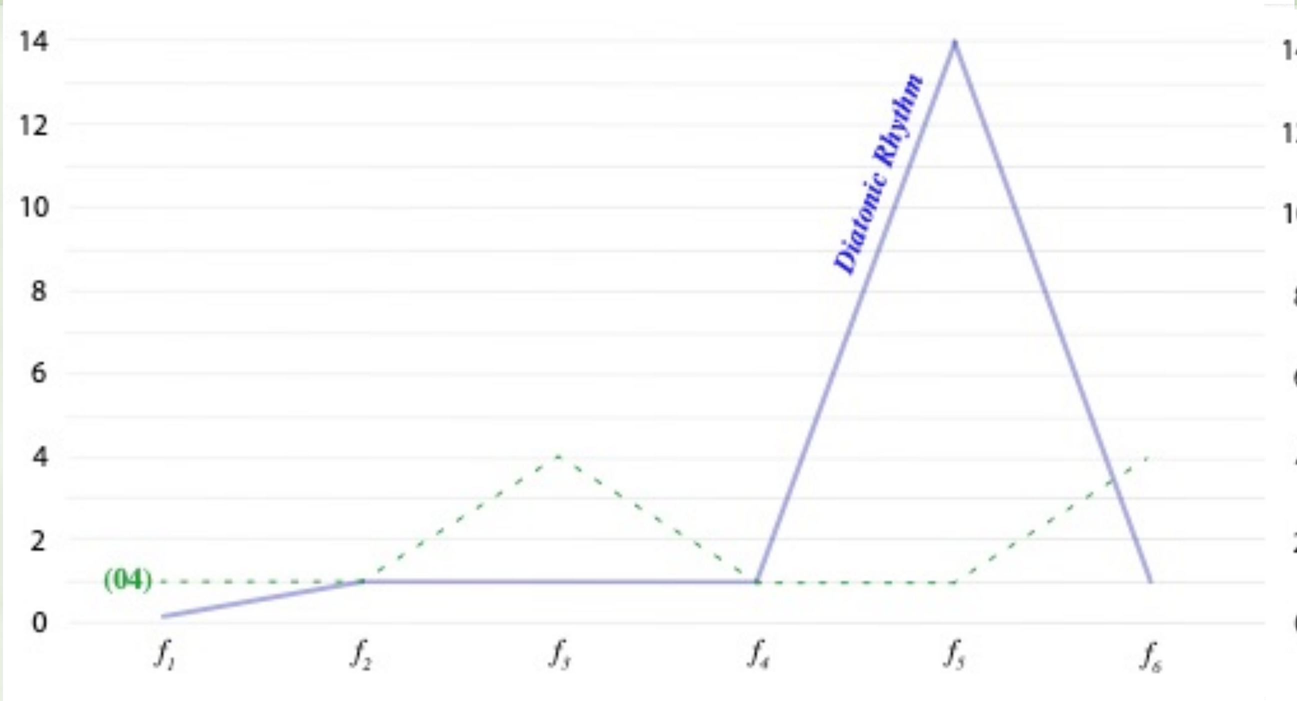
Mathematical properties of the DFT

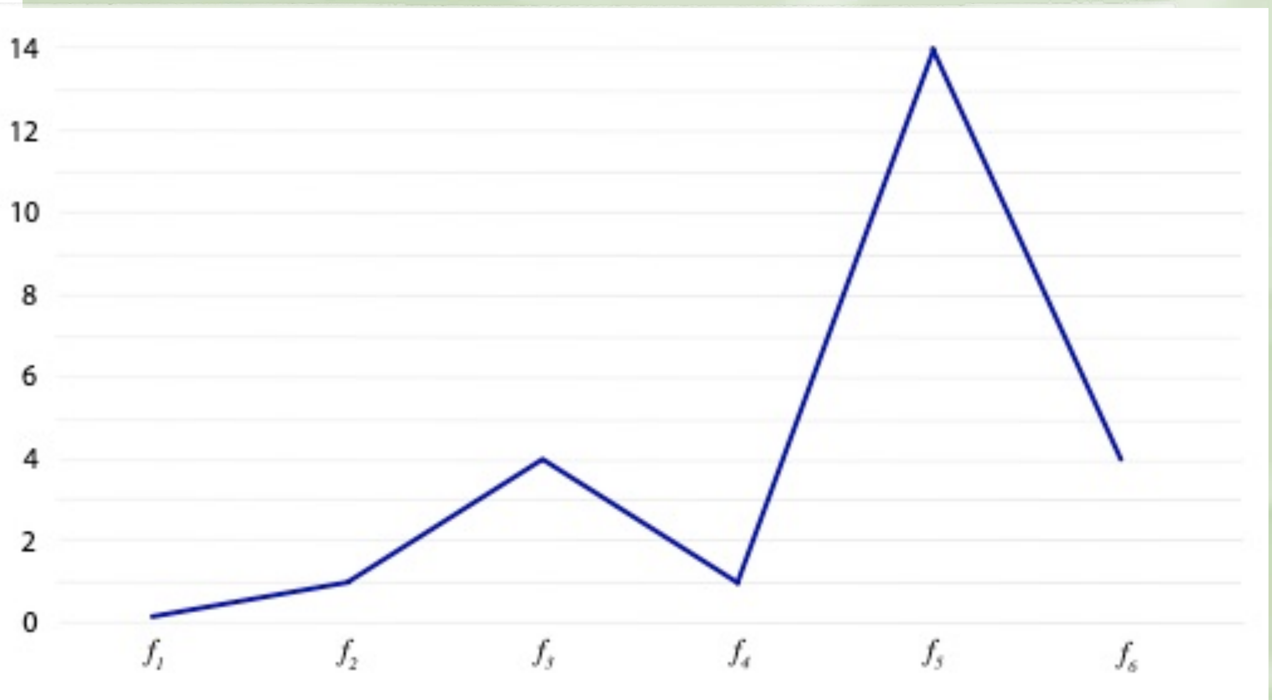
Convolution theorem:

