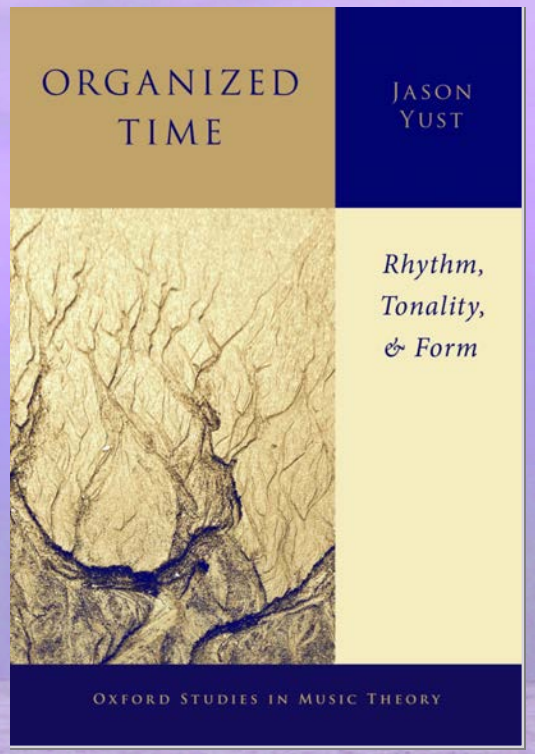
Materials of Tonality: Harmonic Spectra and Phase Spaces for Classical Music, Blues, and Jazz

Jason Yust, BOSTON UNIVERSITY

Northwestern Music Theory and Cognition Colloquium, Nov. 2020



Law of Music Theory Concepts*

Importance : Ambiguity = k

See: Phrase

Sonata form

Meter

Key

Function

Tonality

*This is not entirely serious

What is tonality?

It is sometimes defined as:

- A property of a musical works
- A perceptual act or mode of listening
- A historical or stylistic classification of music

Straus's Features
(*Introduction to Post-Tonal Theory)*

(1) Key (tonic)

(2) Key relations

(3) Diatonic Scales

(4) Triads

(5) Functional harmonies

(6) Voice leading

This is tonality as a **feature of music**. Straus even describes defining a tonic (1) as something the music does.

Tonality: Tymoczko's (2011) Five Features

(1) Conjunct melodic motion

(2) Acoustic consonance

(3) Harmonic consistency

(4) Limited macroharmony

(5) Centricity

These are also largely characterizations of musical **materials**

Like Straus, Tymoczko characterizes centricity (5) as a property of music

From an SMT-discuss thread (July 2020)

"A **force field** of musical tones with varying felt relationships directed to **a central tone.**" —*William Pastille*

"pitch-classes are perceived as relating to one another in a systematic, **hierarchically** arranged manner, especially with respect to **stability/tension** and **implication/realization** relations." —*Frank Lehman*

"a perception of the preeminence of **one tone** and its **force** acting upon other tones." —*Darryl White*

"a **centripetal** organization of the music concerned." —*Nicholas Meeùs*

"certain pitches or chords are heard as **attracted**, in varying degrees, to other pitches and chords." —*Andrew Milne*

" the battle between **tension and release** that creates the musical **grammar** and **syntax** known as tonality." —*Devin Chaloux*

"learned, participatory **expectation.**" —*Stephen Guerra*

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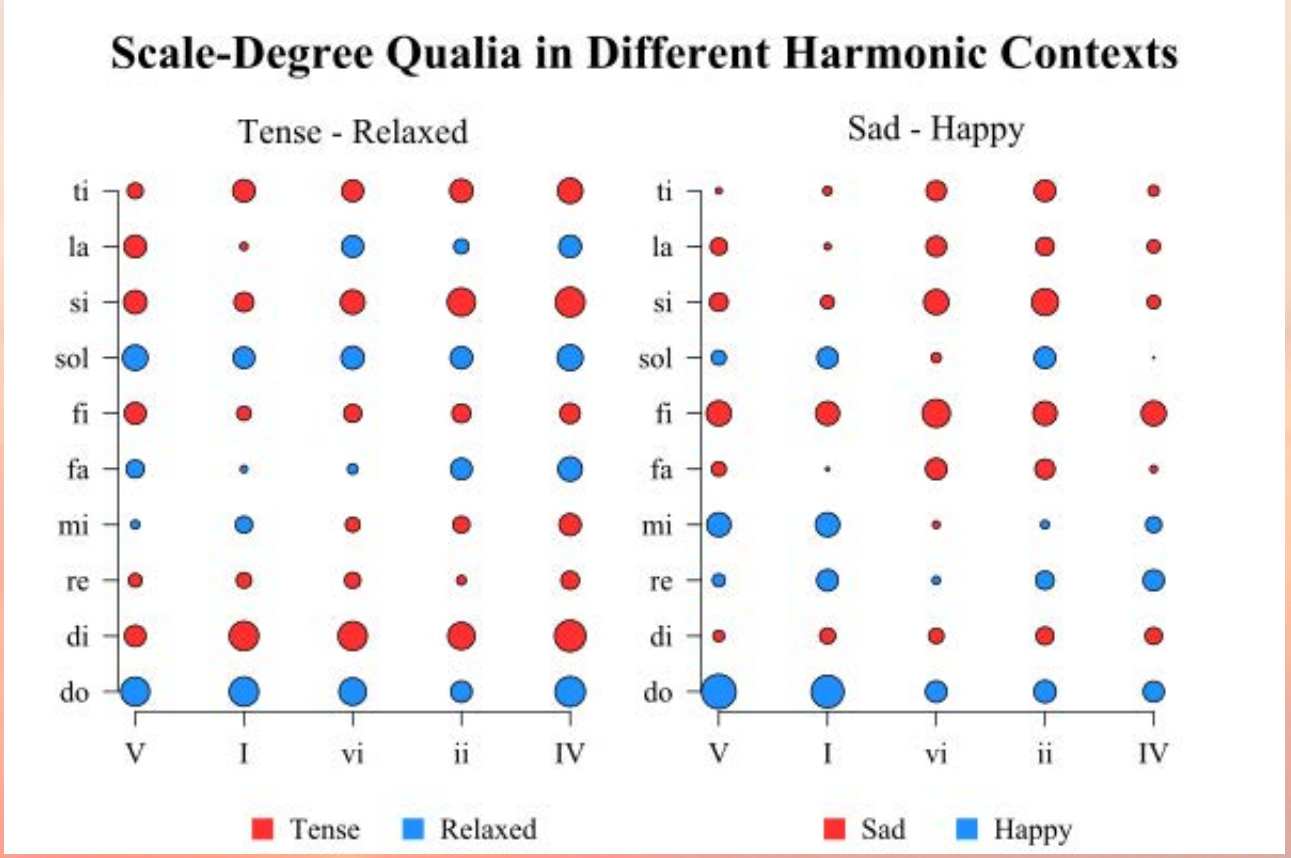
Tonality: Cognitive Definitions

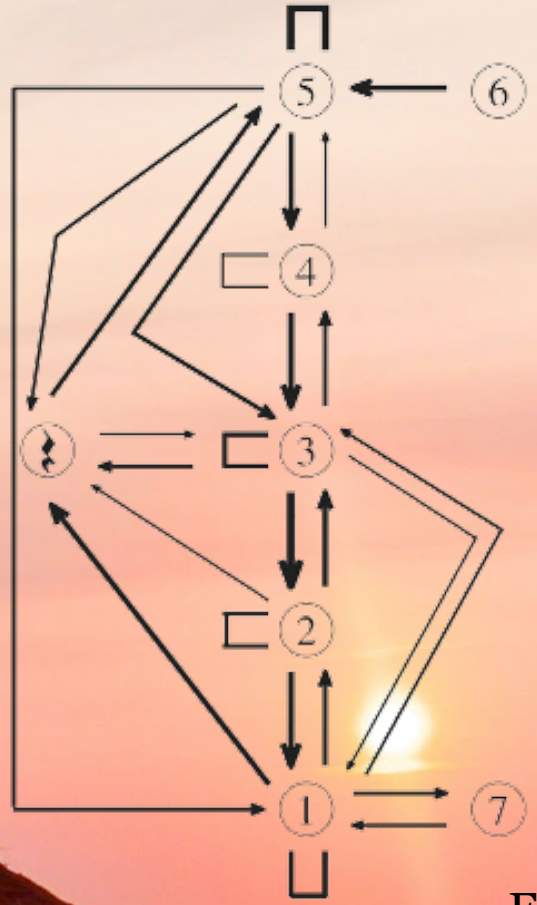
Referential pitch (tonic) → Scale Degree Qualia



—Claire Arthur, 2016. "When the Leading Tone Doesn't Lead: Scale Degree Qualia in Context," PhD Diss., Ohio St. Univ.

Tonality: Cognitive Definitions

Melodic Syntax → Implication → Scale Degree Qualia



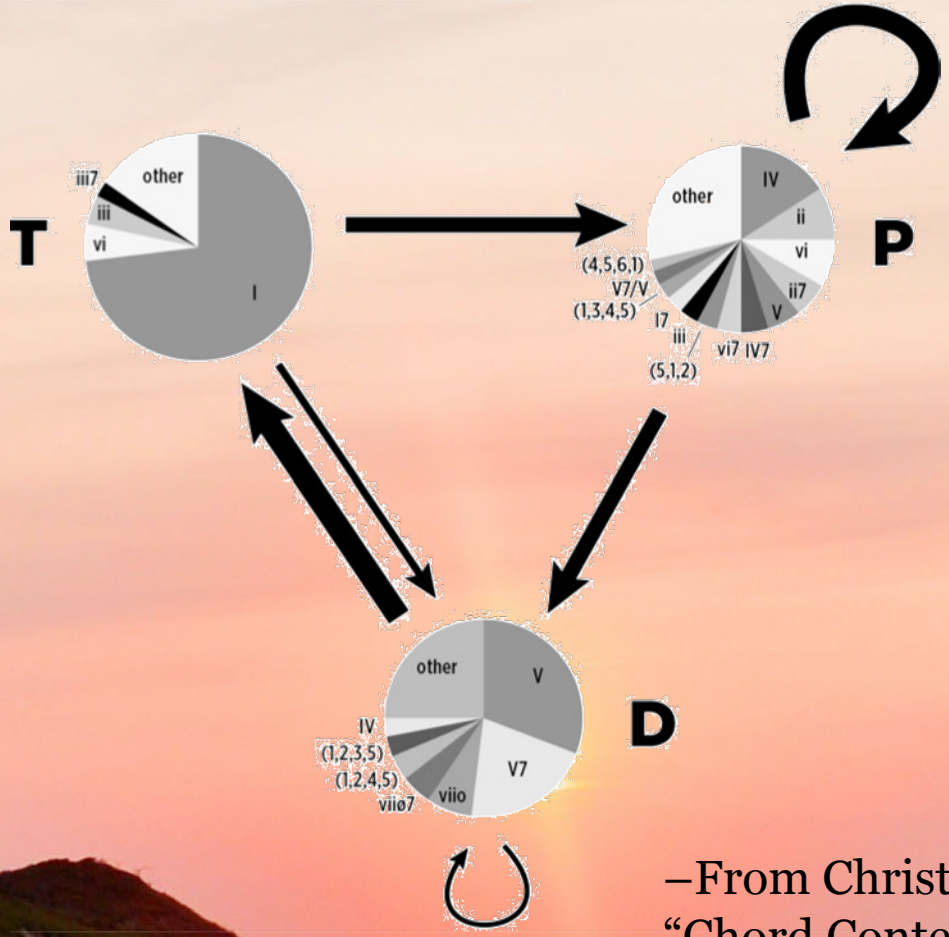
A melodic syntax (represented by **transition probabilities**) creates expectations that lead to qualia.

E.g.: Relaxed tonic, tense leading tone

–From David Huron 2008. *Sweet Anticipation: Music and the Psychology of Expectation* (Bradford Books)

Tonality: Cognitive Definitions

Harmonic Function / Syntax → Implication



Harmonic function can be understood similarly as a syntax.

Triads and **diatonic scales** are elements of this syntax**.**

–From Christopher White and Ian Quinn, 2018. "Chord Context and Harmonic Function in Tonal Music." *Music Theory Spectrum* 40/2.

Tonality

- Cognitive definitions of expectation, syntax, and function are **culturally specific**
- Cognitive definitions also rely on **musical materials**
- Content-based definitions of tonality can be generalized across styles

Can cognitive definitions be generalized?

→ This implies musical universals (intuitively plausible!)

Outline

(1) Tonal space for classical tonality

(2) Fourier theory

—Concepts (triadicity, tetradicity . . .)

—DFT spectra and coefficient spaes

—Tonal space as a phase space

(3) Blues tonality and octatonicity

—Blue notes and blues scale

—DFT of blues melodies

(4) Jazz harmony

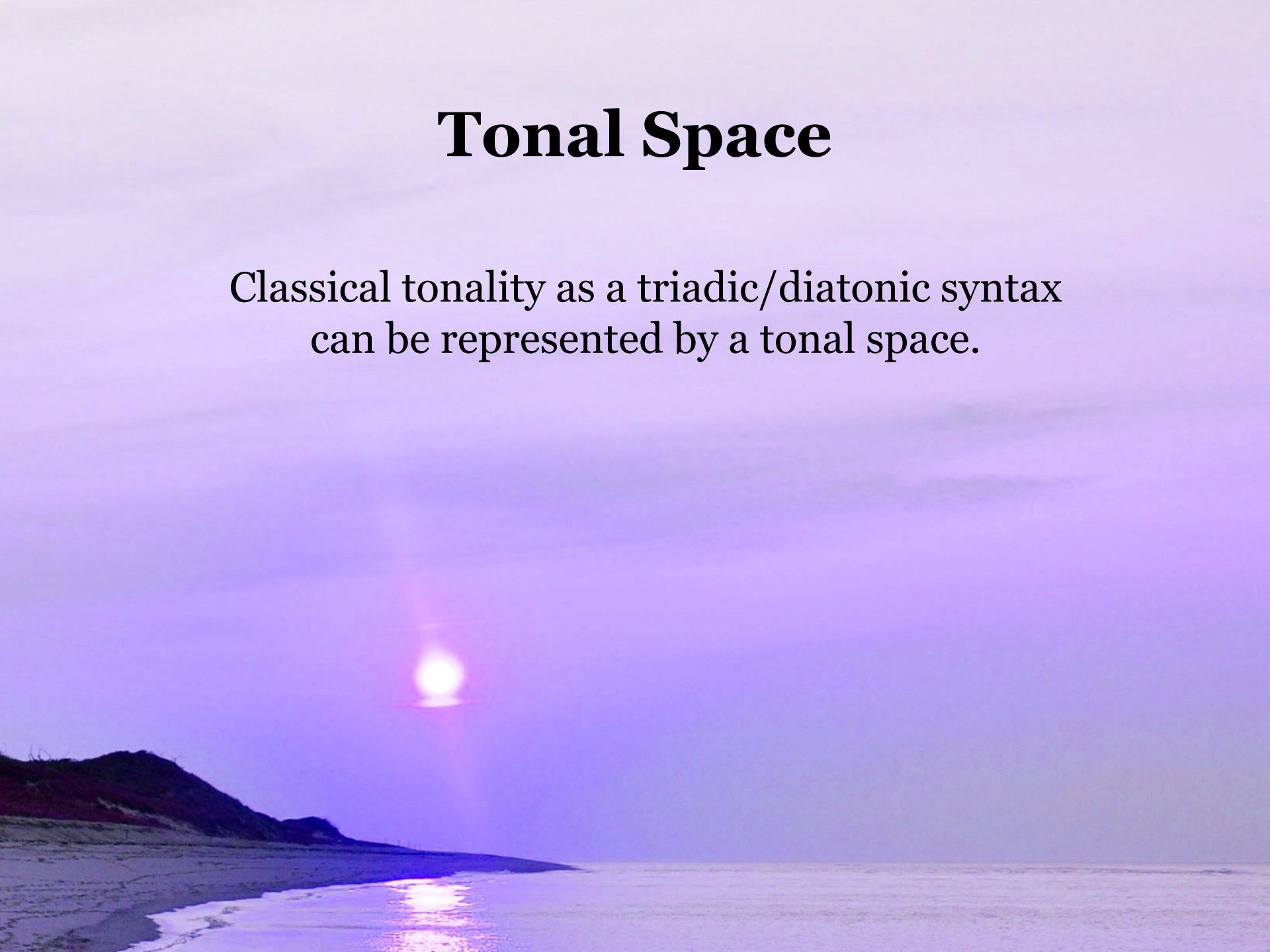
–Spectra for lead-sheet symbols

–Diatonic/octatonic space

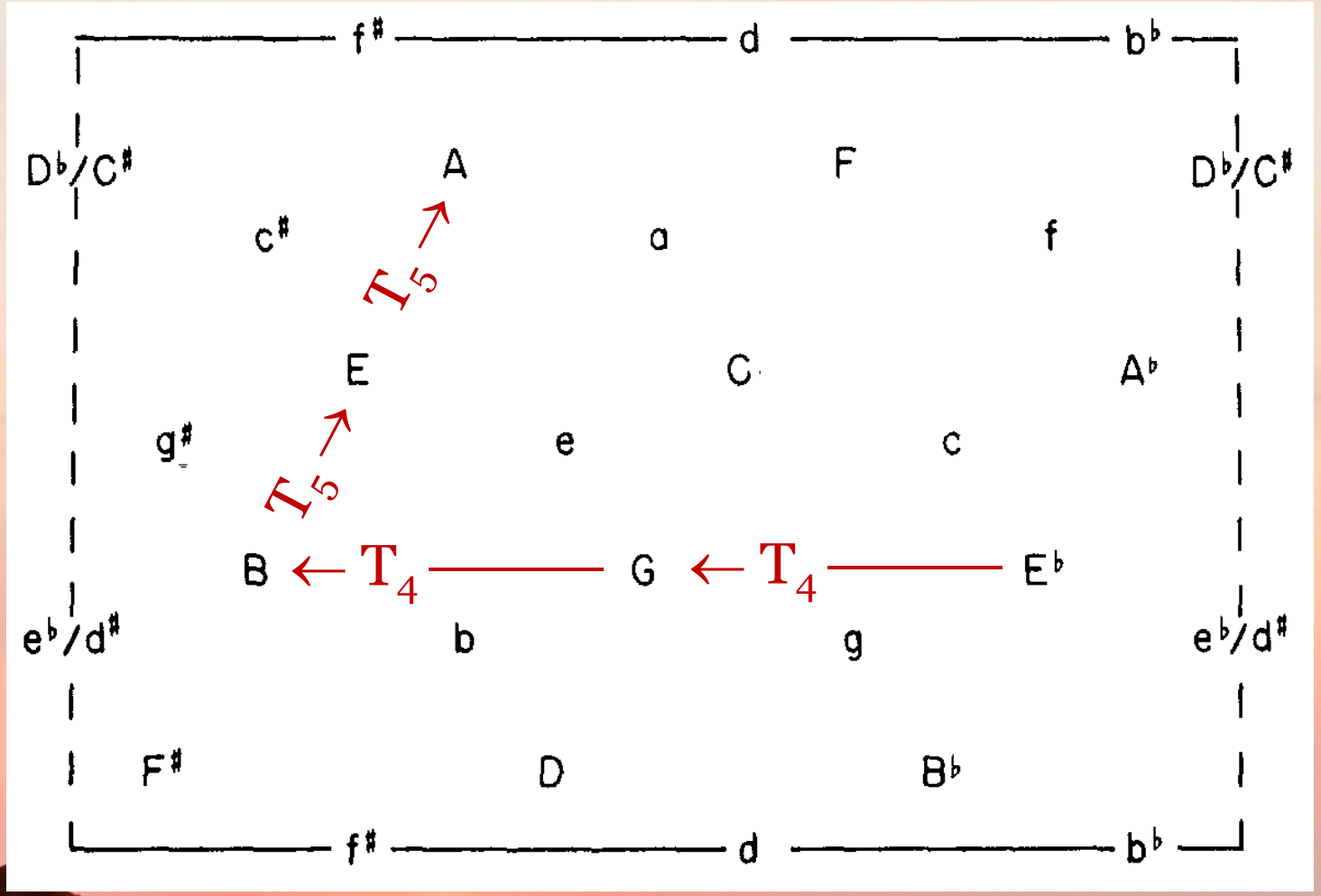
–Bebop examples

Tonal Space

Classical tonality as a triadic/diatonic syntax
can be represented by a tonal space.

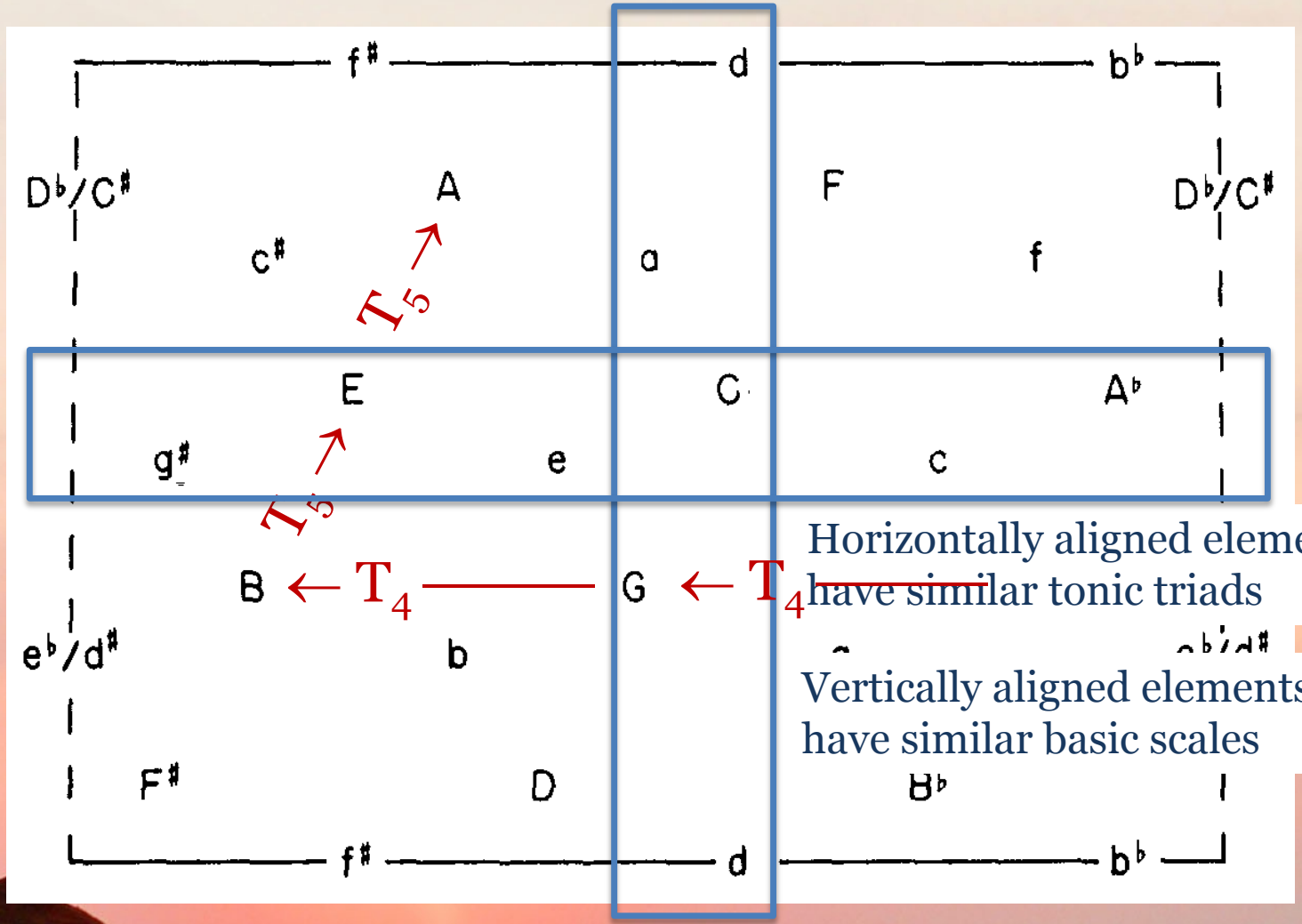


Krumhansl's Tonal Space



Toroidal map of key profiles from Krumhansl and Kessler 1982

Krumhansl's Tonal Space



Horizontally aligned elements have similar tonic triads

Vertically aligned elements have similar basic scales

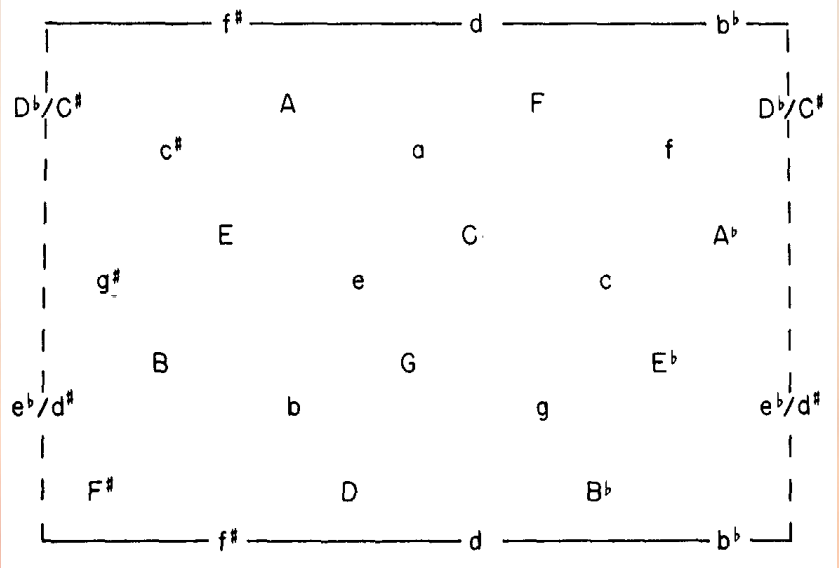
Toroidal map of key profiles from Krumhansl and Kessler 1982

Krumhansl's Tonal Space

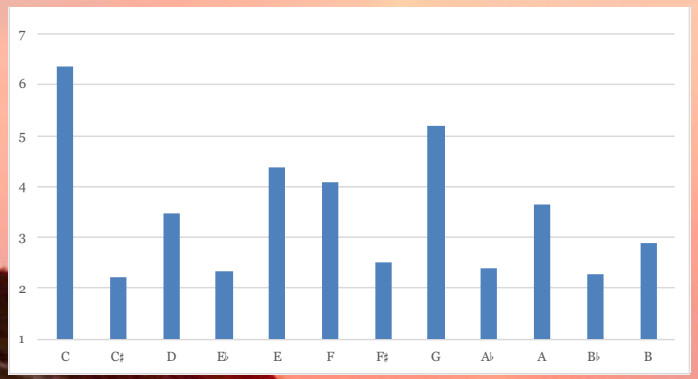
Krumhansl's space is based on **correlations** between ***key profiles.***

Key profiles may be derived from

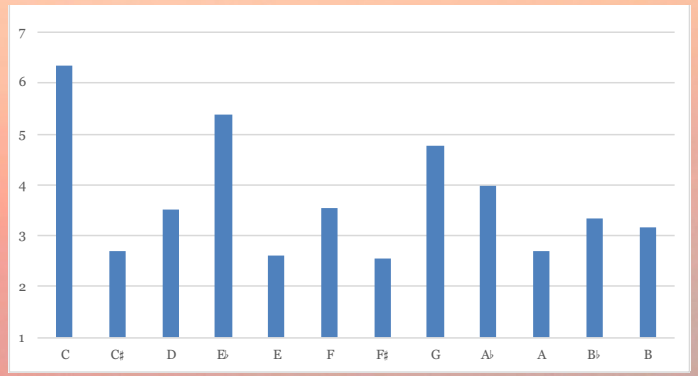
- Listener rankings of stability of tones in a given context,
- Frequency of occurrence of tones in the given key,
- etc.



Krumhansl and Kessler's key profiles based on listener ratings



C major



C minor

Krumhansl's Tonal Space

Considered mathematically . . .

Constraint: **Transposability** (Arrangement of keys invariant under transposition).

