



The Fourier Transform and a Theory of Harmony for the Twentieth Century

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A copy of this talk is available at
people.bu.edu/jyust/

Outline

I. Forte's Project and the DFT

1. A theory of harmony for the 20th century
2. Pc-vectors
3. DFT components and interval content
4. Phase spaces

II. Debussy: “Les sons et les parfums tournent dans l’air du soir”

1. Heptatonic scales and diatonicity
2. Common tones and harmonic qualities

III. Stravinsky and the Octatonic

1. *Rite of Spring*, Introduction and *Augurs*
2. Octatonic scale versus octatonic quality

IV. Feldman, *Palais de Mari*



I. Forte's Project and the DFT

1. A theory of harmony for the 20th century
2. Pc-vectors
3. DFT components and interval content
4. Phase spaces

A Theory of Harmony for the 20th Century

Forte's project:

“It is the intention of the present work to provide a general theoretical framework, with reference to which the processes underlying atonal music may be systematically described.”

The Structure of Atonal Music (1973), Preface



A Theory of Harmony for the 20th Century

Forte's project:

General features of harmony that are largely independent of compositional aesthetic:

- *Interval content* determines *harmonic quality*

Interval content \leftrightarrow DFT components

- *Common pc content* determines *harmonic proximity*

Subset relations \leftrightarrow DFT phase spaces

Discrete Fourier Transform on Pcsets

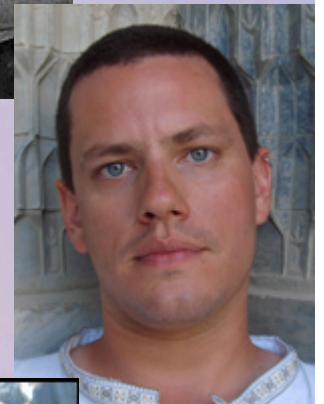
Lewin, David (1959). “Re: Intervallic Relations between Two Collections of Notes,” *JMT* 3/2.

——— (2001). “Special Cases of the Interval Function between Pitch Class Sets X and Y.” *JMT* 45/1.

Quinn, Ian (2006–2007). “General Equal-Tempered Harmony,” *Perspectives of New Music* 44/2–45/1.

Amiot, Emmanuel (2013). “The Torii of Phases.” *Proceedings of the International Conference for Mathematics and Computation in Music, Montreal, 2013* (Springer).

Yust, Jason (2015). “Schubert’s Harmonic Language and Fourier Phase Spaces.” *JMT* 59/1.



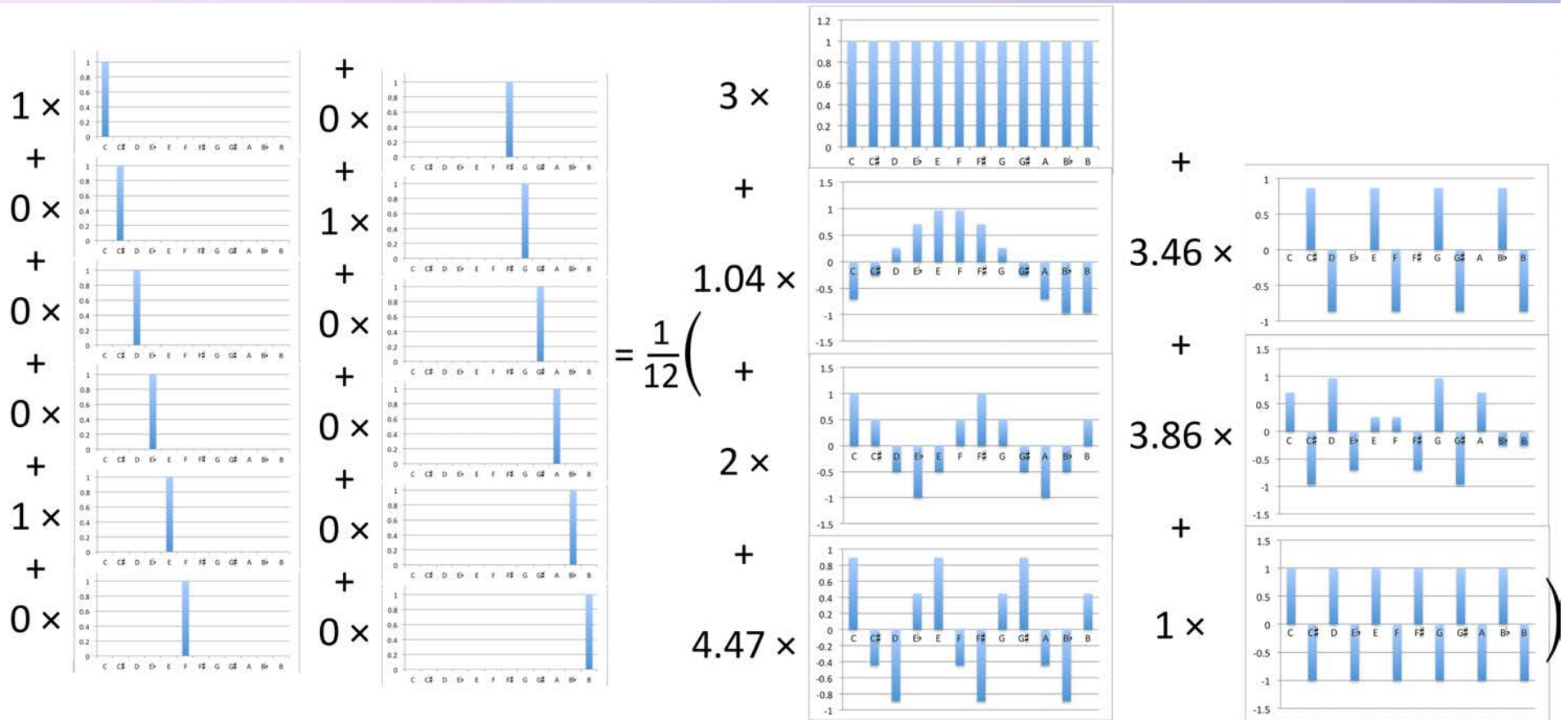
Characteristic Function of a Pcset



The *characteristic function* of a pcset is a **12-place vector** with 1s for each pc and 0s elsewhere:

(1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0)
C C# D E♭ E F F# G G# A B♭ B

DFT Components

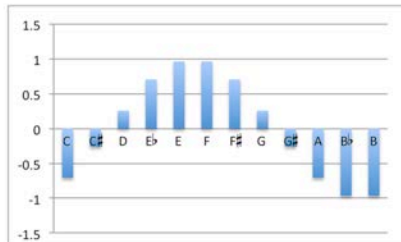


The DFT is a **change of basis** from a sum of pc spikes to a sum of discretized **periodic** (perfectly even) curves.

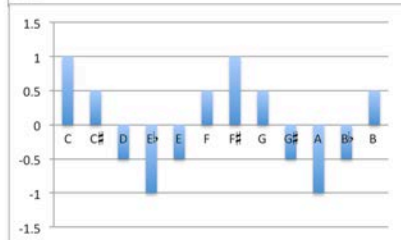
DFT Components

Component

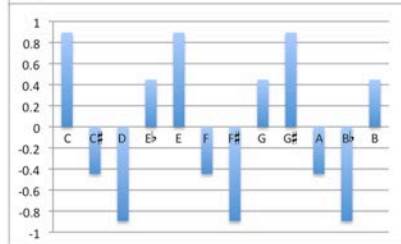
1



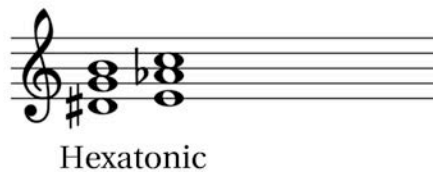
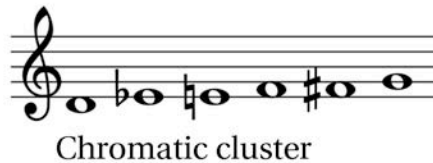
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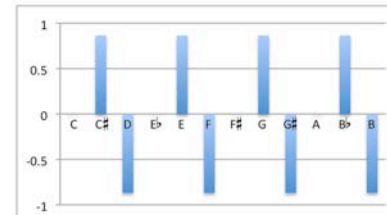


Prototypes

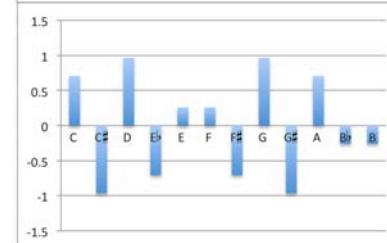


Component

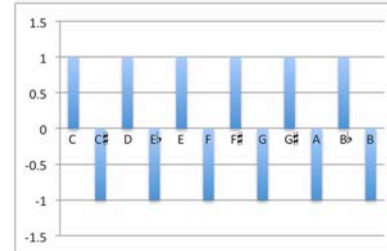
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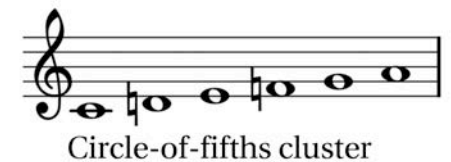
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6



Prototypes



Quinn's **generic prototypes** are pcsets that maximize a given component. **Subsets** and **supersets** of the prototypes are the best representatives of each component

DFT Components

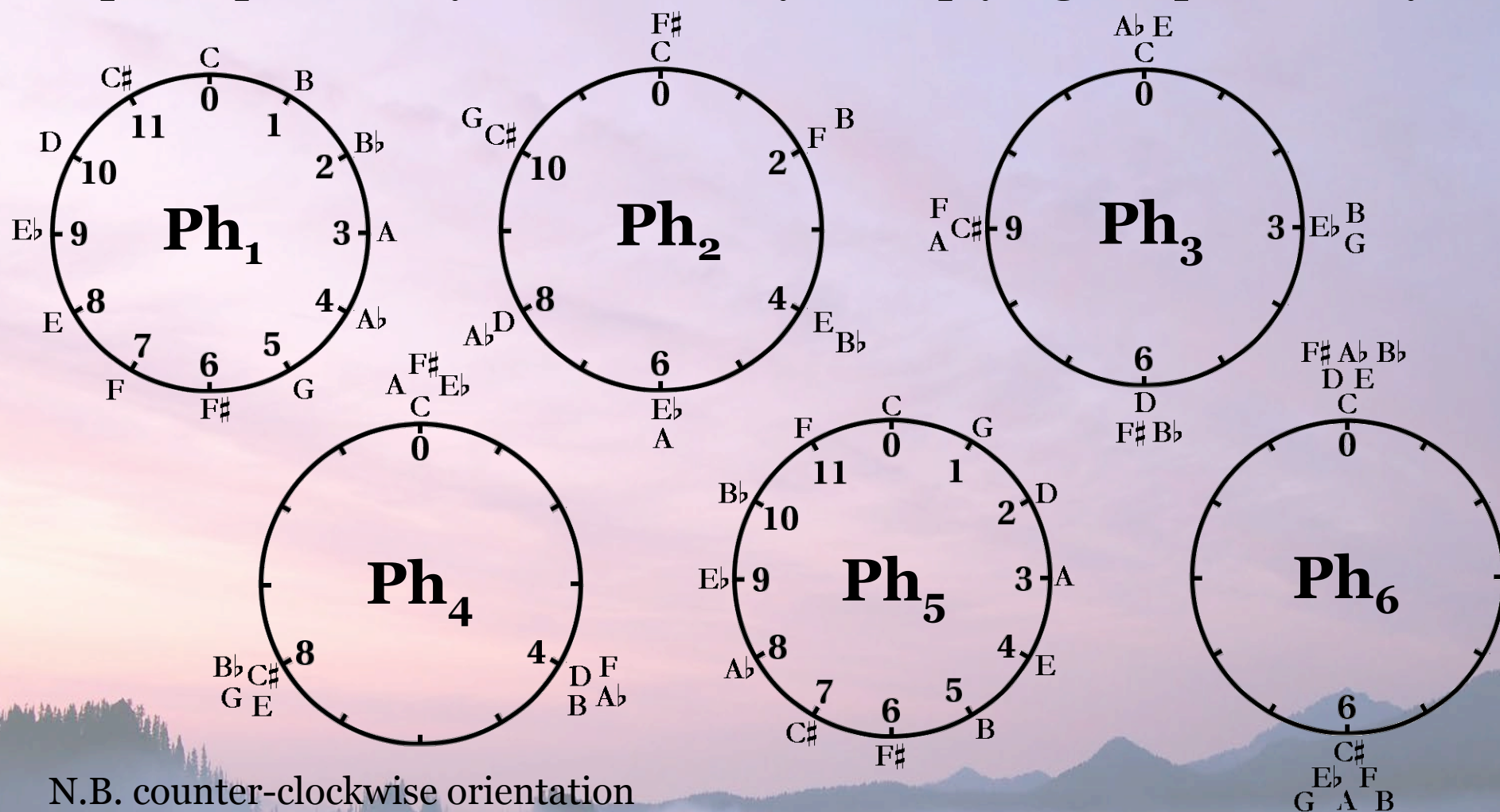
Notation

f_n	The n th DFT component
$ f_n $	The <i>magnitude</i> of the n th component
$ f_n ^2$	Squared magnitude
φ_n	The phase ($0 \leq \varphi_n \leq 2\pi$) of the n th component
\mathbf{Ph}_n	Phase normalized to pc-values: ($0 \leq \mathbf{Ph}_n \leq 12$)

$$\langle\langle (|f_1|^2, \mathbf{Ph}_1), (|f_2|^2, \mathbf{Ph}_2), (|f_3|^2, \mathbf{Ph}_3), (|f_4|^2, \mathbf{Ph}_4), (|f_5|^2, \mathbf{Ph}_5), (|f_6|^2, \mathbf{Ph}_6) \rangle\rangle$$

Phase spaces: One dimensional

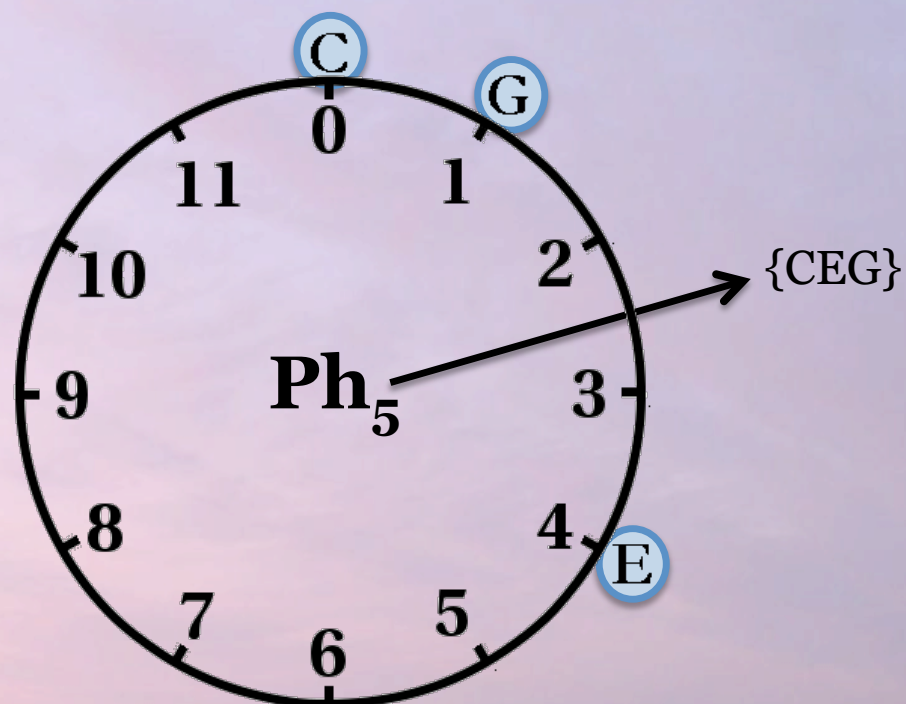
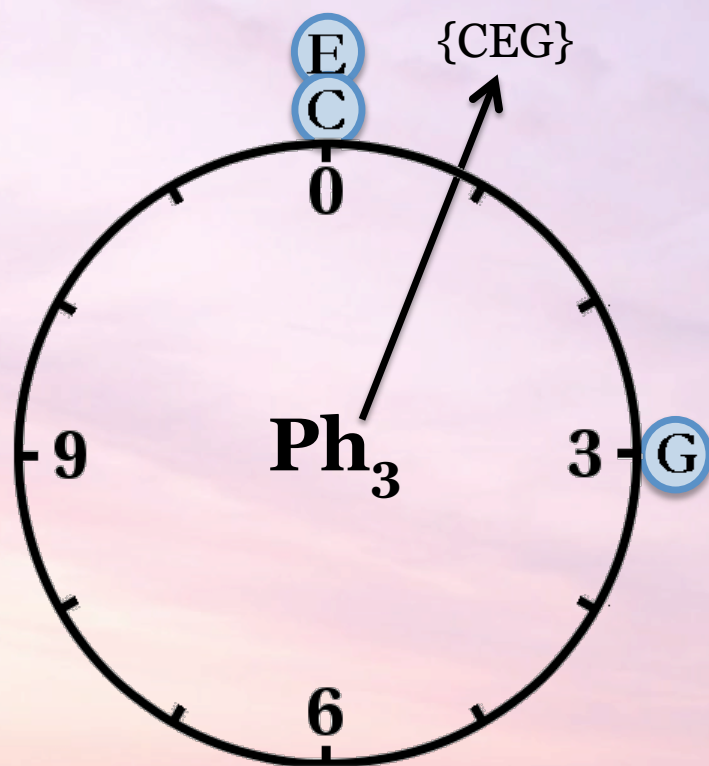
One-dimensional phase spaces are Quinn's *Fourier balances*, superimposed n -cycles created by multiplying the pc-circle by n .