Transpositional combination (convolution) of sets corresponds to *multiplication* of their spectra.

Convolution in the rhythmic domain = ***canon***

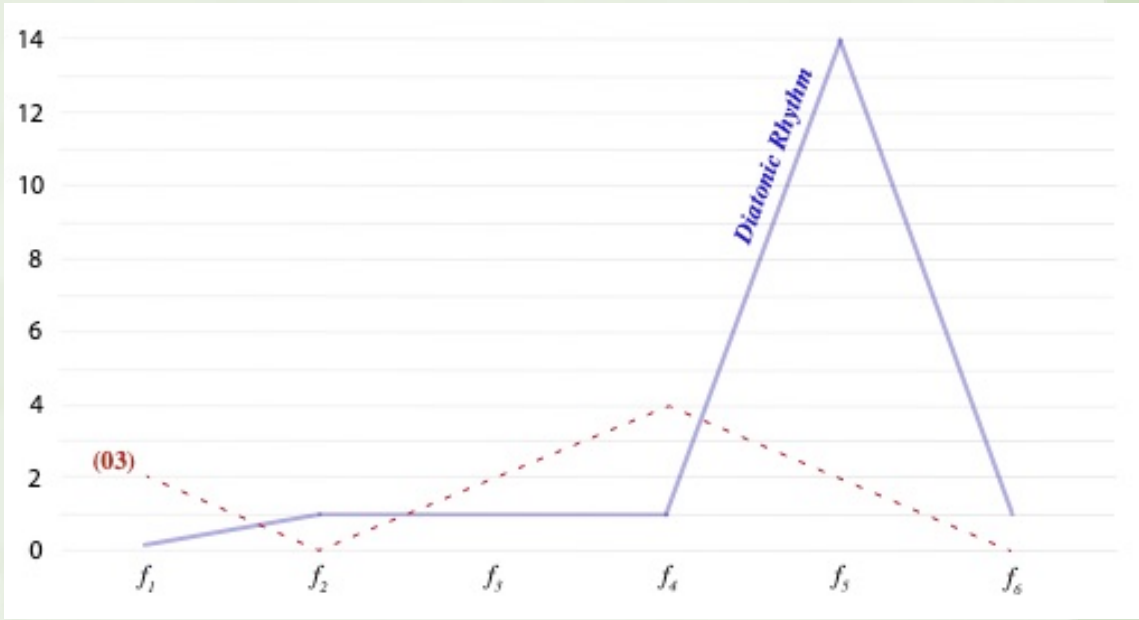
