



Decontextualizing Contextual Inversion

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.



Outline



- Contextual inversion vs. $T_n I$, unordered vs. ordered collections
- Voice-leading-based definitions
- Common-tone based definitions: decomposing common-tones via DFT
- Compositional application of directed inversions (“Not Marble”)



Contextual inversion vs. $T_n I$



Definition: A contextual inversion is an operation mapping pcsets to their inversions that commutes with transpositions

Standard ($T_n I$) inversion:

$$\{C, E, G\} - T_e I \rightarrow \{B, G, E\}$$

$$\{D^b, F, A^b\} - T_e I \rightarrow \{B^b, G^b, E^b\}$$

Contextual inversion (Neo-Riemannian):

$$\{C, E, G\} - L \rightarrow \{B, G, E\}$$

$$\{D^b, F, A^b\} - L \rightarrow \{C, A^b, F\}$$

Contextual inversion on ordered sets

Examples:

$$\begin{aligned} \{C, E, G\} & \text{ -- RICH } \rightarrow \{E, G, B\} \text{ -- RICH } \rightarrow \{G, B, D\} \\ \{E, G, C\} & \text{ -- RICH } \rightarrow \{G, C, E_b\} \text{ -- RICH } \rightarrow \{C, E_b, A_b\} \end{aligned}$$

$$\begin{aligned} \{C, E, G\} & \text{ -- } I_{last}^{first} \rightarrow \{C, E_b, G\} \text{ -- } I_{last}^{first} \rightarrow \{C, E, G\} \\ \{E, G, C\} & \text{ -- } I_{last}^{first} \rightarrow \{E, A, C\} \text{ -- } I_{last}^{first} \rightarrow \{E, G, C\} \end{aligned}$$

Fiore & Noll 2017; Fiore, Noll, & Satyendra 2013a–b; Hall 2009; Hook & Douthett 2008

Contextual inversion on unordered sets

$$\{C, E, G\} - P \rightarrow \{C, E_b, G\}$$

$$\{C, E, G\} - L \rightarrow \{E, G, B\}$$

$$\{C, E, G\} - R \rightarrow \{A, C, E\}$$

$$\{C, D_b, G\} - P \rightarrow ?$$

$$\{C, D_b, G\} - L \rightarrow ?$$

$$\{C, D_b, G\} - R \rightarrow ?$$

Contextual inversion on unordered sets

Straus (2011):

$$\{C, E, G\} - P \rightarrow \{C, E\flat, G\}$$

$$\{C, E, G\} - L \rightarrow \{E, G, B\}$$

$$\{C, E, G\} - R \rightarrow \{A, C, E\}$$

$$\{C, D\flat, G\} - P \rightarrow \{C, F\#, G\}$$

$$\{C, D\flat, G\} - L \rightarrow \{D\flat, G, A\flat\}$$

$$\{C, D\flat, G\} - R \rightarrow \{G\flat, C, D\flat\}$$

... but:

$$PR^3 = e?, \quad PR^2 = e?$$

etc. ...

(See Fiore & Noll 2017)



Voice-leading-based definitions



- Minimum voice-leading
 - Choice of metric (L_1 , L_2 , L_∞)
 - Uniqueness: often not well-defined, hard to predict
- Sum class (balanced voice leading)
 - Well-defined for all pentachords and heptachords, but not trichords, tetrachords, hexachords
 - Not necessarily correlated with minimum voice leading



Another proposal

From Dmitri Tymoczko, circa 17th of June, 2019

A set of inversions defined as follows has a general interpretation relating to the **homology of set-class space**:

I_A : Inversion preserving the smallest interval of the chord

I_B : Inversion exchanging the smallest interval with the one directly above in scalar order (preserving the sum of these)

I_C : Inversion exchanging the smallest interval with the one two positions above.

etc. . . .

Ex: I_A : $CD\flat FA\flat B\flat \rightarrow CD\flat E\flat FA\flat$

I_B : $CD\flat FA\flat B\flat \rightarrow CEFGA$

I_C : $CD\flat FA\flat B\flat \rightarrow CE\flat GA\flat B\flat$



Common-tone theorem



The convolution theorem (zeroeth case) gives:

$$\#CTs (\mathcal{A}-\mathcal{I}\mathcal{A}) = \frac{1}{12} \sum_{k=0}^{11} |f_k(\mathcal{A})|^2 \cos(\varphi_k(\mathcal{A}) - \varphi_k(\mathcal{I}\mathcal{A}))$$

where $|f_k(\mathcal{A})|$ is the size of the k th Fourier coefficient of \mathcal{A}
and $\varphi_k(\mathcal{A})$ is its phase

I.e., the number of common tones (intersection) can be decomposed
into contributions from 6 non-trivial DFT components ($f_1 - f_6$).