N.B. counter-clockwise orientation

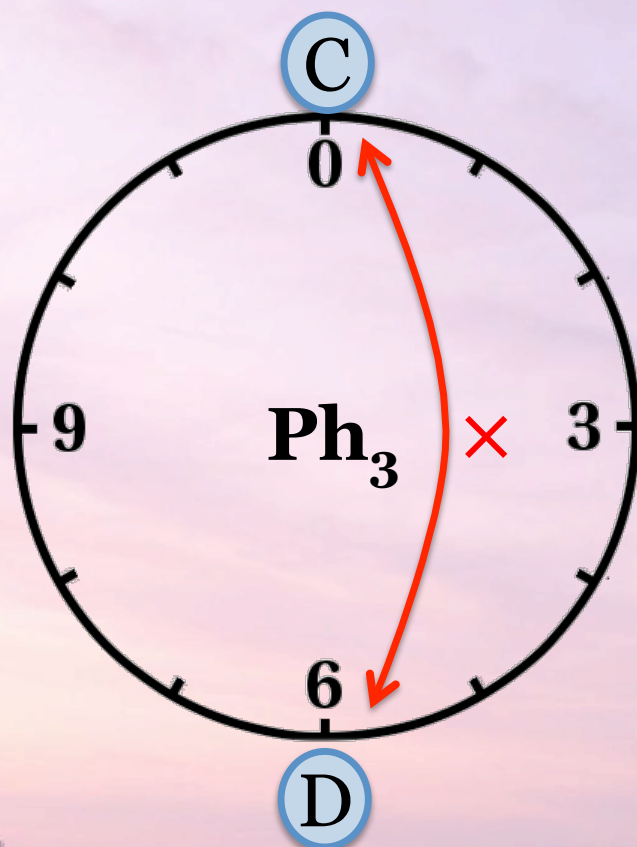
Phase spaces: One dimensional

The position of the pcset in the phase space is the circular average of the individual pcs



Phase spaces: One dimensional

Opposite phases cancel one another out.
Therefore pcsets can have undefined phases.



$\{\text{CD}\}$ has undefined Ph_3 ,
 $|f_3| = 0$

This is a kind of
Generalized Complementation:
Complements balance one another
in *all* phase spaces.

Phase spaces: One dimensional

An analytical proto-methodology:

Each Fourier component measures an independent musical quality: (1) chromaticism, (2) quartal harmony, (3) triadic harmony, (4) octatonicism, (5) diatonicism, (6) whole-tone balance.

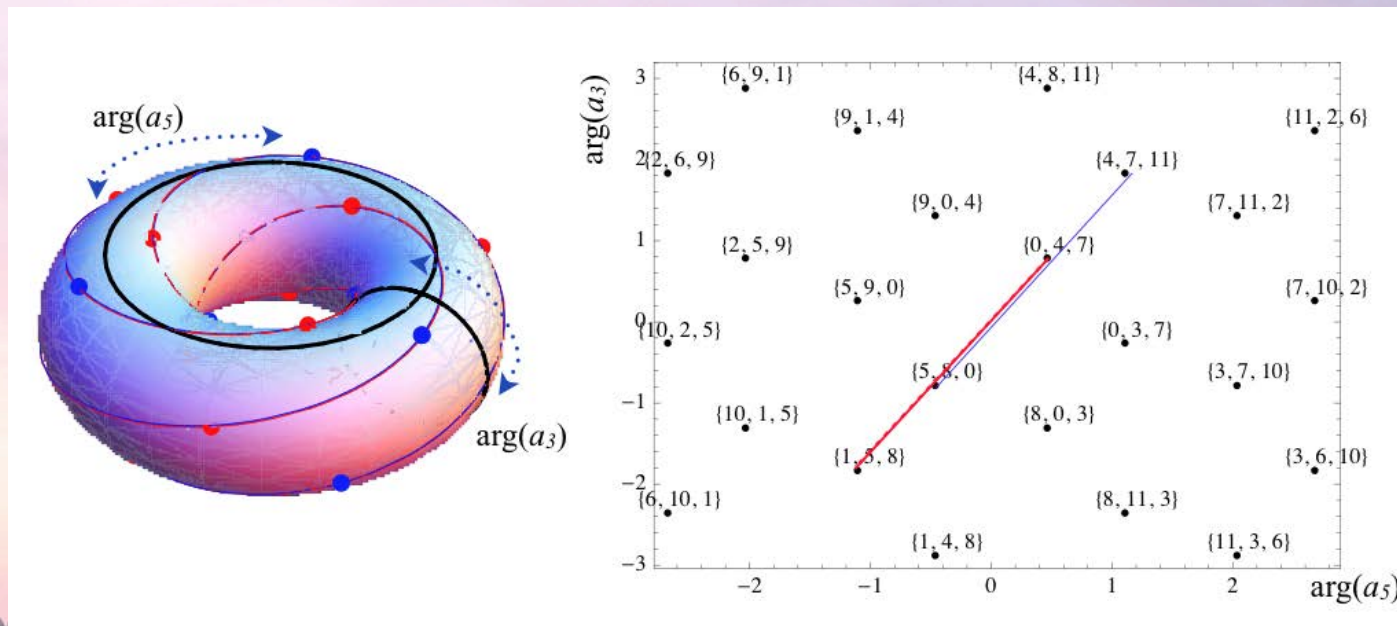
Distances in phase spaces indicate:

- Relatedness of harmonies on the given dimension
- Whether the harmonies **reinforce** one another or **weaken** one another on the given dimension when combined.

Phase Spaces: Two dimensional

A two-dimensional phase space tracks the phases of two components, and is topologically a *torus*.

Amiot (2013) and Yust (2015) use Ph_{3-5} -space to describe tonal harmony.

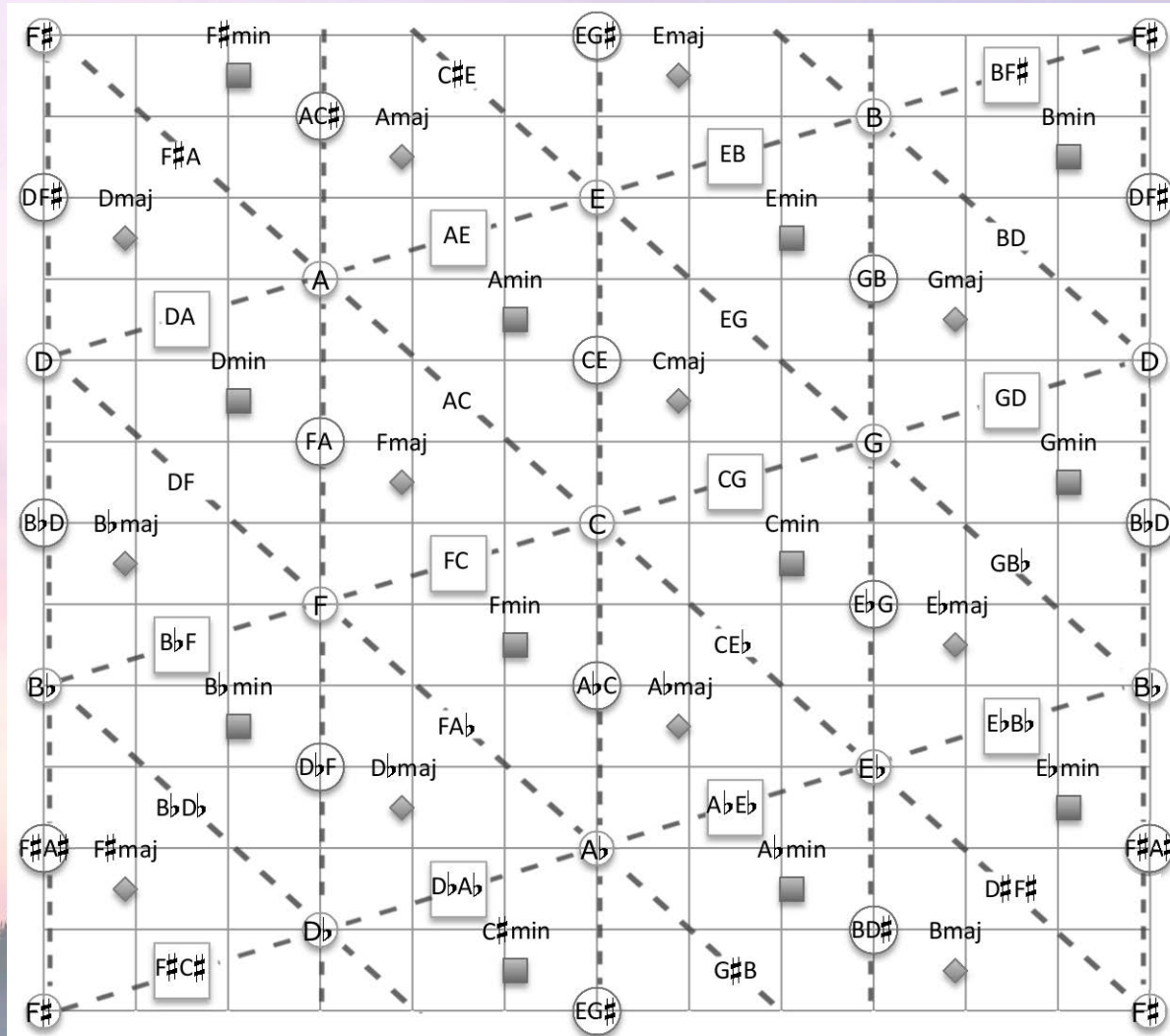


from
Amiot,
MCM 2013
proceedings

Phase Spaces: Two dimensional

Amiot (2013) and Yust (2015) use $\text{Ph}_{3,5}$ -space to describe

tonal harmony



→ Pcs, consonant
dyads and triads,
and *Tonnetz*
in $\text{Ph}_{3,5}$ -space,
from Yust (2015)
(*JMT* 59/1)



II. Debussy, “Les sons et les parfums tournent dans l’air du soir”

1. Heptatonic scales and diatonicity
2. Common tones and harmonic qualities

Scale Theory, Subsets, and Phase Space

Problems in the analytical application of scale theory:

- (1) The status of **subsets** of multiple scales and **supersets** of multiple scales.
- (2) The range of variability in what counts as a scalar set.

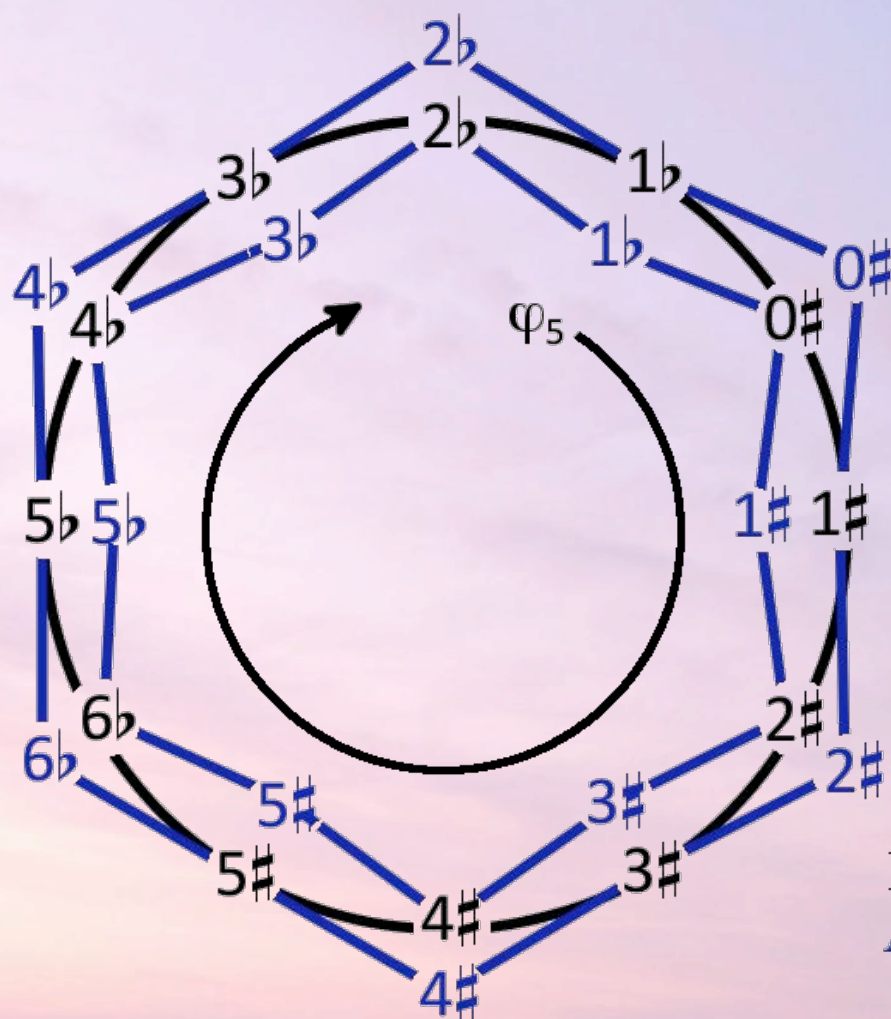
A possible solution: Phase space 5

Debussy and Scale-Network Wormholes

The image displays a musical score for Debussy's *Preludes I, no. 4*, "Les sons et les parfums tournent en l'air de soir". The score is in 3/4 time, marked "Modéré (♩ = 84)" and "(harmonieux et souple)". The key signature is one sharp (F#). The score is divided into two systems. The first system features a treble and bass staff. The treble staff has a tempo marking "Modéré (♩ = 84)" and a performance instruction "(harmonieux et souple)". The bass staff has a performance instruction "pp" and a dynamic marking "m.d.". A green oval highlights a section of the first system, with the annotation "1st harmonic major" in green. A purple oval highlights a section of the first system, with the annotation "A pedal" in purple. The second system also features a treble and bass staff. The treble staff has a performance instruction "m.d.". A blue oval highlights a section of the second system, with the annotation "C# dim7" in purple. A purple oval highlights a section of the second system, with the annotation "A pedal" in purple. A blue oval highlights a section of the second system, with the annotation "Subset of 3rd harmonic minor" in blue. The score includes various musical notations such as notes, rests, and dynamic markings.