Convolution

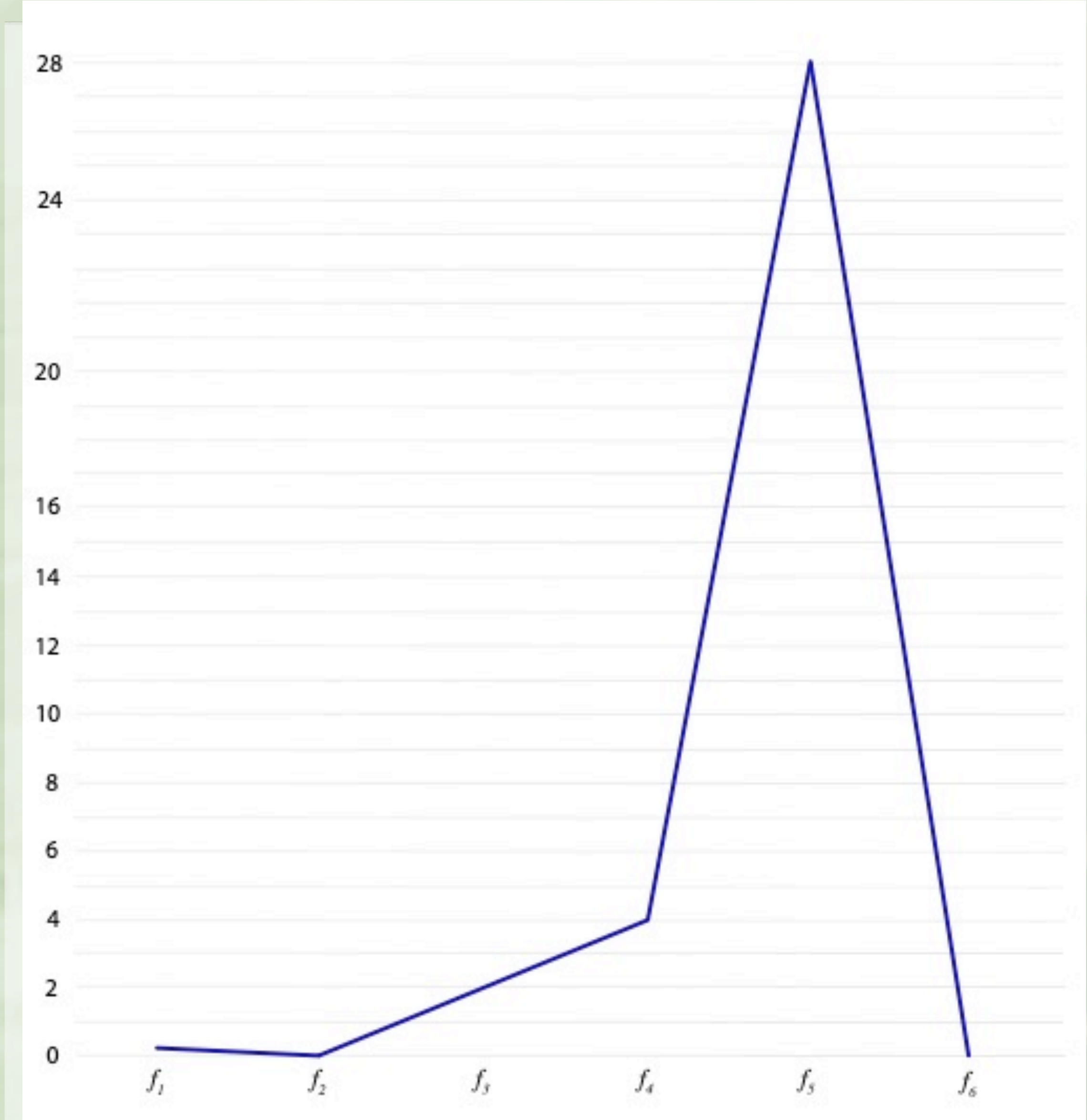




(0,2,4,5,7,9,11) x (0,8) = (0,0,1,2,3,4,5,5,7,7,8,9,10,11)

Convolution





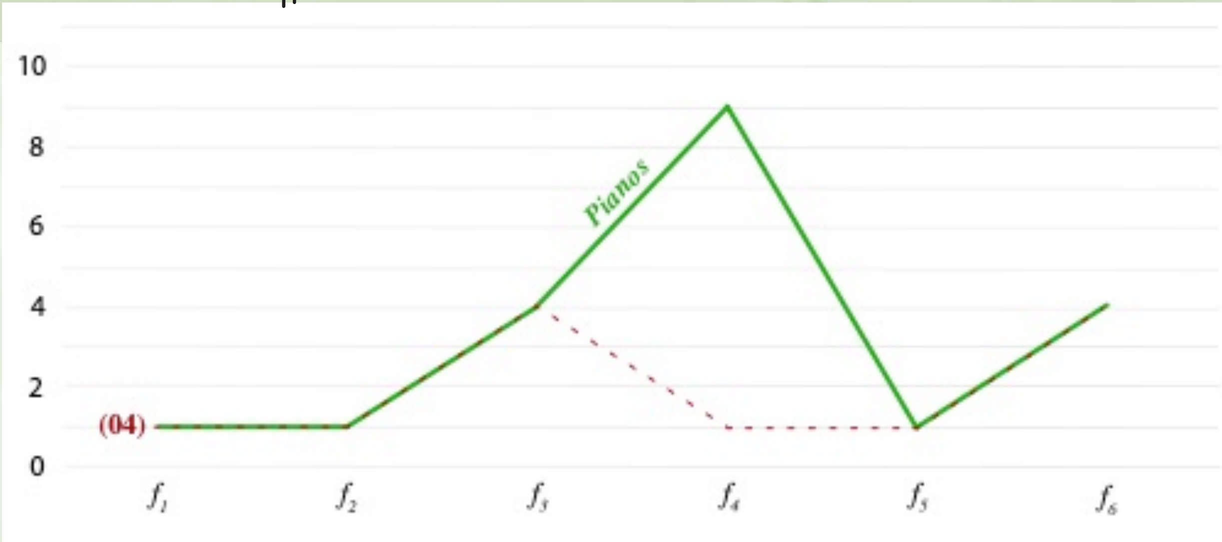
(0,2,4,5,7,9,11) x (0,3)