$\Rightarrow$

—The space is **toroidal** (circular in two dimensions)

—Each dimension represents an **interval cycle**

Possible interval cycles:

Interval class:	1	2	3	4	5	6
8^{ve} division:	1	6	4	3	5	2

Towards Fourier Theory, Some Concepts

Triadicity

Tetradicity

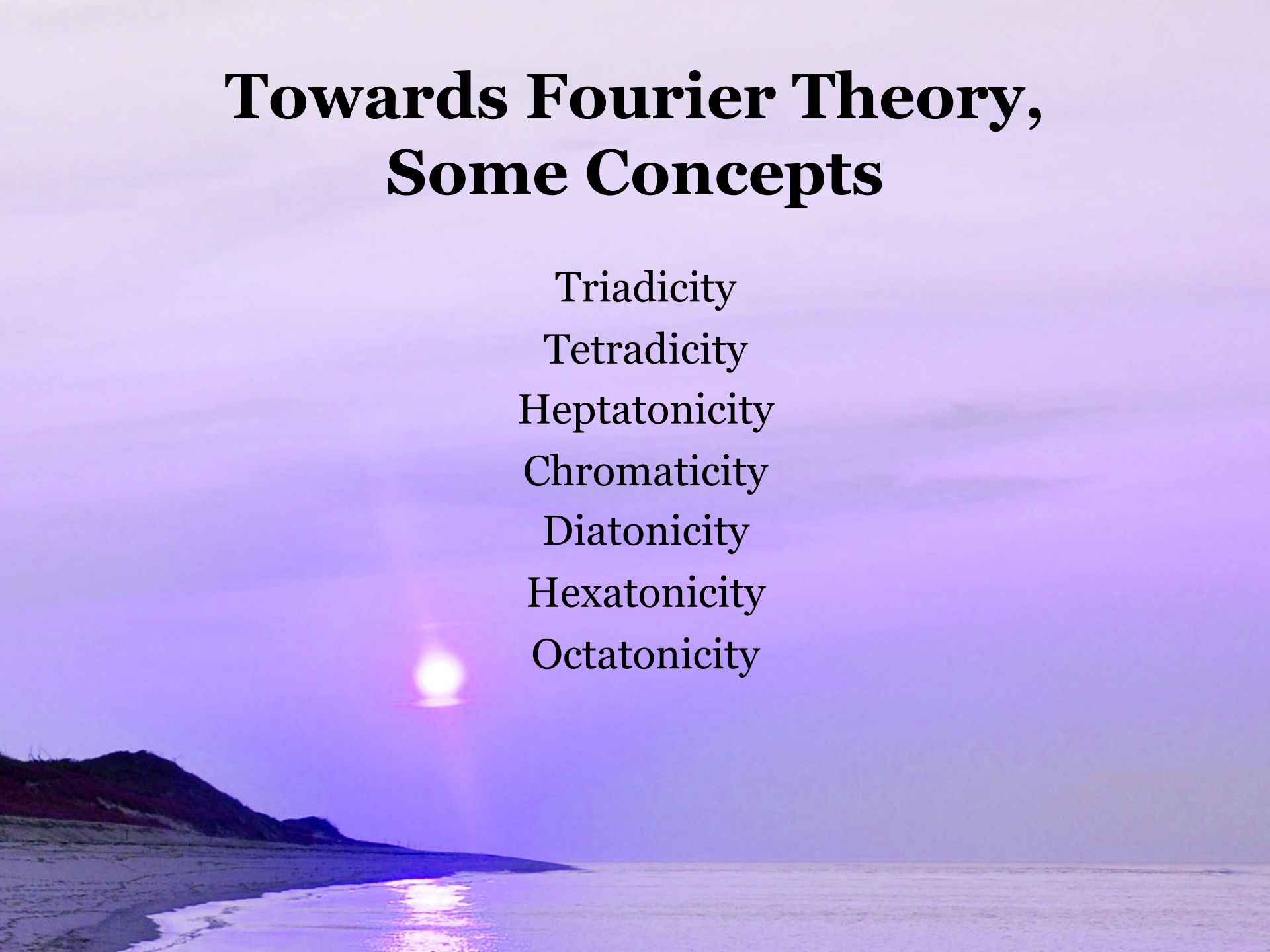
Heptatonicity

Chromaticity

Diatonicity

Hexatonicity

Octatonicity

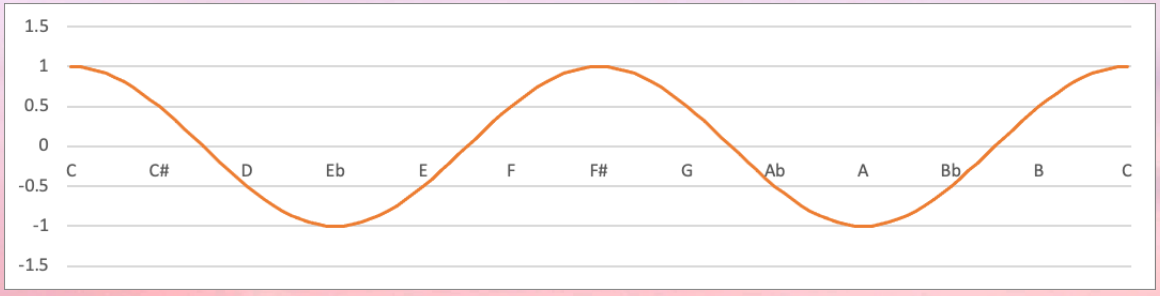


Triadicity

Triad: Relatively even spacing of three pitch-classes

A cardinality/pitch-flexible formulation (*triadicity*):

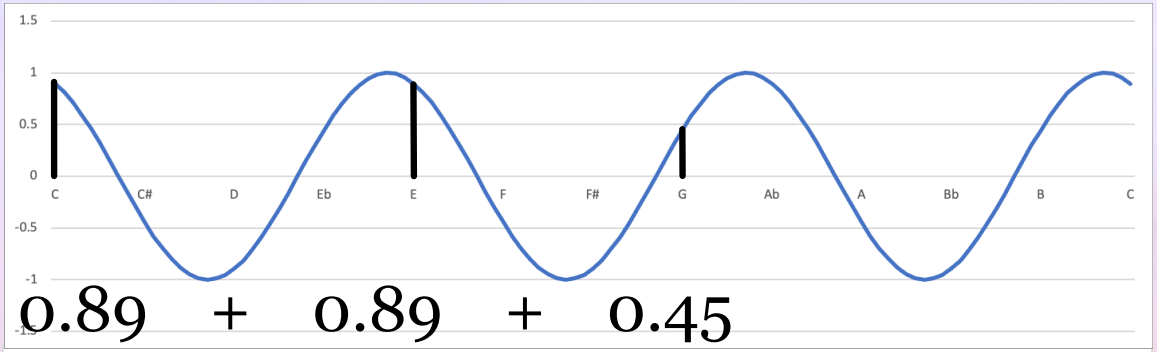
Triadicity: A cosine function over the pitch-classes with frequency 8ve/3:



- Positive values are good representatives of the triadicity; negative values are poor representatives.
- The curve can vary in phase (different triadicities).

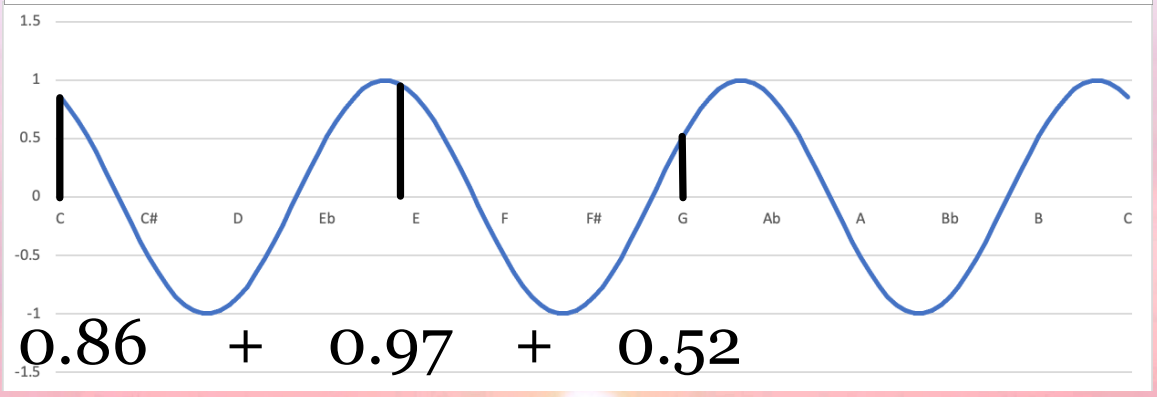
Triadicity

Example: Triadicity of some triads



0.89 + 0.89 + 0.45 = 2.23

12-t C major triad



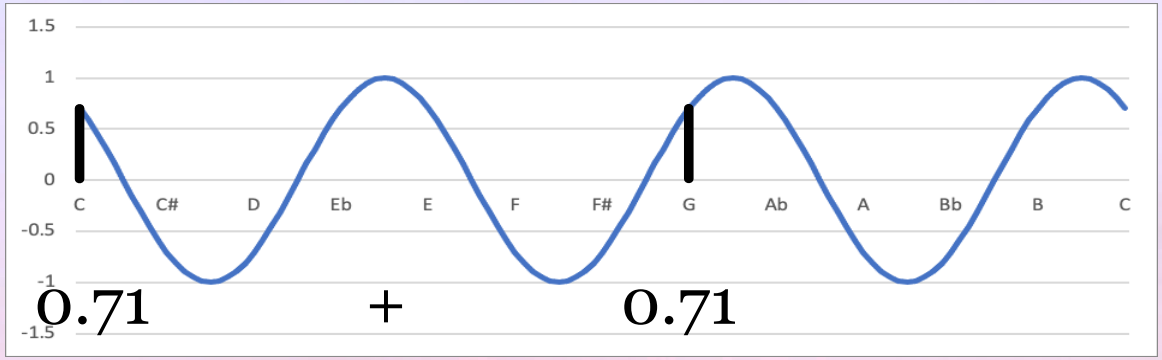
0.86 + 0.97 + 0.52 = 2.35

JI C major triad

- No assumption is made of a 12-tone grid
- The phase of the triadicity curve can be set to maximize triadicity for the given pitch-class set

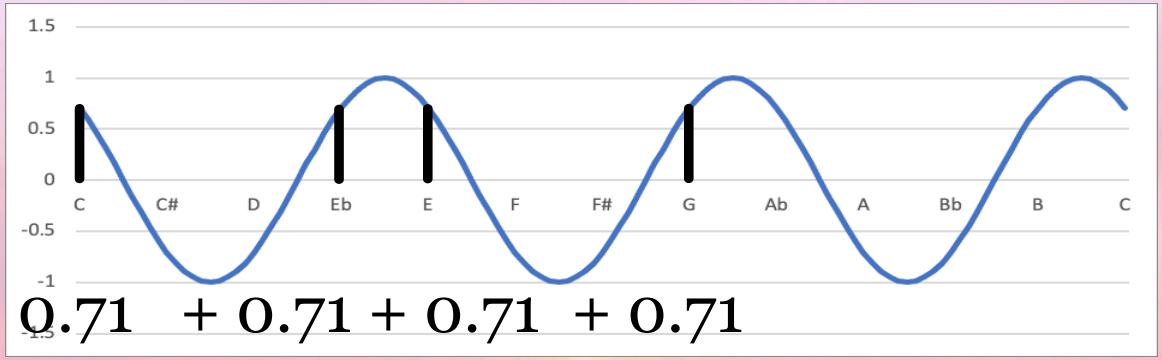
Triadicity

Example: Triadicity of non-triads



Perfect 5th

= 1.42



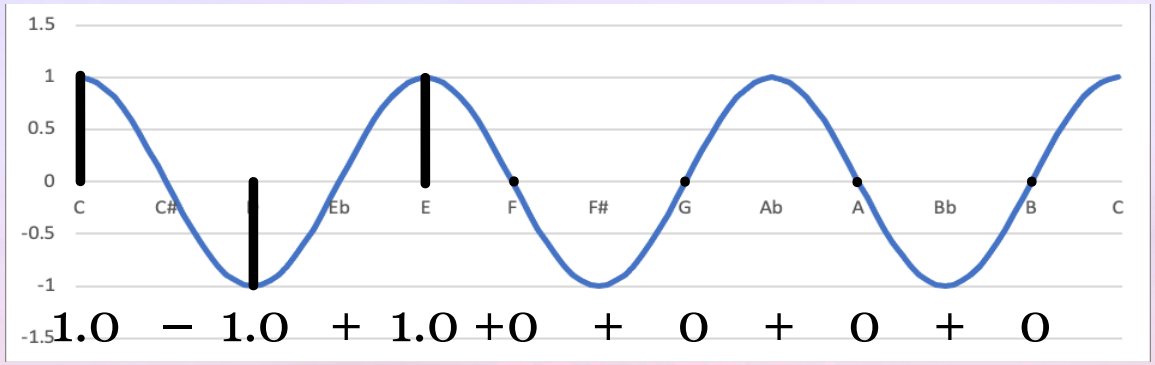
(0347) tetrachord

= 2.84

- Cardinality-flexible: applies to chords of any size
- Not all triad positions need to be represented; multiple notes can represent a single category

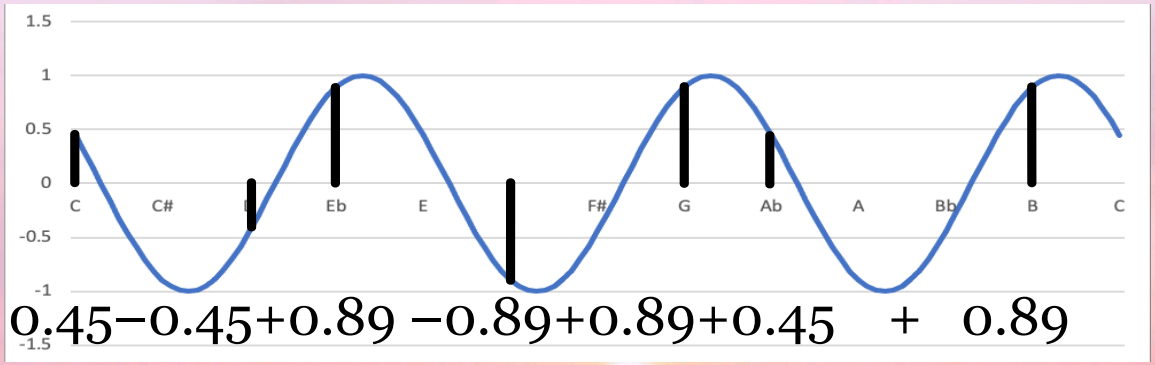
Triadicity

Example: Triadicity of scales



C diatonic scale

1.0 − 1.0 + 1.0 +0 + 0 + 0 + 0 = 1.0



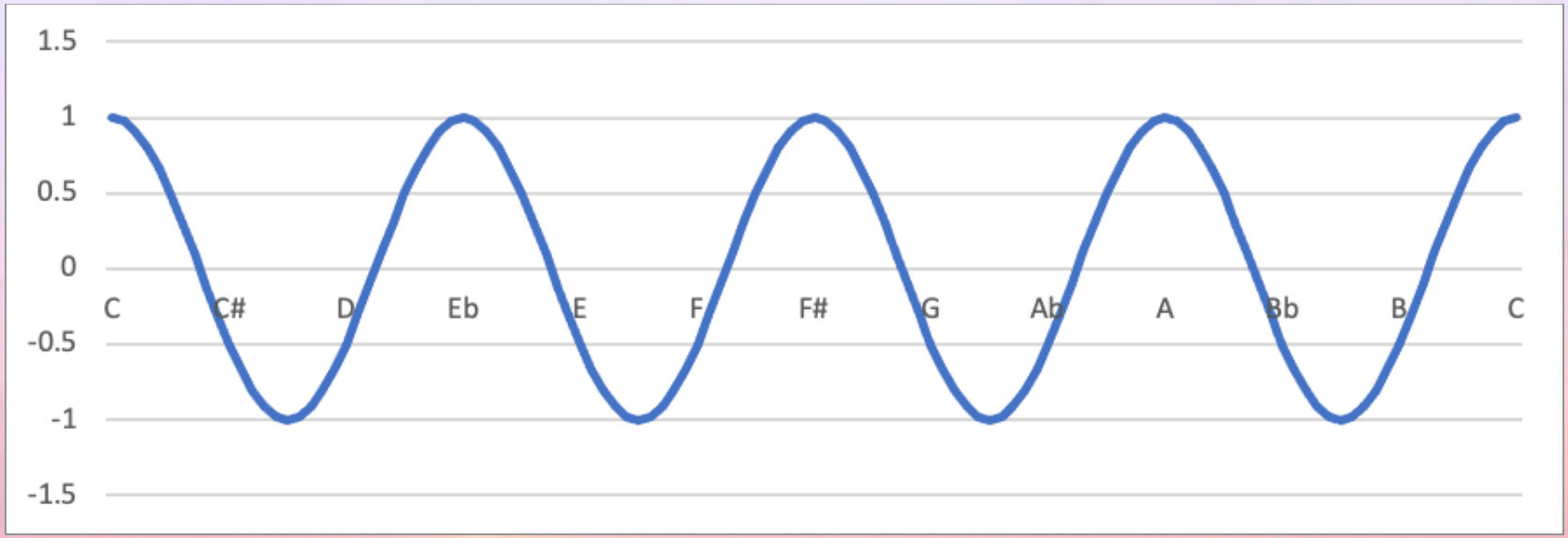
C harmonic minor

0.45−0.45+0.89 −0.89+0.89+0.45 + 0.89 = 2.23

- Cardinality-flexible: applies to chords of any size
- Not all triad positions need to be represented; multiple notes can represent a single category

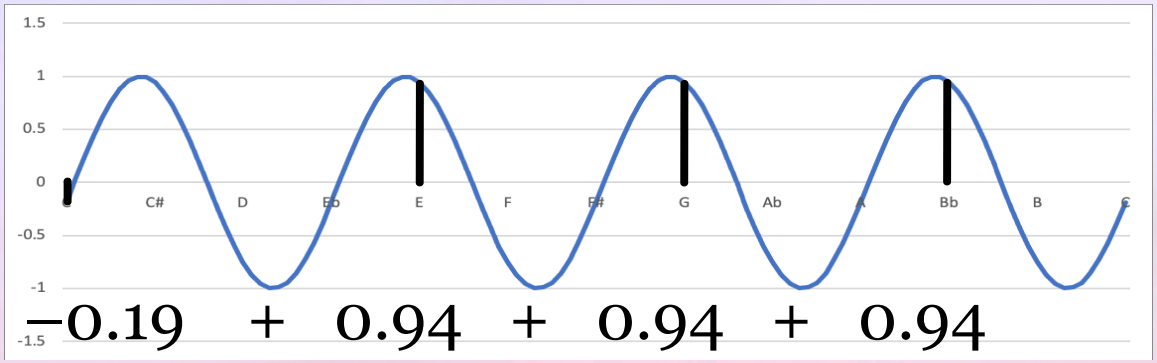
Tetradicity

An analogous concept of *tetradicity* (8ve/4):



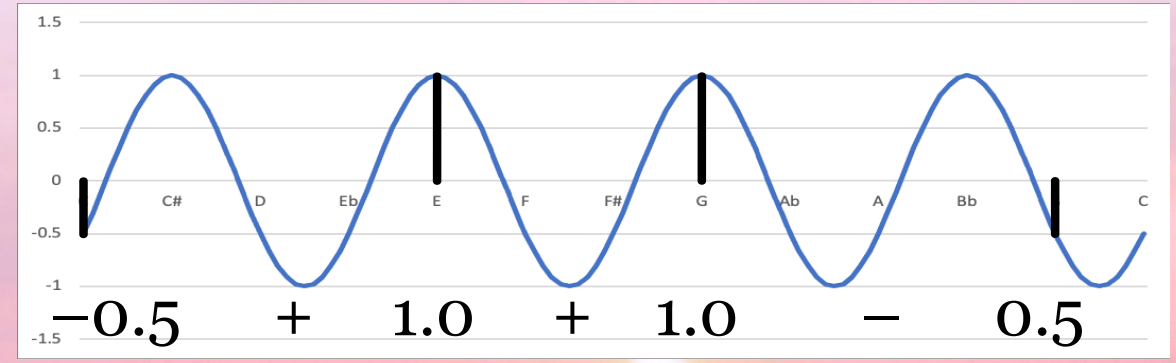
Tetradicity

Examples: Tetradicity of seventh chords



−0.19 + 0.94 + 0.94 + 0.94 = 2.65

Dominant seventh

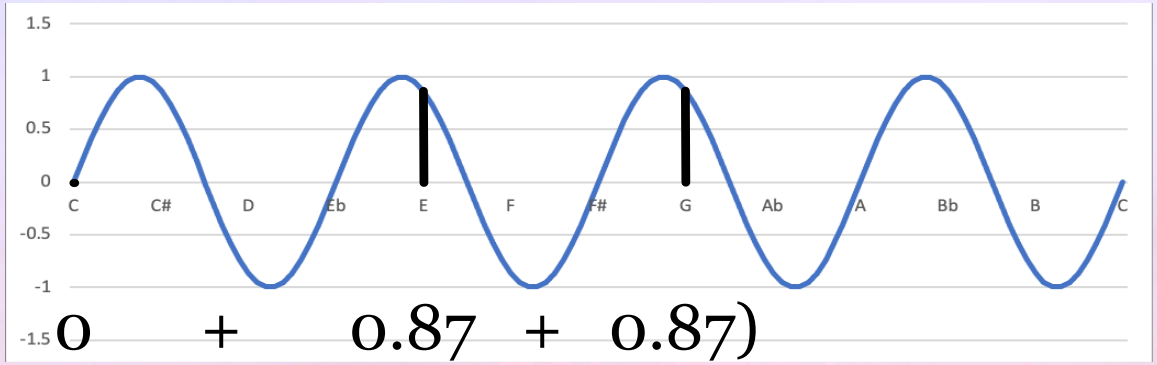


−0.5 + 1.0 + 1.0 − 0.5 = 1.0

Major seventh

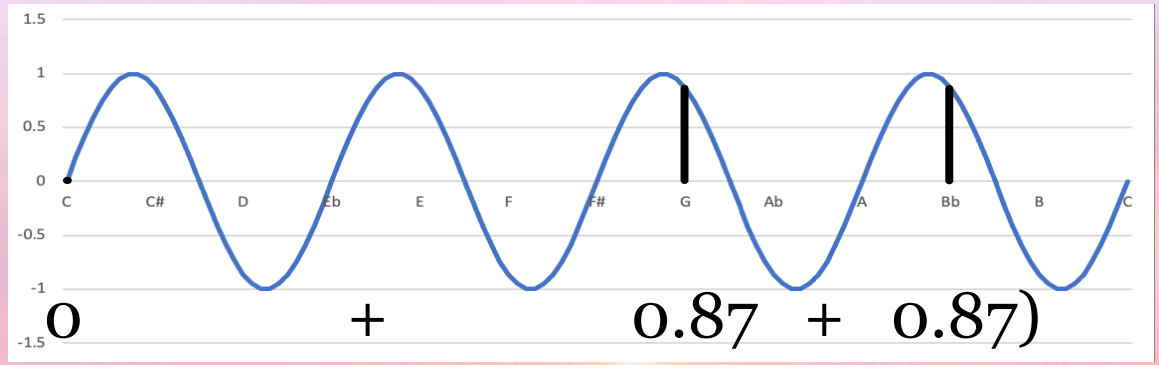
Tetradicity

Examples: Tetradicity of incomplete seventh chords



0 + 0.87 + 0.87) = 1.73

C major triad

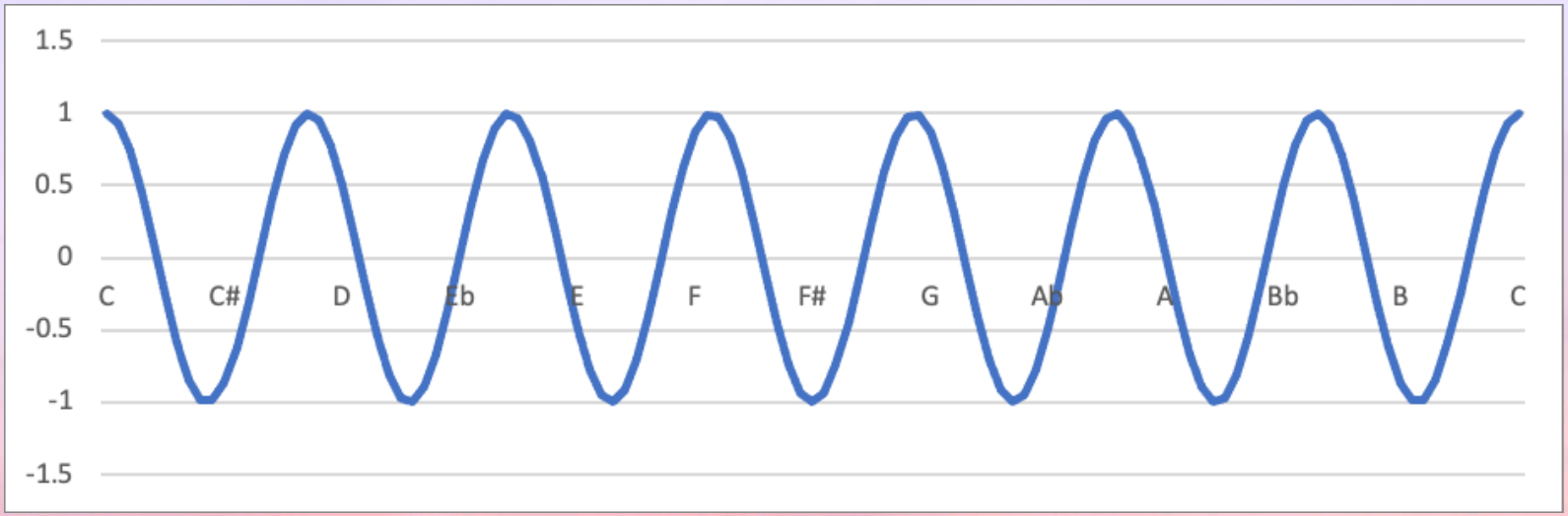


0 + 0.87 + 0.87) = 1.73

C7 (no 3rd)

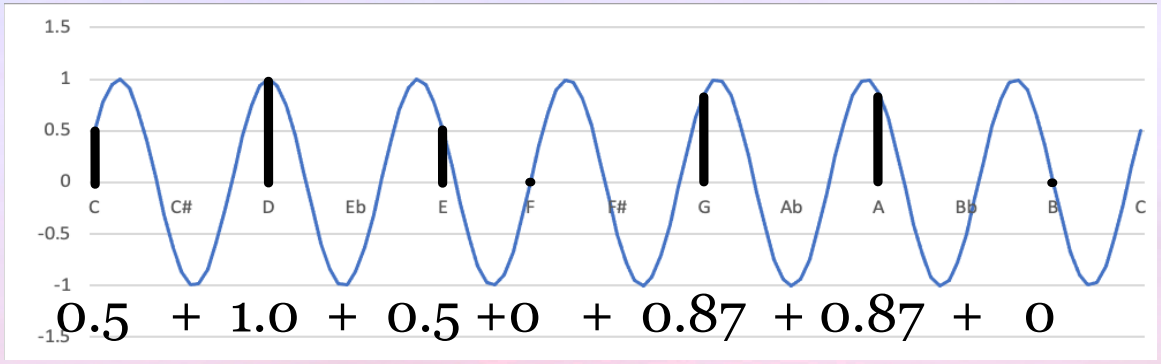
Heptatonicity

An analogous concept of *heptatonicity* (8ve/7):