Debussy, *Preludes I*, no. 4, "Les sons et les parfums tournent en l'air de soir"

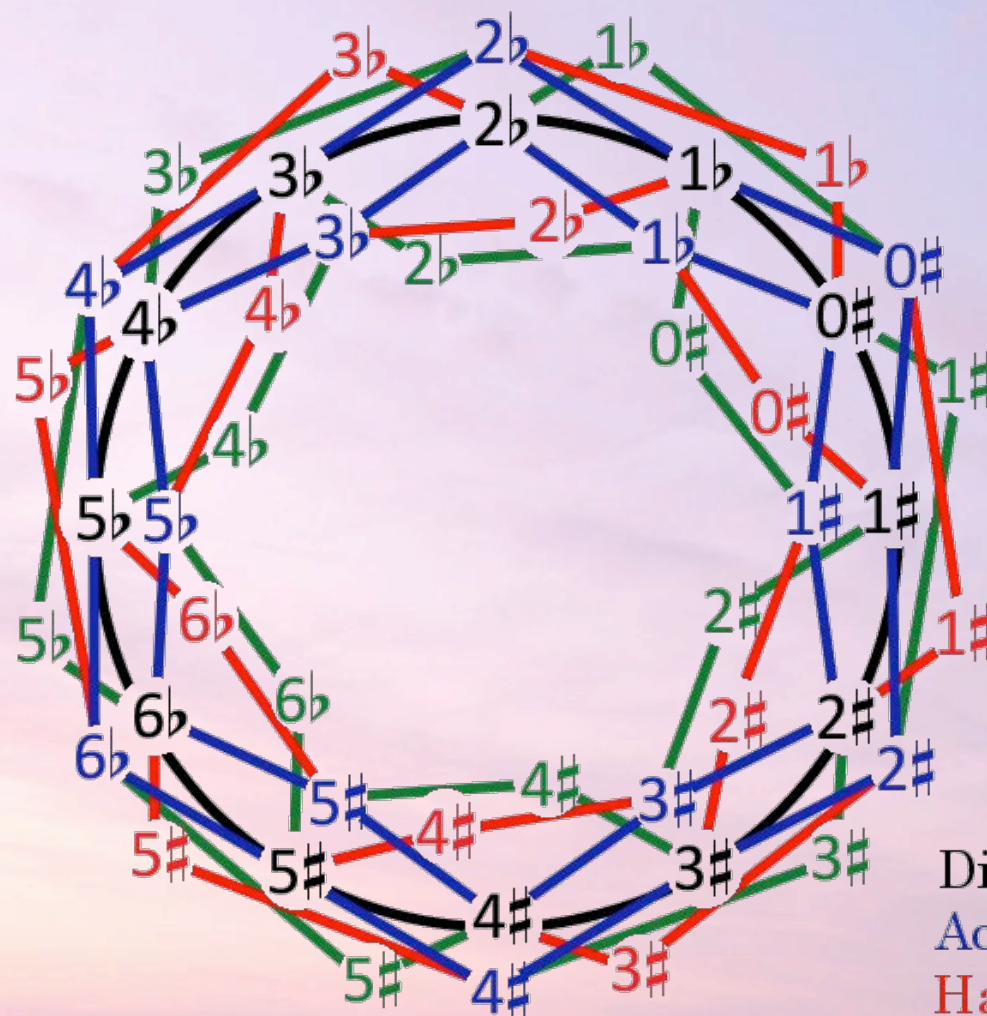
Debussy and Scale-Network Wormholes



A scalar network
for relatively even
heptatonic scales
(after Tymoczko
2004, 2011)
corresponds to
positions on a
 Ph_5 cycle.

Diatonic
Acoustic

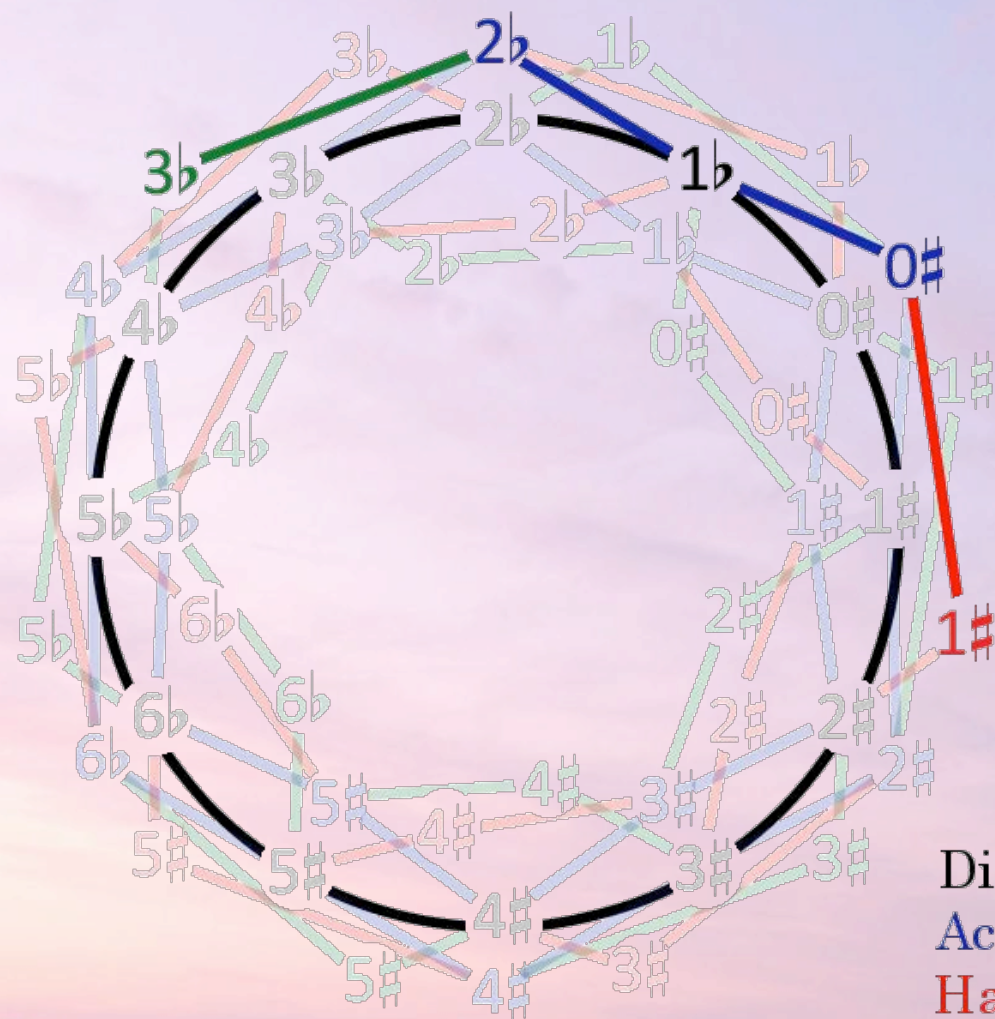
Debussy and Scale-Network Wormholes



A scalar network
for relatively even
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corresponds to
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Diatonic
Acoustic
Harmonic major
Harmonic minor

Debussy and Scale-Network Wormholes



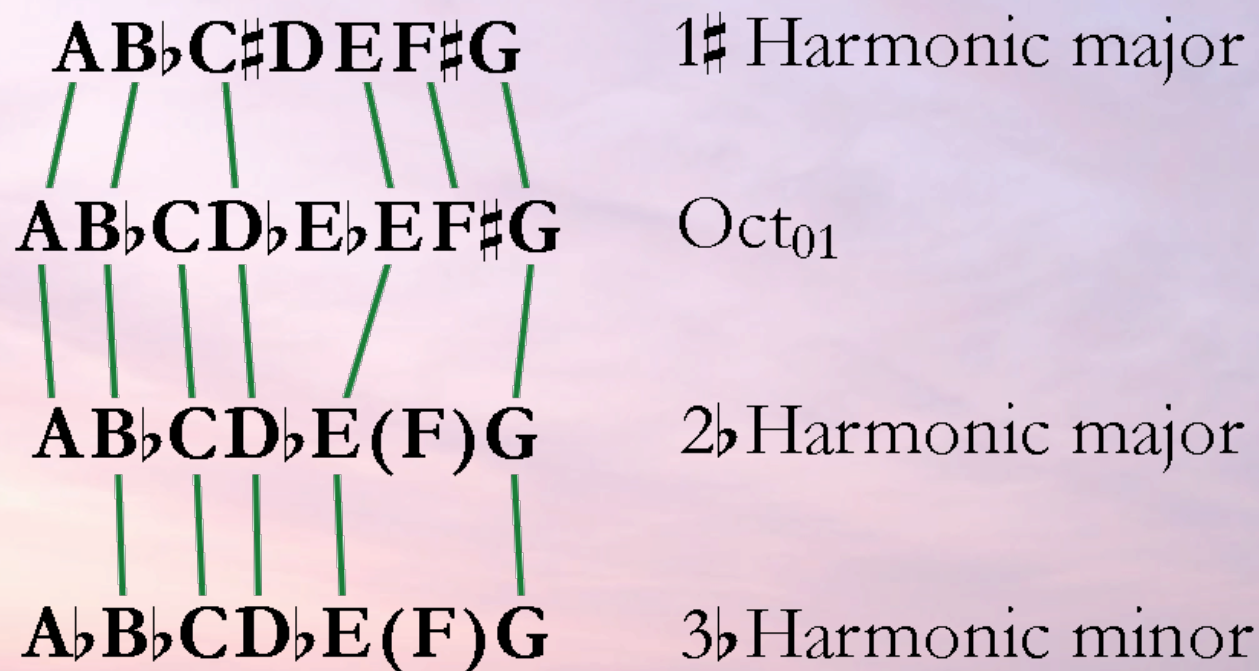
The scales in the opening of the Prelude require four moves in the scale network

Diatonic
Acoustic
Harmonic major
Harmonic minor

Debussy and Scale-Network Wormholes

But . . .

These scales can also be connected with just three moves
By using Oct_{01}



Debussy and Scale-Network Wormholes

Or . . .

By using Oct₁₂

A B_b C[#] D E F[#] G

1[#] Harmonic major

A B_b C[#] D E F G

0[#] Harmonic minor

A_b B_b B C[#] D E F G

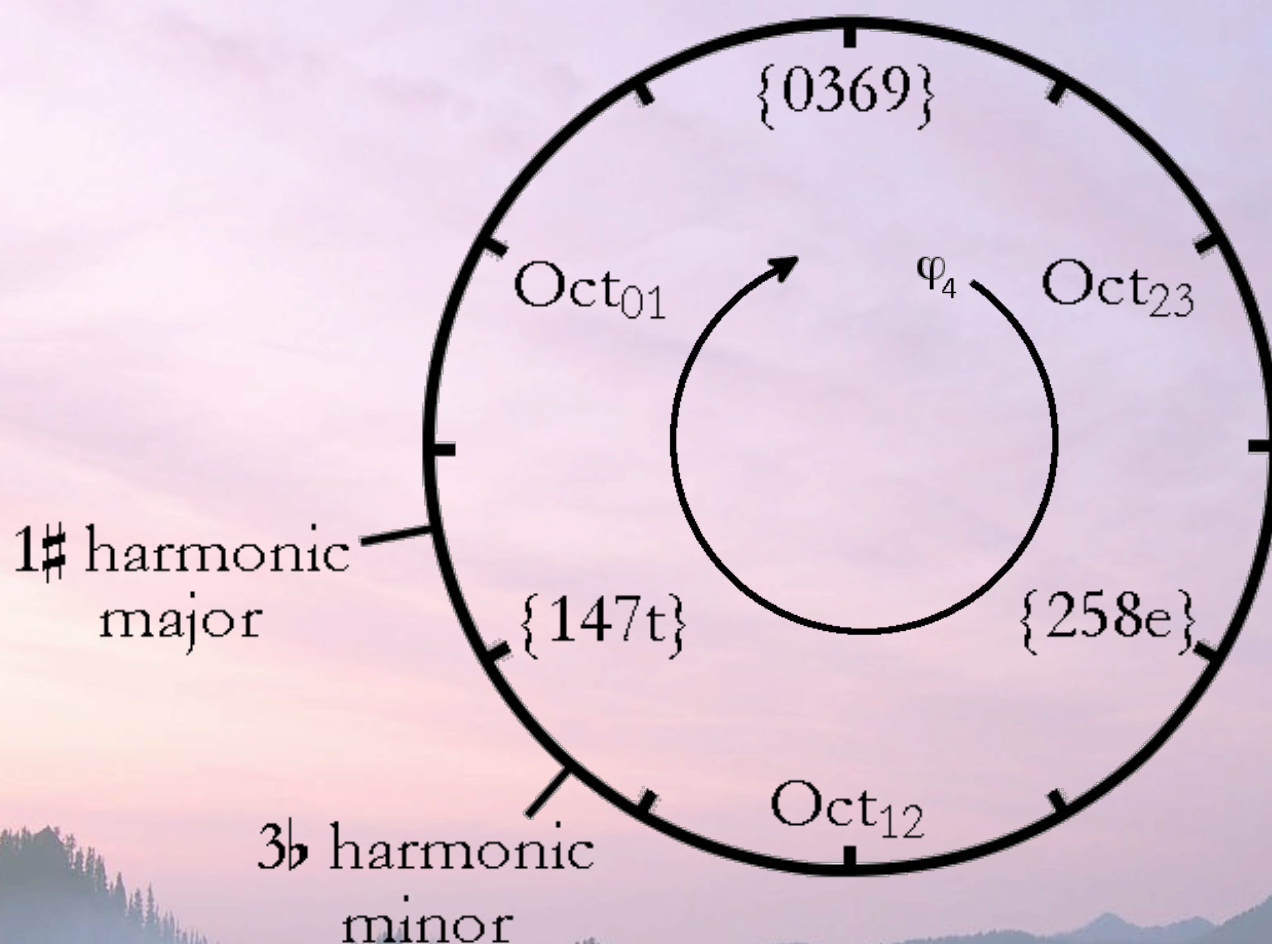
Oct₁₂

A_b B_b C D_b E (F) G

3_b Harmonic minor

Debussy and Scale-Network Wormholes

Why? Although *far apart* in Ph_5
They are *close together* in Ph_4



Debussy and Scale-Network Wormholes

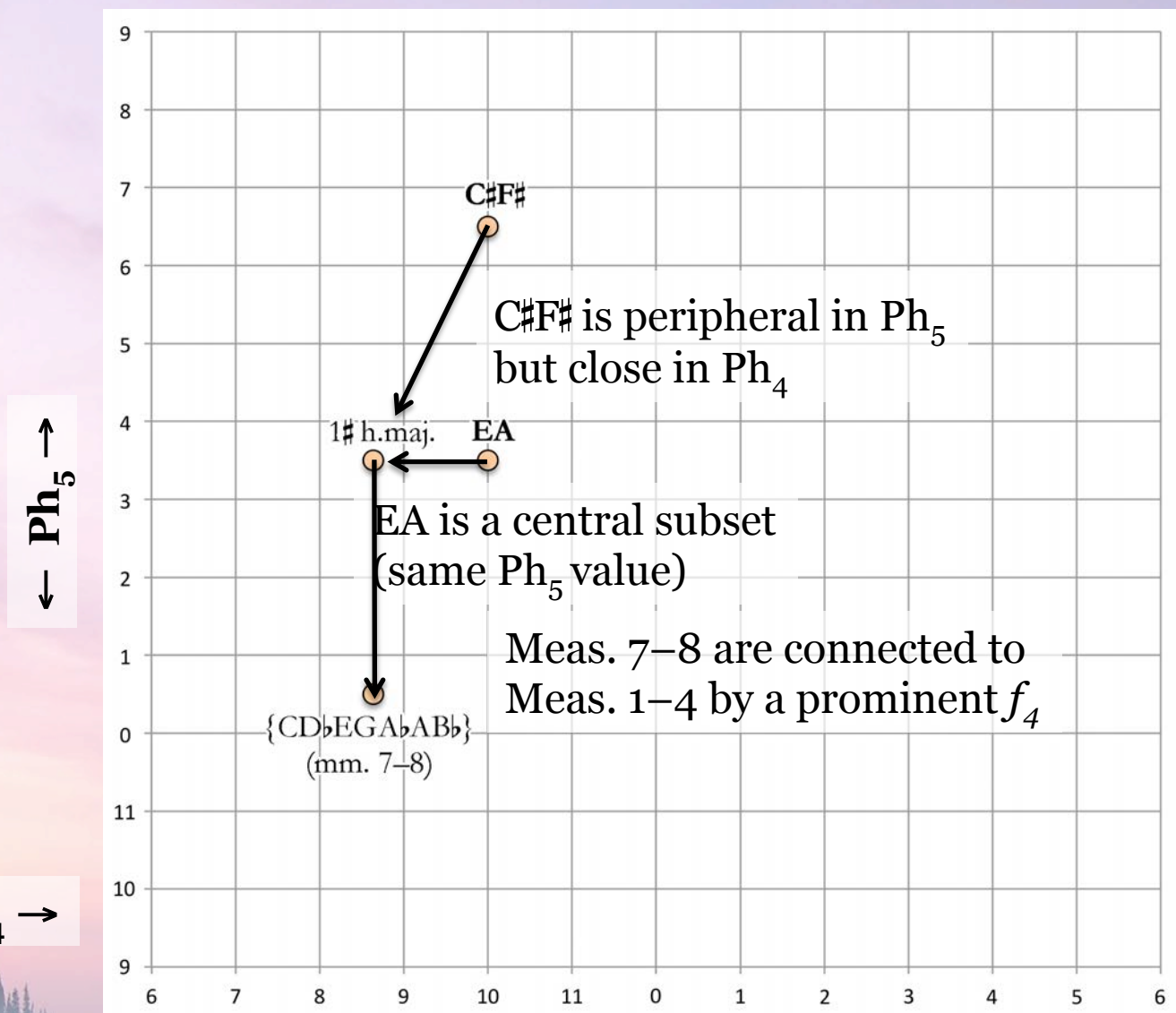
The image displays a musical score for Debussy's *Preludes I, no. 4, "Les sons et les parfums tournent en l'air de soir"*. The score is in 3/4 time, marked "Modéré (♩ = 84)" and "(harmonieux et souple)". The key signature is one sharp (F#). The score is divided into two systems. The first system includes a treble and bass staff. The second system also includes a treble and bass staff. Annotations include:

- E-A motive**: A red box with an arrow pointing to the first measure of the first system.
- 1# harmonic major**: A green oval highlights the first measure of the first system.
- F#-C# motive**: A red box with an arrow pointing to the second measure of the first system.
- A pedal**: A purple label with an arrow pointing to the first measure of the first system.
- C# dim7**: A purple label with an arrow pointing to the second measure of the first system.
- Subset of 3b harmonic minor**: A blue oval highlights the second measure of the first system.