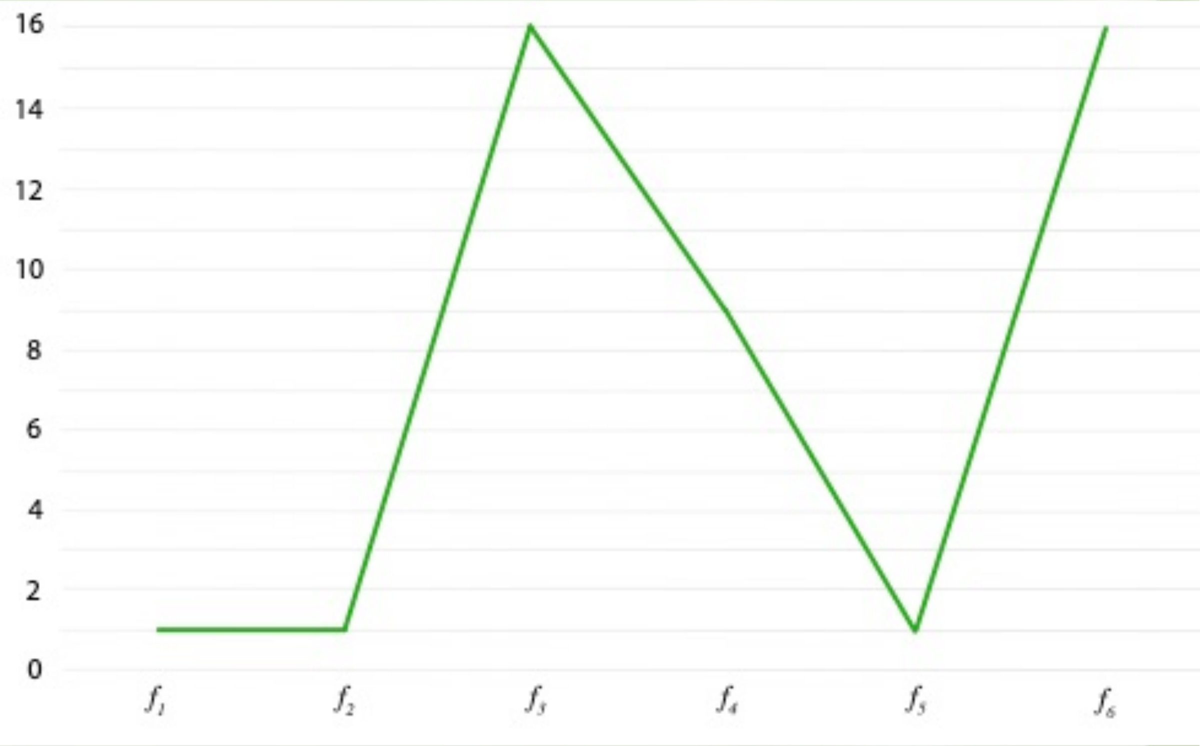
= (0,0,2,2,3,4,5,5,7,7,8,9,10,11)

Changing meter in the finale of the *Sextet*

Piano part, *Sextet,* mvt. 5

Changing meter in the finale of the *Sextet*





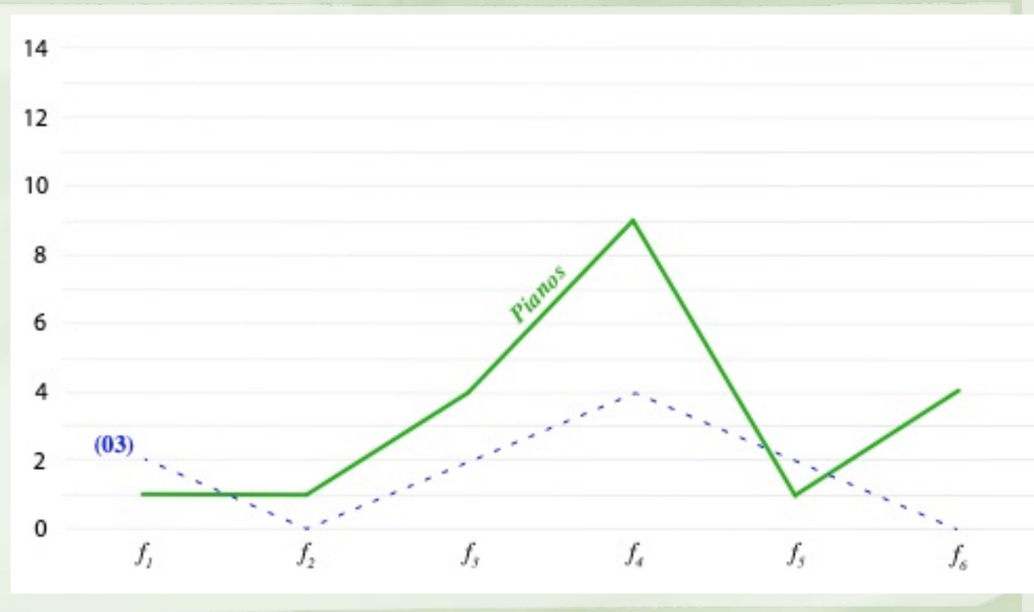
(0,3,4,6,8,9) x (0,8)