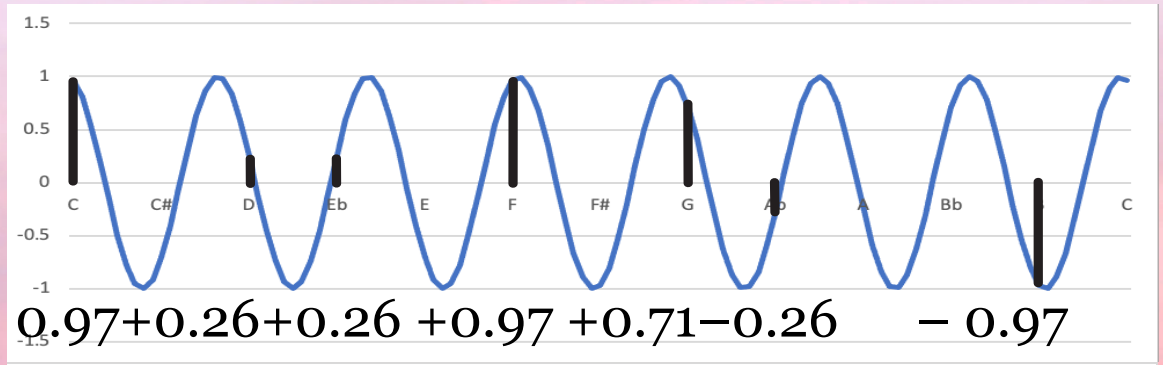
Heptatonicity

Examples: Heptatonicity of scales



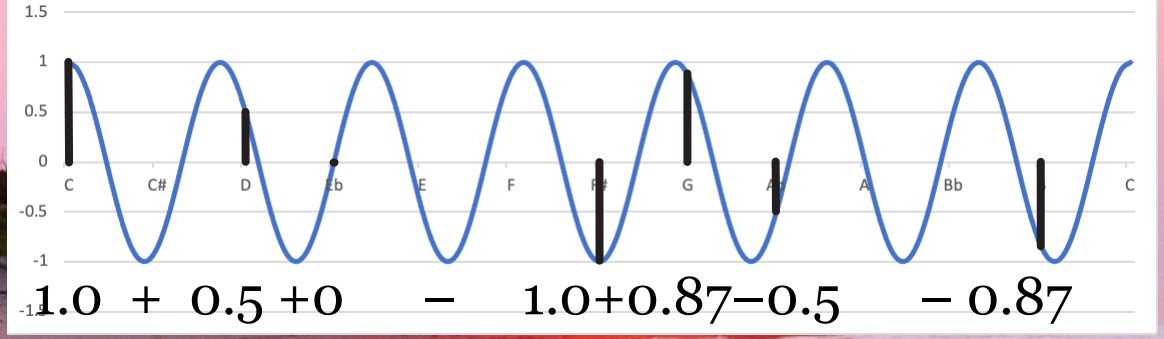
0.5 + 1.0 + 0.5 +0 + 0.87 + 0.87 + 0 = 2.28

C diatonic scale



0.97+0.26+0.26 +0.97 +0.71–0.26 – 0.97 = 1.93

C harmonic minor

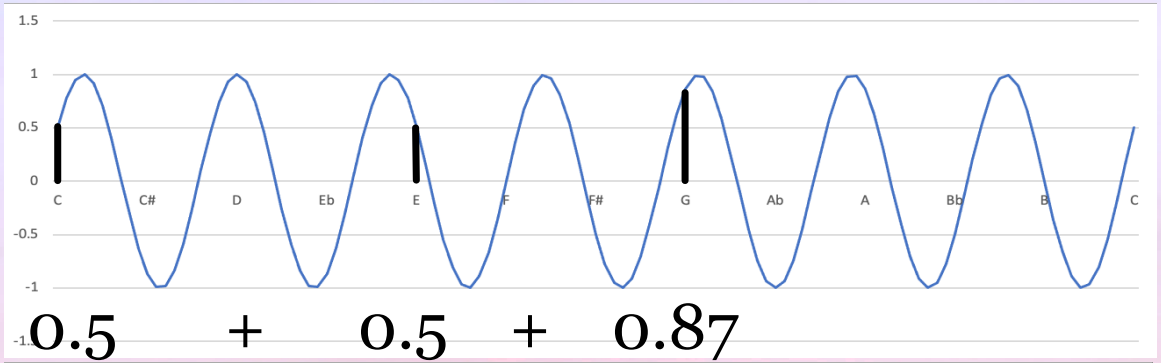


1.0 + 0.5 +0 – 1.0+0.87–0.5 – 0.87 = 0

C Hungarian minor

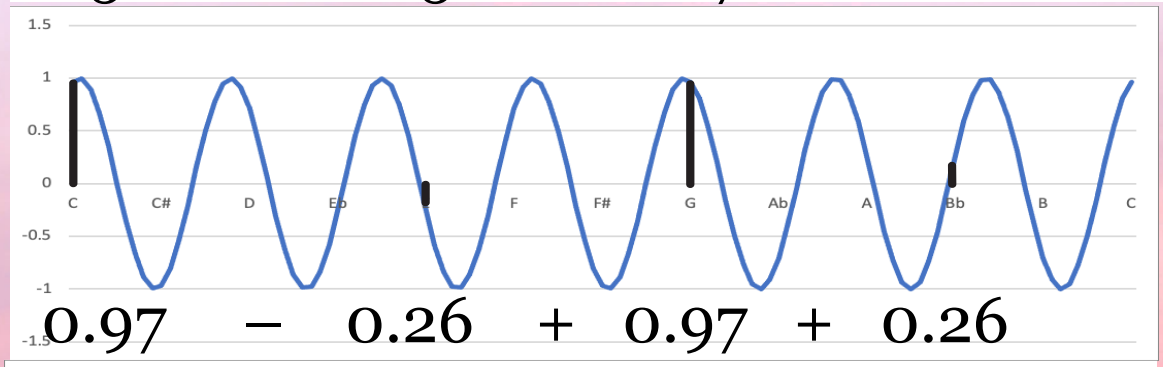
Heptatonicity

Examples: Heptatonicity of chords



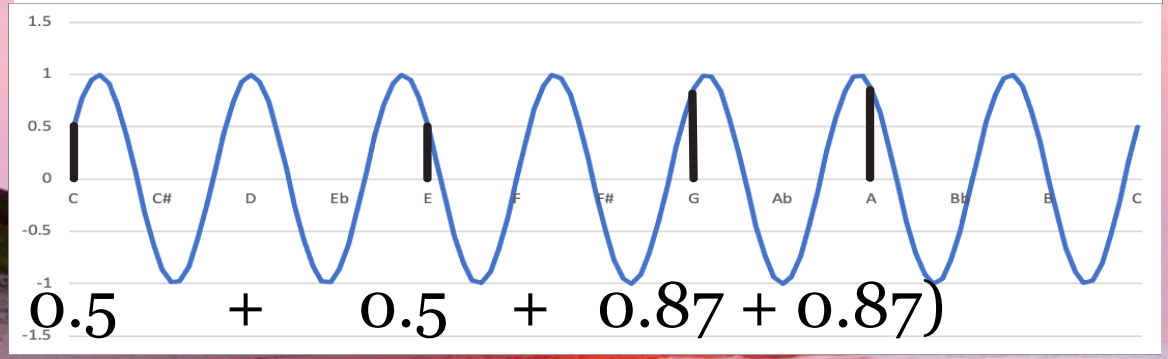
C major triad

0.5 + 0.5 + 0.87 = 1.87



C dominant seventh

0.97 – 0.26 + 0.97 + 0.26 = 2.46

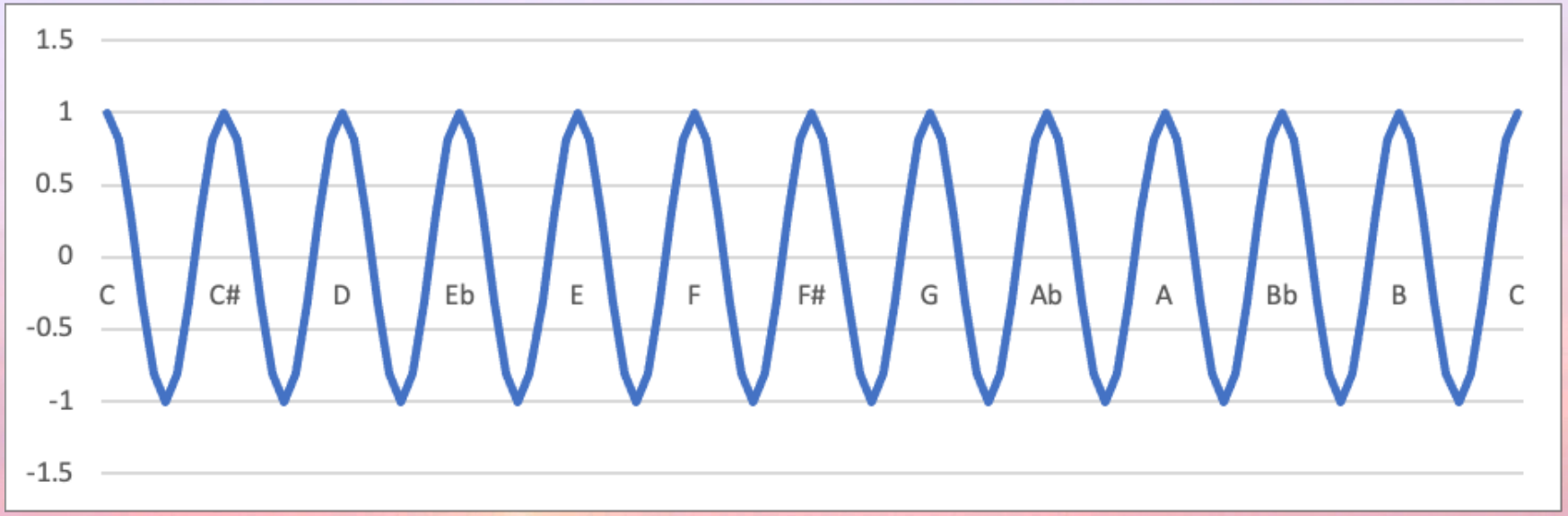


C add 6

0.5 + 0.5 + 0.87 + 0.87) = 2.73

Chromaticity

We can similarly define ***chromaticity*** as **approximation to a subset of a 12-tone equal tuning**



All ordinary pcsets have perfect chromaticity

Heptatonicity vs. Diatonicity

The diatonic scale is a ***prototype* of heptatonicity:** it maximizes heptatonicity for a 7-note subset of 12-tET.

Therefore heptatonicity **in a 12-tET context** equates to similarity to characteristic diatonic subsets.

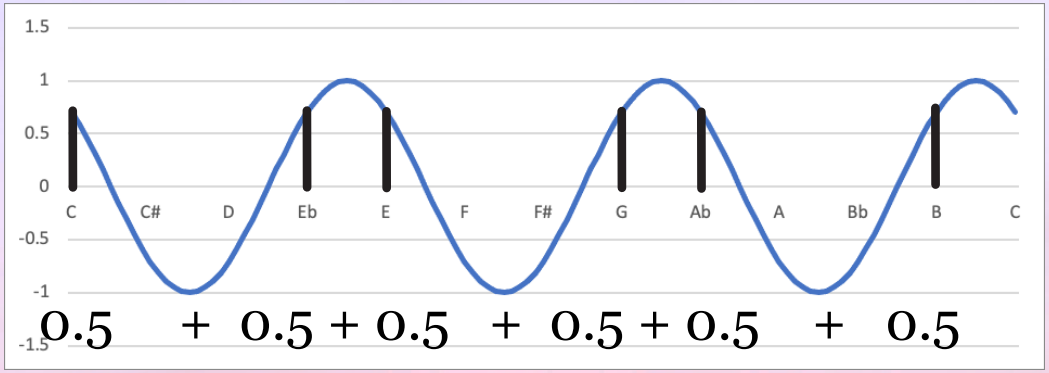
i.e.

Diatonicity = Heptatonicity + chromaticity

The distinction is only relevant in the context of alternate or flexible tuning, but it is also conceptually important.

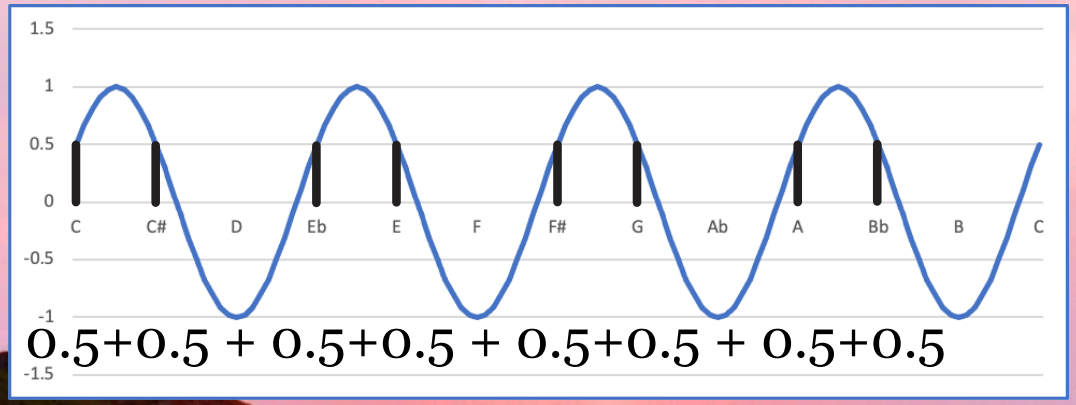
Hexatonicity and Octatonicity

The 12-tET prototype of **triadicity** is a ***hexatonic scale:***



Hexatonicity = Triadicity + chromaticity

The 12-tET prototype of **tetradicity** is an ***octatonic scale:***



Octatonicity = Tetradicity + chromaticity

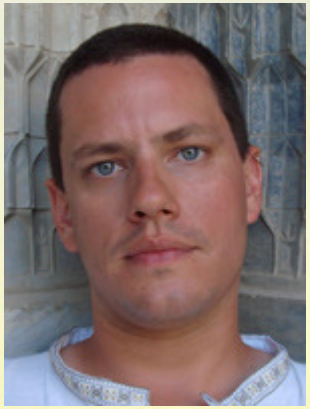
Fourier Transform on Pitch-Class Sets and Distributions

Using discrete Fourier transform, we can

- Identify harmonic qualities, through **spectra**
- Relate harmonies of similar quality through **phase spaces**

Krumhansl's tonal space is a phase space.

Fourier Transform on Pitch-Class Vectors: A brief history



Lewin, David (1959). "Re: Intervallic Relations between Two Collections of Notes," *JMT* 3/2.

——— (2001). "Special Cases of the Interval Function between Pitch Class Sets X and Y." *JMT* 45/1.

Quinn, Ian (2006–2007). "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1.

Callender, Cliff (2007). "Continuous Harmonic Spaces," *JMT* 51/2

Amiot, Emmanuel (2007). "David Lewin and Maximally Even Sets." *Journal of Mathematics and Music* 1/3.

——— (2016). *Music Through Fourier Space: Discrete Fourier Transform in Music Theory*. (Springer)

Yust, Jason (2015). "Schubert's Harmonic Language and Fourier Phase Spaces." *JMT* 59/1.

——— (2016). "Special Collections: Renewing Forte's Set Theory." *JMT* 60/2.

Pitch-Class Vectors

Key profiles are *pitch-class vectors*

Example:	C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
Krumhansl-Kessler C major:	(6.4,	2.2,	3.5,	2.3,	4.4,	4.1,	2.5,	5.2,	2.4,	3.7,	2.3,	2.9)
Krumhansl-Kessler C minor:	(6.3,	2.7,	3.5,	5.4,	2.6,	3.5,	2.5,	4.8,	4.0,	2.7,	3.3,	3.2)

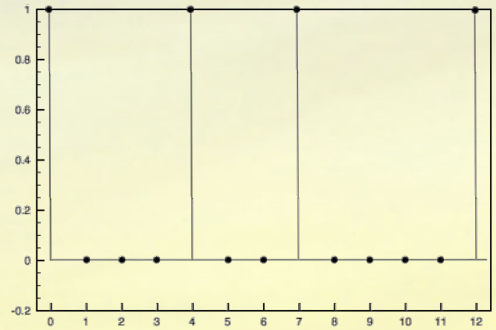
Pitch-class sets and multi-sets are also pitch-class vectors

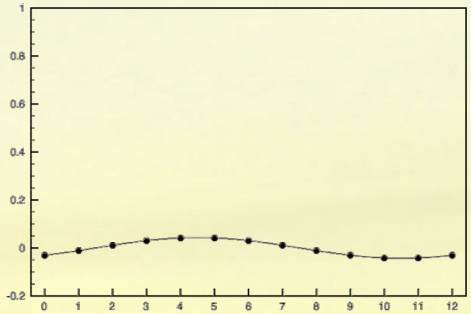
Example:	C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
C major triad:	(1,	0,	0,	0,	1,	0,	0,	1,	0,	0,	0,	0)
C major scale + tonic triad:	(2,	0,	1,	0,	2,	1,	0,	2,	0,	1,	0,	1)

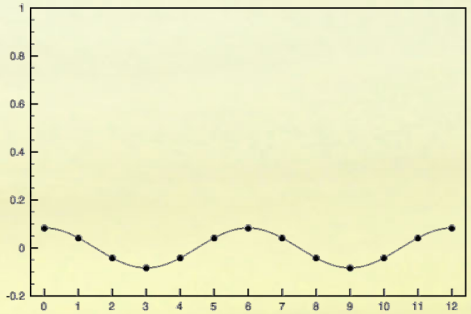
Other names: “Characteristic function,” “pitch-class distribution”

Fourier Transform of a Pitch-Class Vector

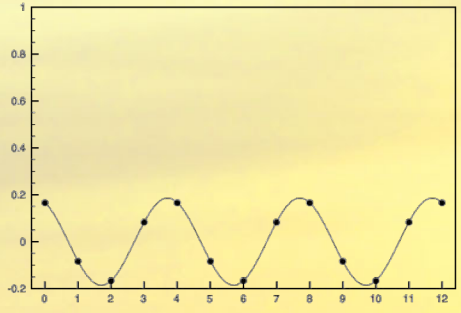
C major triad = f_0 + f_1 + f_2

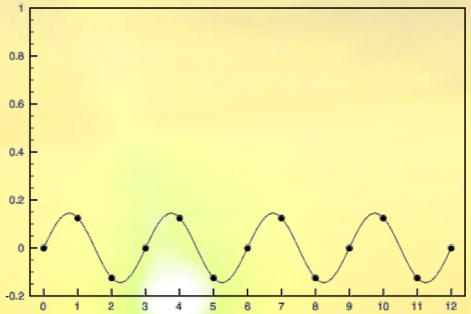


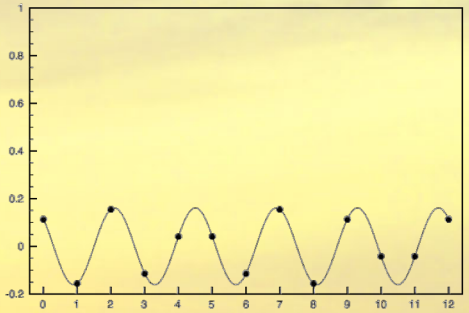


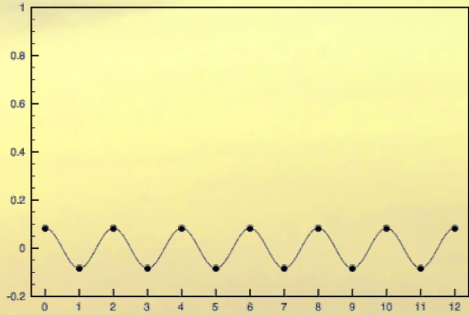


+ f_3 + f_4 + f_5 + f_6 + . . .



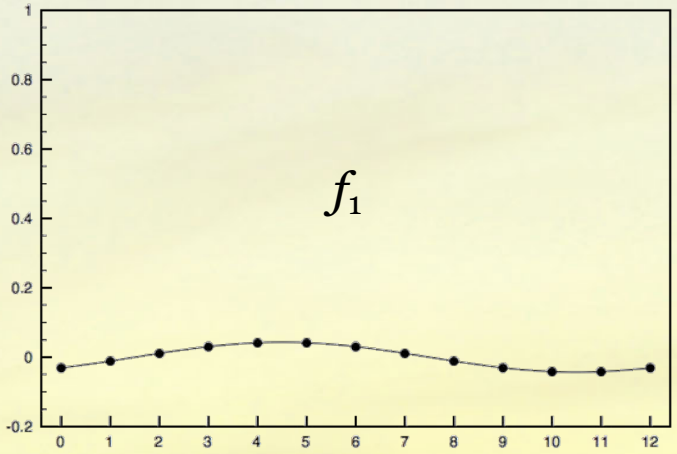


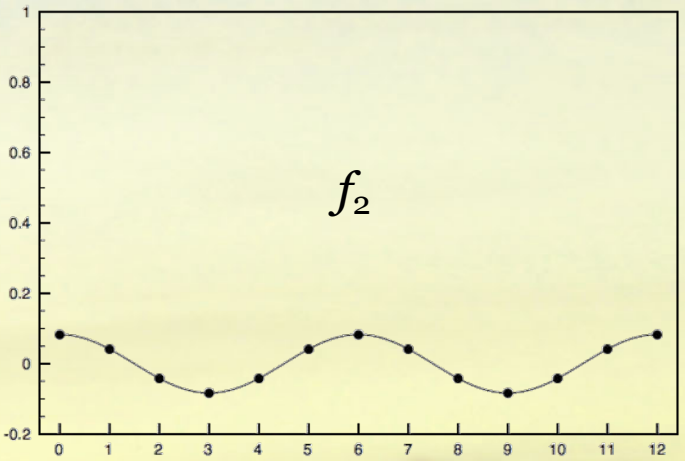




The DFT is a **lossless transformation** from a pitch-class vector to a sum of **periodic functions** dividing the octave into 1–6 parts.

Fourier Qualities

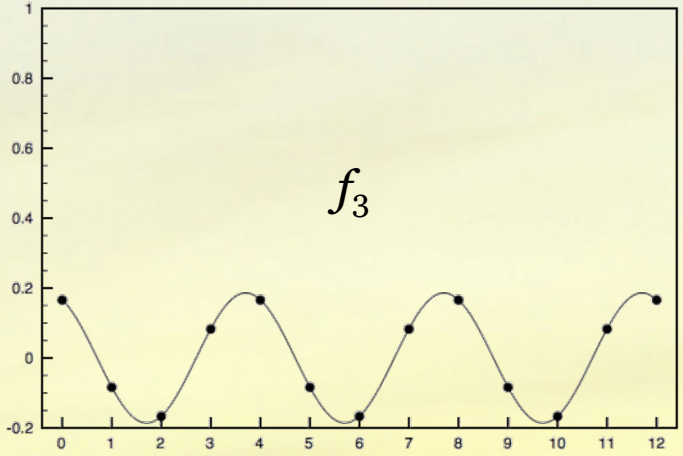


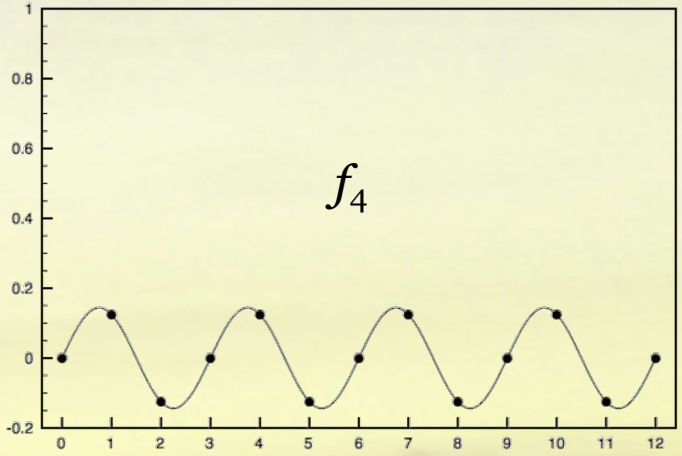


f_1 represents a concentration of pitch-class weight on the full pc circle.

f_2 represents ***Dyadicity*** concentration of pitch-class weight on a half-octave (tritone) cycle.

Fourier Qualities

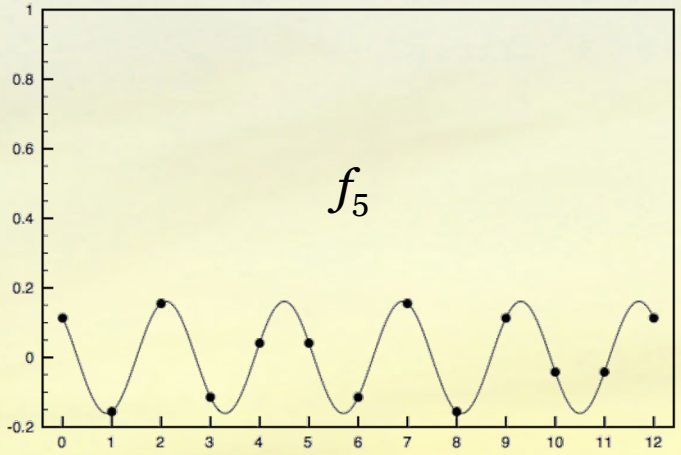




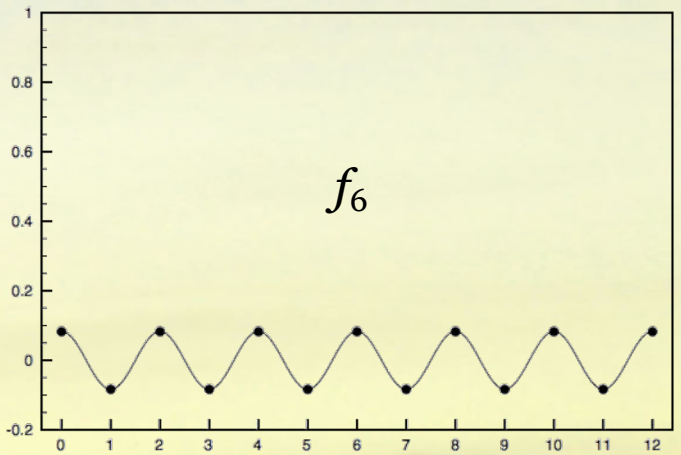
f_3 gives the ***triadicity*** or ***hexatonicity***.

f_4 gives the ***tetradicity*** or ***octatonicity***.

Fourier Qualities



f_5 give the balance on the ***circle of fifths***, which is the ***diatonicity*** or ***heptatonicity***



f_6 gives the weighting on one of the two ***whole-tone*** collections.