The score includes dynamic markings such as *pp* (pianissimo) and *m.d.* (mezzo-dolce). The notation includes various musical symbols such as notes, rests, and accidentals.

Debussy, *Preludes I*, no. 4, "Les sons et les parfums tournent en l'air de soir"

Debussy and Scale-Network Wormholes



Debussy *Airs*: Change of Quality

The image displays two systems of musical notation for Debussy's *Airs*. The first system features a treble and bass staff with a melody in the treble and a harmonic accompaniment in the bass. The melody is marked *m.d.* (moderato). The bass staff contains a series of chords that are part of a chromatic sequence. Below the first system, the text "En animant un peu" is written. The second system continues the piece, with the melody marked *m.d.* and *p* (piano), and the bass staff marked *p* and *mf* (mezzo-forte). The bass staff in the second system shows a change in the harmonic sequence, indicated by a blue arrow pointing from the first system's sequence to the second. The sequences are labeled as {AC#D#E#} and {ABbCDbEGAb}.

m.d.

En animant un peu

m.d. *p* *expressif*

m.d. *p* *mf*

p

{AC#D#E#}

{ABbCDbEGAb} →

Debussy *Airs*: Change of Quality

DFT magnitudes²

$$\{AB\flat CD\flat EGA\flat\}: \begin{matrix} |f_1|^2 & |f_2|^2 & |f_3|^2 & |f_4|^2 & |f_5|^2 & |f_6|^2 \\ \langle\langle 3.73, 1, & 5, & 7, & 0.27, 1 \rangle\rangle \end{matrix}$$

$$\{AC\sharp D\sharp E\sharp\}: \langle\langle 1, 1, 4, 1, 1, 16 \rangle\rangle$$

Common-Tone Theorem

The number of common tones between sets X and $Y =$

$$\frac{1}{12} \sum_{n=0}^{11} |f_n(A)| |f_n(B)| \cos(\varphi_n(A) - \varphi_n(B))$$

For each component
(sum over components)

Cosine of the phase difference
(ranges from -1 for opposite
phases to 1 for same phase)

Weighted by the component
magnitudes

Common-Tone Theorem

The number of common tones between sets X and $Y =$

$$\frac{1}{12} \sum_{n=0}^{11} |f_n(A)| |f_n(B)| \cos(\varphi_n(A) - \varphi_n(B))$$

In other words:

Distances in phase space for the most prominent components determine the number of common tones

Common-Tone Theorem

Example:

$$|f_0| (|f_1|, \text{Ph}_1) (|f_2|, \text{Ph}_2) (|f_3|, \text{Ph}_3) (|f_4|, \text{Ph}_4) (|f_5|, \text{Ph}_5) (|f_6|, \text{Ph}_6)$$

$$\{\text{AB}\flat\text{CD}\flat\text{EGA}\flat\}: \langle\langle 7, (1.93, 2.5) (1, 8) (2.24, 11.1), (2.65, 8.64) (0.52, 0.5), (1, 0) \rangle\rangle$$

$$\{\text{AC}\sharp\text{D}\sharp\text{E}\sharp\}: \langle\langle 4, (1, 9) (1, 6) (2, 9) (1, 0) (1, 9) (4, 6) \rangle\rangle$$

$$\begin{array}{l} \text{Multiply mag.,} \\ \text{Cosine of Ph. diff.} \end{array} \quad 28, (1.9, -1.0) (1, 0.5) (4.5, 0.45) (2.7, -0.19) (0.52, -0.26) (4, -1)$$

$$\begin{array}{l} \text{Multiply and} \\ \text{divide by twelve} \end{array} \quad 2.33 + -0.31 + 0.08 + 0.33 + -0.08 + -0.02 + -0.33$$

$$\text{Sum} = 2 \text{ common tones (A and C}\sharp\text{)}$$



Common-tone Linkage: Debussy

a Tempo

pp *pp* *pp m.d.*

1# Harmonic Major subset AC#EF#G

Plus lent *En animant*

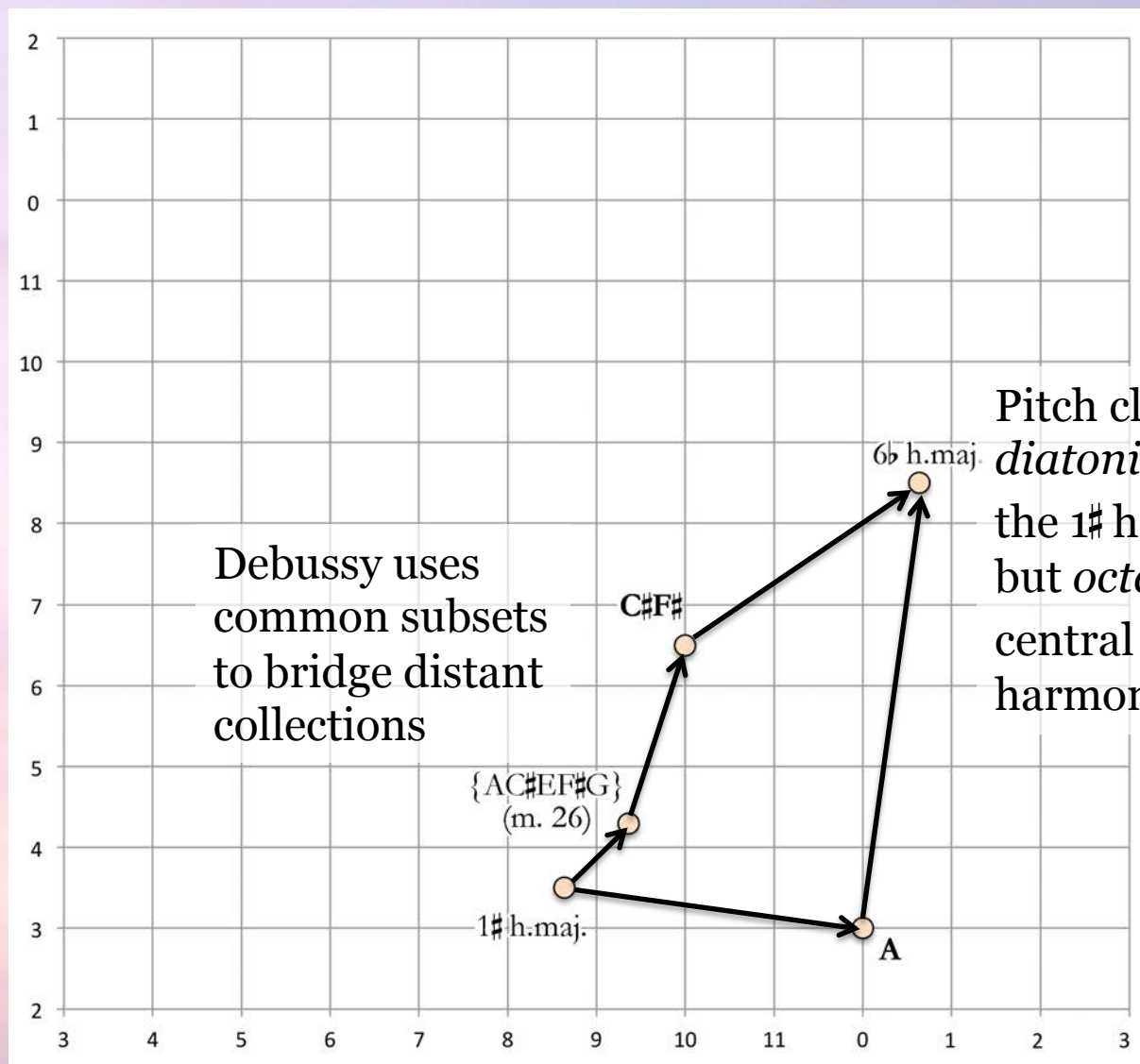
pp *p* *mf*

F#C# 6b Harmonic Major

Common-tone Linkage: Debussy

← Ph₅ →

← Ph₄ →





Common-tone Linkage: Debussy

8 En retenant - - - // a Tempo
égal et doux

p dim. *pp*

en dehors

Serrez un peu - - - // Retenu - - //

p *p*

a Tempo

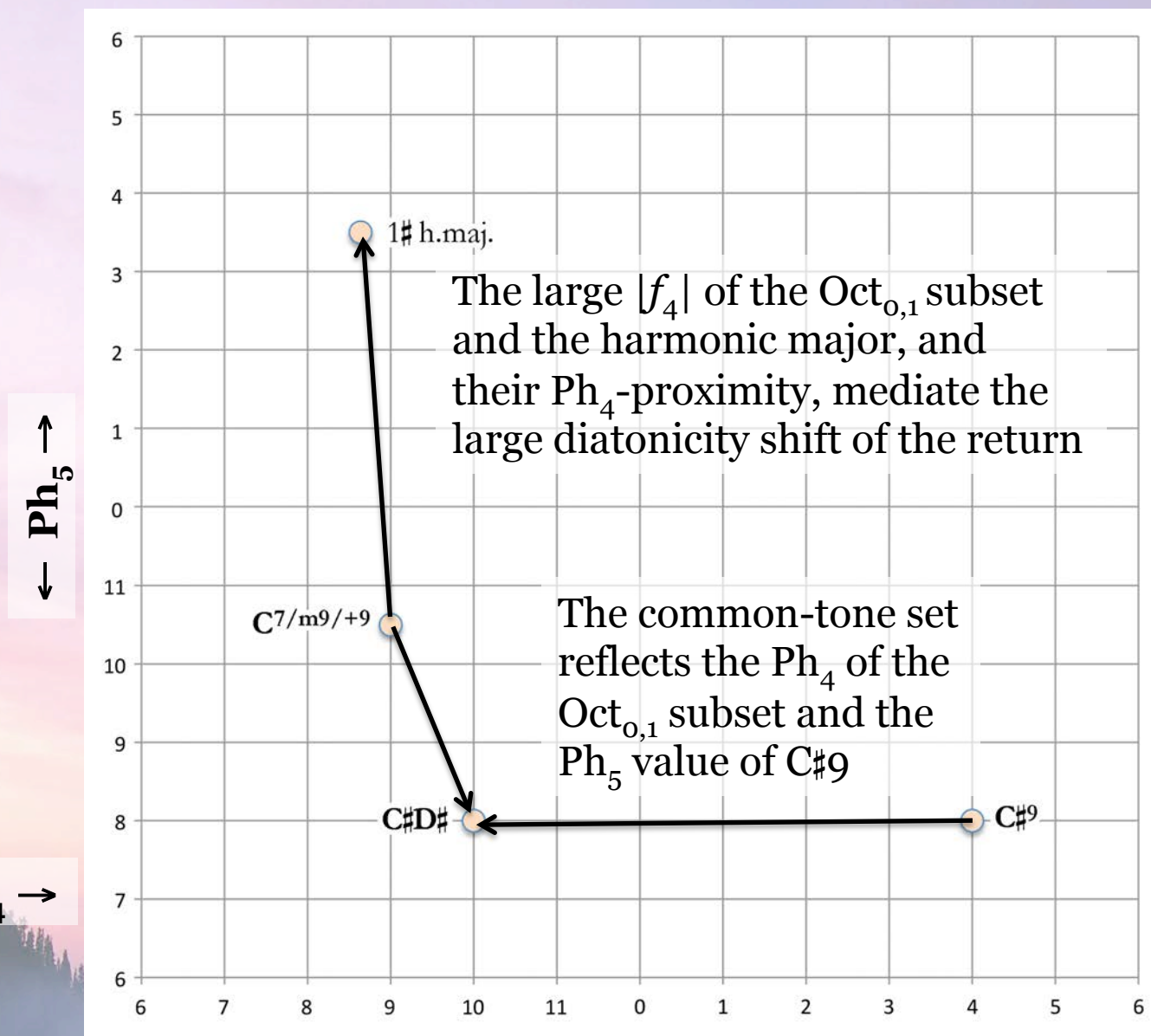
pp *pp* *pp m.d.*

Oct₀₁ subset C^{#9}

etc.

1[#] Harmonic Major

Common-tone Linkage: Debussy



Common-Tone Theorem

Example:

	$ f_0 $	(f_1 , Ph_1)	(f_2 , Ph_2)	(f_3 , Ph_3)	(f_4 , Ph_4)	(f_5 , Ph_5)	(f_6 , Ph_6)
$\text{C}7/\text{m}9/+9$	$\langle\langle$	7, (0.52, 5.5)	(1, 8)	(2.2, 6.9)	(2.6, 8.6)	(1.9, 3.5)	(1, 0) $\rangle\rangle$
1^\sharp h.maj.	$\langle\langle$	6, (1.4, 10.5)	(0, -)	(1.4, 1.5)	(3.5, 9)	(1.4, 10.5)	(0, -) $\rangle\rangle$
Multiply mag., Cosine of Ph. diff.	42,	(0.7, -0.9)	(0, -)	(3.2, -0.95)	(2.7, -0.19)	(0.52, -0.26)	(0, -)
Multiply and divide by twelve	3.5	+ -0.1	+ 0	+ -0.5	+ 1.5	+ -0.4	+ 0
Sum	= 4 common tones (C \sharp , E, G, B \flat)						



Stravinsky and the Octatonic



1. *Rite of Spring*, Introduction and *Augurs*

Stravinsky and the Octatonic

Is this music octatonic?

The image displays a musical score for a piano piece, likely from Igor Stravinsky's 'Les Noces'. The score is written for three systems of staves. The first system consists of a grand staff (bass and treble clefs) and a single bass staff. The tempo is marked 'Tempo giusto' with a quarter note equal to 56 (♩ = 56). The key signature has two flats (B-flat and E-flat). The first system features a strong octatonic pattern in the bass staff, marked with a forte (*f*) dynamic and accents. The second system shows a more complex texture with a mezzo-forte (*mf*) melody in the treble staff and a bass staff with octatonic accompaniment. The third system continues the octatonic patterns, with dynamics ranging from mezzo-forte (*meno f*) to forte (*f*). The score is set against a background of a misty, mountainous landscape.

Stravinsky and the Octatonic



Global
0-5
(0 2 5/0 3 5/0 2 3 5)
tetrachord

0-11 "major 7th"
interval span

0-5, 11

0 2 3 5 6 8 9 11
oct. scales

Local
0, 3, 6, 9

(0 2 3 5) tetrachords;
(0 3 7/0 4 7/0 4 7 10)
triads, "dom. 7ths"

0 2 3 5 6 8 9 11
oct. scales;
0 2 3 5 7 9 10 (0)
D-scales



Van den Toorn:
5 of the 7 notes in
the "Augurs" chord
and the C triad
come from Oct_{0,1}
(E + E^{b7}).
The G[#] and B
reinforce E

Stravinsky and the Octatonic

What are the chances?!?!

100%

All 8-note collections overlap at least one octatonic by six or more pitch classes.

Stravinsky and the Octatonic

Joseph Straus (review of *Music of Stravinsky*):

Van den Toorn “never provides and systematic criteria for determining the presence of the octatonic collection; as a result, a number of his attributions are suspect. Almost any passage containing nine to twelve pitch classes can be discussed as ‘diatonic interpenetration’ of an octatonic context.”



Stravinsky and the Octatonic

Dmitri Tymoczko (on the analyses in *Music of Stravinsky*):

“If even *these* passages can be understood as the result of ‘octatonic-diatonic interpenetration,’ then we should rightly ask whether there is any music that *cannot* be understood in this way.