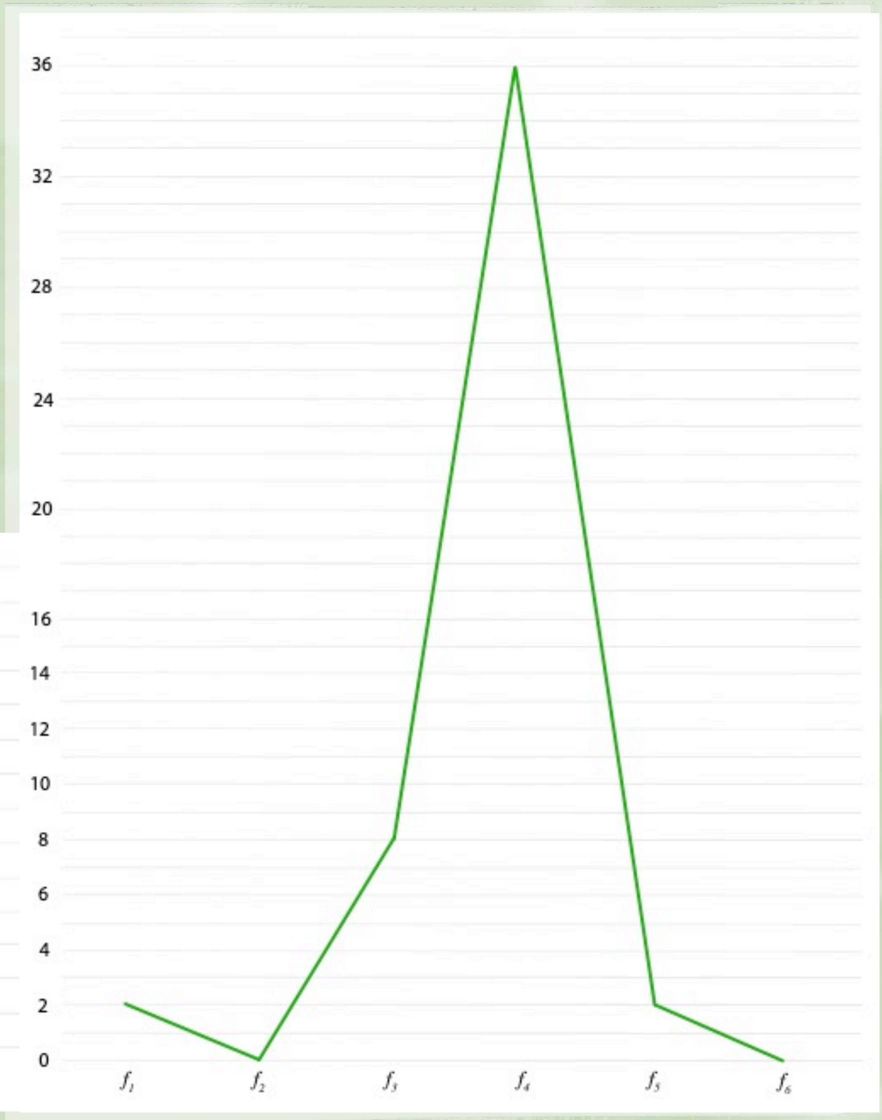
= (0,0,2,3,4,4,5,6,8,8,9,11)

Changing meter in the finale of the *Sextet*

(0,3,4,6,8,9) x (0,3)

= (0,0,3,3,4,6,6,8,9,9,11)





Canons on the signature rhythm

Sextet mvt. 1:

(0,1,2,4,5,7,9,10) x (0,2)

Canons on the signature rhythm

Sextet mvt. 1:

(0,1,2,4,5,7,9,10) x (0,2,5)