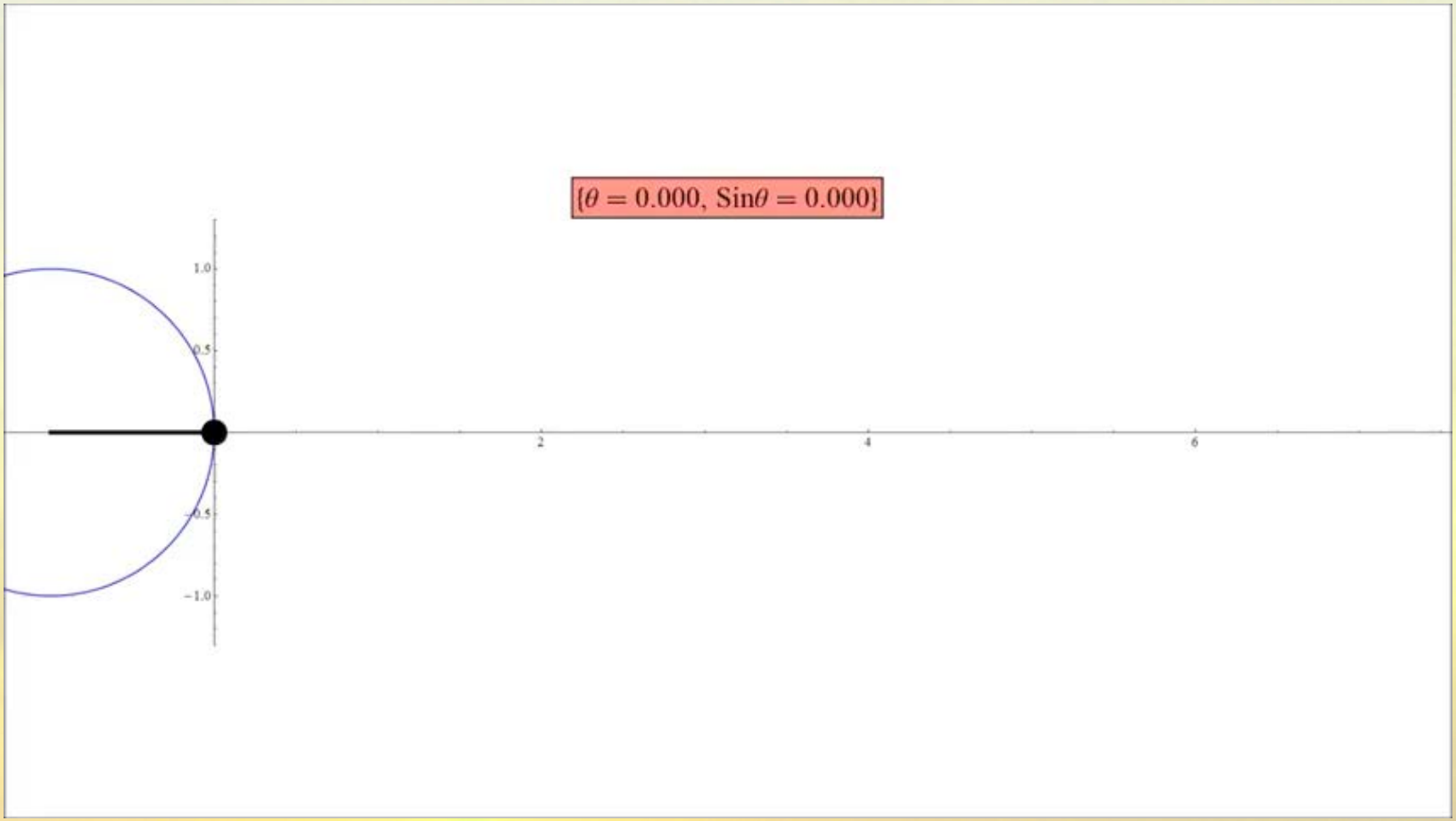
Fourier Qualities

Coefficients 7–11 are equivalent to 1–5 (with opposite phases)

$\Rightarrow$

Pentatonicity in a 12-tET context is **equivalent to diatonicity**

Fourier coefficients on the complex plane



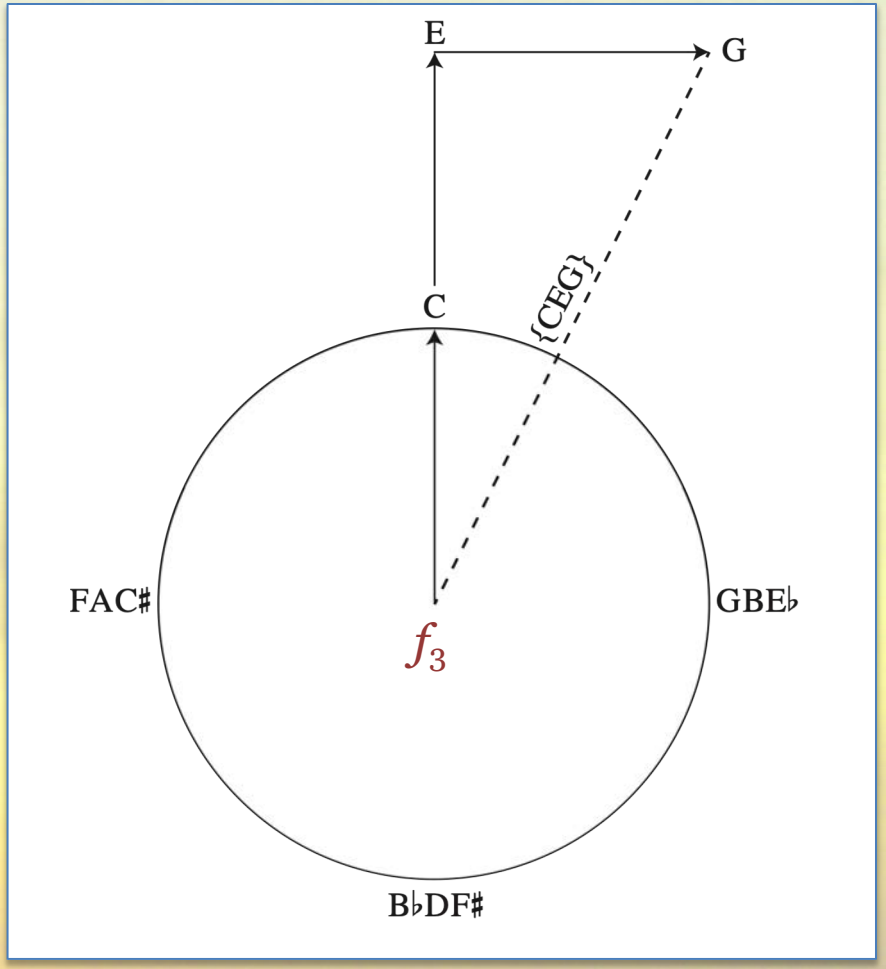
by Arkady Etkin

Fourier Transform as Vector Sums

Fourier component f_k can be derived as a vector sum with each pitch class as a unit vector, where the unit circle is the 8ve/k.

The length of the resulting vector is the **magnitude** of the component, and the angle is its **phase.**

Example: C maj. triad, k = 3



Fourier Transform as Vector Sums

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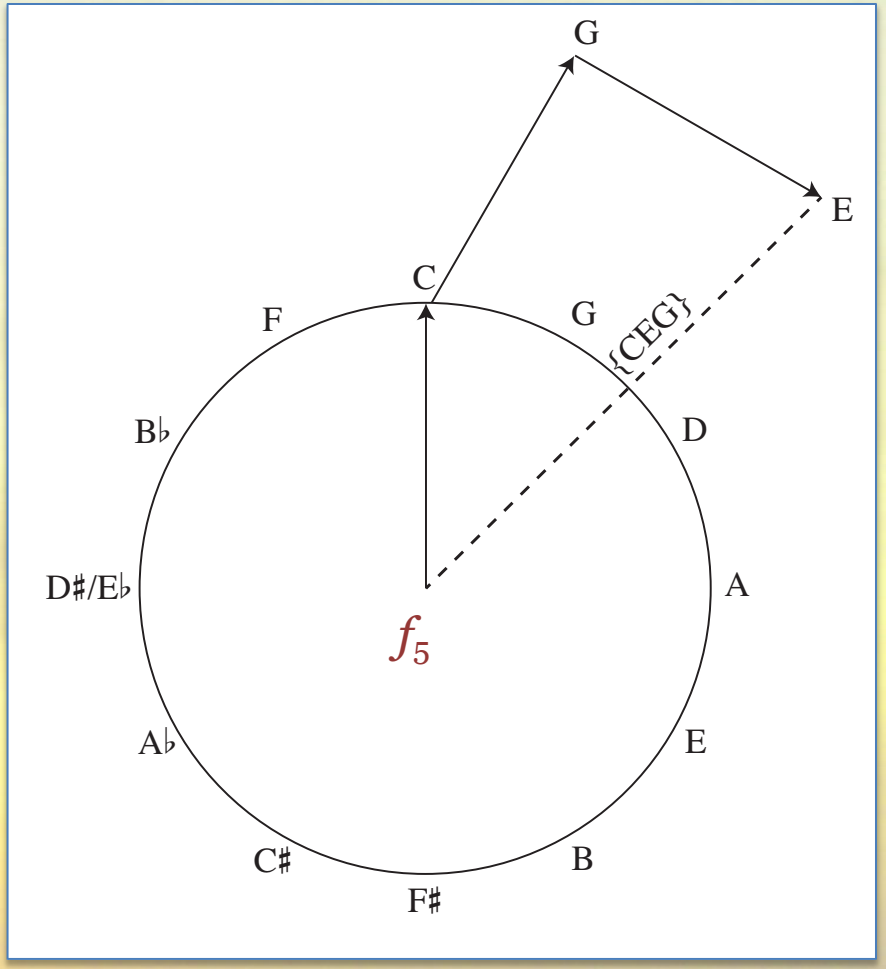
Example: C maj. triad, k = 5

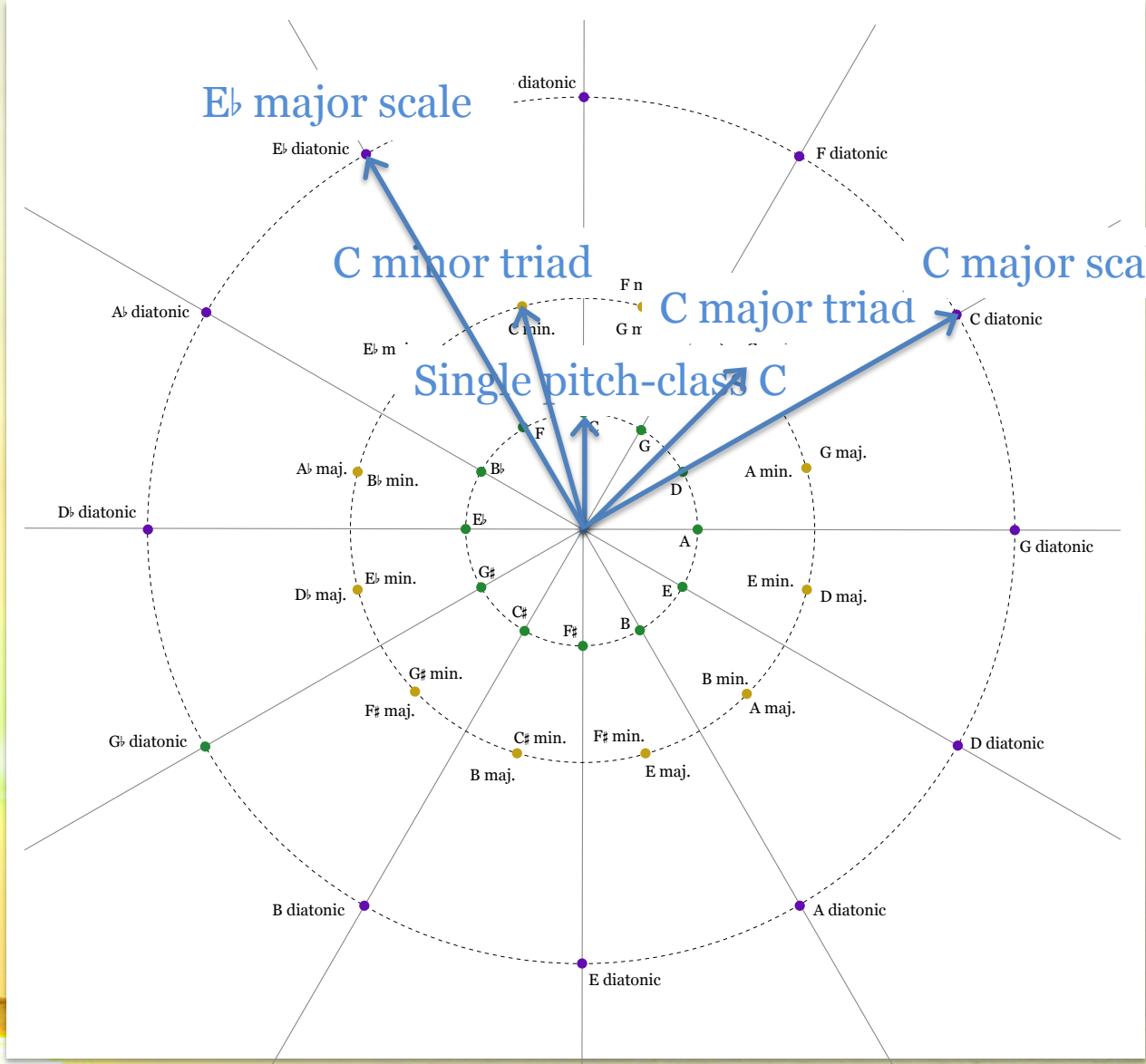


Single component spaces (complex plane)

Example: F_5 space

Distance from the center is the ***magnitude*** of f_5

Angle is the ***phase*** of f_5



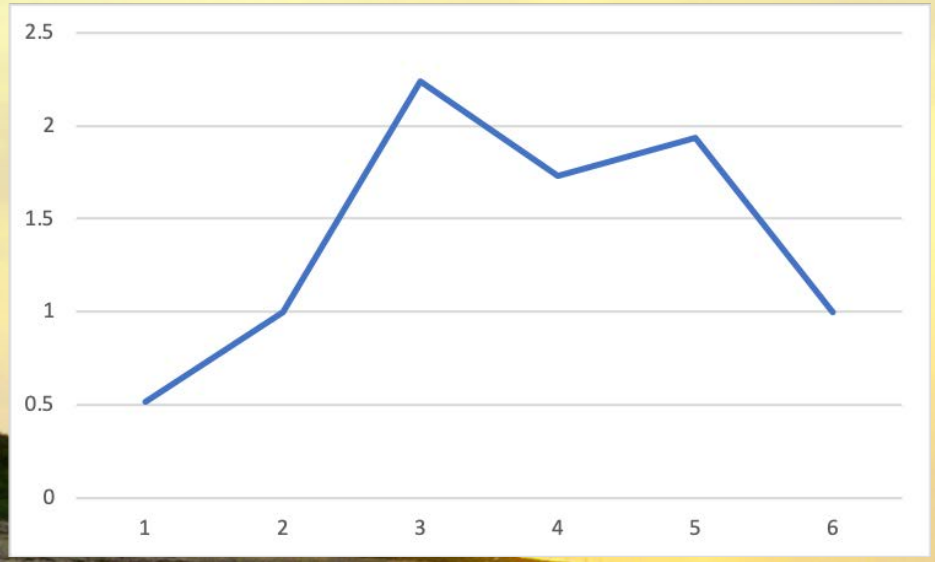
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

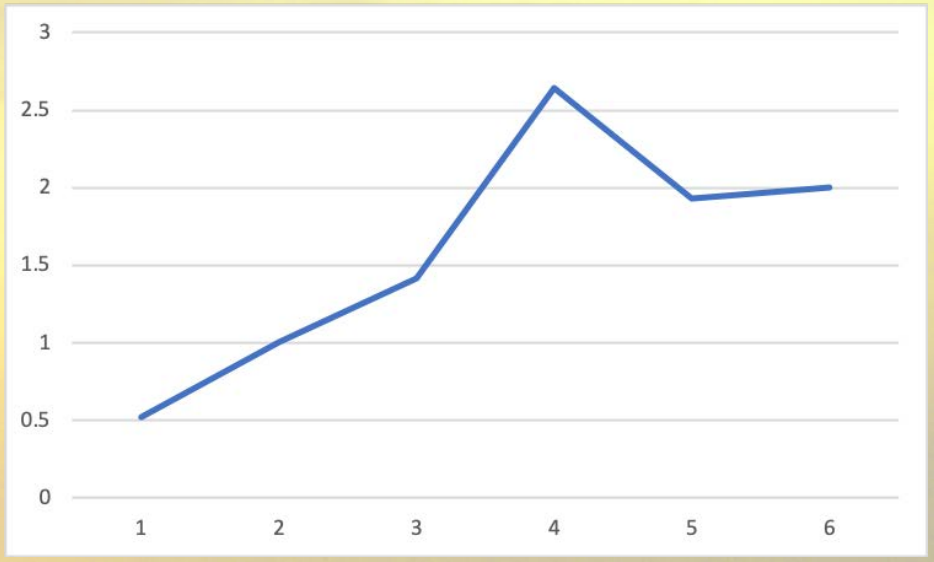
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Major/minor triad



Dominant 7th



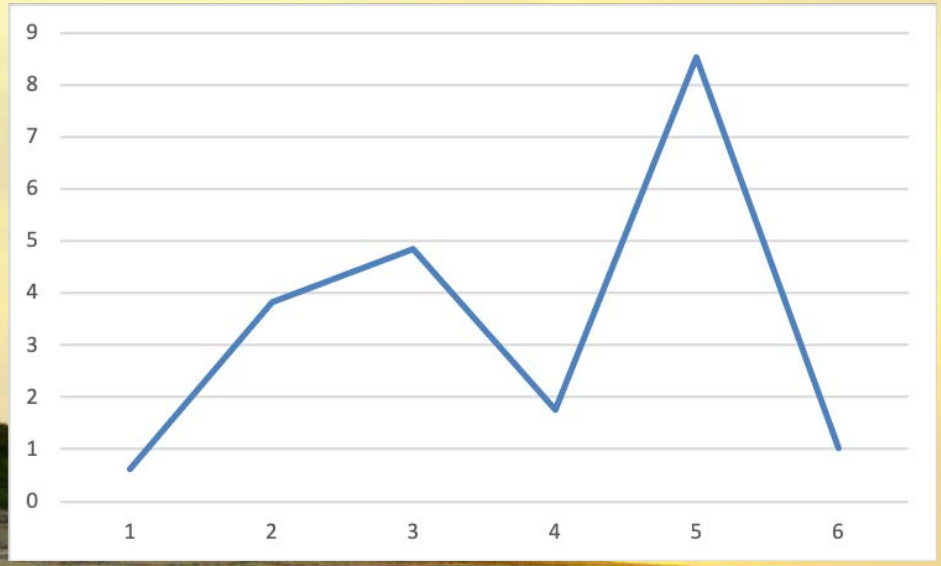
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

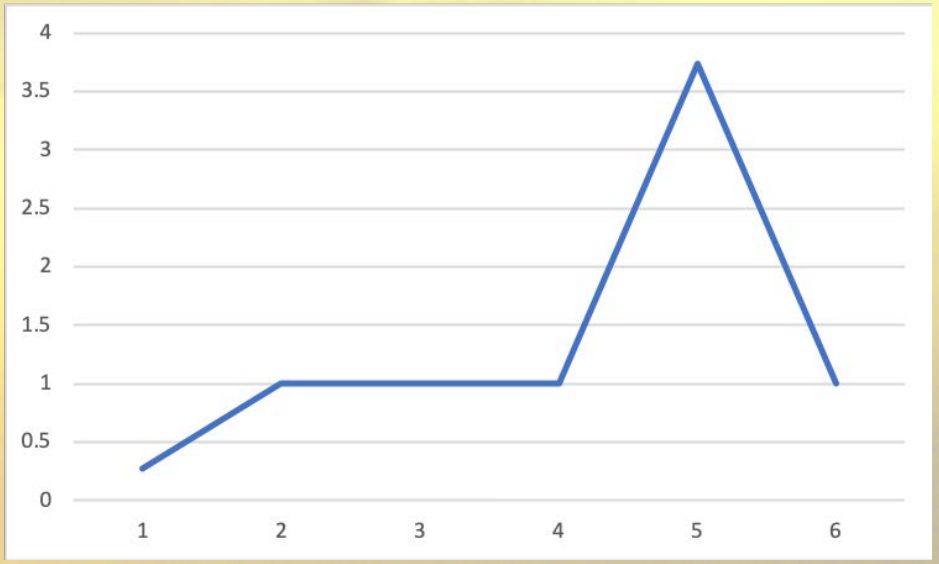
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Krumhansl-Kessler major key

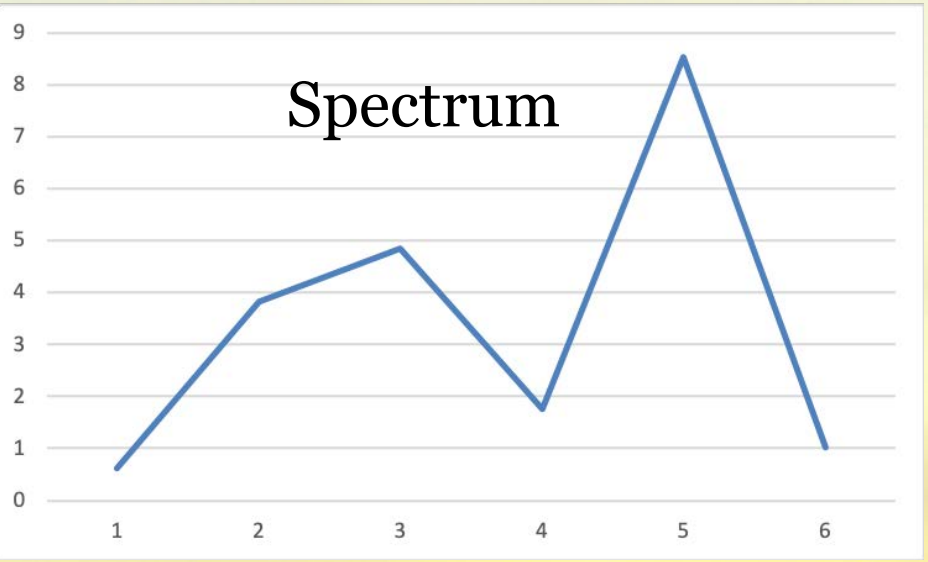


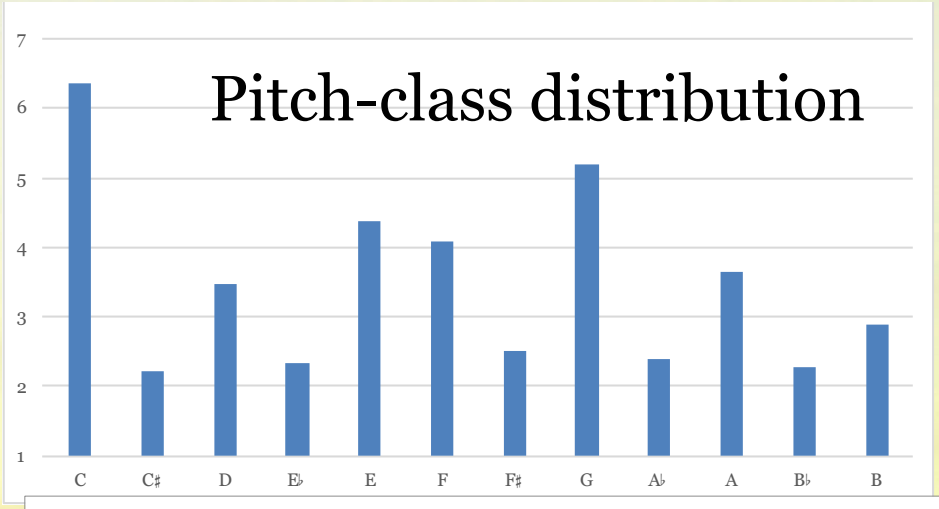
Diatonic scale



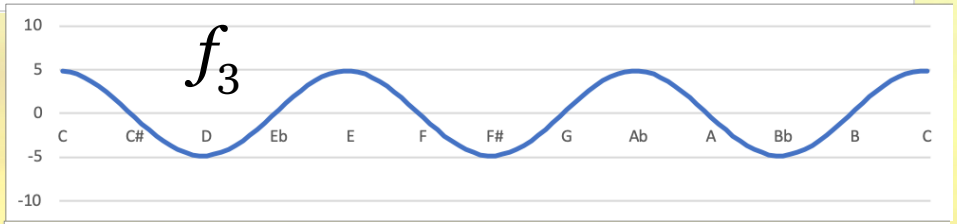
From Spectra to Phase Spaces

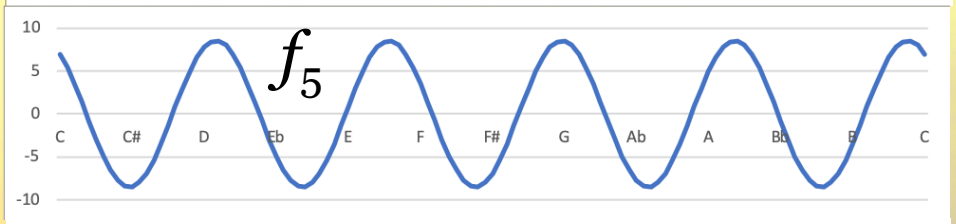
Krumhansl-Kessler major key



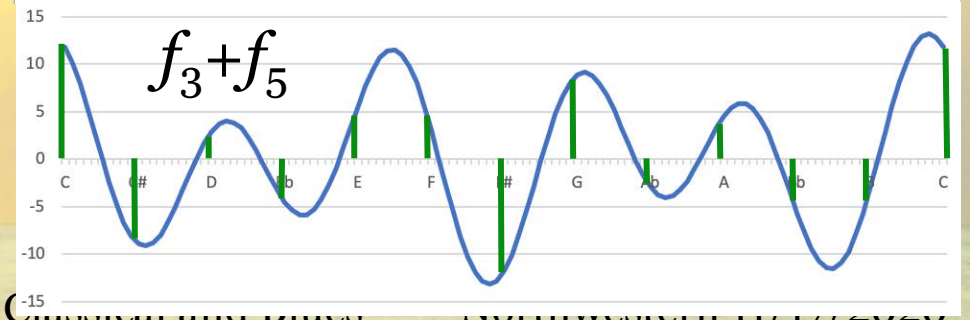


Principal components:



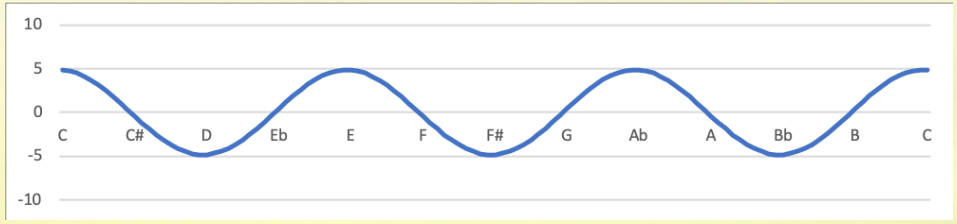


The sum of these gives a good approximation:

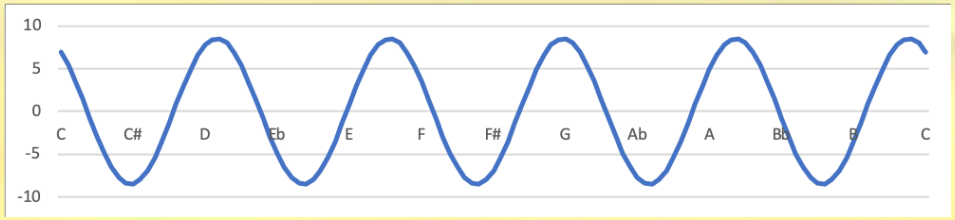


From Spectra to Phase Spaces

Transpositions of the key correspond to phase shifts of the components

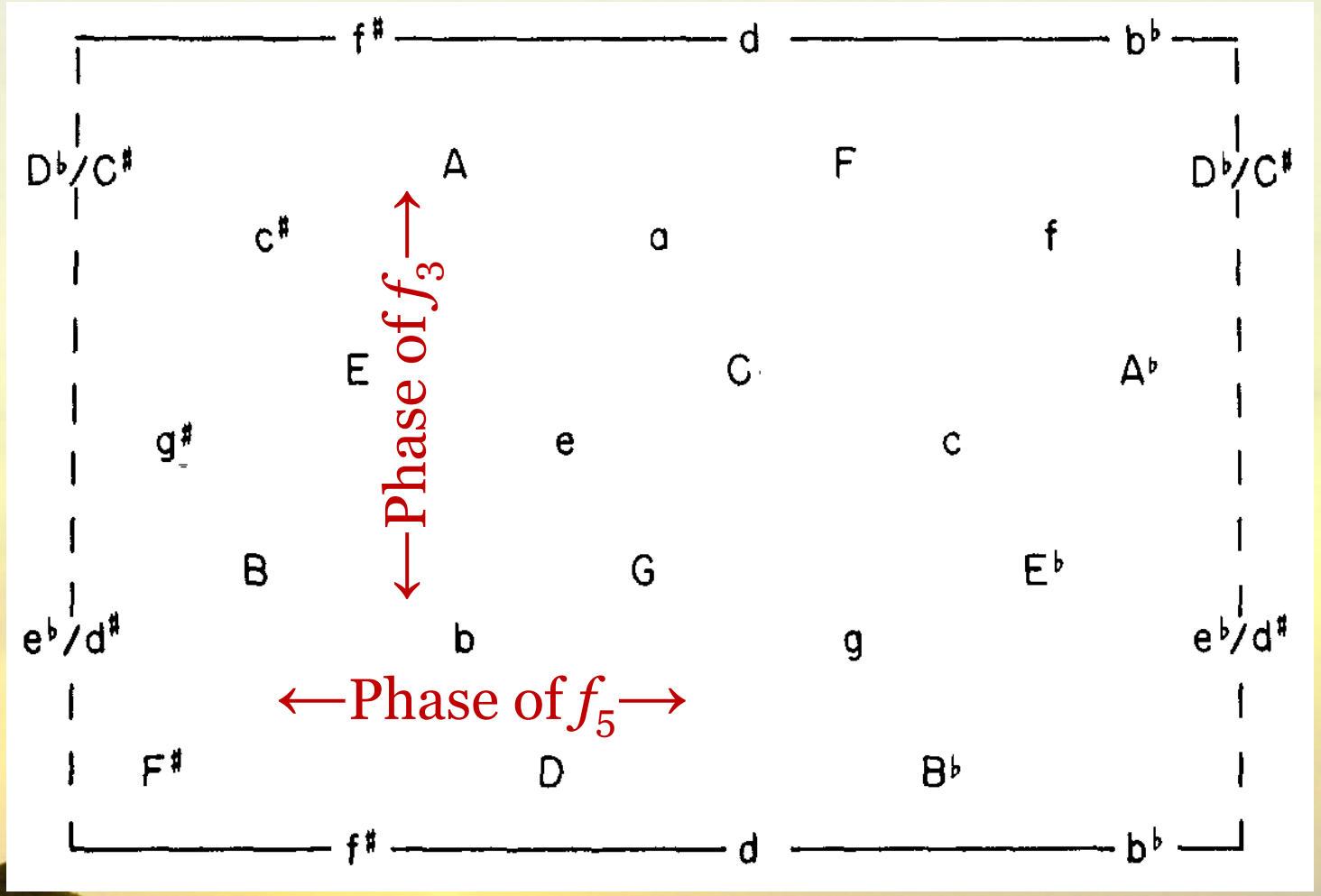


←Phase of f_3→



←Phase of f_5→

From Spectra to Phase Spaces



Krumhansl and Kessler Tonal Space

Blues Tonality

Blue notes and heptatonicity vs. diatonicity

Octatonicity in blues tonality:

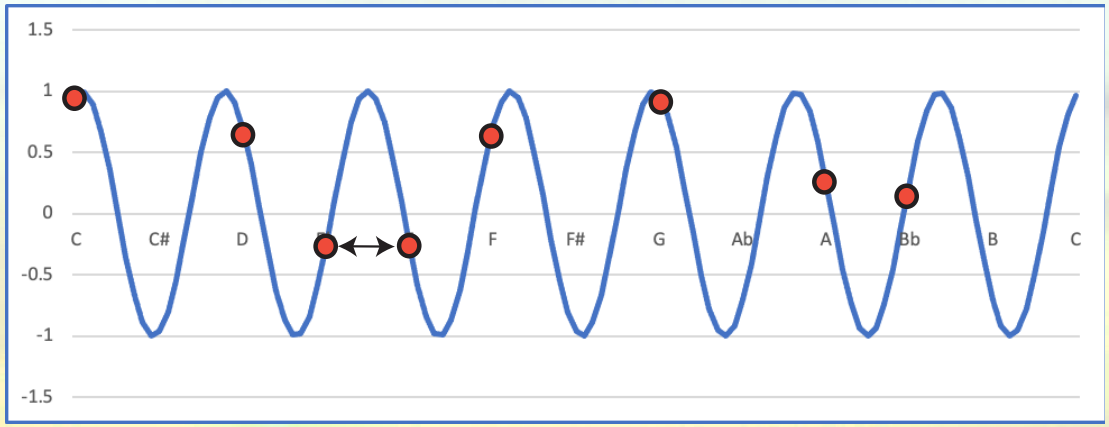
—Downhome blues (Titon transcriptions)

—Early published blues (W.C. Handy)

Blue notes and heptatonicity

Traditional African tone systems approximate **equiheptatonic** with **intonational flexibility** (see e.g., Kubik 2010)

Heptatonicity phase-locked to C-G:

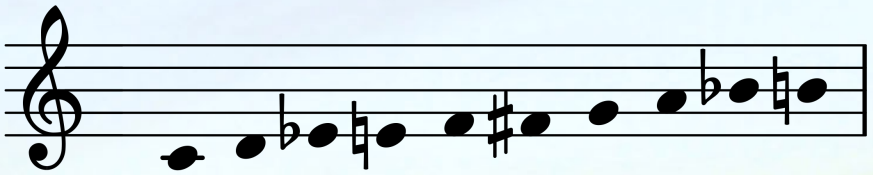


Some pitch-classes are good representatives (B♭, F, D, A), E♭/E are poor representatives.

This could explain why flexibility of intonation would be especially strong around E – E♭.

“Blues Scale”

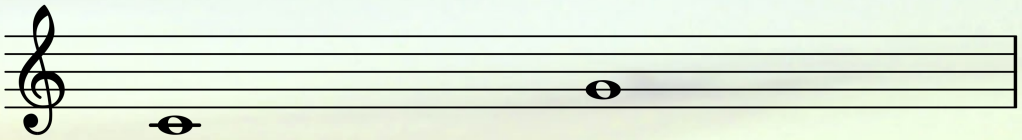
A scale gives a poor description of blues tonality:



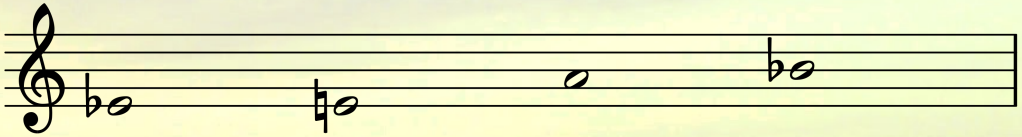
Too many notes!
(Only C♯ and A♭ are omitted.)

We get a better description with more levels:

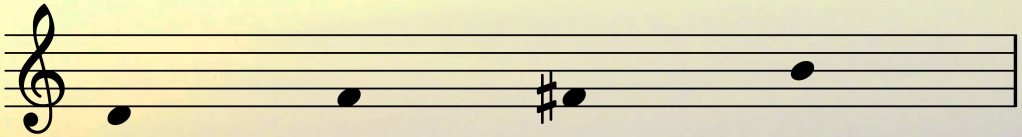
Anchors:



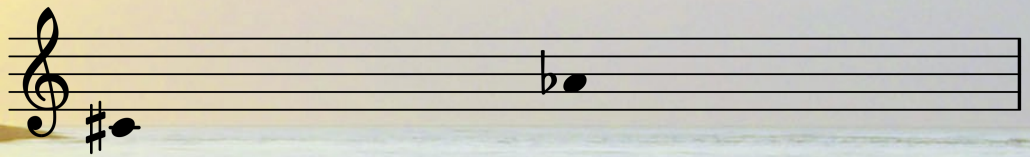
Principal notes:



Other usable:

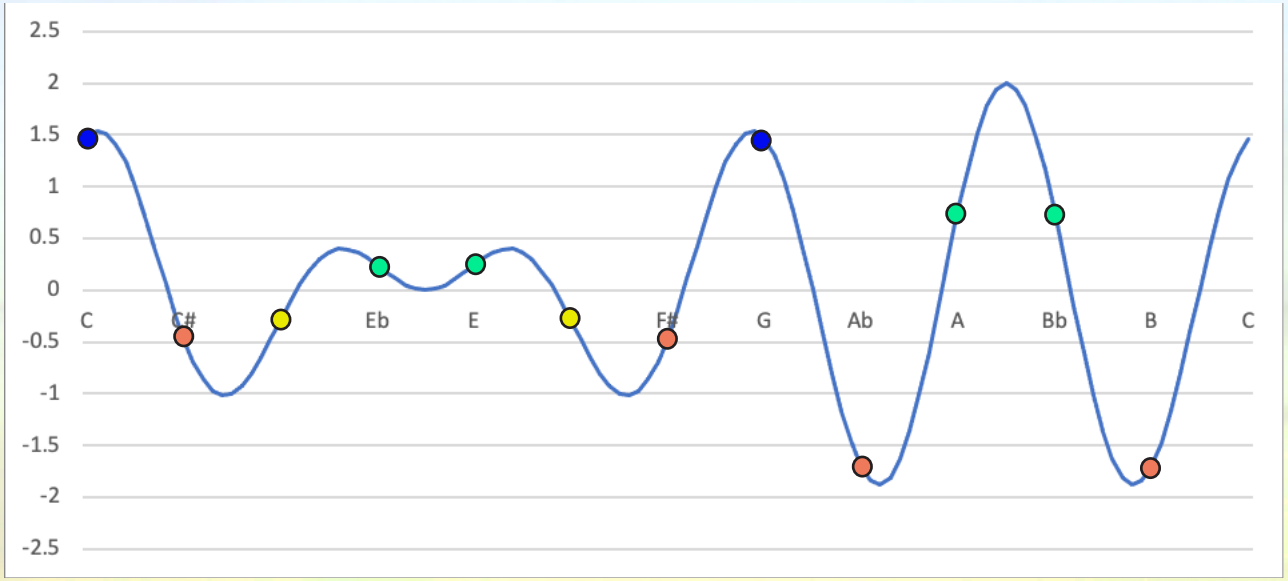


Not used:



“Blues Scale”

A good model: diatonic and octatonic distances from CG;

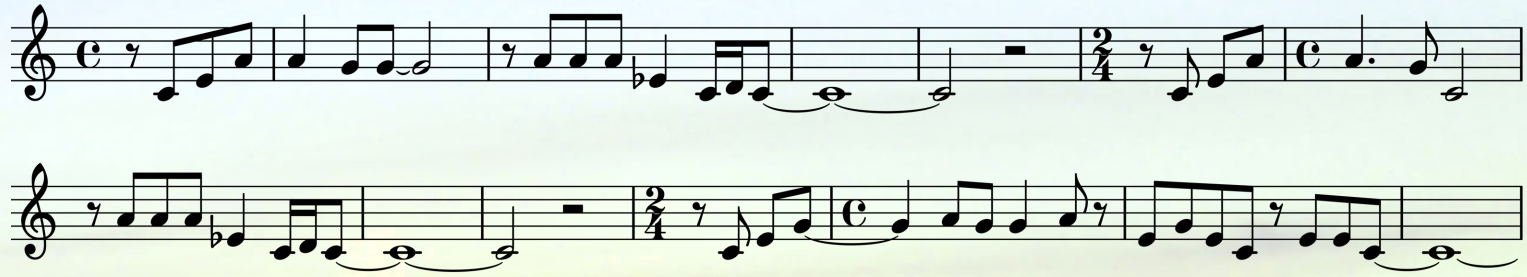


This is the sum of two sine curves of period 4 (octatonic) and period 5 (diatonic)

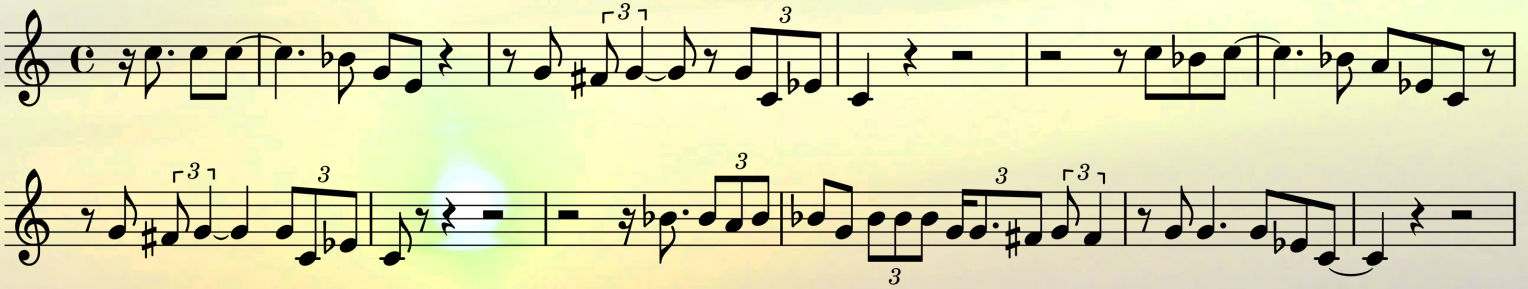
Downhome Blues Melodies

Transcriptions from Titon *Early Downhome Blues*

“Pony Blues” (Charlie Patton)



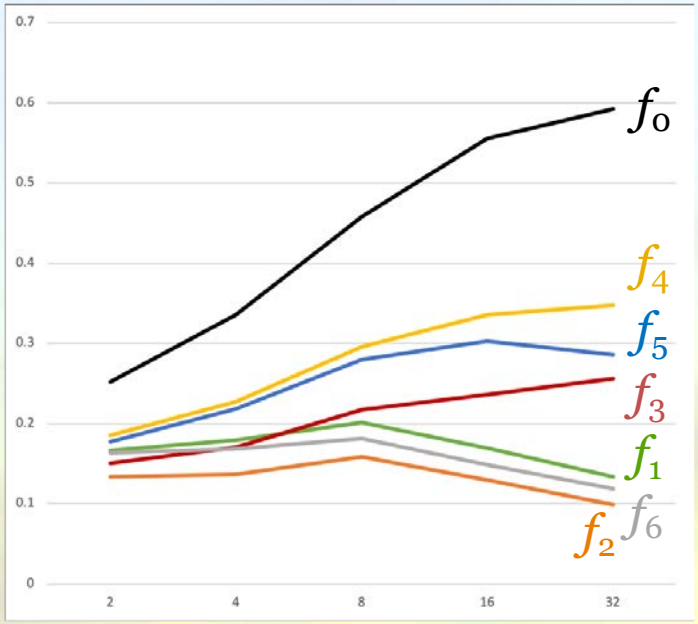
“Stranger Blues” (Rosie Mae Moore)



DFT of Blues Melodies

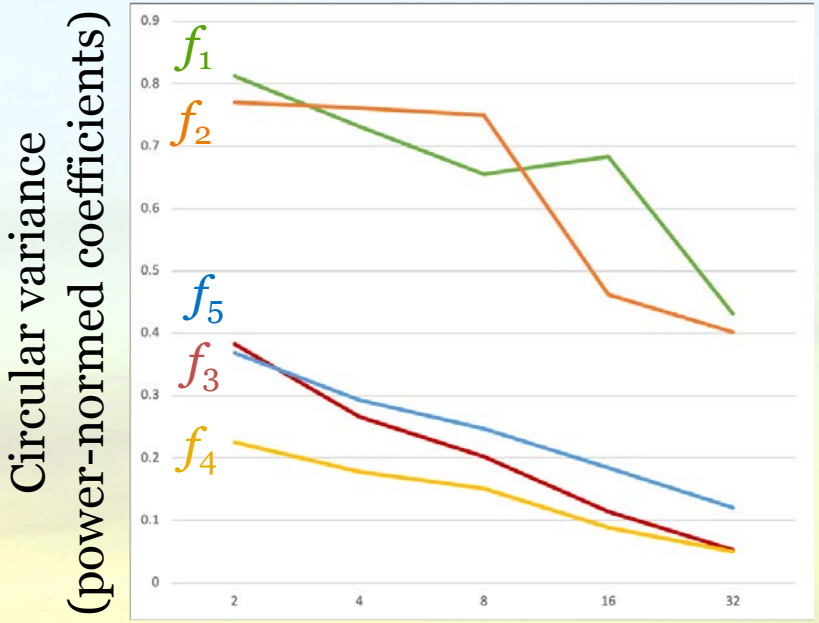
Example: “Pony Blues”

Coefficient magnitudes



Window in quarter-notes

Circular variance



Window in quarter-notes

Magnitudes measure the size of each quality.

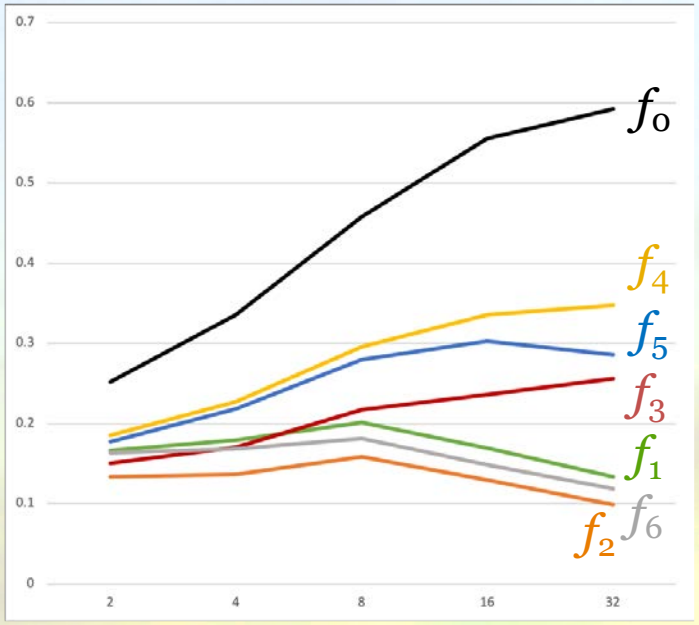
f_0 indicates the number of pcs used.

Values are normalized to vary from 0–1

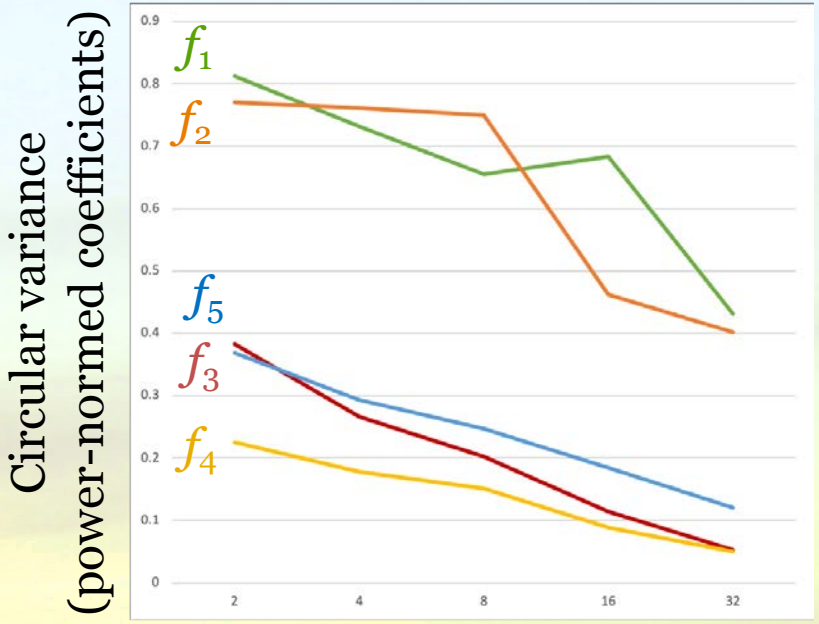
DFT of Blues Melodies

Example: "Pony Blues"

Coefficient magnitudes



Circular variance



Circular variance measures variability around an average phase value.

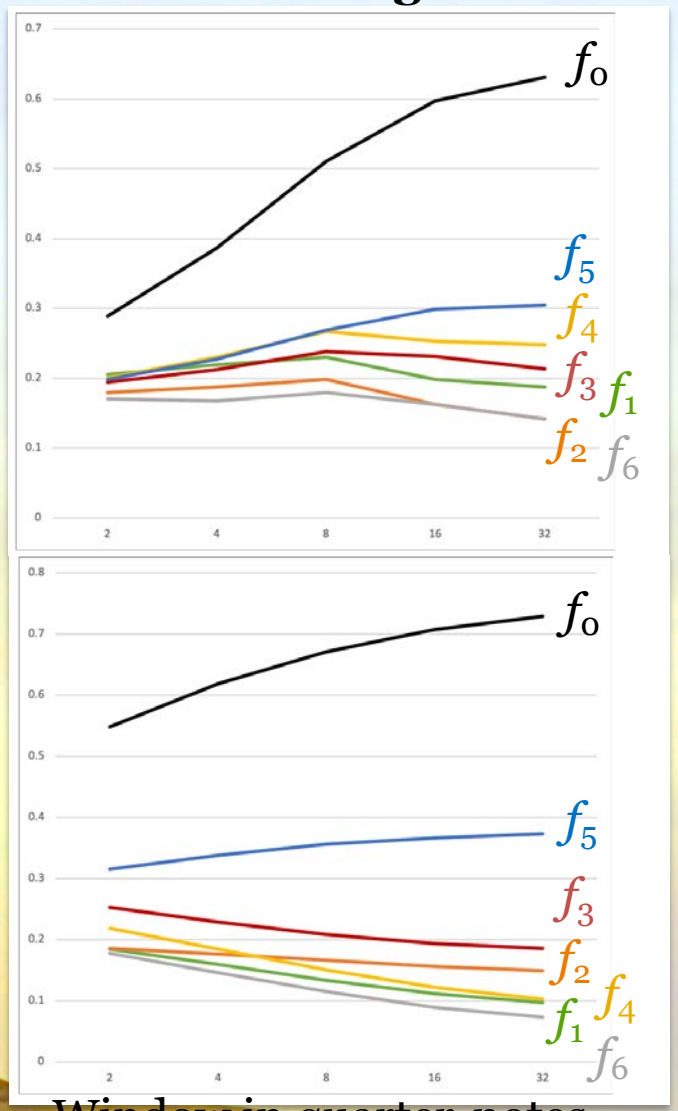
Larger magnitudes are typically associated with lower variances

DFT of Blues Melodies

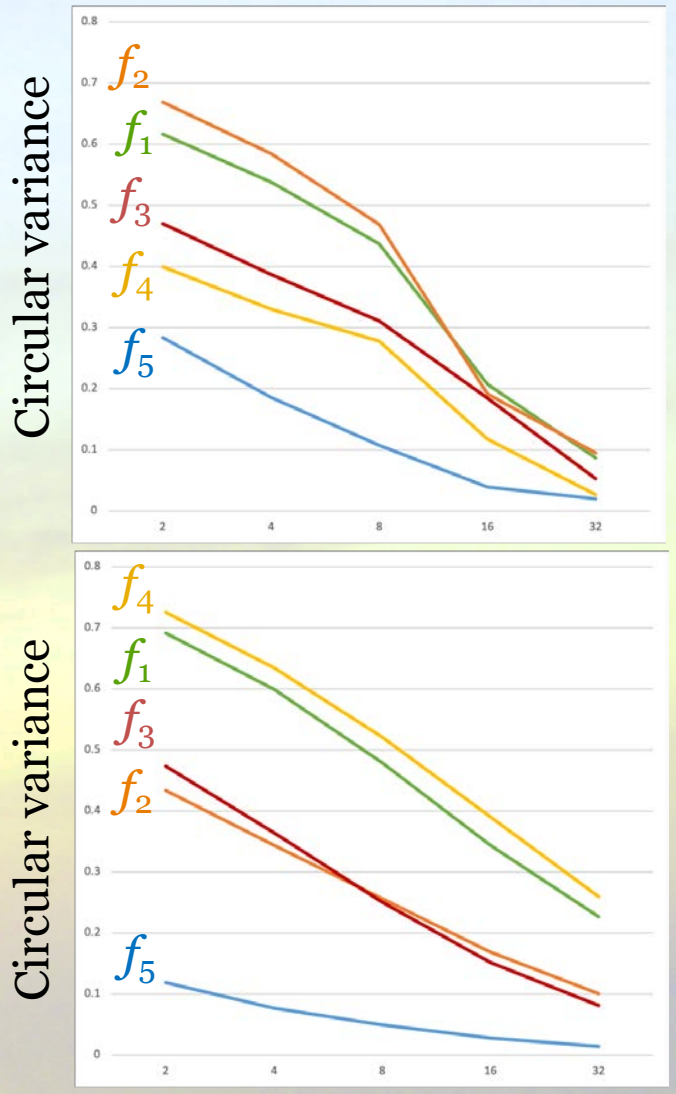
Windowed DFT averaged across 7 downhome blues transcriptions (Titon 1977)

Compare to a corpus of 18th-century minuets:

Coefficient magnitudes



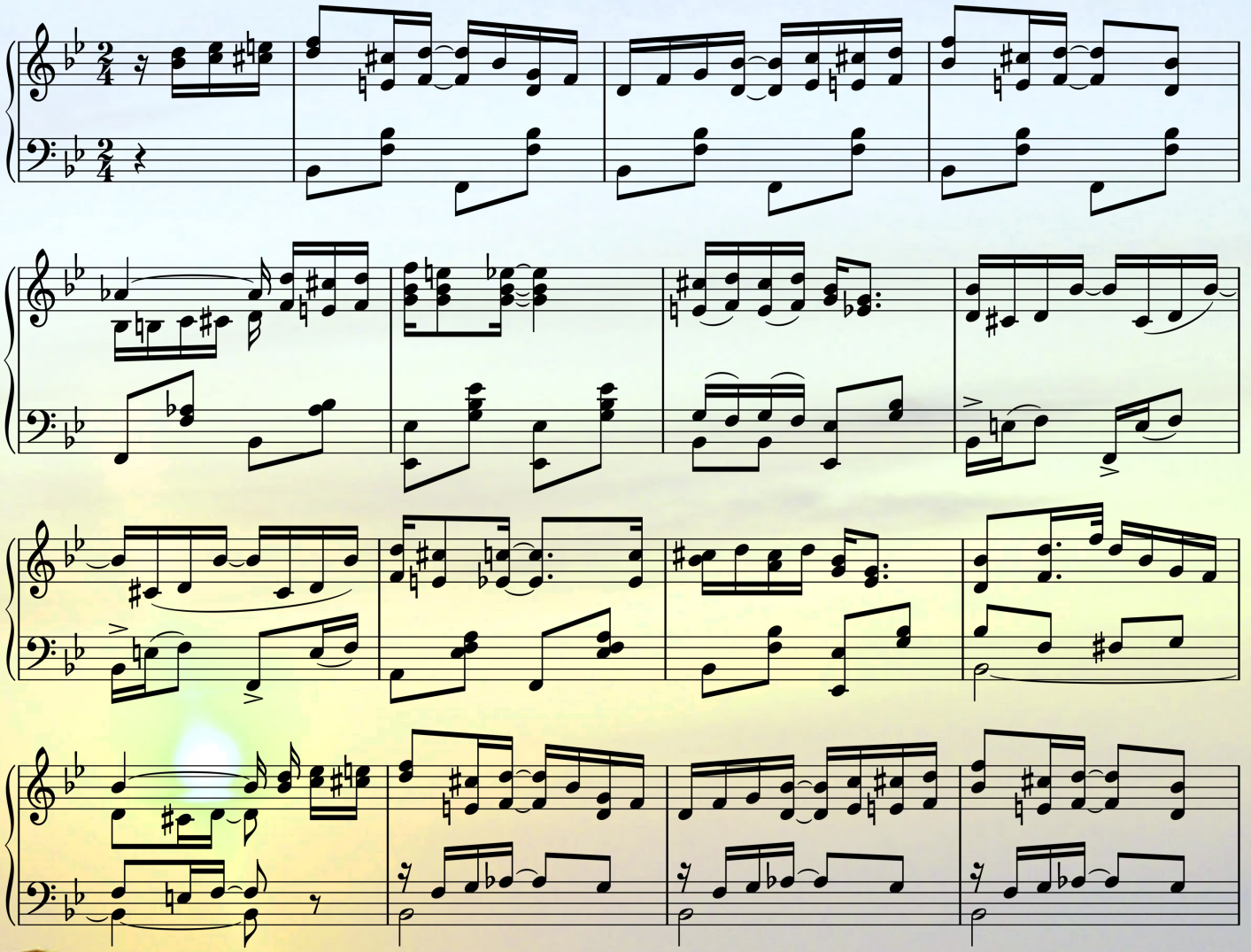
Circular variance



Memphis Blues (W.C. Handy)