In a sense, there is not: any proper subset can be decomposed into diatonic and octatonic components.”



Stravinsky and the Octatonic

Let's try again . . .

E \flat 7

F \flat maj.

C maj.

Tempo giusto ♩ = 56

f

mf

meno f

f

The image displays a musical score for Stravinsky's 'The Octatonic'. It consists of three systems of staves. The first system has a bass staff with a treble clef and a key signature of two flats (B-flat and E-flat). It contains two staves of music. The first staff has a tempo marking 'Tempo giusto' and a metronome marking '♩ = 56'. The second staff has a dynamic marking 'f'. The second system has a bass staff with a treble clef and a key signature of two flats. It contains two staves of music. The first staff has a dynamic marking 'mf'. The third system has a bass staff with a treble clef and a key signature of two flats. It contains two staves of music. The first staff has a dynamic marking 'meno f'. The second staff has a dynamic marking 'f'. The score is annotated with three blue circles and labels: 'E \flat 7' points to the first circle in the first system, 'F \flat maj.' points to the second circle in the first system, and 'C maj.' points to the third circle in the third system. The background of the slide features a misty, mountainous landscape with evergreen trees.

Stravinsky and the Octatonic

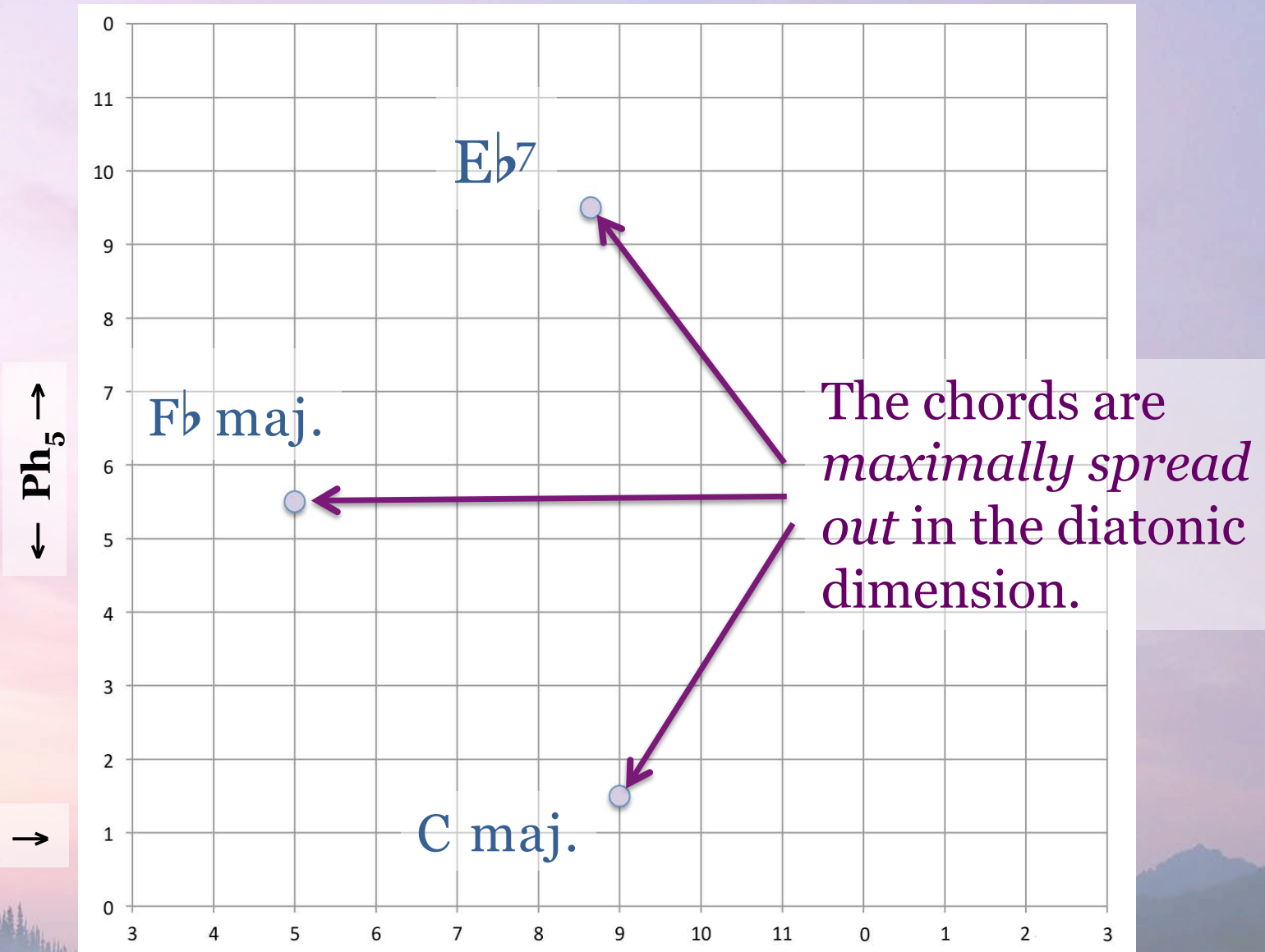
Components f_4 and f_5 are the largest
across the set-types

DFT magnitudes²

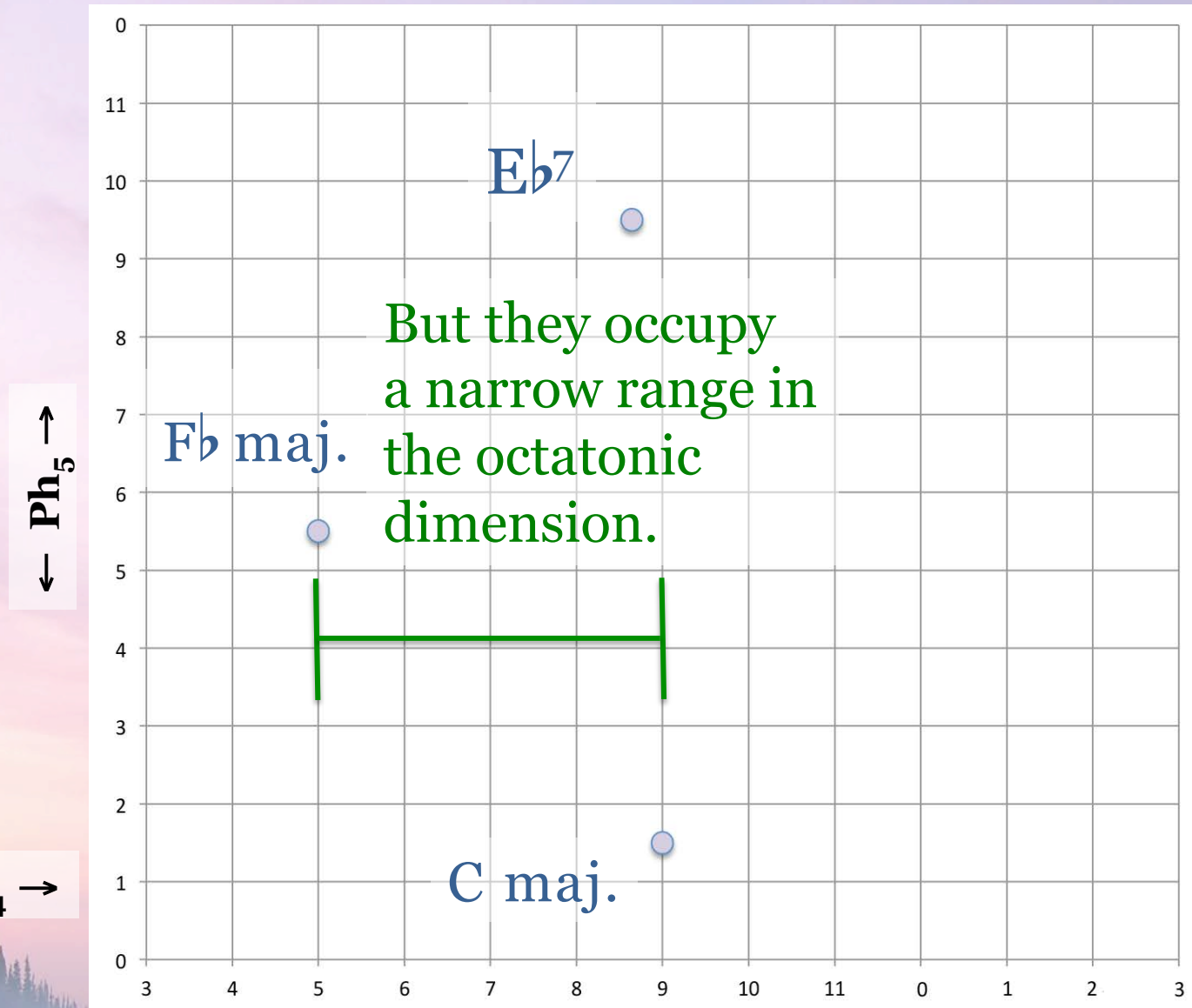
Major/minor triad: $\langle\langle |f_1|^2 \ |f_2|^2 \ |f_3|^2 \ |f_4|^2 \ |f_5|^2 \ |f_6|^2 \rangle\rangle$
 $\langle\langle 0.27, 1, 5, 3, 3.73, 1 \rangle\rangle$

Dominant 7th: $\langle\langle 0.27, 1, 2, 7, 3.73, 4 \rangle\rangle$

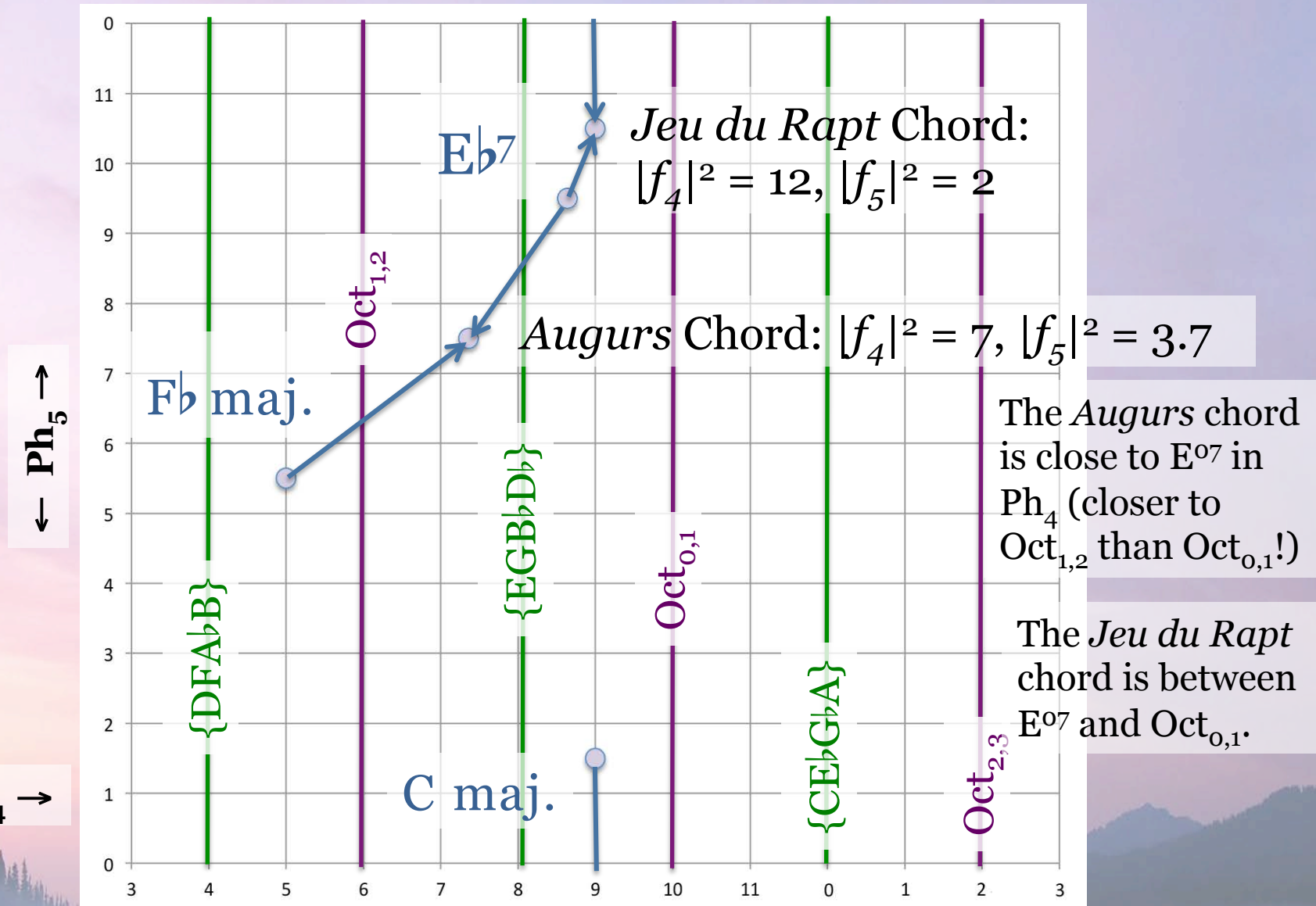
Stravinsky and the Octatonic



Stravinsky and the Octatonic



Stravinsky and the Octatonic



Stravinsky and the Octatonic: *Jeu du Rapt*

Presto ♩ = 132

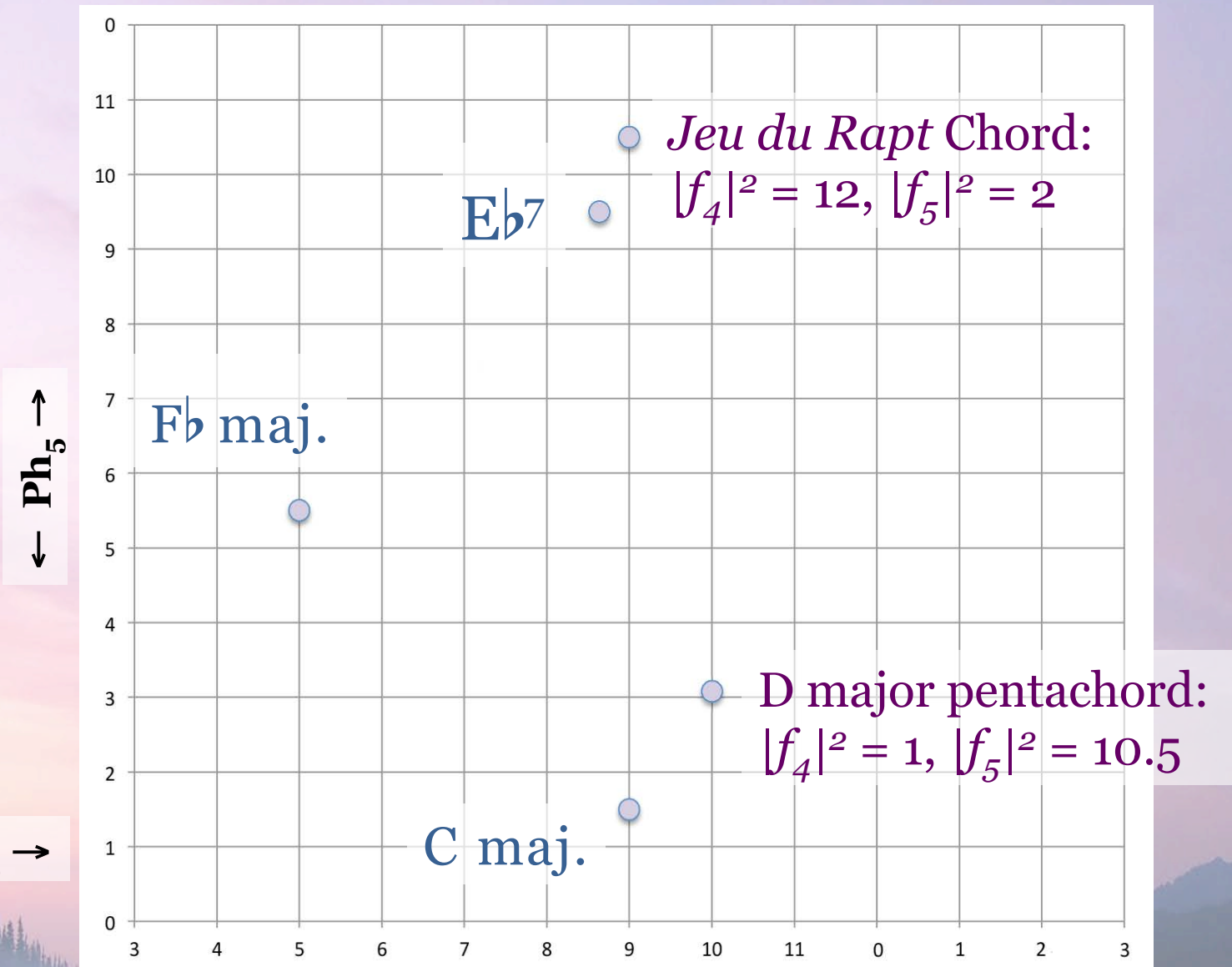
The image displays a musical score for Igor Stravinsky's 'Jeu du Rapt'. It features four systems of staves. The first system shows a piano part with a treble and bass clef, marked 'Presto' and '♩ = 132'. The second system includes a timpani part, indicated by '(Timpani)' and 'f', with a dynamic marking of 'f'. The third and fourth systems continue the piano part. Annotations in blue text identify specific harmonic elements: 'Eb7 + C maj.' and 'D maj. pentachord'.