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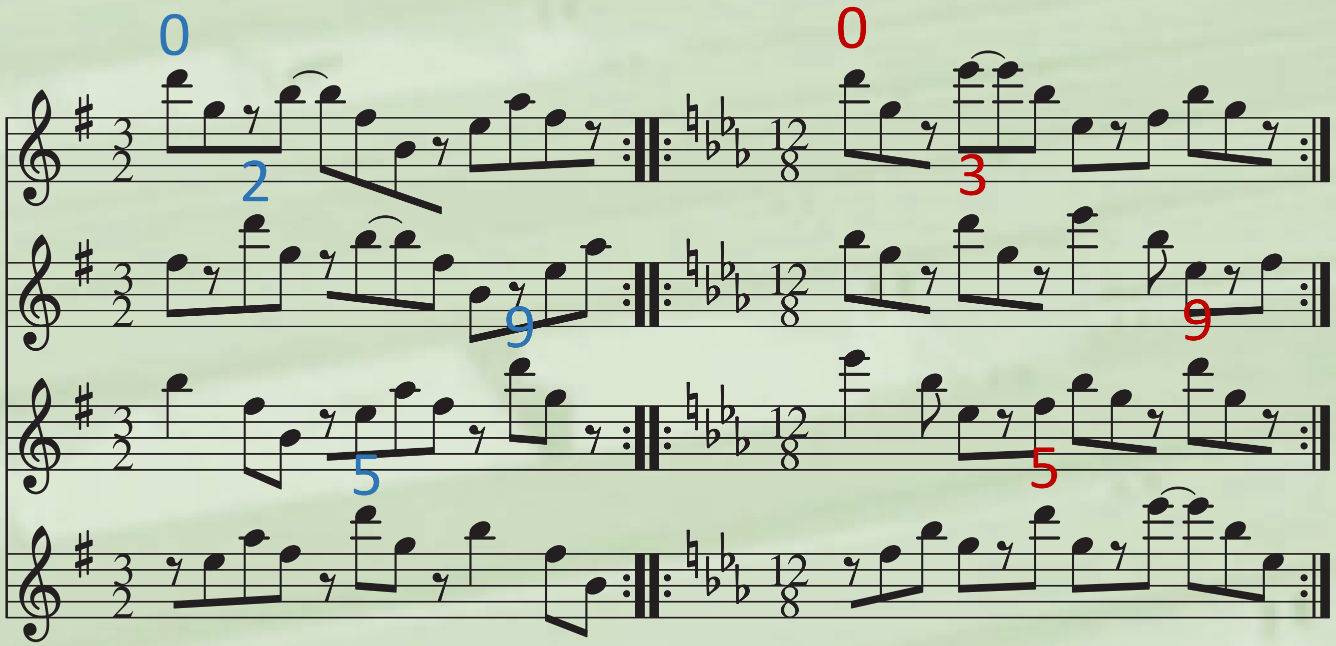
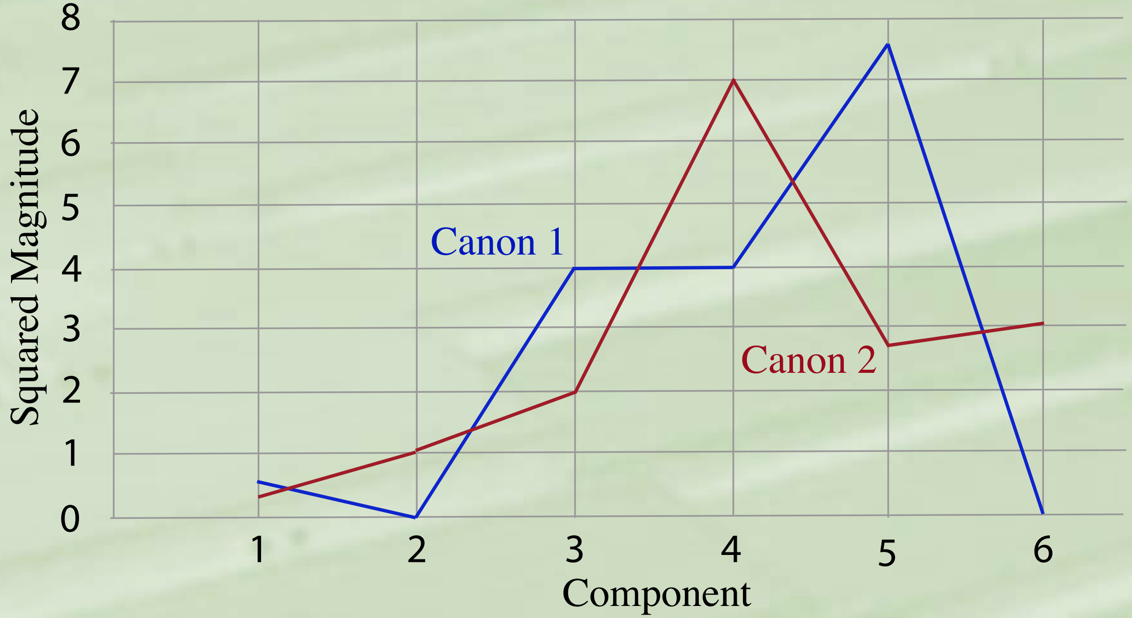
Canons on the signature rhythm

Electric Counterpoint mvt. 3:



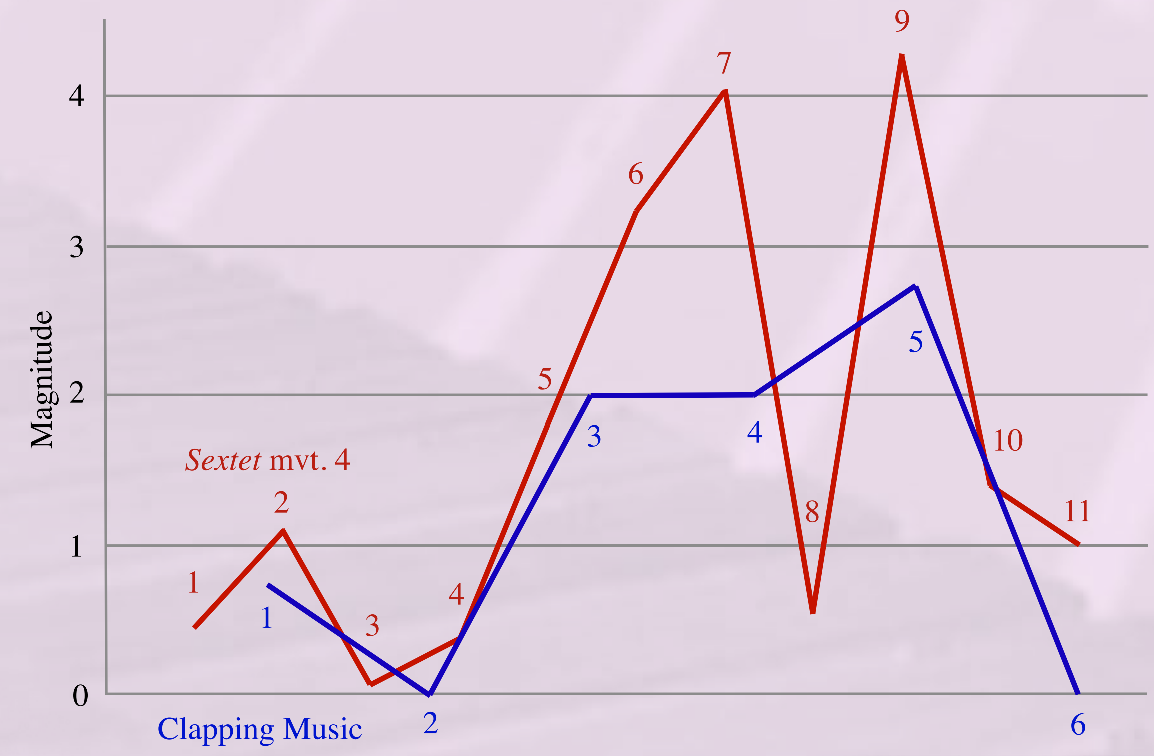
(0,2,5,9) ⟶ (0,3,5,9)