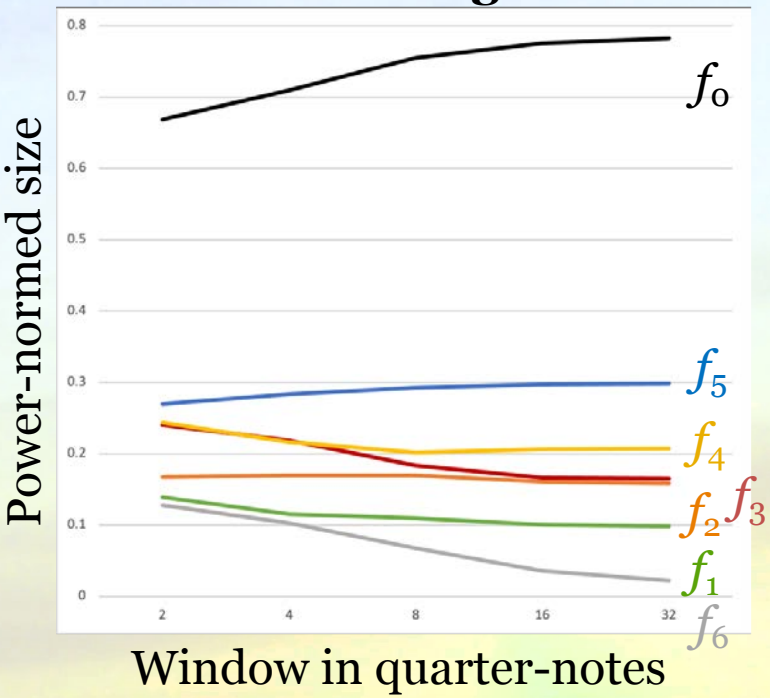
The first published blues (1912)

By a classically trained composer, devoted to African American music

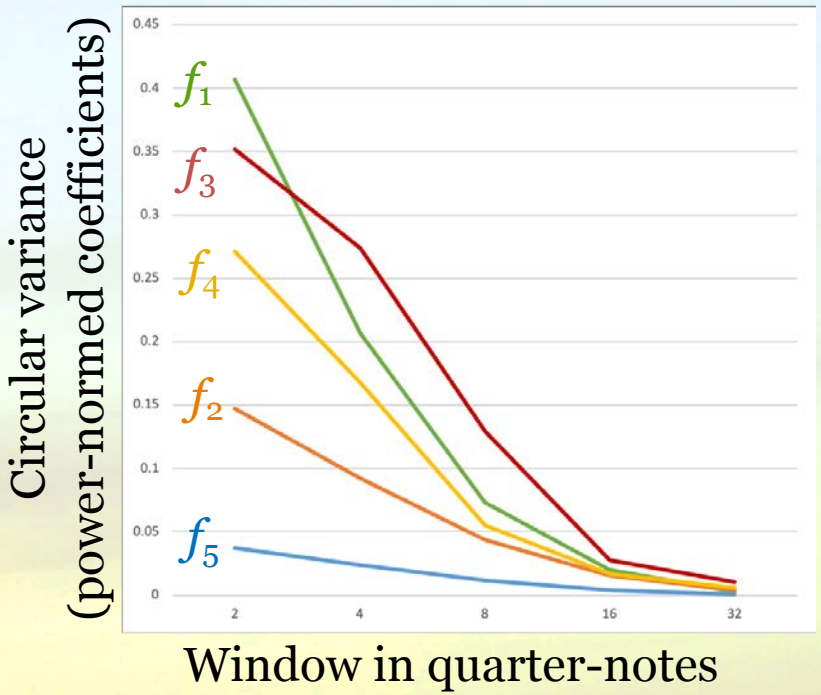


Memphis Blues

Coefficient magnitudes



Circular variance



- More chromatic (higher f_0) than blues melodies
- Same diatonic/octatonic harmonic quality

Jazz Harmony

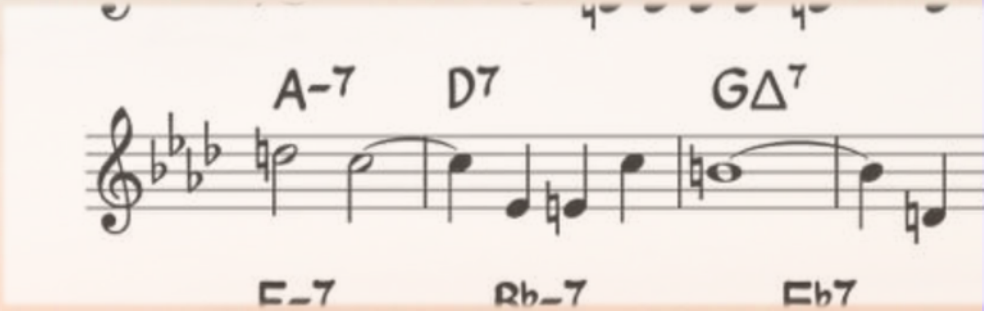
PC vectors are good for interpreting lead-sheet harmony symbols

To better understand jazz harmony, consider:

- Spectra of typical chord types
- Progressions in phase space

Jazz Chords as Pitch-Class Vectors

What is the meaning of a lead-sheet chord symbol?



- A **pitch-class set is a poor model**, because in practice not all notes need to be played, and certain notes may be added.
- A **pitch-class vector** can account for the varying probabilities of each pitch-class occurring.

—The **root** has a special significance in determining the bass line.

—**Guide tones** (3^{rd} and 7^{th}) are especially important harmonic tones

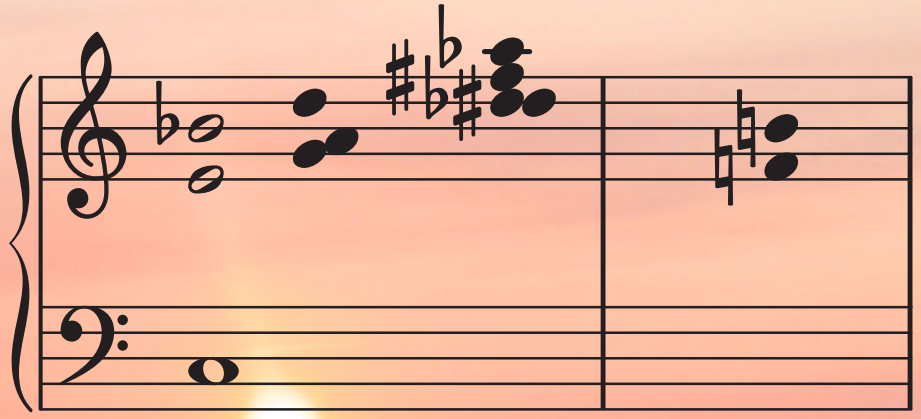
—Each chord also has optional **extensions** or "tensions"

—Chords also have certain **"avoid tones"**

Jazz Chords as Pitch-Class Vectors

Dominant

Guide tones	Extensions:		Avoid:
7	9	♭13	♯7
3	6	♯11	4
	5	♭9 / ♯9	



Root

C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(3,	1,	1,	1,	2,	0,	1,	1,	1,	1,	2,	0)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

Jazz Chords as Pitch-Class Vectors

Minor

Guide tones

7
3

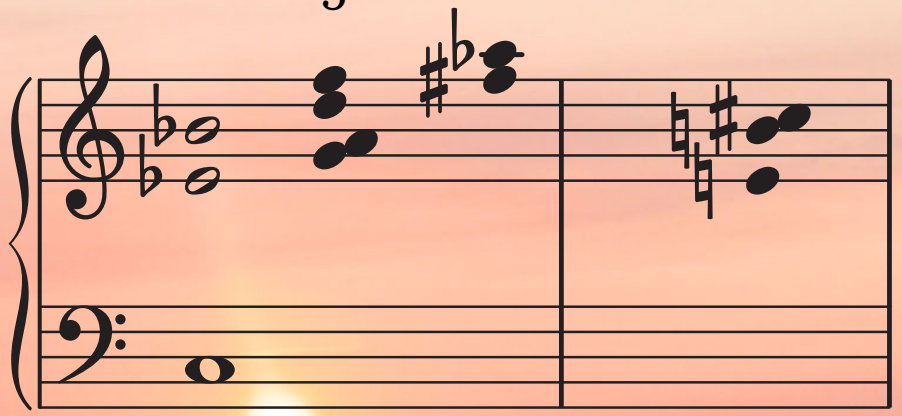
Extensions:

11
9
6
5

♭13
♯11

Avoid:

♭9
♯7
♯3



Root

C C♯ D E♭ E F F♯ G A♭ A B♭ B

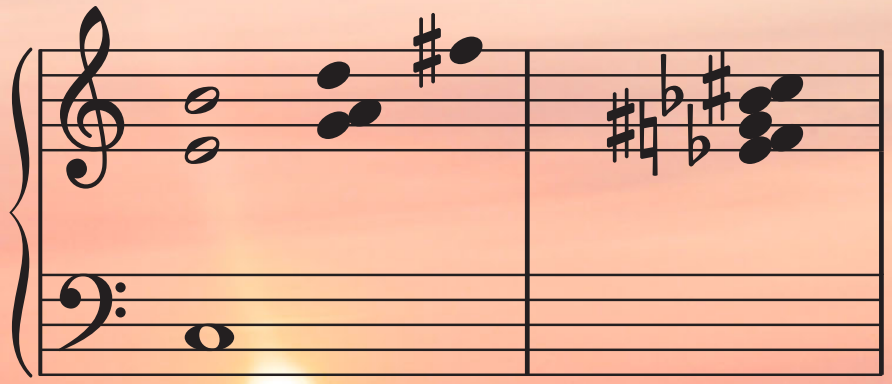
(3, 0, 1, 2, 0, 1, 1, 1, 1, 1, 2, 0)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

Jazz Chords as Pitch-Class Vectors

Major

Guide tones	Extensions:		Avoid:
7	9		♭9
3	6	♯11	♭7 ♭6
	5		♭3 4



Root

C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(3,	0,	1,	0,	2,	0,	1,	1,	0,	1,	0,	2)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

Jazz Chords as Pitch-Class Vectors

Minor flat-5

Guide tones

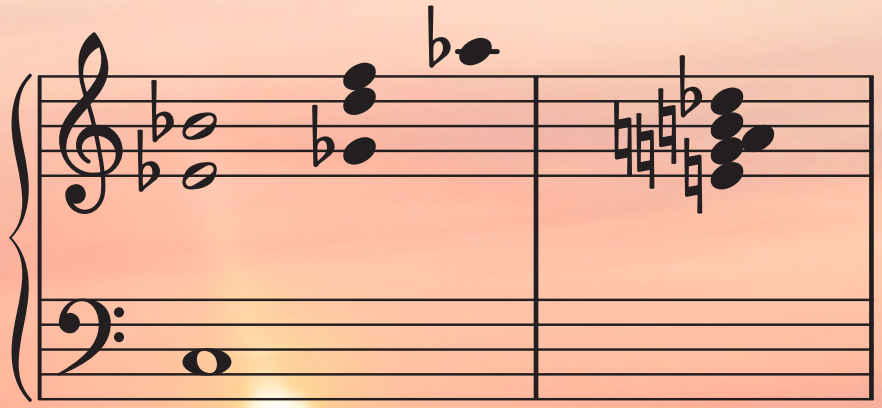
7
3

Extensions:

11
9
♭5
♭13

Avoid:

♯7 ♭9
♯5 6
♯3



Root

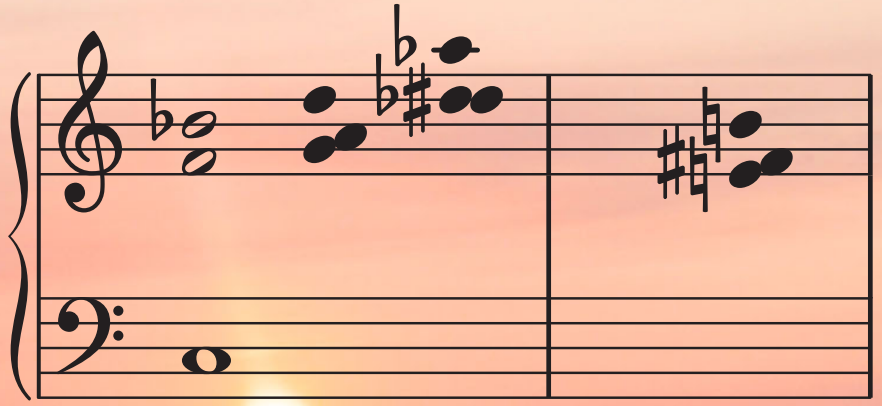
C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(3,	0,	1,	2,	0,	1,	1,	0,	1,	0,	2,	0)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

Jazz Chords as Pitch-Class Vectors

Sus7

Guide tones	Extensions:		Avoid:
7	9	♭13	♯7
3	6	♭9 / ♯9	♯4
	5		♯3



Root

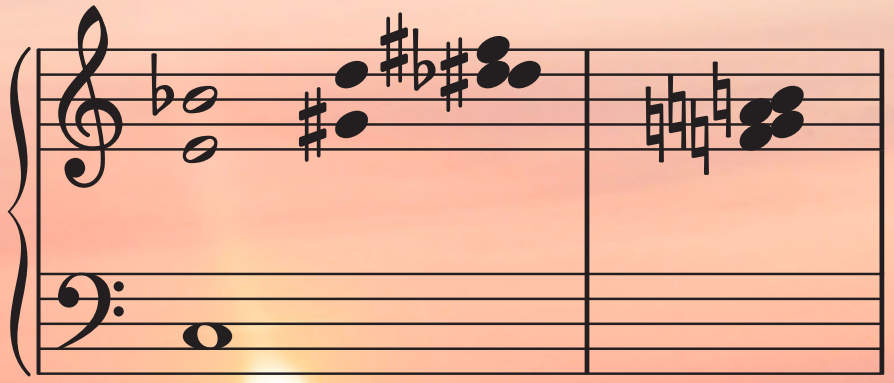
C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(3,	0,	1,	1,	0,	1,	0,	1,	1,	1,	2,	0)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

Jazz Chords as Pitch-Class Vectors

Alt7

Guide tones	Extensions:		Avoid:	
7	9	♯11	♯7	6
3	♯5	♭9 / ♯9	♯3	4



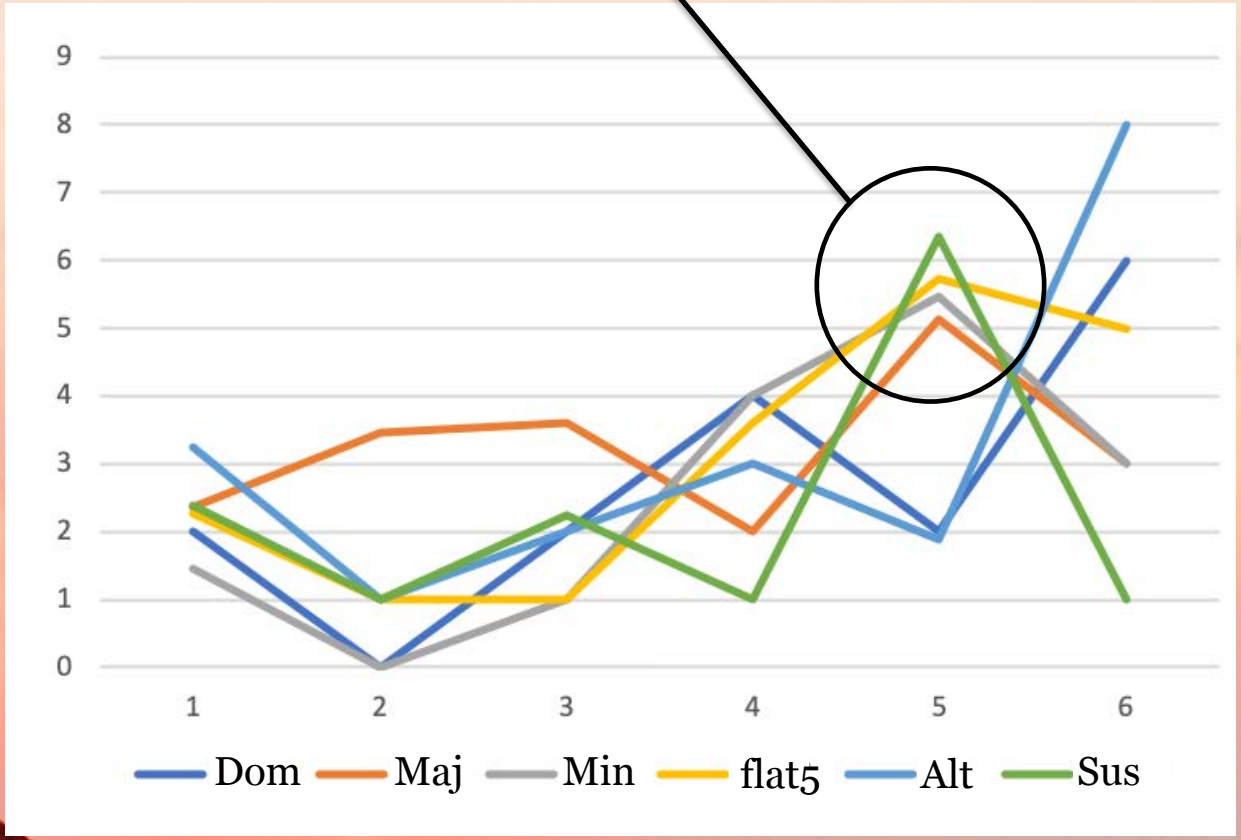
Root

C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
(3,	1,	1,	1,	2,	0,	1,	0,	1,	0,	2,	0)

Source:
Terefenko, D. 2014. *Jazz Theory: From Basic to Advanced Study* (Routledge).

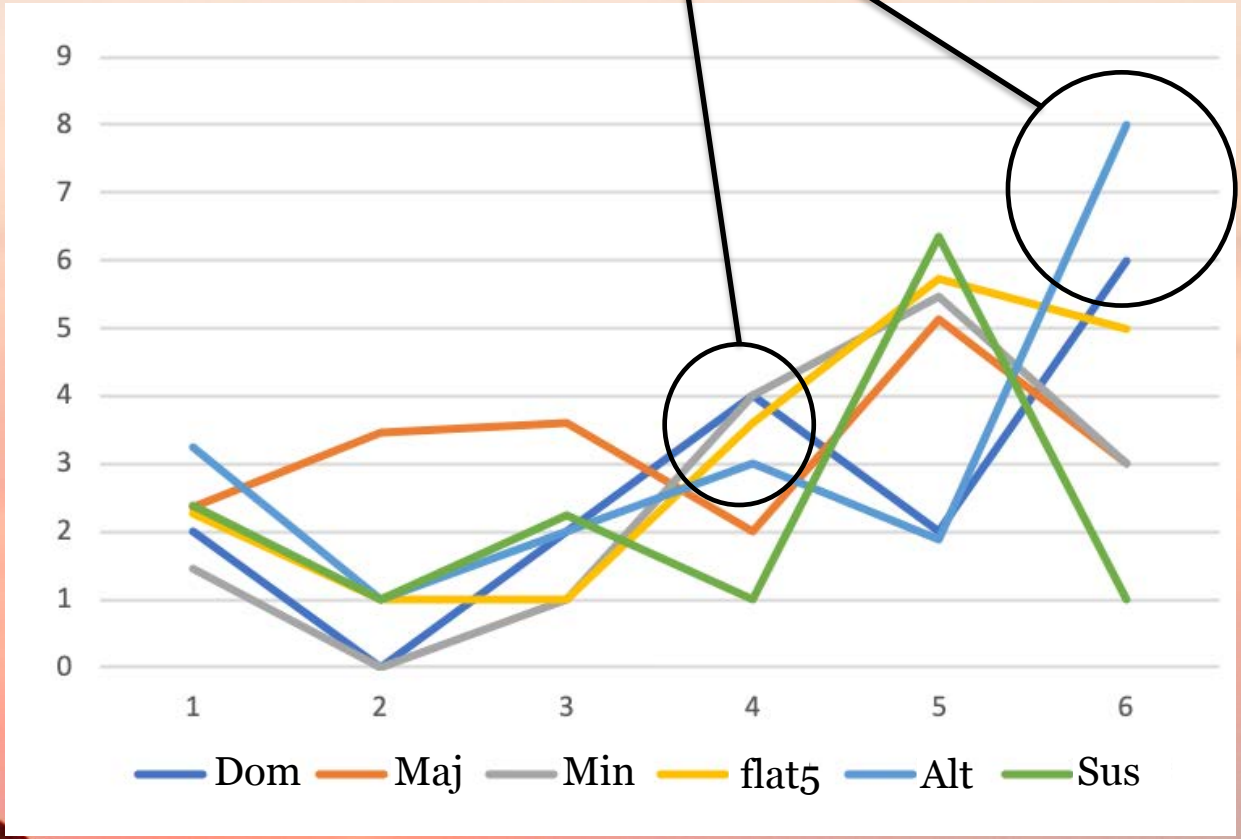
Spectra of Jazz PC Vectors

Most chords peak at f_5 (exceptions: Dom. & Alt.)



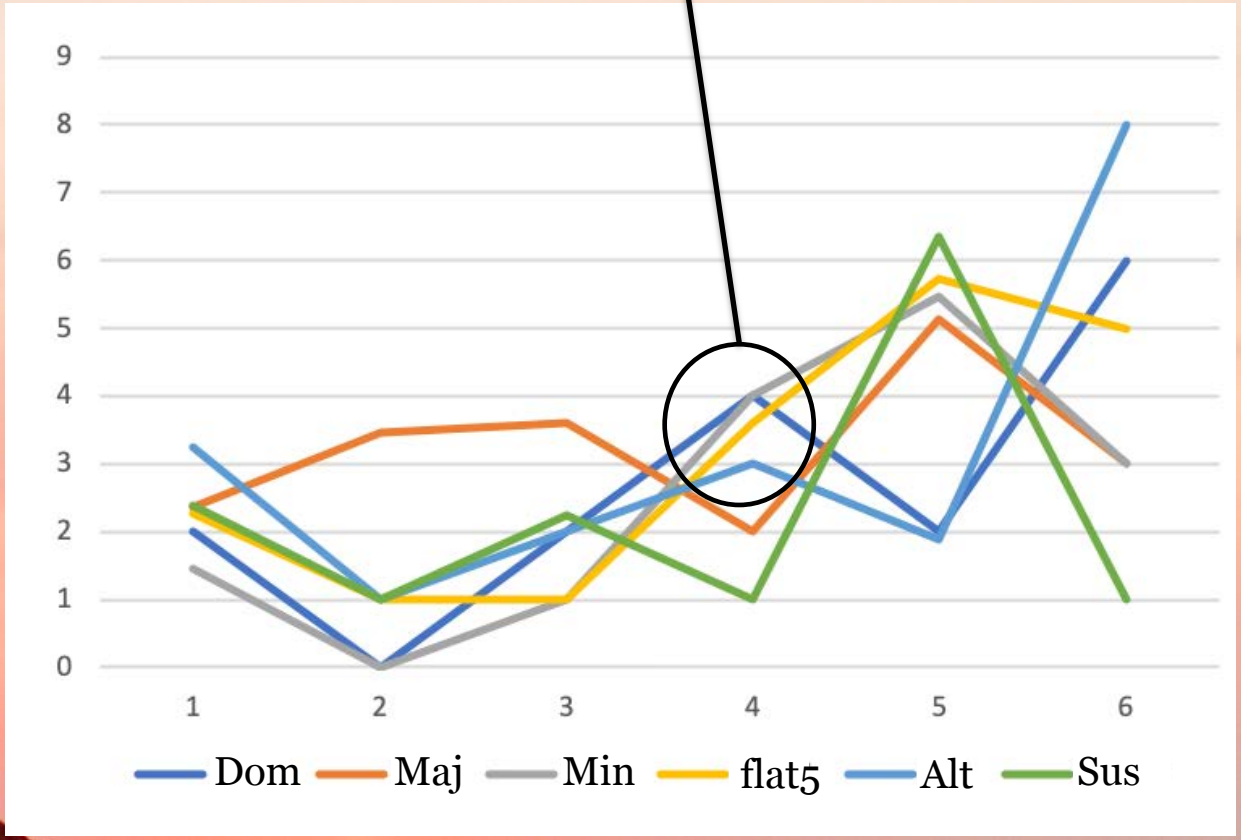
Spectra of Jazz PC Vectors

Dom. & Alt. chords instead peak at f_4 and f_6



Spectra of Jazz PC Vectors

Most chords have a secondary peak at f_4 (except: Maj. & Sus)



Phase Space for Jazz Harmony

Relationships between jazz harmonies will be mostly determined by f_4, f_5, and f_6.

However, phase of f_6 is degenerate: it only takes two values, 0 and π.

$\Rightarrow$

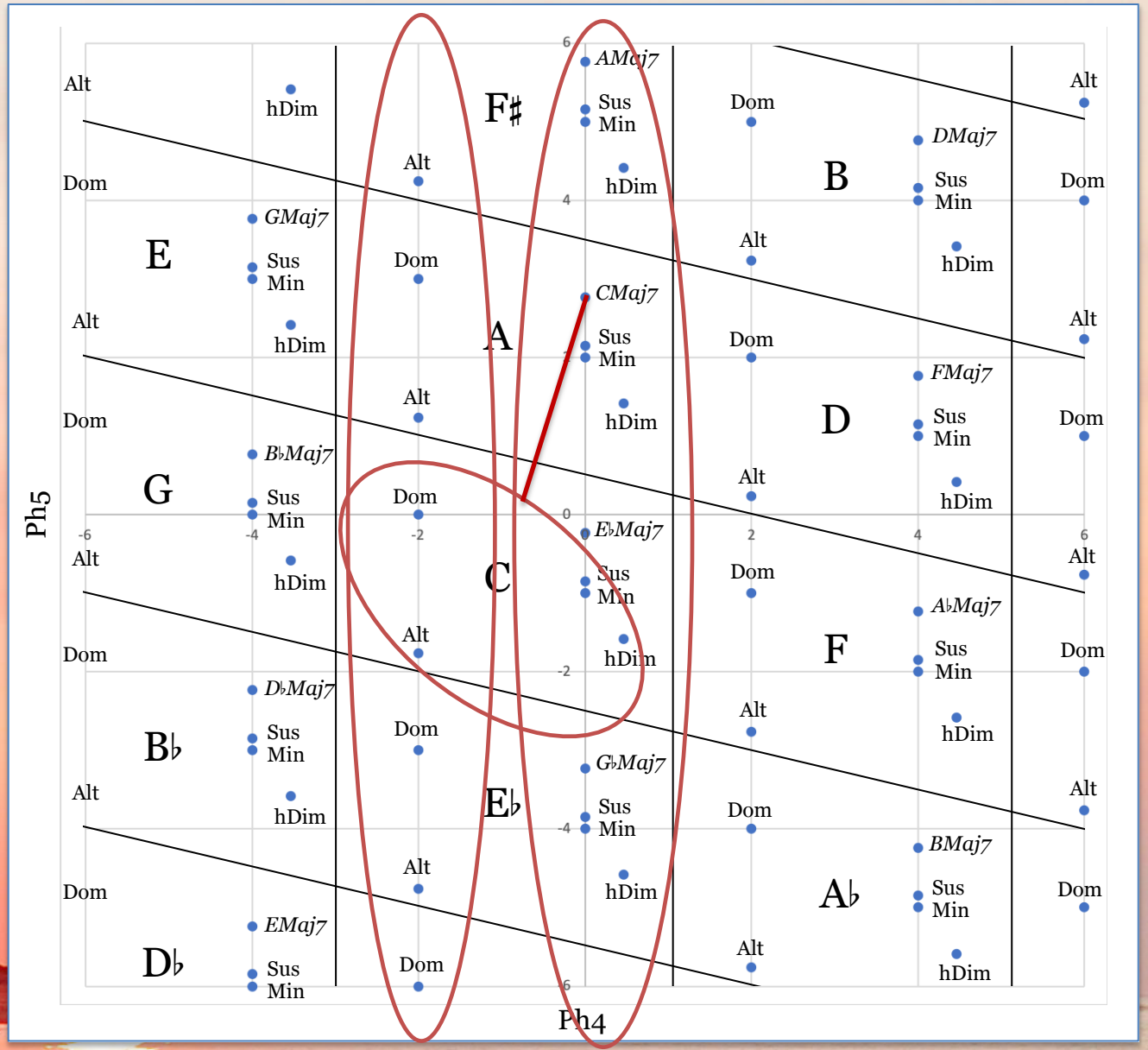
Chords relationships can be mapped in **a phase space for f_4 and f_5**, with an added consideration of whole-tone relations (WT_0 vs. WT_1: the phase of f_6 always corresponds to the root)

Phase Space for Jazz Harmony (Ph_4-Ph_5)

Proximity in phase space = **similarity of pc content.**

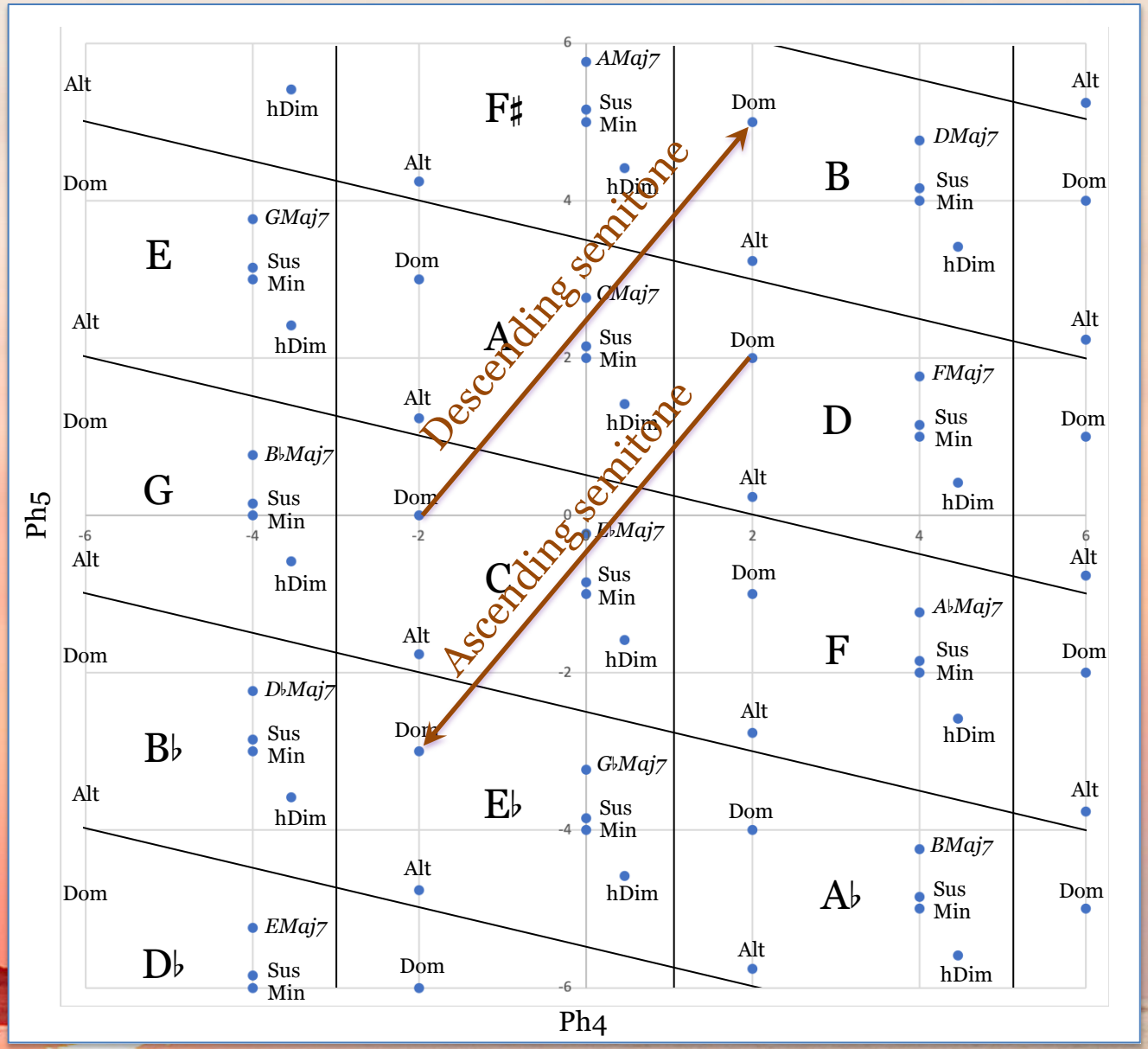
Chords with the same root group together *except major 7ths*.