$E_b7 + C \text{ maj.}$

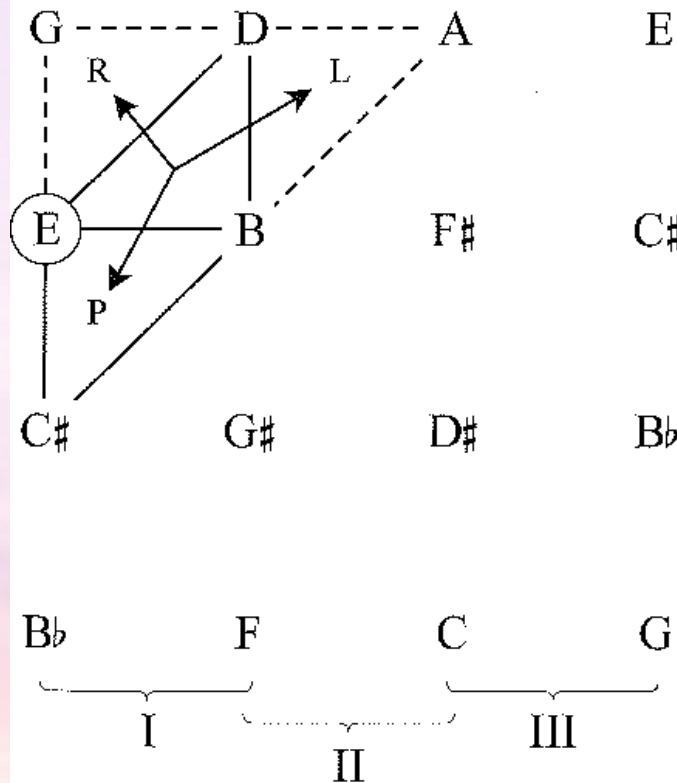
D maj. pentachord

(Timpani)

Stravinsky and the Octatonic

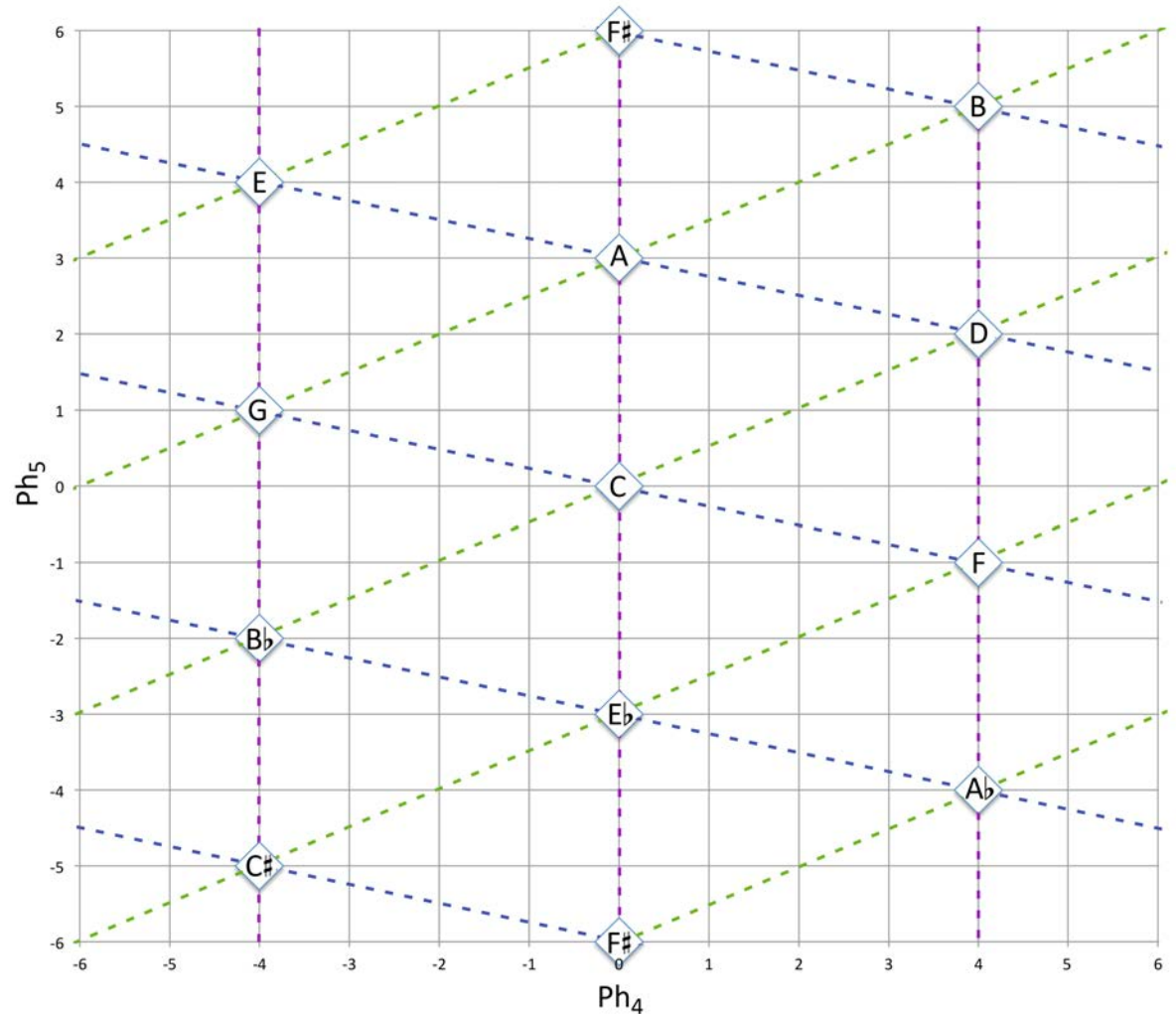


Stravinsky and the Octatonic



octatonic collections

Octatonic Tonnetz
Van den Toorn and
McGuinness 2012



Pitch classes in $Ph_{4,5}$ -space

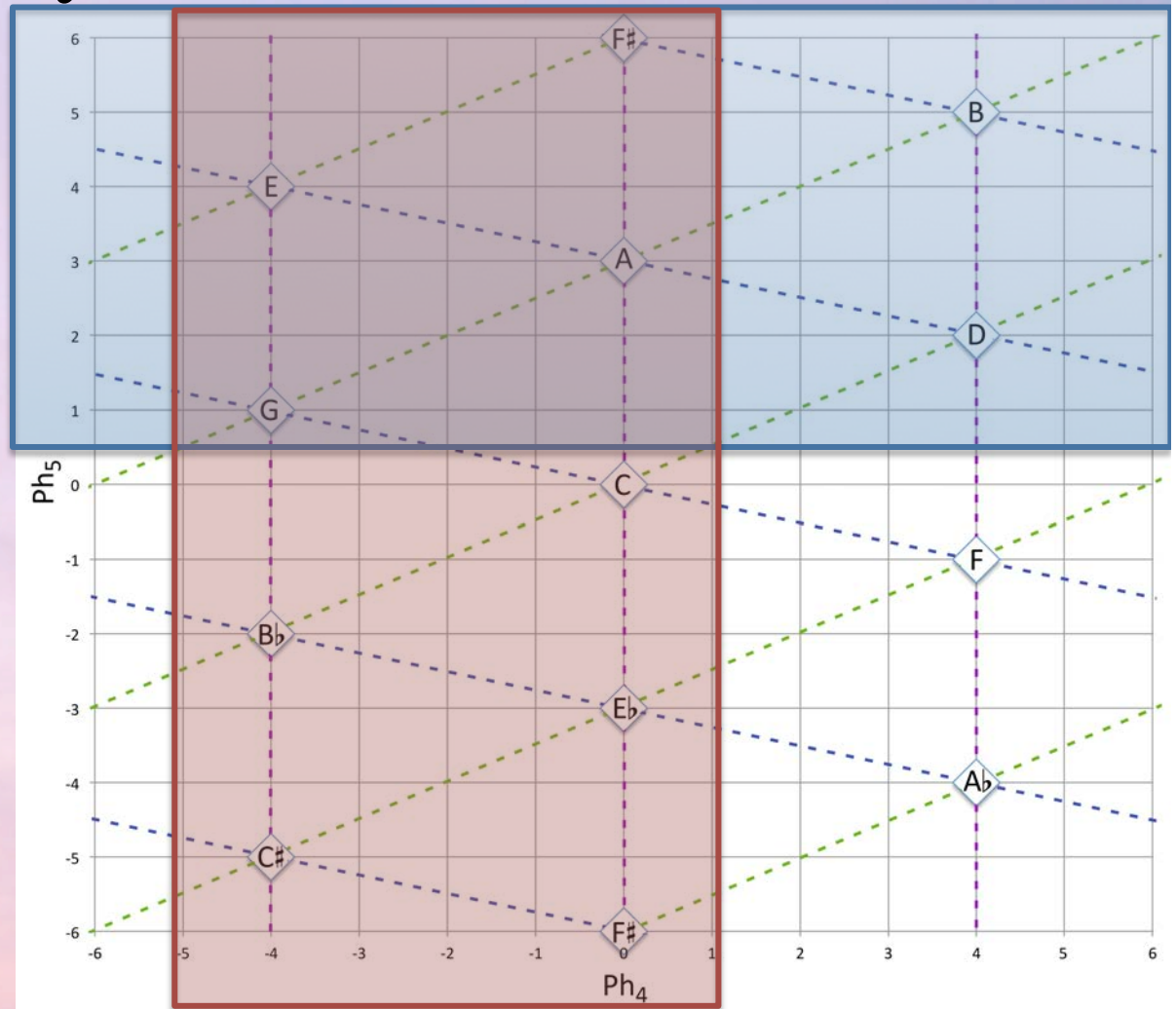
Stravinsky and the Octatonic

Diatonic materials:
D major hexachord

Overlap:

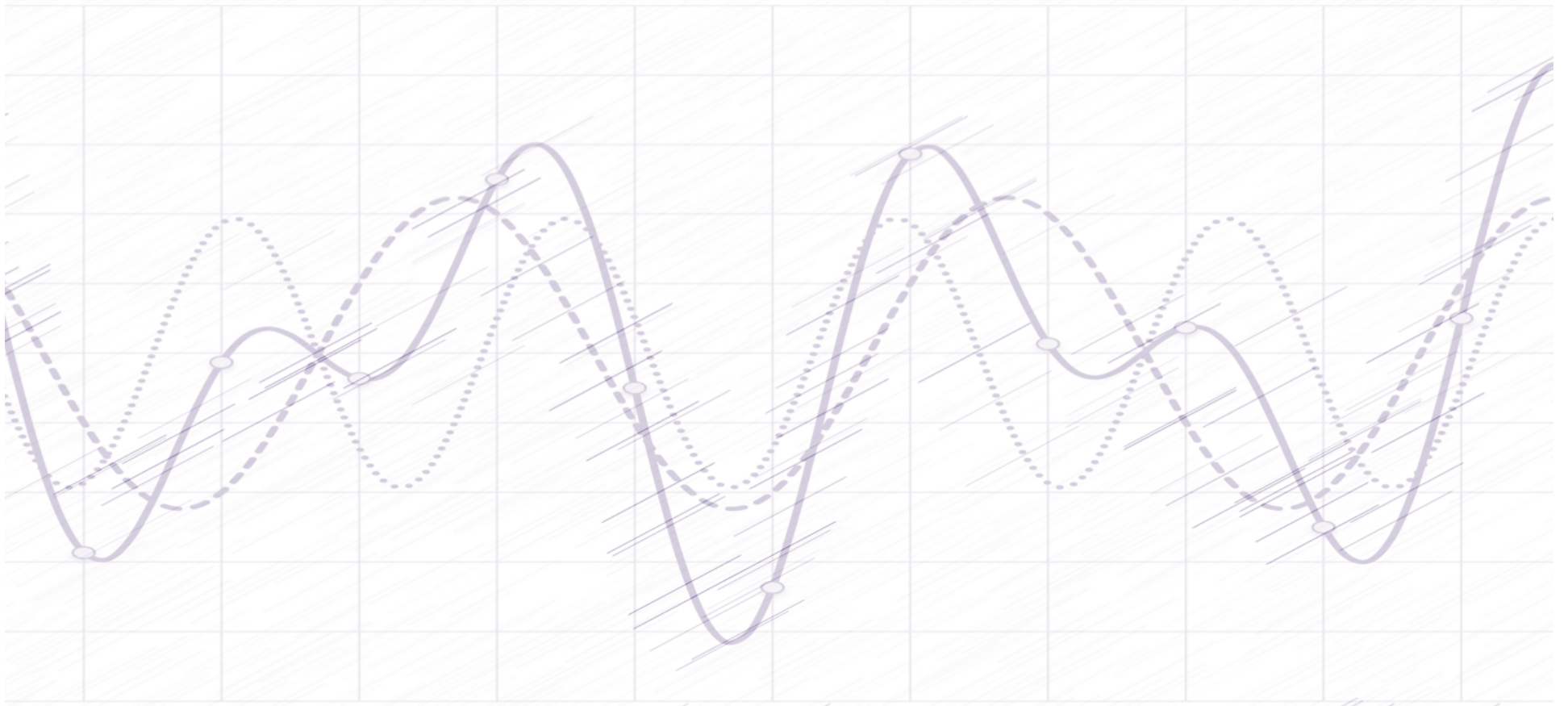
A-dorian tetrachord
(cf. *Jeu du Rapt*)

Octatonic materials:
 $\text{Oct}_{0,1}$



Pitch classes in $\text{Ph}_{4,5}$ -space

Feldman, *Palais de Mari*



Feldman, *Palais de Mari*

♩ = 63-66

ppp

22. →

7

14

22

The image displays a musical score for the piece 'Palais de Mari' by Morton Feldman. The score is written for piano and consists of four systems of staves. The first system (measures 1-6) begins with a tempo marking '♩ = 63-66' and a dynamic marking 'ppp'. The second system (measures 7-13) includes a '22.' marking with an arrow pointing to the right. The third system (measures 14-21) and the fourth system (measures 22-28) continue the composition. The score features complex rhythmic patterns and a variety of dynamic markings, including 'ppp' and '6:5' (likely indicating a 6:5 ratio or a specific interval). The notation includes treble and bass clefs, key signatures, and various note values and rests.

Feldman, *Palais de Mari*

Features of the piece:

- Composed in 1986, Feldman's last work for solo piano.
- Long but sparse: the 9-page score takes ca. 25 minutes to play.
- Made up of discrete gestures, frequently repeated and varied (often in subtle ways).
- Pedal is held continuously throughout most of the piece. This blurs the distinction between *successive* and *simultaneous* sounds.
- Extended segmentational analysis in Hanninen, *A Theory of Music Analysis* (2012).

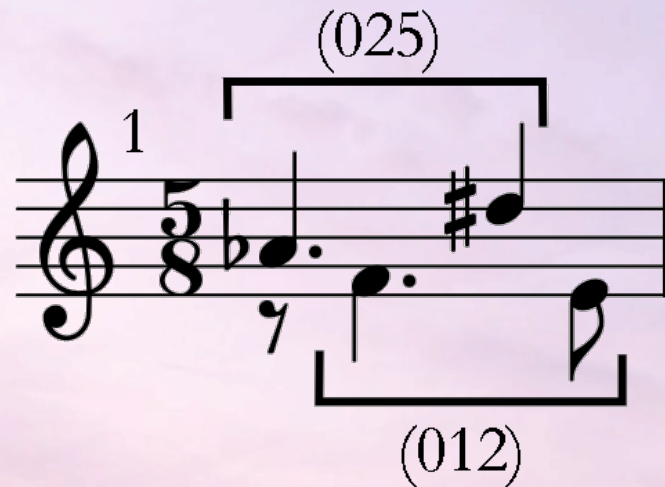
Feldman, *Palais de Mari*

Features of the piece:

- Long sections on the piece tend to dwell on a limited set of gestures, giving the piece a sense of trajectory that is nonetheless non-teleological.
- Composed around the same time as his *Second String Quartet*, which Feldman described as “a dialectic of sorts between such elements as . . . chromaticism/consonance.”
- “Reverse Development”: Gestures often appear *before* the idea from which they are derived, replacing a process of development with one of *revelation*.