Fourier-phase-based definitions



Possibilities:

- Proximity in φ_1 or φ_5
- Proximity in a two-dimensional phase space (e.g. $\varphi_3 \times \varphi_5$)
- Weighted proximity in a two- or higher-dimensional phase space, e.g. $\min(|f_3| \cos \varphi_3 + |f_4| \cos \varphi_4 + |f_5| \cos \varphi_5)$
- Directed proximity in a two-dimensional phase space



Fourier-phase-based definitions



Minimum distance in a phase space gives inversions of order 2 (involutions).

However, this is not necessary:

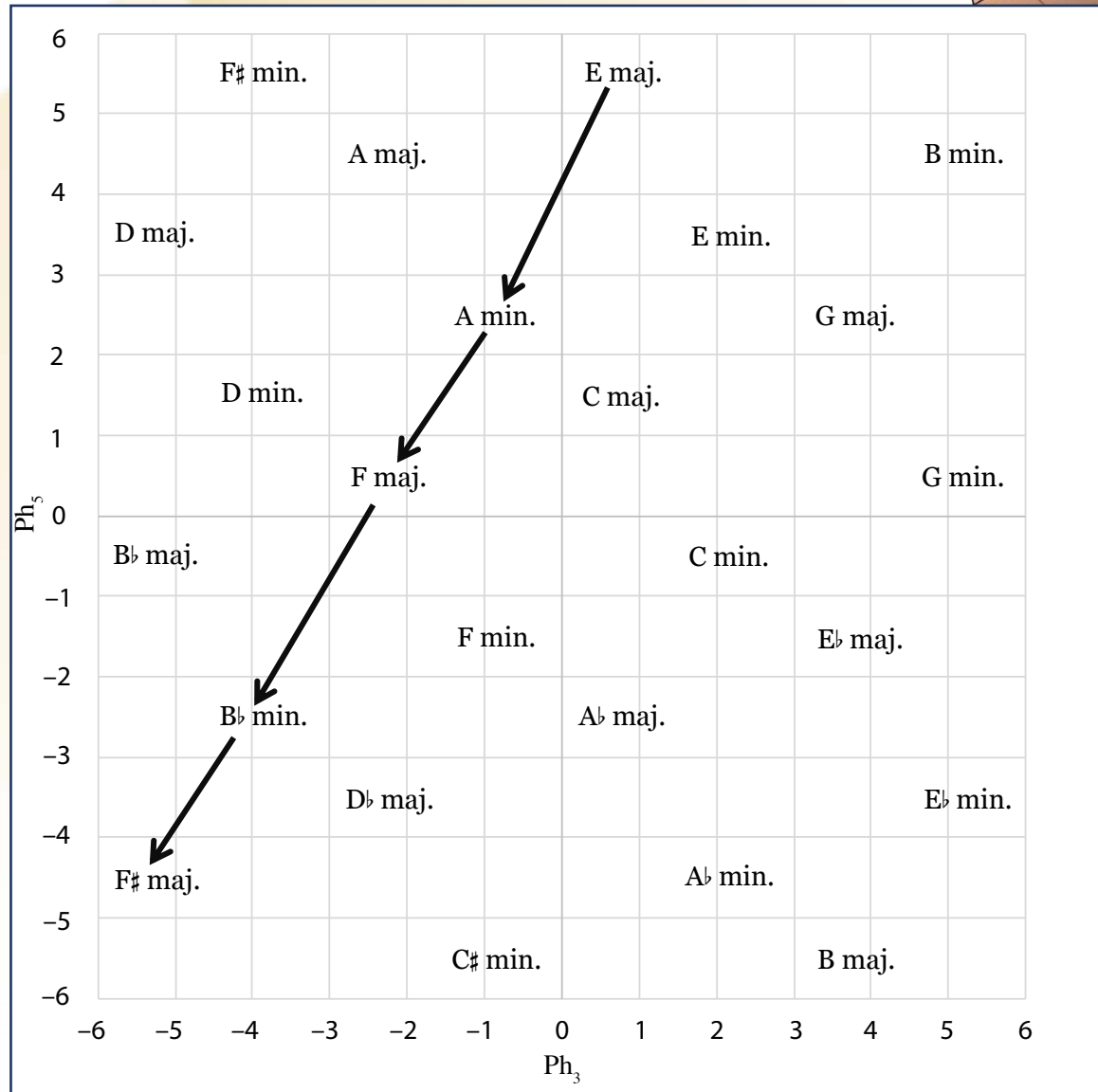
Directed inversions that compose to non-zero transpositions are also possible.