Dominant and alt-dominant have distinct Ph_4 values.



Phase Space for Jazz Harmony (Ph_4-Ph_5)

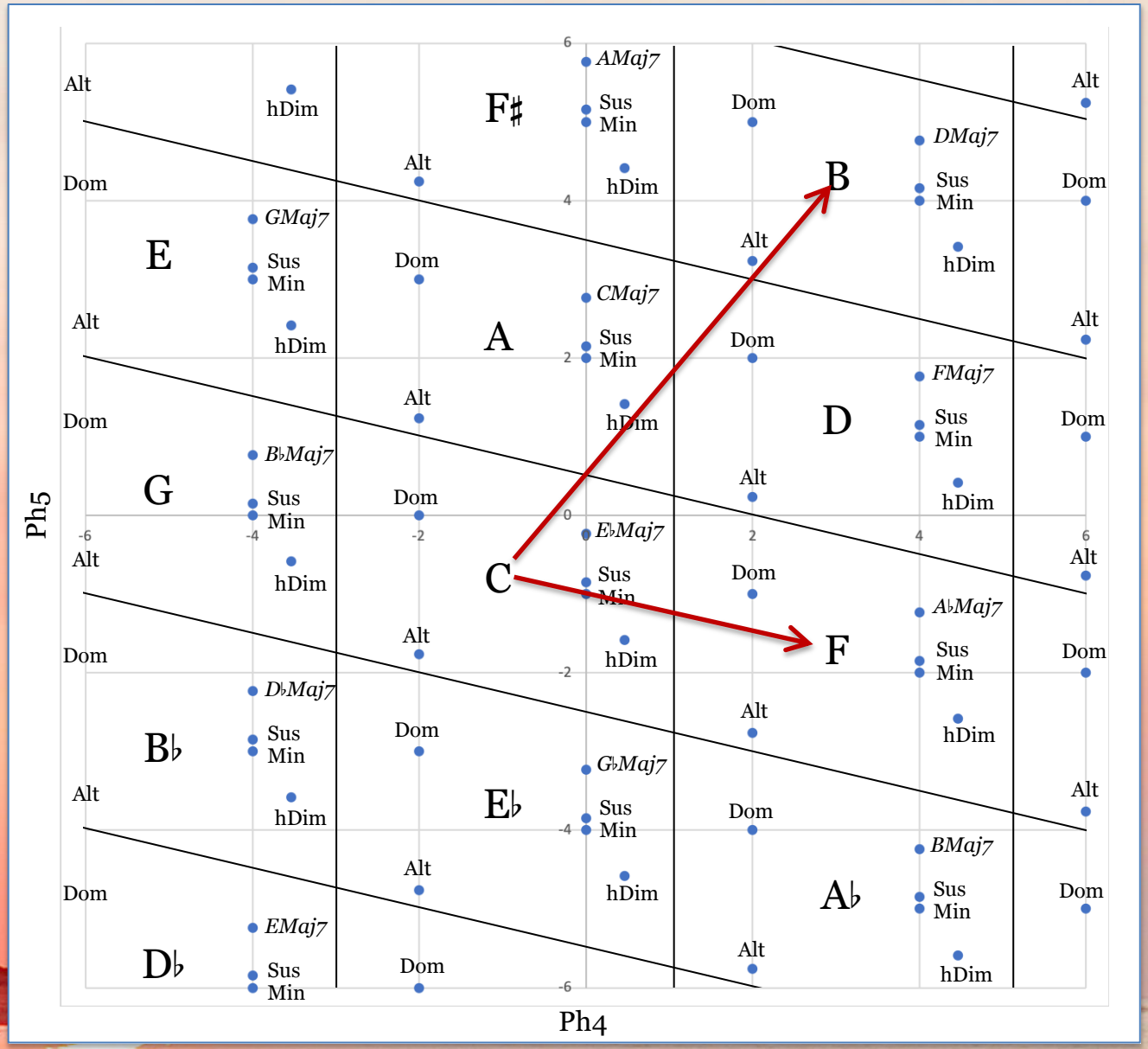
Similarity to the T_1 and T_{-1} vectors indicates the number of **ascending** or **descending semitones**.



Phase Space for Jazz Harmony (Ph_4-Ph_5)

Typical functional motion (such as descending fifths) is to the left

Restricted vertical motion produces **diatonicism,** while **chromaticism** results from large vertical motion.

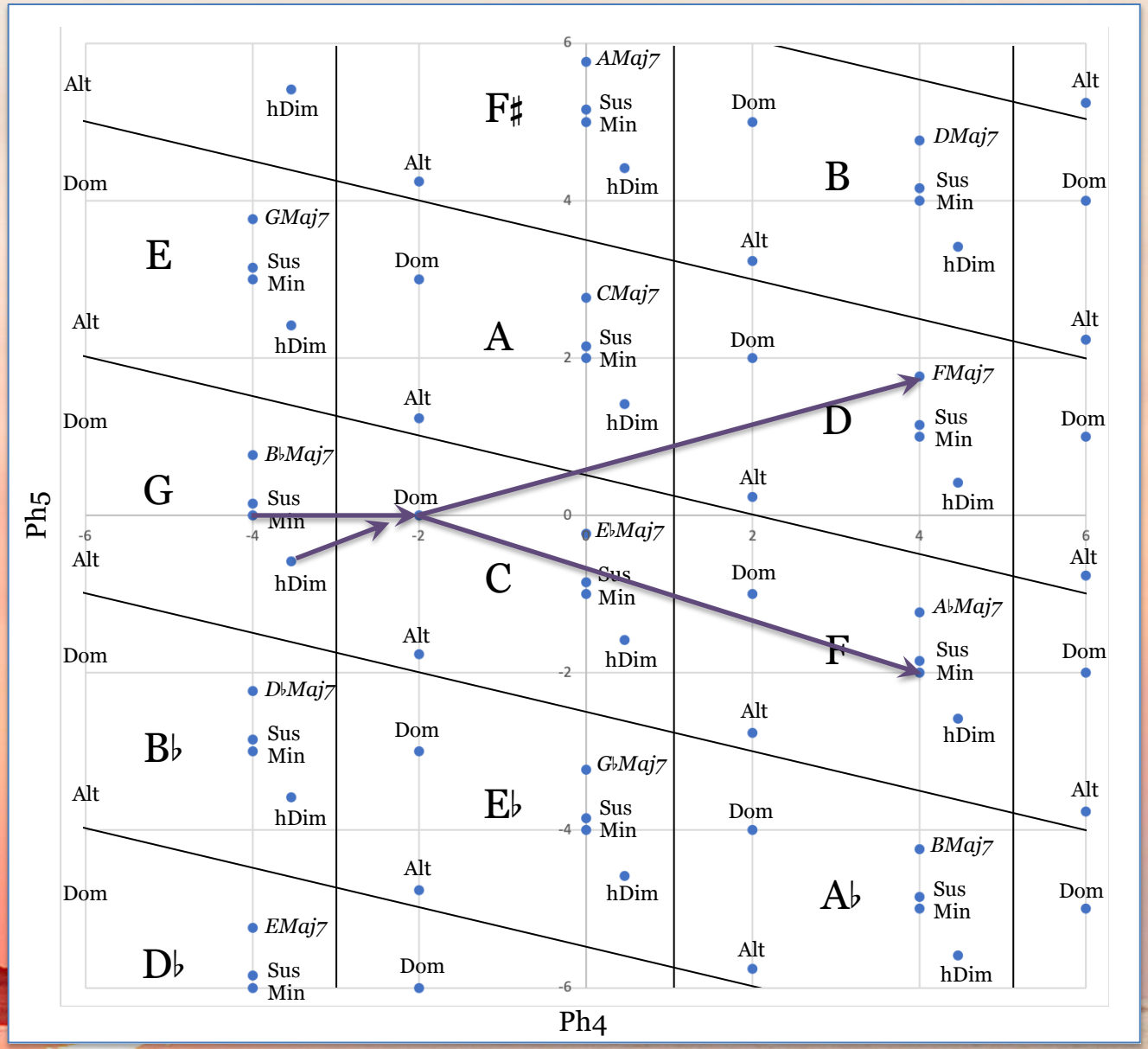


Phase Space for Jazz Harmony (Ph_4-Ph_5)

Typical ii-V-I progression

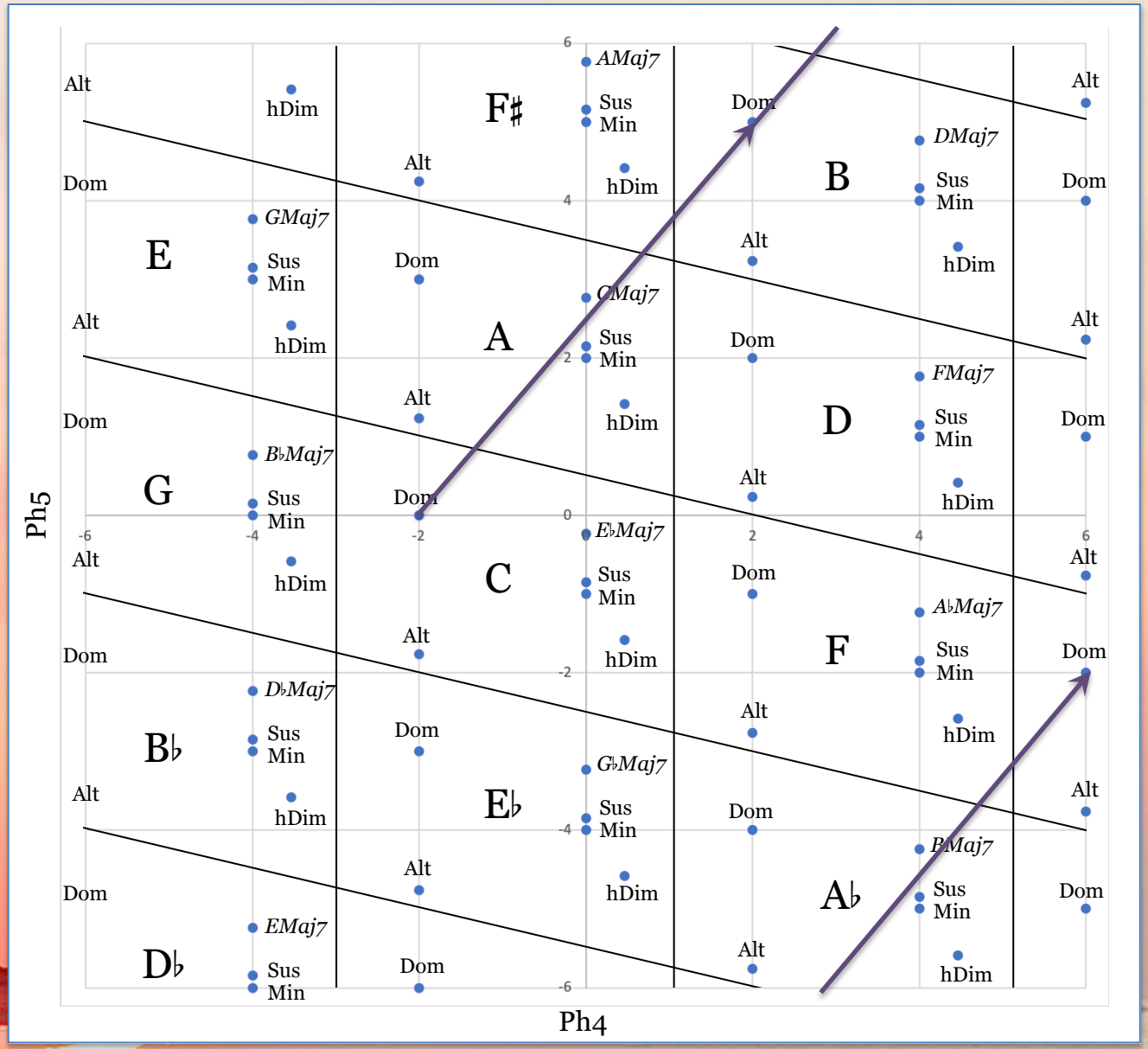
(F major)

(F minor)



Phase Space for Jazz Harmony (Ph_4-Ph_5)

Descending fifths progression with tritone substitution

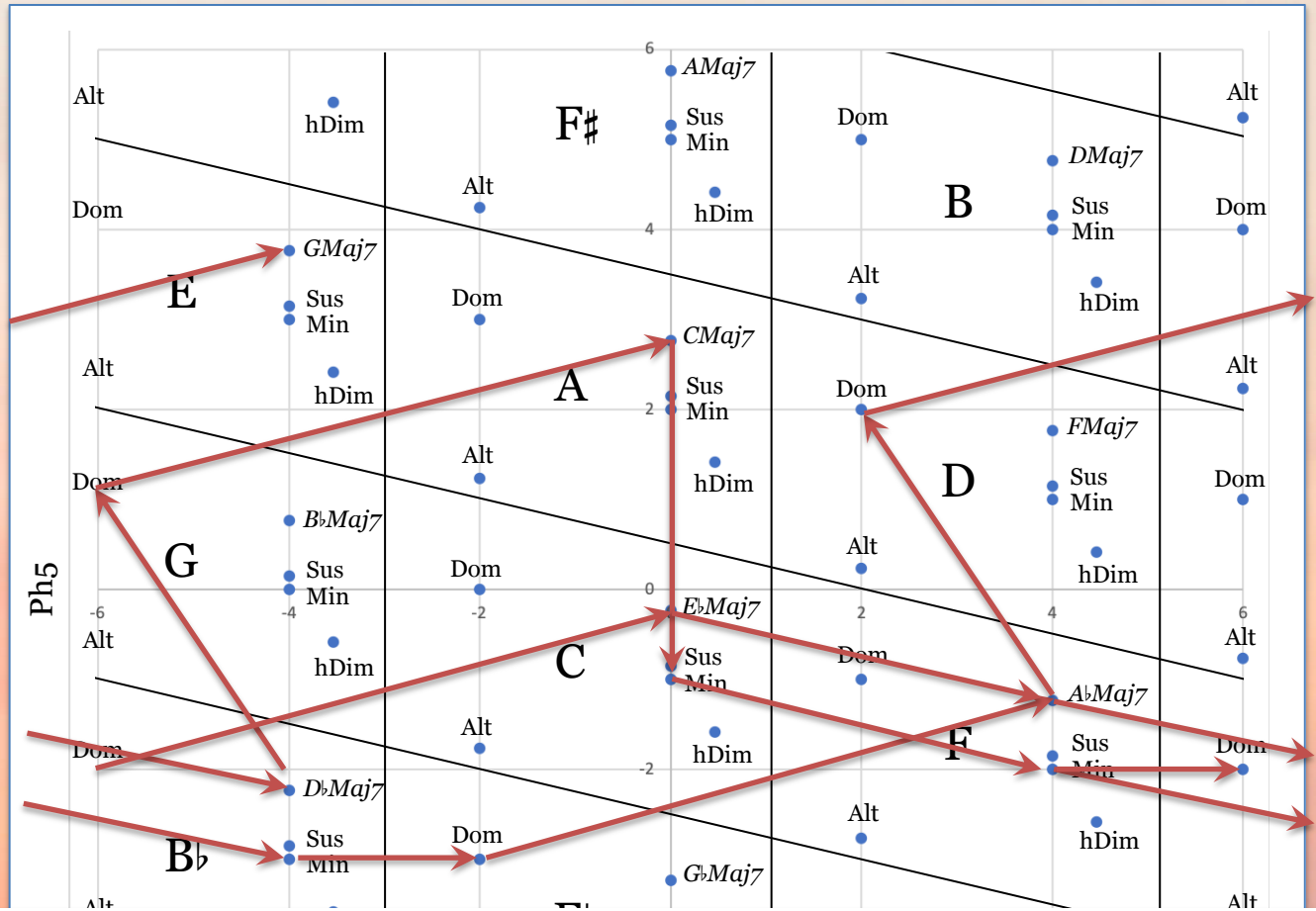


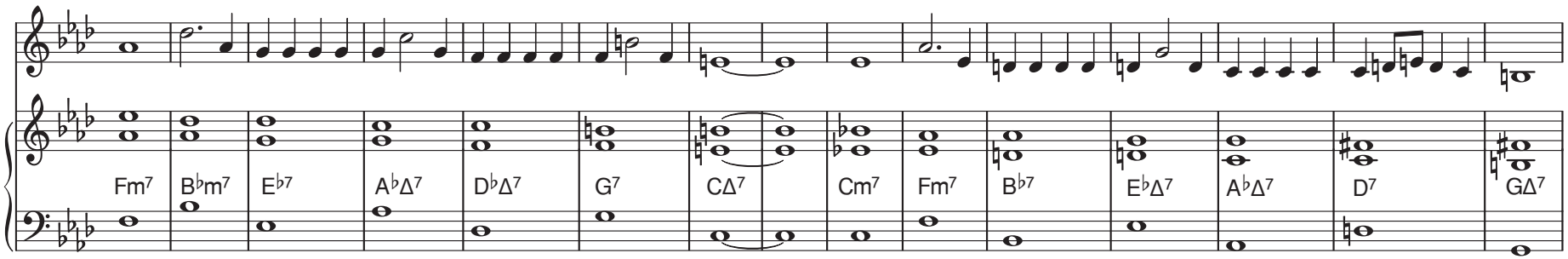
“All the Things You Are” in Ph_4-Ph_5 space

Meas. 1–4:
ii-V-I in A♭

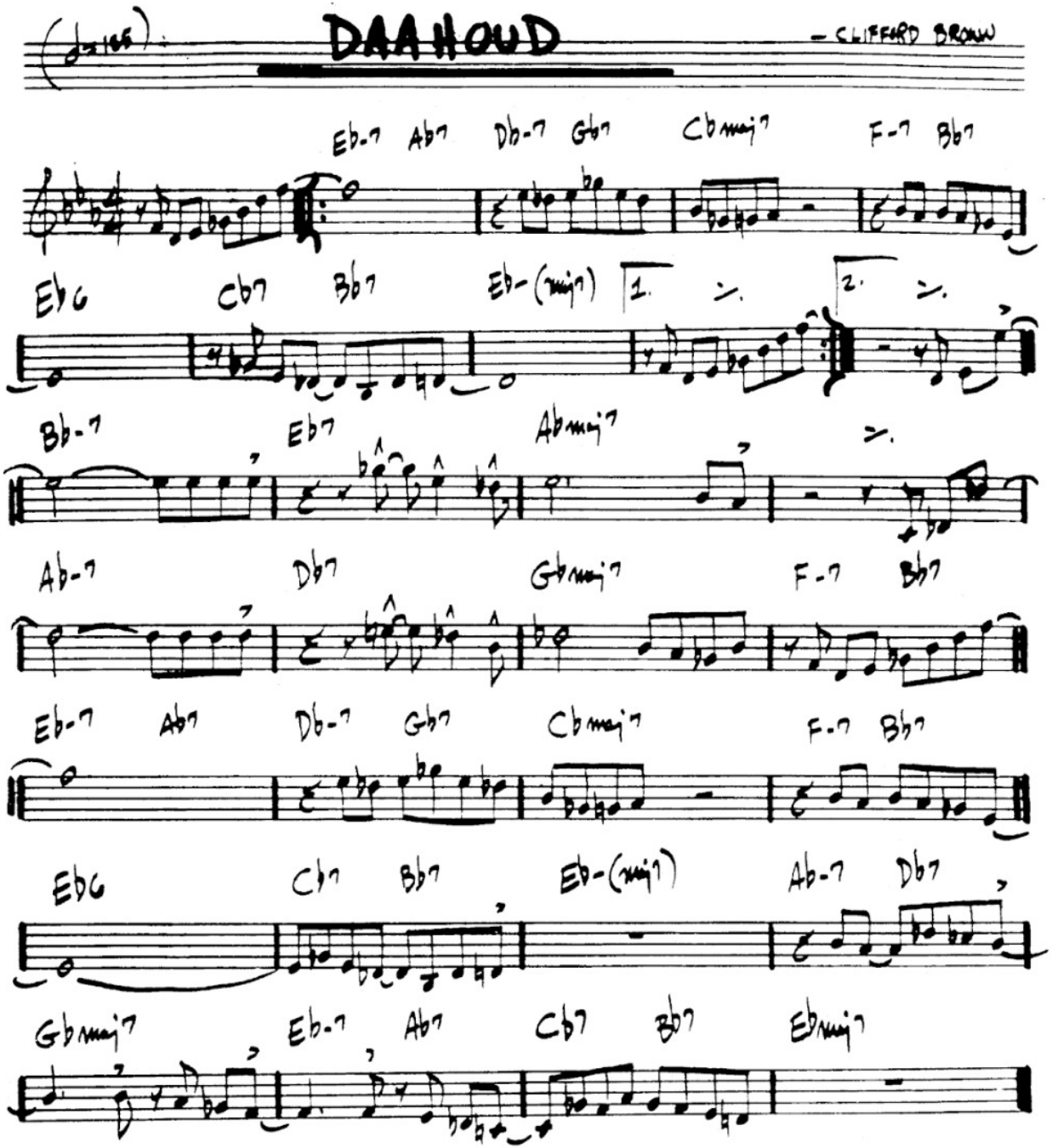
Meas. 5–8:
Tritone shift
takes us to C

Meas. 9–16:
Parallel shift leads
to sequence of
mm. 1–8





Example: "Daahoud" (Clifford Brown)