Feldman, *Palais de Mari*

The initial gesture (Hanninen set A) stages a chromatic–diatonic conflict



(025) $\langle\langle 2.27, 1, 1, 3, \mathbf{5.73}, 1 \rangle\rangle$

(012) $\langle\langle \mathbf{7.46}, 4, 1, 0, \mathbf{0.54}, 1 \rangle\rangle$

(0125) $\langle\langle \mathbf{5.73}, 3, 4, 1, 2.27, 0 \rangle\rangle$

The first three notes are highly diatonic,
but the final E introduces a concentrated chromaticism.

Hanninen: “The contrast between harmonies rich in ics 2 and 5,
versus those rich in ic1, resonates throughout the piece.”

Feldman, *Palais de Mari*

The initial gesture (Hanninen set A) stages a chromatic–diatonic conflict



The gesture can also be divided by part into a fourth and a minor second.

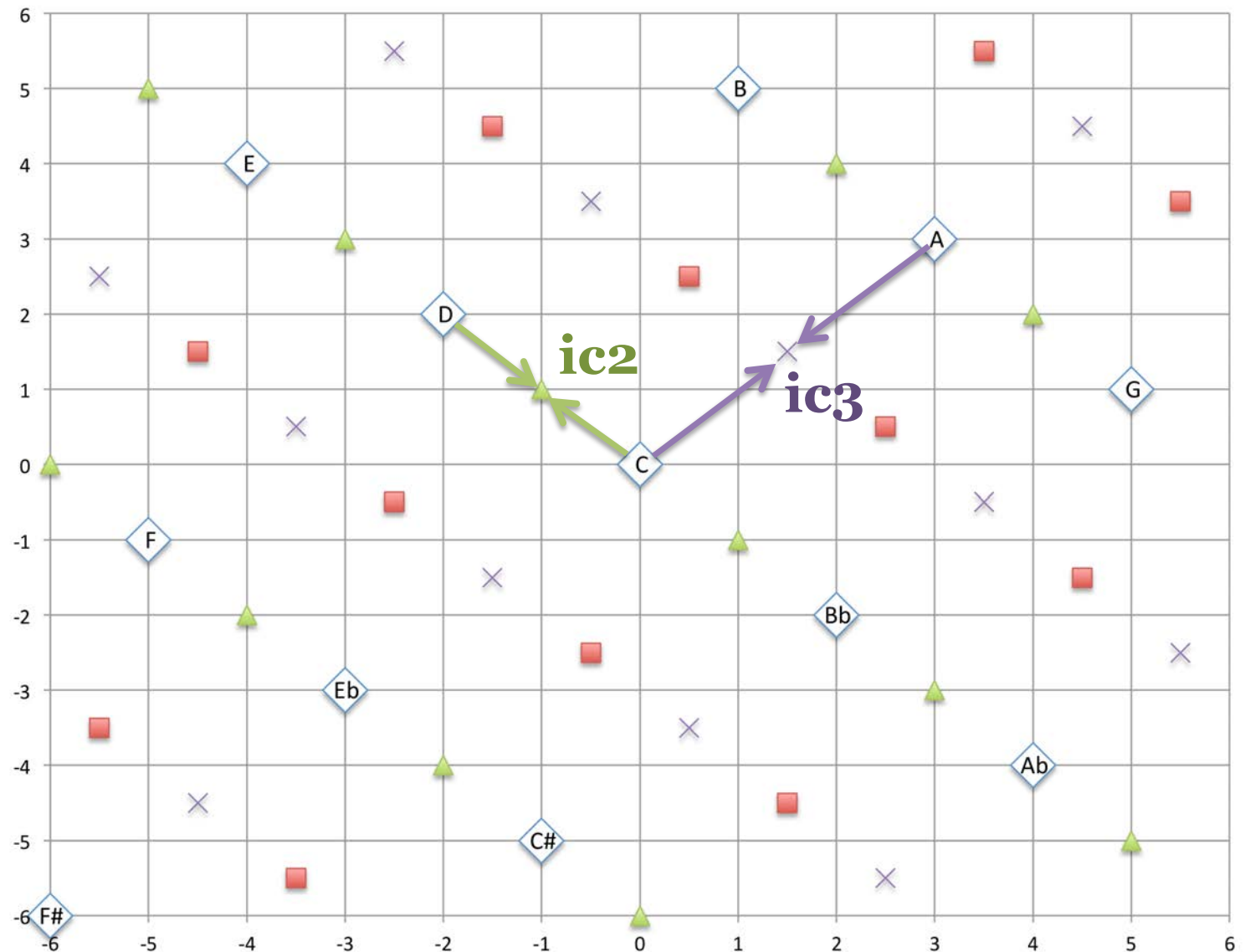
Feldman, *Palais de Mari*

Ph_{1,5}-space

ic2 and ic3 are the highest-magnitude ics in the space and are balanced between f_1 and f_5

← Ph₅ →

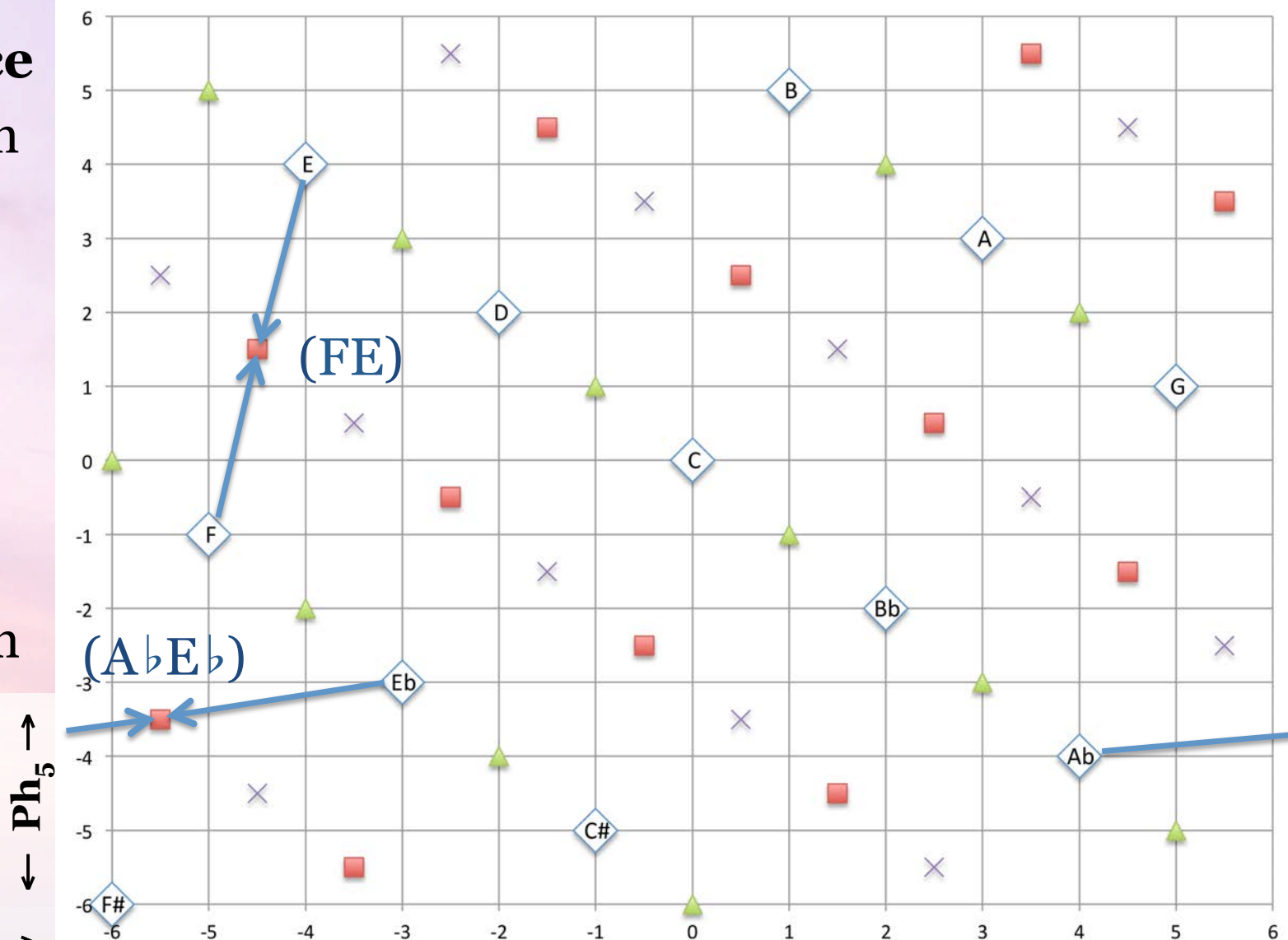
← Ph₁ →



Feldman, *Palais de Mari*

Ph_{1,5}-space

Dyads from the upper and lower voices of m. 1—
ic1 and ic5 occupy the same
positions in
the space.

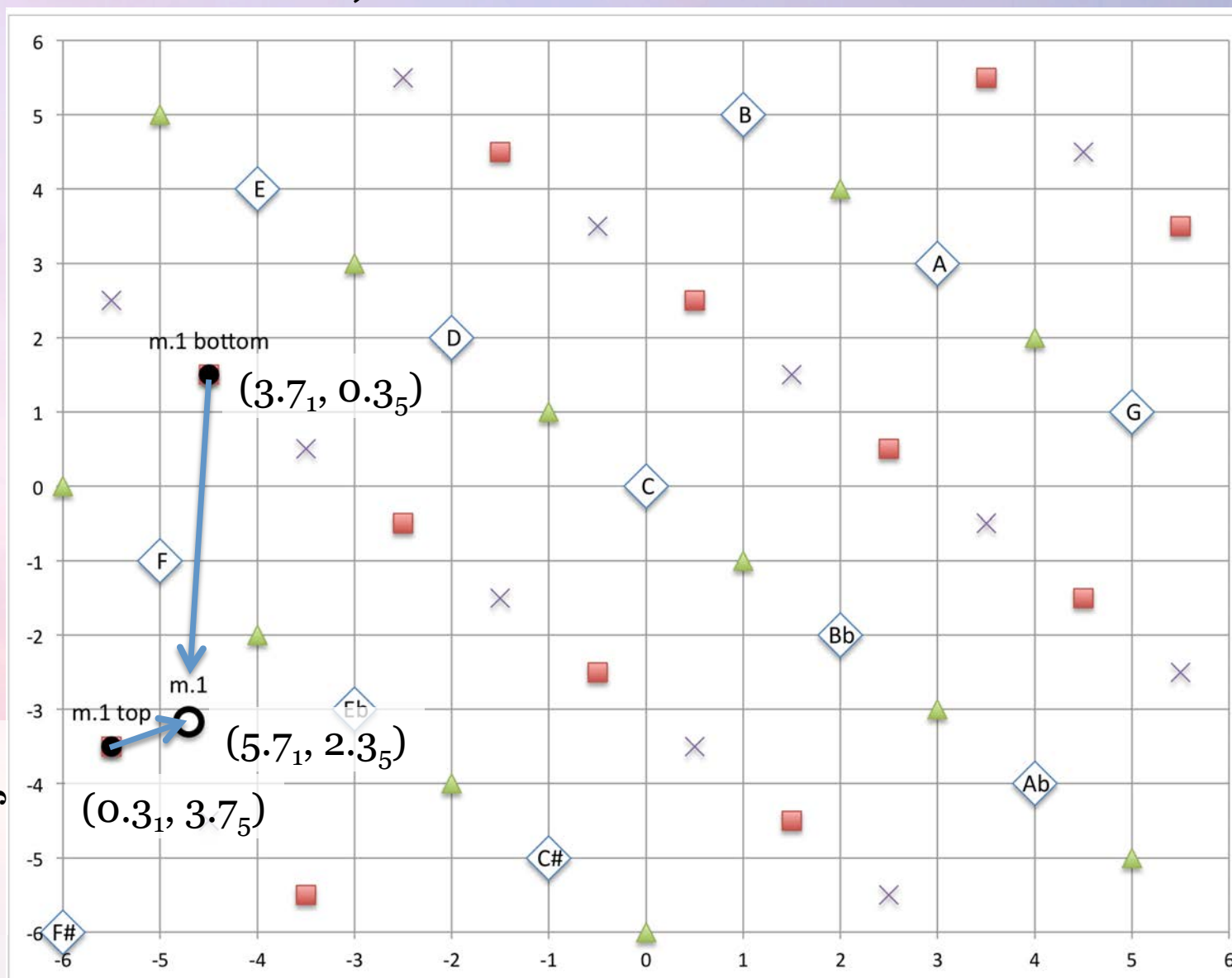


Feldman, *Palais de Mari*

Ph_{1,5}-space

The upper and lower voice dyads are **close** in Ph₁ and **distant** in Ph₅.


N.B.: They are also imbalanced (bottom: high f_1 ; top: high f_5).



Feldman, *Palais de Mari*

One of Feldman's basic harmonic techniques is
Transpositional Combination

m. 20:



(B \flat F)

T_2



(Hanninen set *C*)

m. 41:



(CD)

T_1

$ic5*ic2$



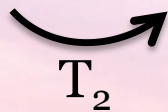
(Hanninen set *E/d*)

Feldman, *Palais de Mari*

Convolution Theorem: Transpositional combination (with doublings retained) is the same as **multiplying DFT magnitudes** and adding the phases.



(B \flat F)



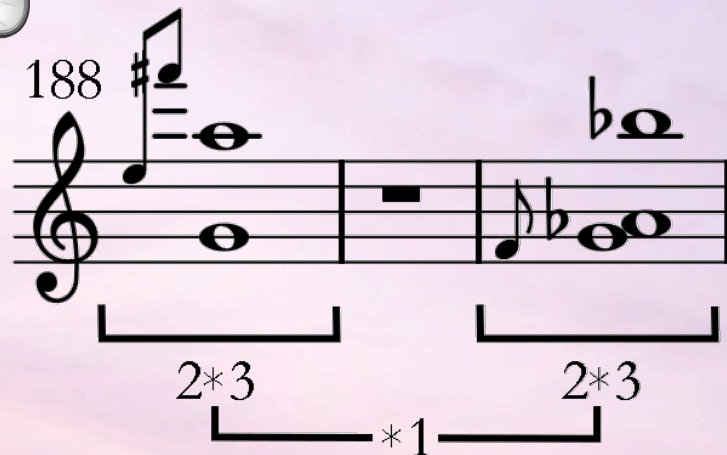
= (t7) \times (o2)

$$\begin{array}{r} (t7) \\ \times (o2) \\ \hline = (t079) \end{array}$$

$$\begin{array}{l} \langle\langle (0.27, 4.5), (3, 3), (2, 7.5), (1, 6), (3.73, 10.5), (0, -) \rangle\rangle \\ \times \langle\langle (3, 11), (1, 10), (0, -), (1, 2), (3, 1), (4, 0) \rangle\rangle \\ \hline = \langle\langle (0.8, 3.5), (3, 1), (0, -), (1, 8), (11.2, 11.5), (0, -) \rangle\rangle \end{array}$$

Feldman, *Palais de Mari*

Transposition of entire gestures by semitone
reinforces component 1 and cancels out component 5



$(0235) = 2*3$ is balanced
between f_1 and f_5 , but
multiplying by (01)
weakens f_5 in favor of f_1

$$\begin{aligned} & (46) \\ & \times (03) \\ & = (4679) \end{aligned}$$

$$\begin{aligned} & (4679) \\ & \times (01) \\ & = (4567^289) \end{aligned} \quad \begin{aligned} & \langle\langle (3, 7)_1, (3, 5)_5 \rangle\rangle \\ & \times \langle\langle (2, 10.5)_1, (2, 10.5)_5 \rangle\rangle \\ & \hline & = \langle\langle (6, 5.5)_1, (6, 3.5)_5 \rangle\rangle \\ & \langle\langle (6, 5.5)_1, (6, 3.5)_5 \rangle\rangle \\ & \times \langle\langle (3.73, 11.5)_1, (0.27, 9.5)_5 \rangle\rangle \\ & \hline & = \langle\langle (22.4, 5)_1, (1.6, 8)_5 \rangle\rangle \end{aligned}$$

Feldman, *Palais de Mari*

This idea is repeated frequently throughout the last part of the piece.