Spectra:



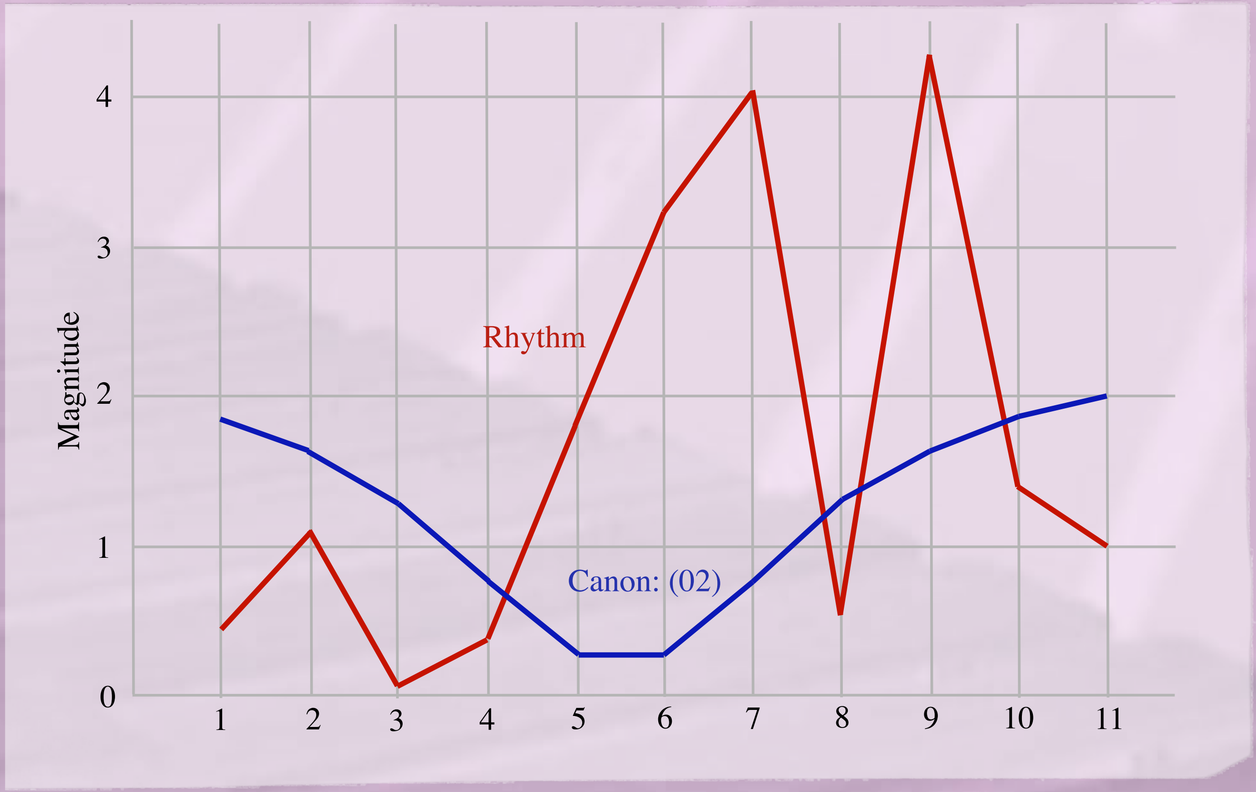
Sextet Mvt. 4

A modification of the signature rhythm to fit in an irregular 22-onset universe

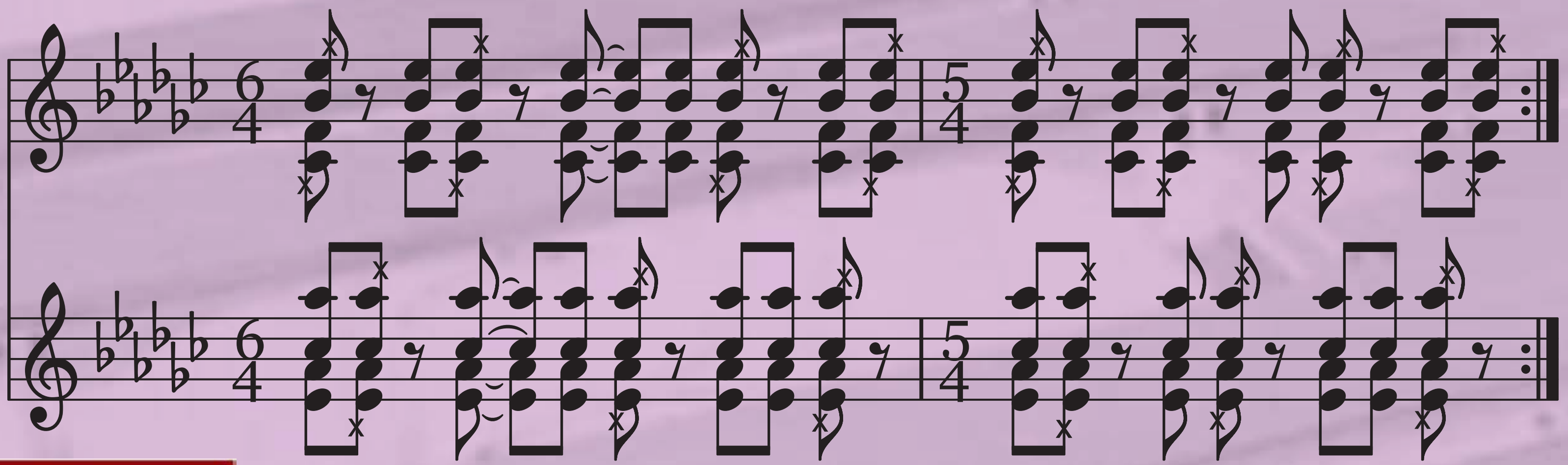


The relationship between these rhythms can be seen by scaling their spectra to reflect the common shape

Sextet Mvt. 4

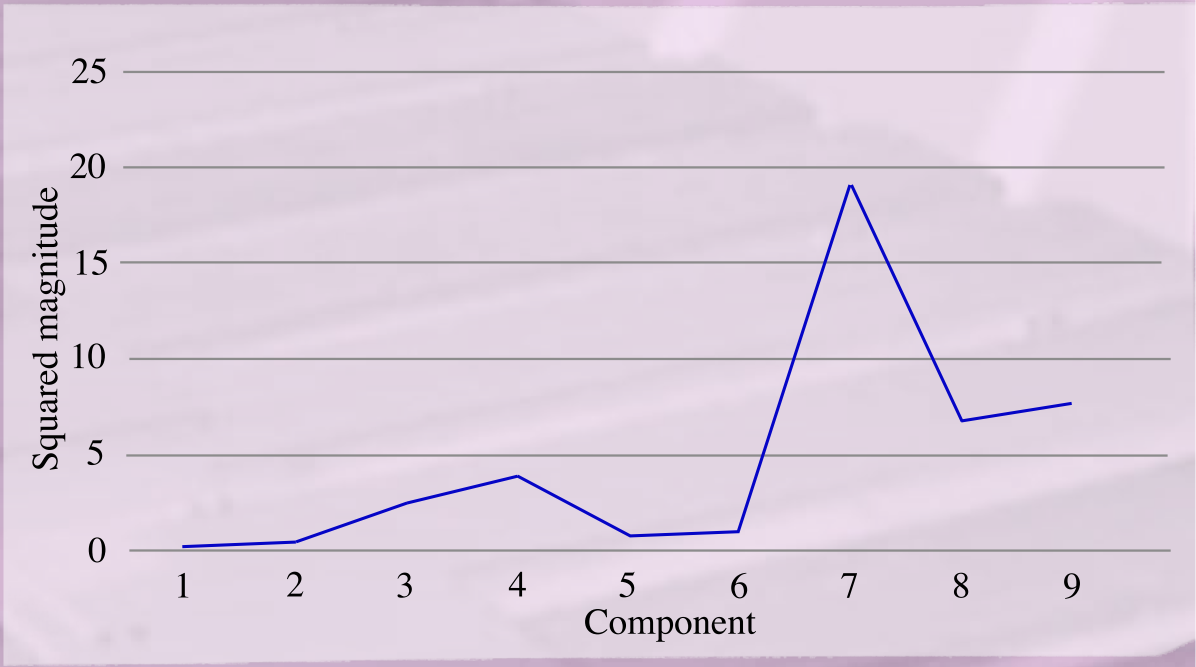


One of Reich's canons on this rhythm



Electric Counterpoint, Mvt. 2

	0	1	3	5	6	7		9	11	12	14	16	17
ME(12, 19)	0	1	3	5	6		8	9	11	12	14	16	17
ME(4, 19)	0			5				9			14		
		1			6				11			16	
			3			7				12			17



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Presentation to SMT 2019, Columbus, Ohio

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