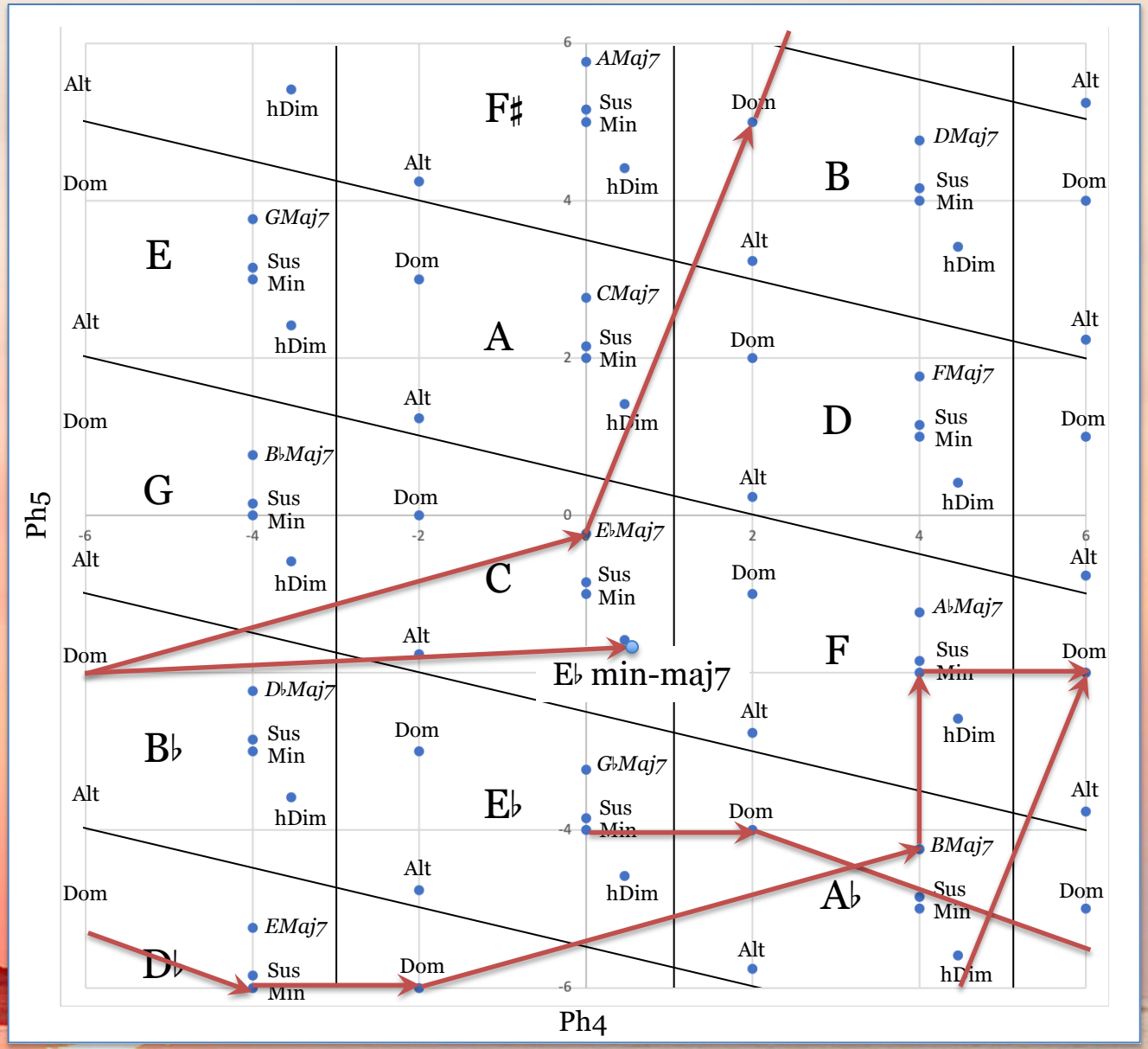
“Daahoud” in Ph_4-Ph_5 space

Chorus

Meas. 1–3:
Desc. 5ths (ii-V)
to C♭

Meas. 4–6:
ii-V-I in E♭

Meas. 6–7:
Tritone sub. and
resolution to $E♭_{-\Delta 7}$



“Daahoud” in Ph_4-Ph_5 space

Bridge

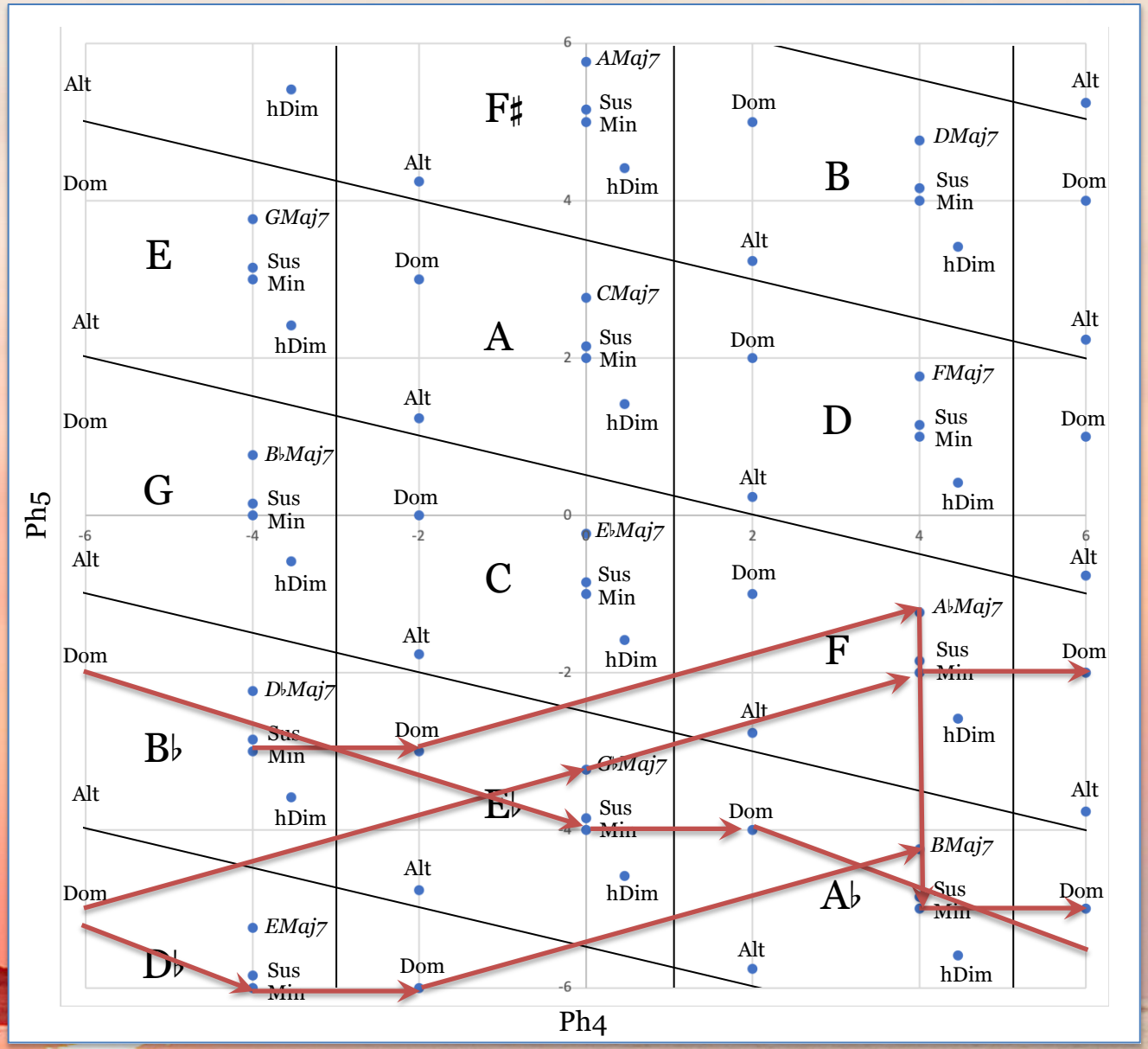
Meas. 9–12:
ii-V-I in A♭

Meas. 13–15:
ii-V-I in G♭
(sequence)

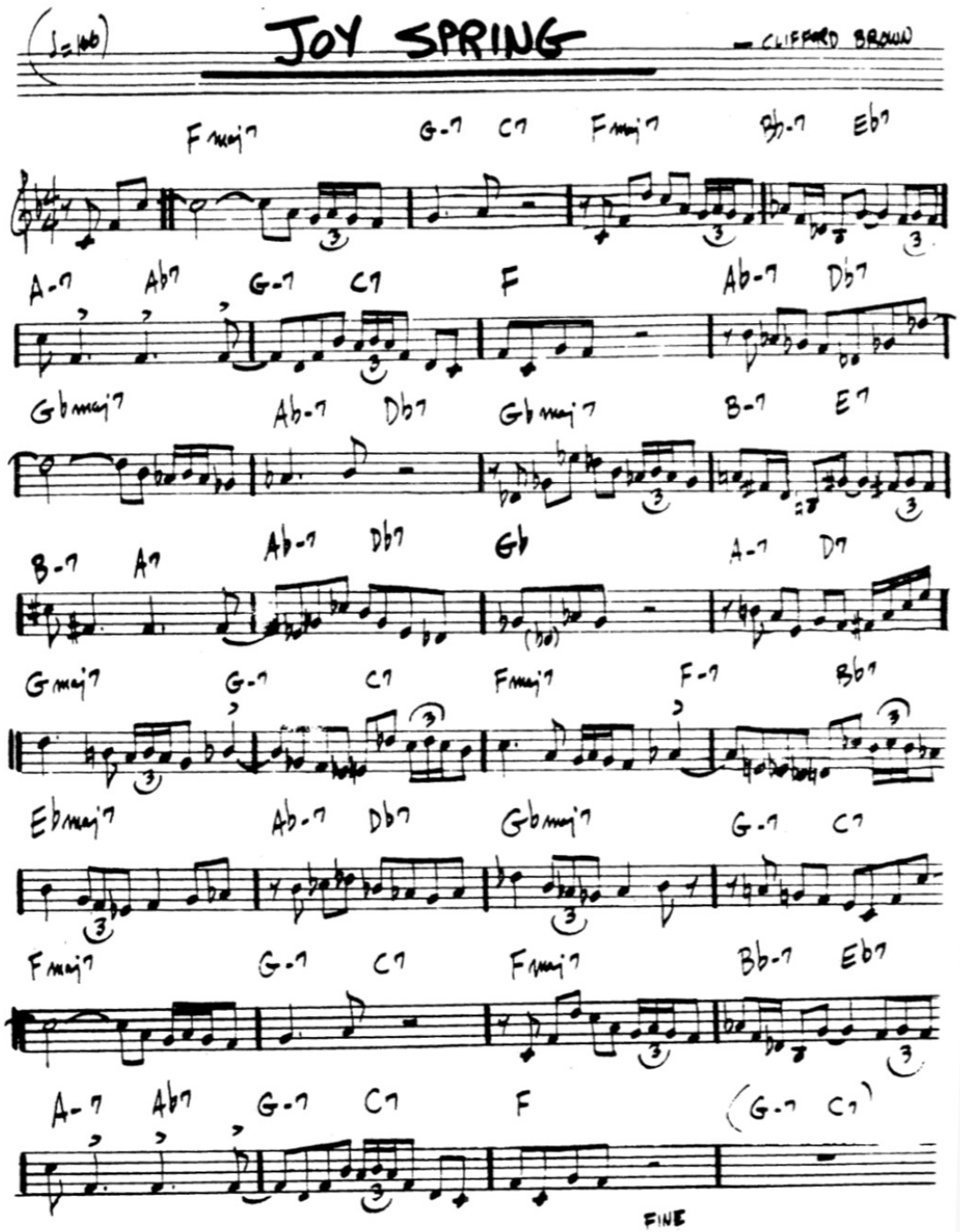
Meas. 16:
ii-V in E♭

Repeat chorus

etc. . . .



Example: "Joy Spring" (Clifford Brown)



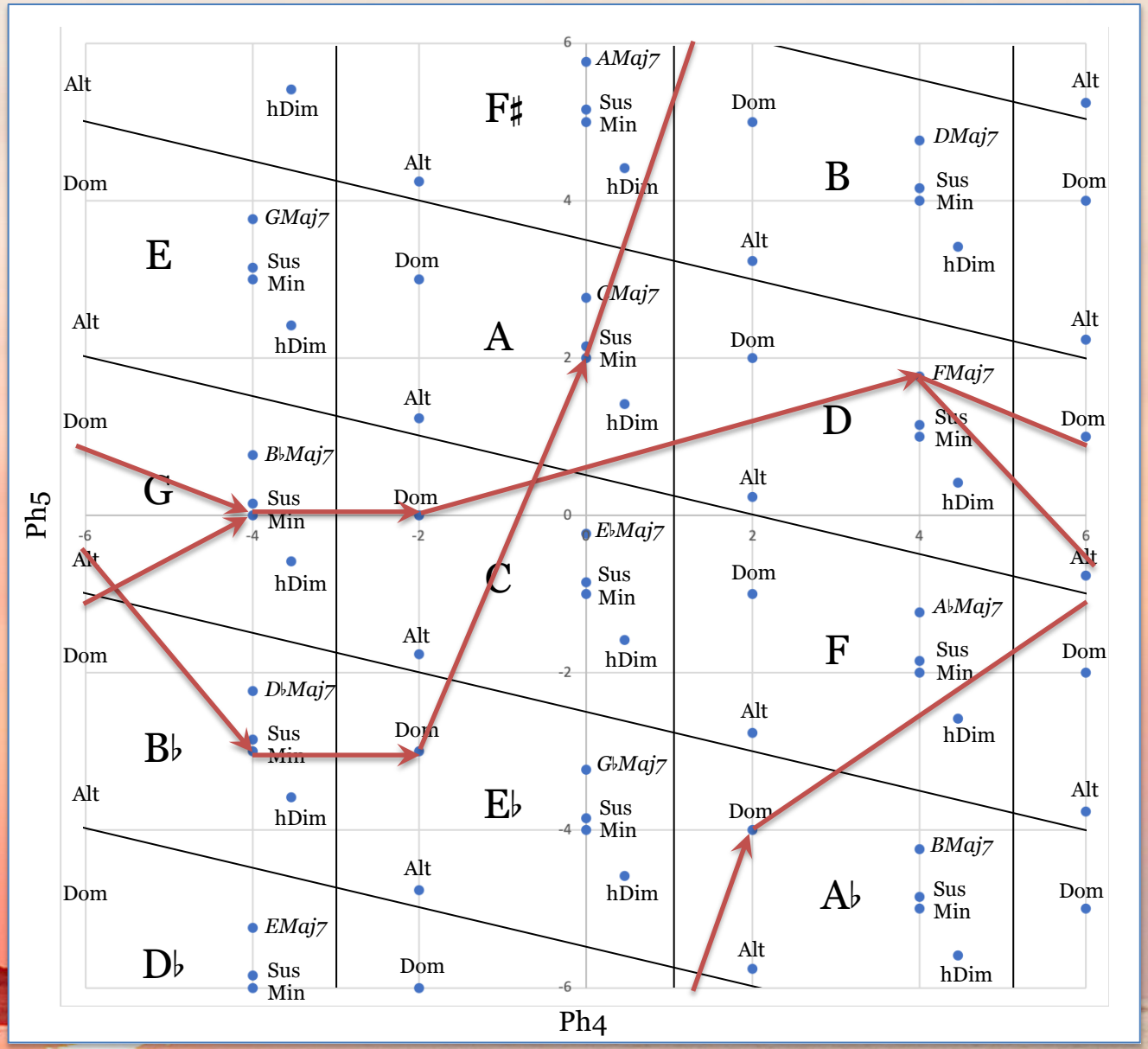
“Joy Spring” in Ph_4-Ph_5 space

First phrase

Meas. 1–3:
ii-V-I in F

Meas. 4–7:
ii-V in A♭

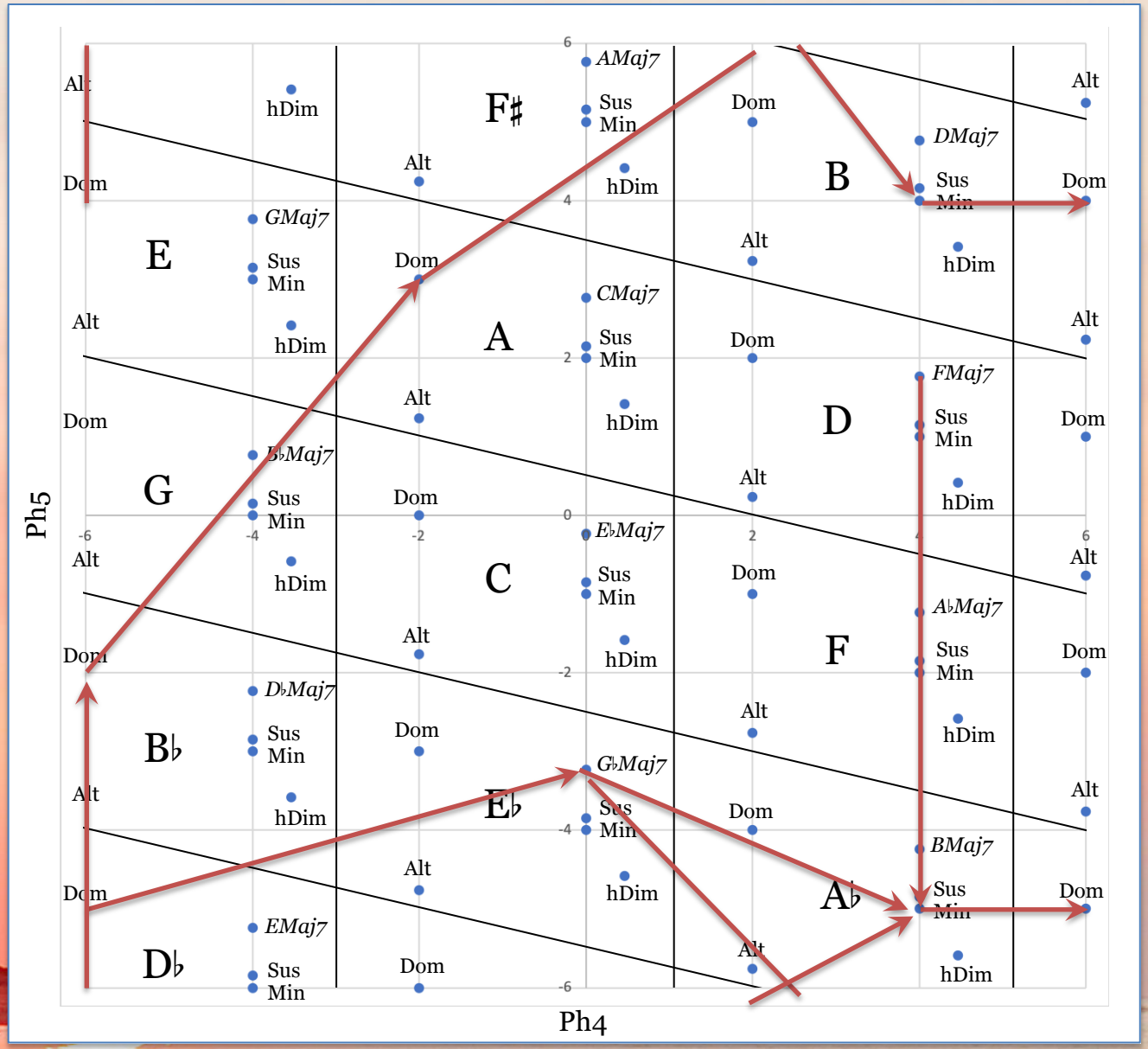
Tritone sub. progression back to F



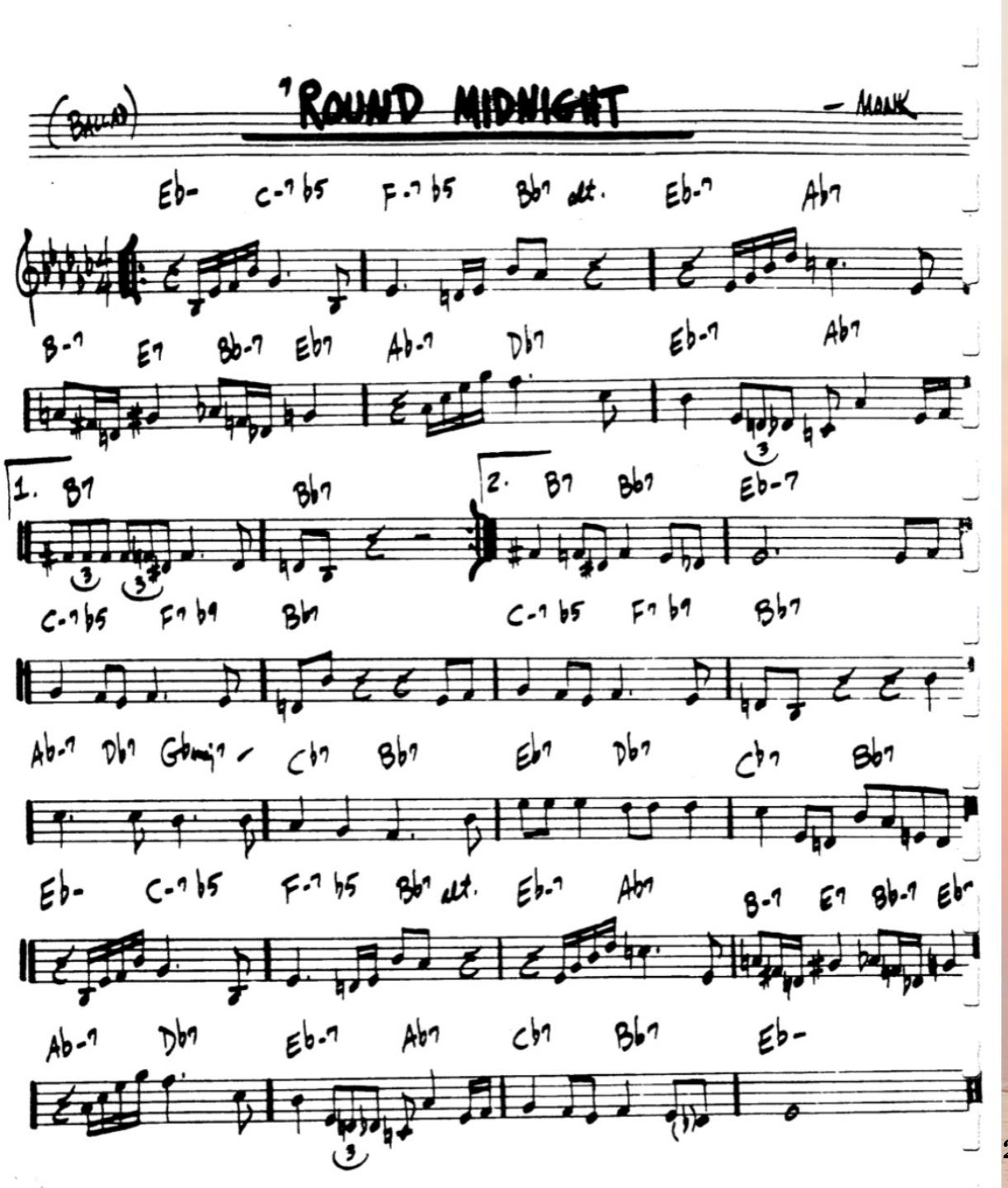
“Joy Spring” in Ph_4-Ph_5 space

Meas. 7–15:

Sequence up by semitone (G♭)



Example: "'Round Midnight" (Thelonious Monk)



“Round Midnight” in Ph_4-Ph_5 space

Meas. 1–3:
i-vi-ii-V-i in E♭ minor

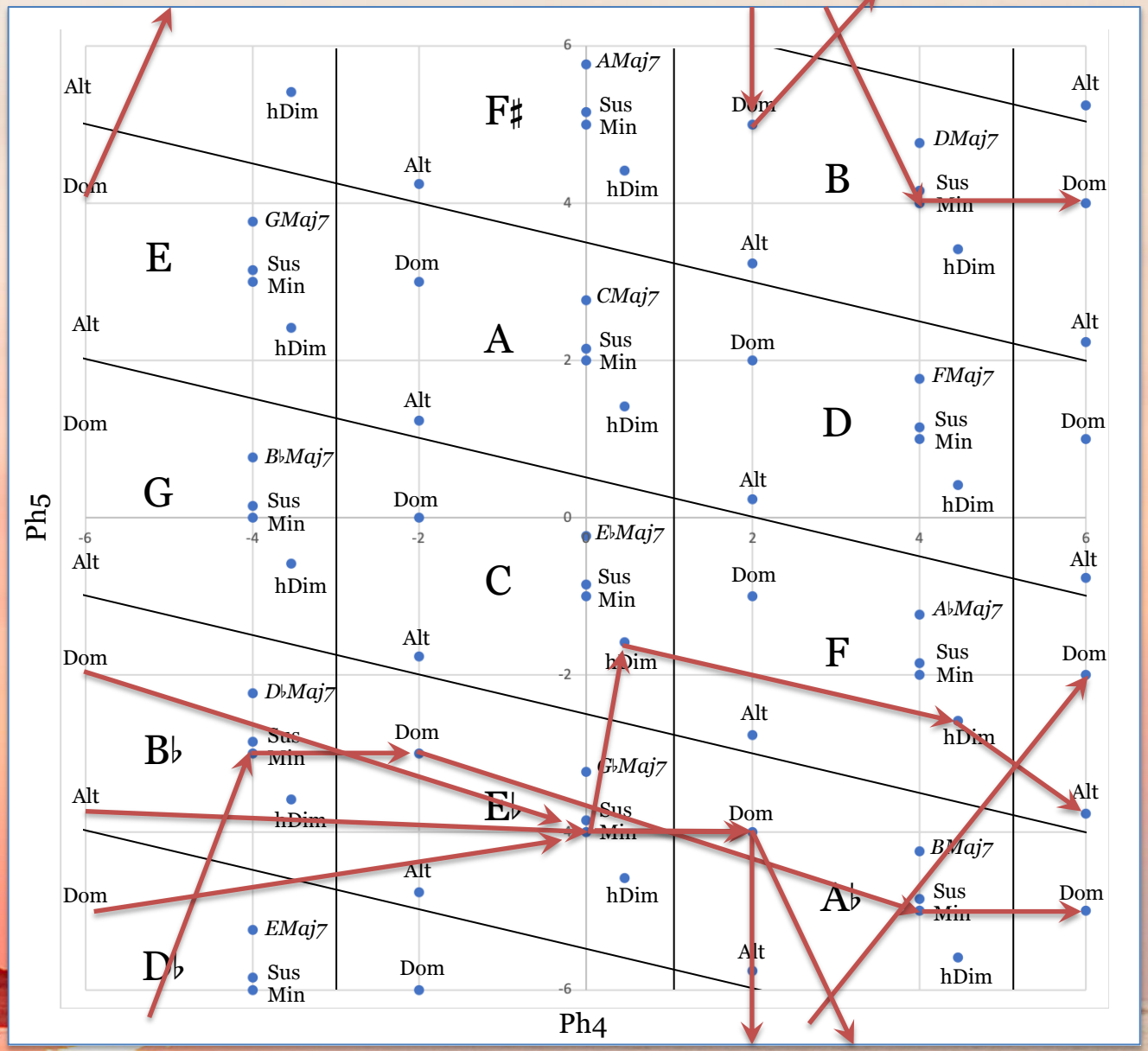
Meas. 3:
ii-V in D♭

Meas. 4:
Chromatic prog.

Meas. 5:
ii-V in G♭

Meas. 6:
ii-V in D♭

Meas. 7–8:
Tritone sub. of
ii-V in E♭



Commentary

Music and musicking

Tonality/Tonalities

Universals and Differencing

Musical Materials and Musicking

"Musicking" (Small 1998) refers to music as an activity as opposed to an object.

Replacing ***music*** with ***musicking*** breaks down barriers between musical materials and perception.

Activities of composing, performing, and listening exist on a continuum and are not independent.

Tonal hearing is conditioned by **tonal materials** and vice versa.

Tonality/Tonalities

Tonality is both one thing (**universals**) and many (**differences**)

Example: Classical music and blues have **distinct tonalities**

But both are **tonal.**

Possible Universals

Shared features of classical and blues/jazz tonalities:

Definition of a basic space

➥ Allows for the location of **centers**/reference points

Use of diatonicity as an axis

➥ **Acoustic** salience of the perfect fifth

Use of higher index numbers (f_3, f_4, f_5)

➥ Allows for **voice-leading** relationships

These resemble (and generalize) features frequently used to define tonal music.

Differences

There is value to the **distinctiveness** of blues tonality.

Within the constraints on defining a basic space, blues tonality establishes a distinctiveness from classical tonality through its use of a **tetradic** axis in place of a **triadic** axis.

This is a property of the music as composed, performed, and experienced, i.e. it is a feature of blues/jazz musicking.

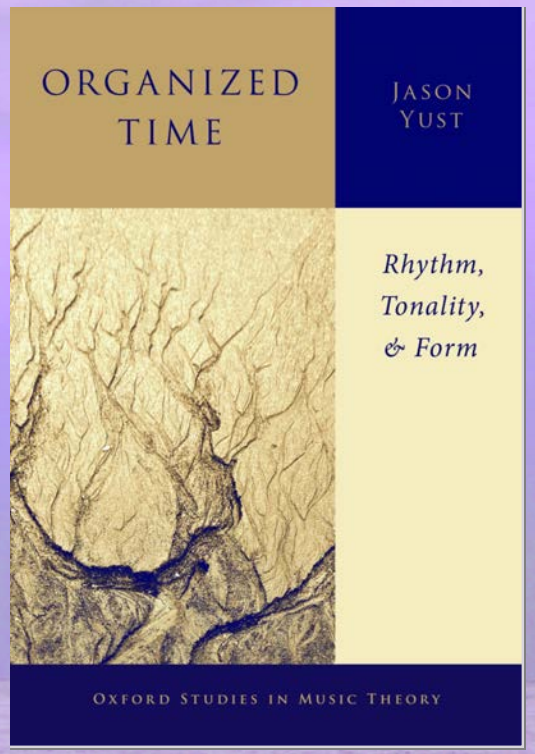
On *differencing* in ethnomusicology and music analysis, see Kofi Agawu (2003) *Representing African Music*.

Materials of Tonality: Harmonic Spectra and Phase Spaces for Classical Music, Blues, and Jazz

Jason Yust, BOSTON UNIVERSITY

You can download my slides at
http://people.bu.edu/jyust

Thanks for listening!



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Fourier Qualities across Tonal Styles

Why do tonal chords and scales tend to emphasize certain qualities (f_3, f_4, f_5, f_6) regardless of style?

For example:

- Classical tonality: f_5, f_3
- Jazz, Ravel: f_5, f_4, f_6
- Late Scriabin: f_4, f_6
- Debussy: f_5, f_6, f_4, f_3

Fourier Qualities across Tonal Styles

Why do tonal chords and scales tend to emphasize certain qualities (f_3, f_4, f_5, f_6) regardless of style?

These qualities (f_3, f_4, f_5, f_6) are precisely those in which **semitones have relatively large phase distances.**

$\Rightarrow$

Semitonal voice leading is distinct from common tones. That is, semitones between chords tend to not overlap with common tones between chords.