(027) (025)

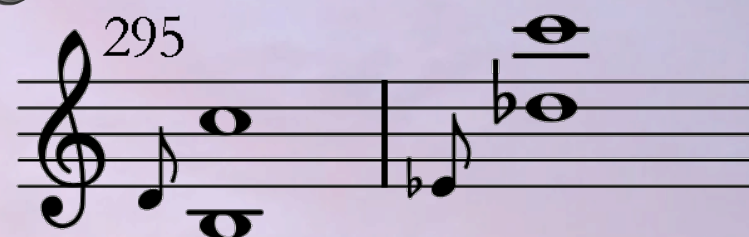


2 * 3

RH

LH

This variant of the idea makes it evident that *both* are products of (013) with ic5 or ic1:



(013)

(013)



2 * 1

RH

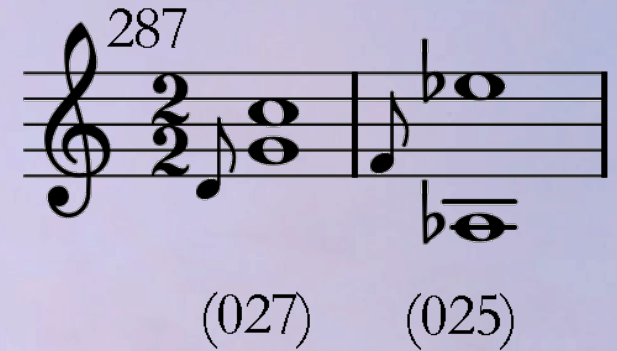
LH

(Hanninen *G/a*)

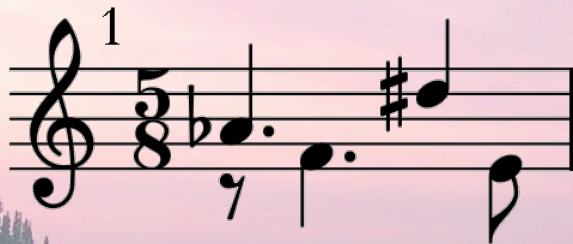
Feldman, *Palais de Mari*

Hanninen:

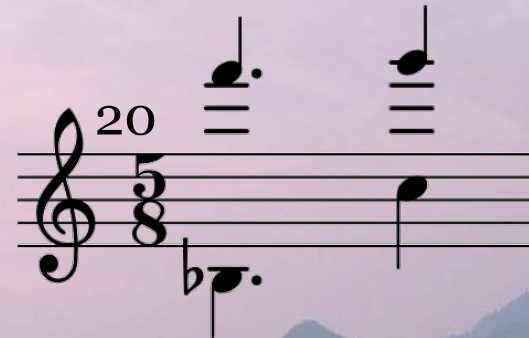
“The arrival of G/a 287–88 . . . is the keystone in a remarkable confluence of events. First, it defines the center of subset G/a , and also of set G . Second, it forms a bridge to set A , recalling and rearranging intervals and key pcs of set A . Third, it forms a second, and stronger, bridge to set C .”



Gesture from set A:



Gesture from set C:



Ph_{1,5}-space

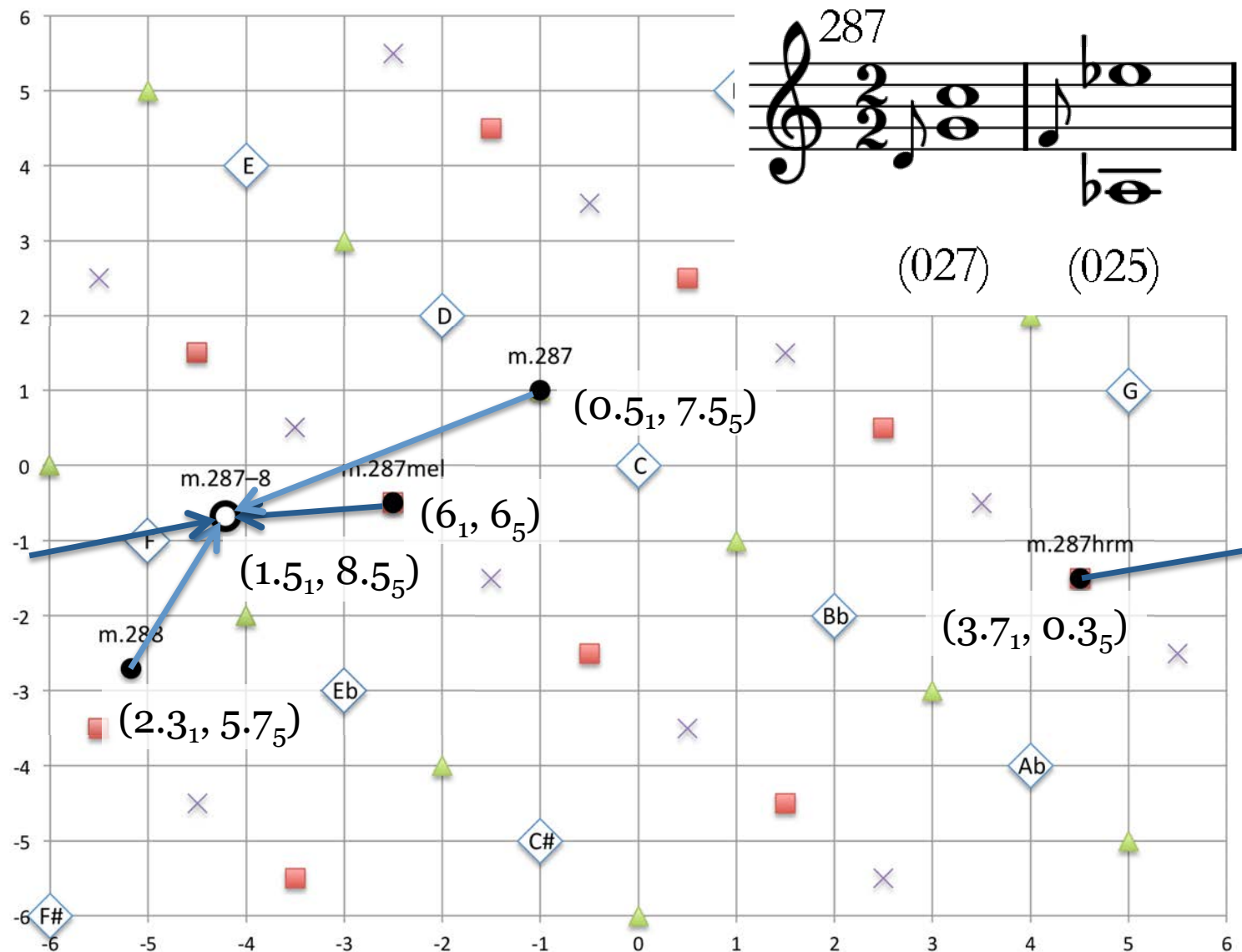
The (0235) and G–A_b subsets are spread out in Ph₁ and close in Ph₅

The individual chords are more spread out in Ph₅, meaning they have stronger f_5

← Ph₅ →

← Ph₁ →

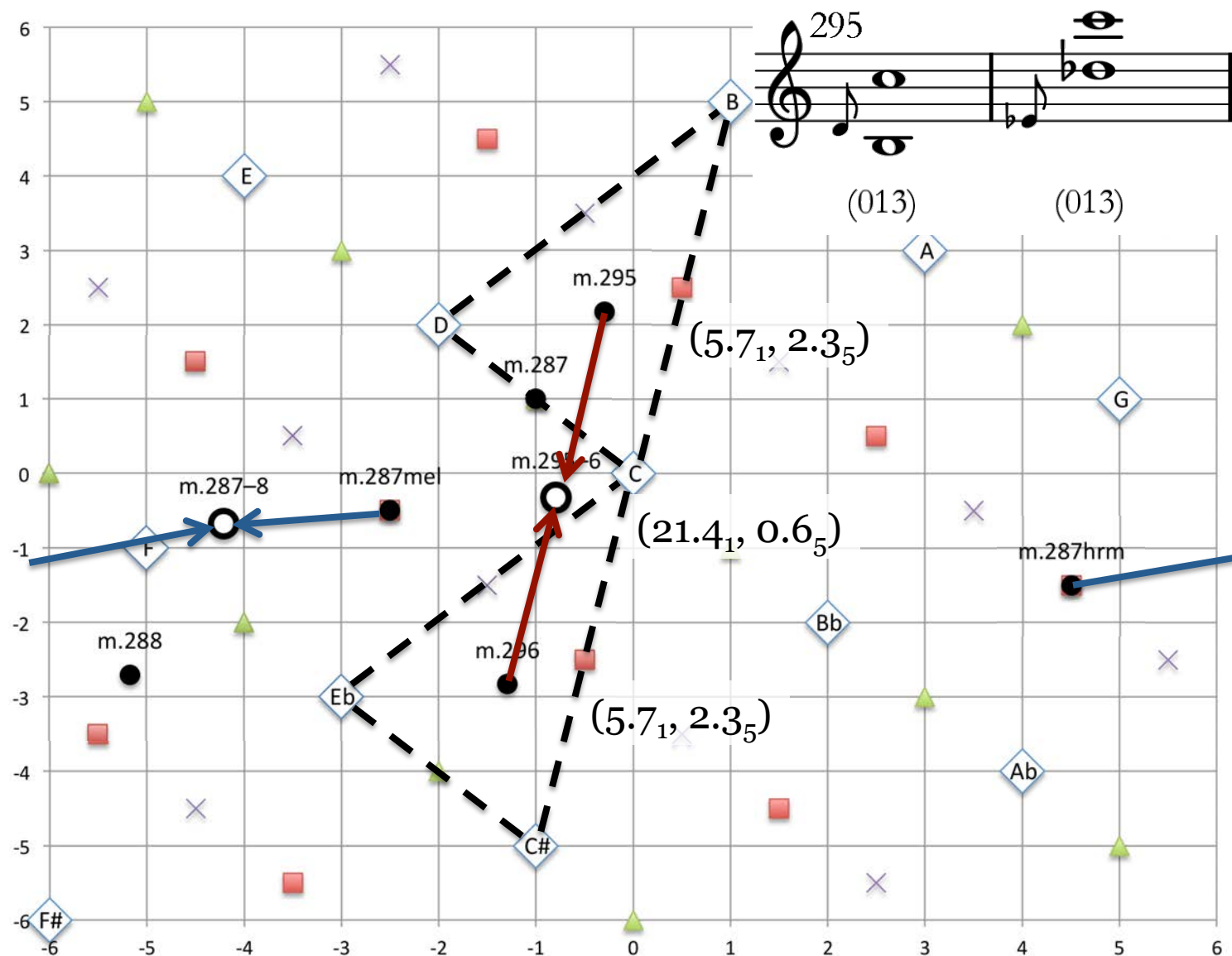
Feldman, *Palais de Mari*



Feldman, *Palais de Mari*

Ph_{1,5}-space

The (013)s of the variant gesture are spread out in Ph₅ and close in Ph₁, making the sum strongly chromatic.



Feldman, *Palais de Mari*

One important gesture reveals how Feldman “cripples” symmetries by asymmetrically dividing a symmetric entity



(0126)

(01237)

(F#GG#D)

+ (ABbBCE)

= (F#GG#ABbBCDE)

Symmetric
source:



=



(024) × (036)

$\langle\langle (3.73, 5.5)_1, (0.27, 3.5)_5 \rangle\rangle$

$\langle\langle (5.73, 1.3)_1, (2.27, 2.8)_5 \rangle\rangle$

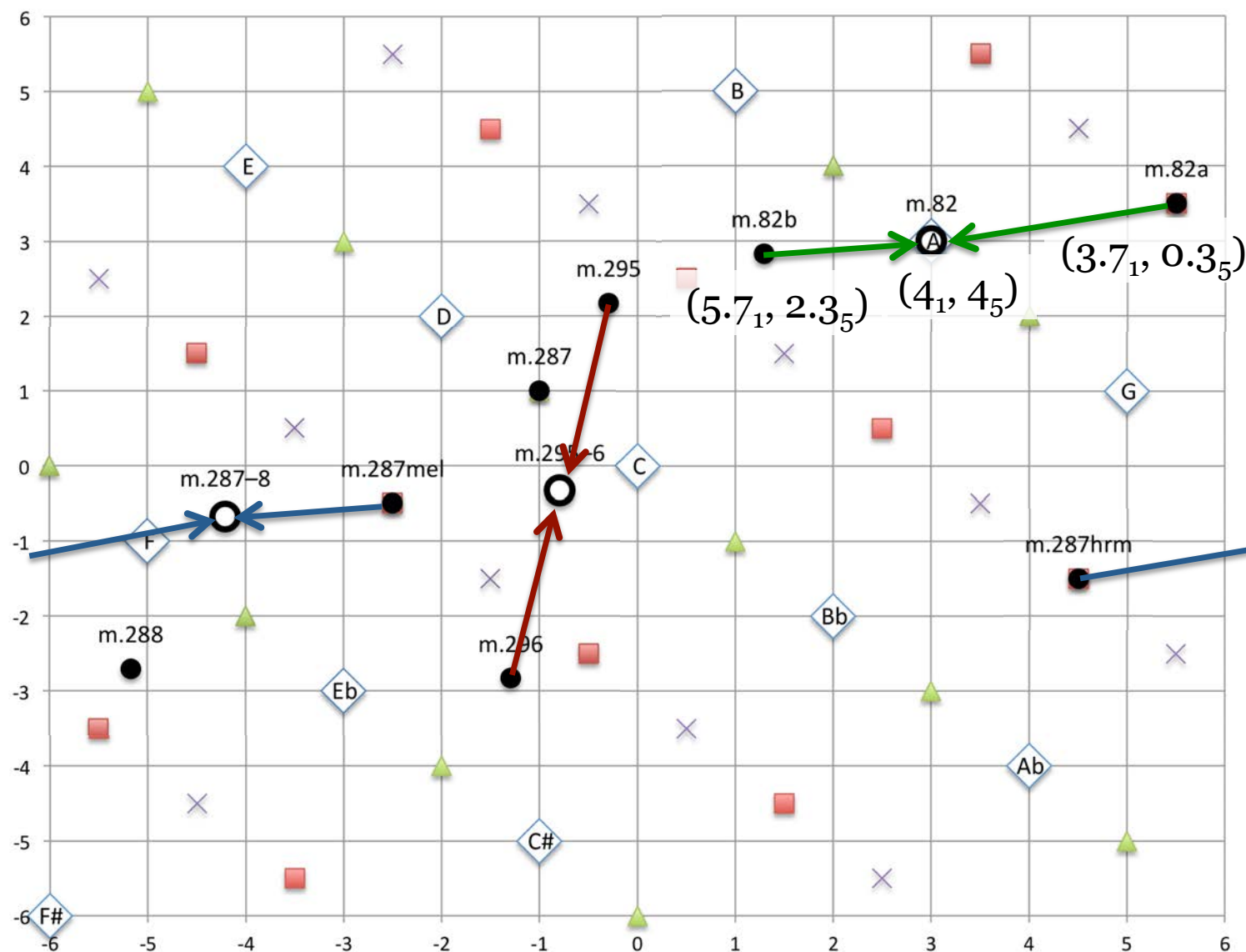
$\langle\langle (4, 3)_1, (4, 3)_5 \rangle\rangle$

The symmetric source chord is balanced between f_1 and f_5 , but it is split into more heavily chromatic chords.

Feldman, *Palais de Mari*

Ph_{1,5}-space

The Ph₁ spread shows how more chromatic chords are extracted from a balanced parent chord.



Summary

- DFT is a **change of basis** applied to the domain of pc-distributions.
- Each DFT component measures a musically interpretable quality relating to a type of **periodicity**.
- **DFT magnitudes** can replace much of pcset-theory's use of interval content to relate harmonic entities.
- The **fifth Fourier component** measures **diatonicity**, and provides a more systematic approach to reconciling **subsets and supersets** with scale theory.
- The **fourth Fourier component** represents **octatonicity** and is used by composers like Debussy and Stravinsky to relate diatonically distant harmonies.
- Distances in **phase space** provide a common-tone-based measure of relatedness between collections of any size.

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