Directed inversions can be defined by proximity to an intervallic axis in phase space.

Directed Inversions

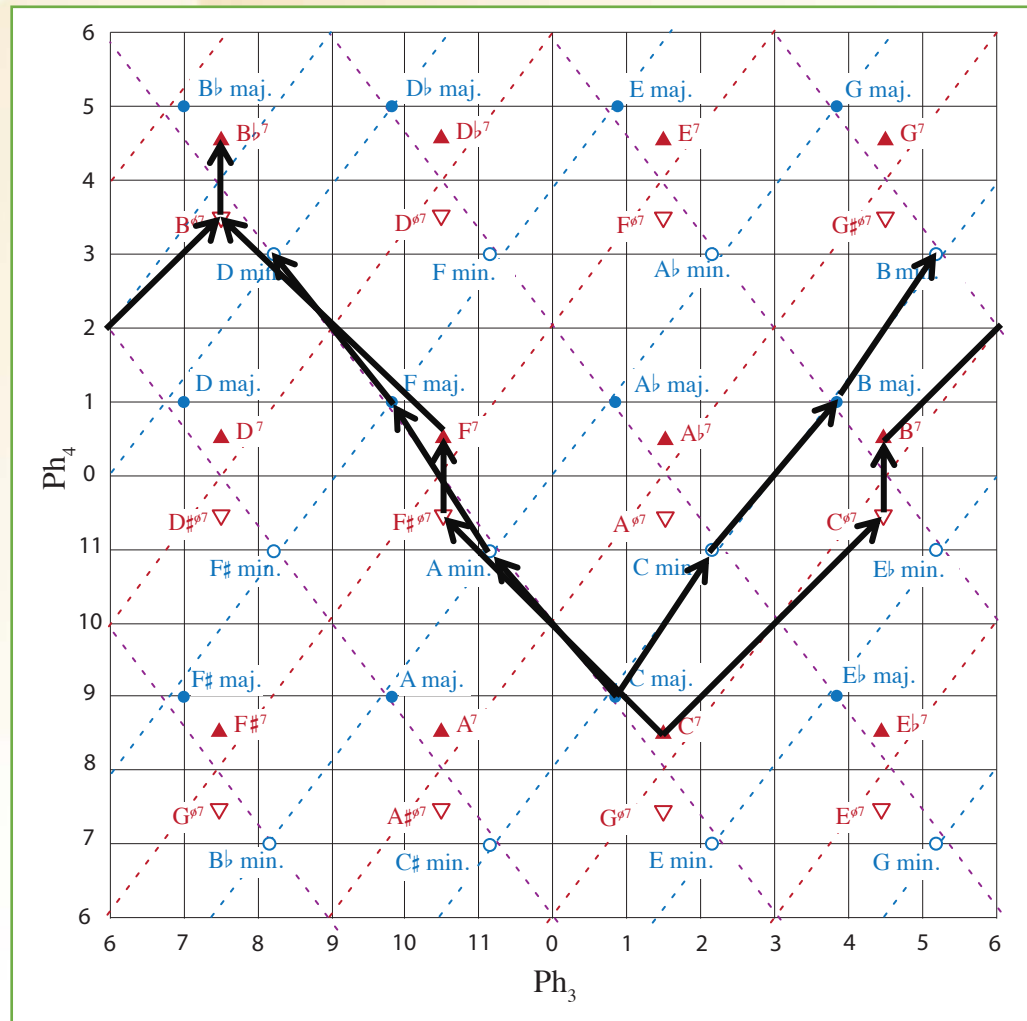
Motivating
example:

Schubert's favorite
sequence,
ic1 axis
in $Ph_{3,5}$ -space



Directed Inversions

Example:
ic1 and ic5 axes
in $Ph_{3,4}$ -space





Compositional Application: “Not Marble”



Shakespeare Sonnet no. 55

Not marble, nor the gilded monuments
Of princes, shall outlive this powerful rhyme;
But you shall shine more bright in these contents
Than unswept stone besmear'd with sluttish time.

When wasteful war shall statues overturn,
And broils root out the work of masonry,
Nor Mars his sword nor war's quick fire shall burn
The living record of your memory.

'Gainst death and all oblivious enmity
Shall you pace forth; your praise shall still find room
Even in the eyes of all posterity
That wear out this world to the ending doom.

So, till the judgment that yourself arise
You'll live in this, and dwell in lover's eyes.

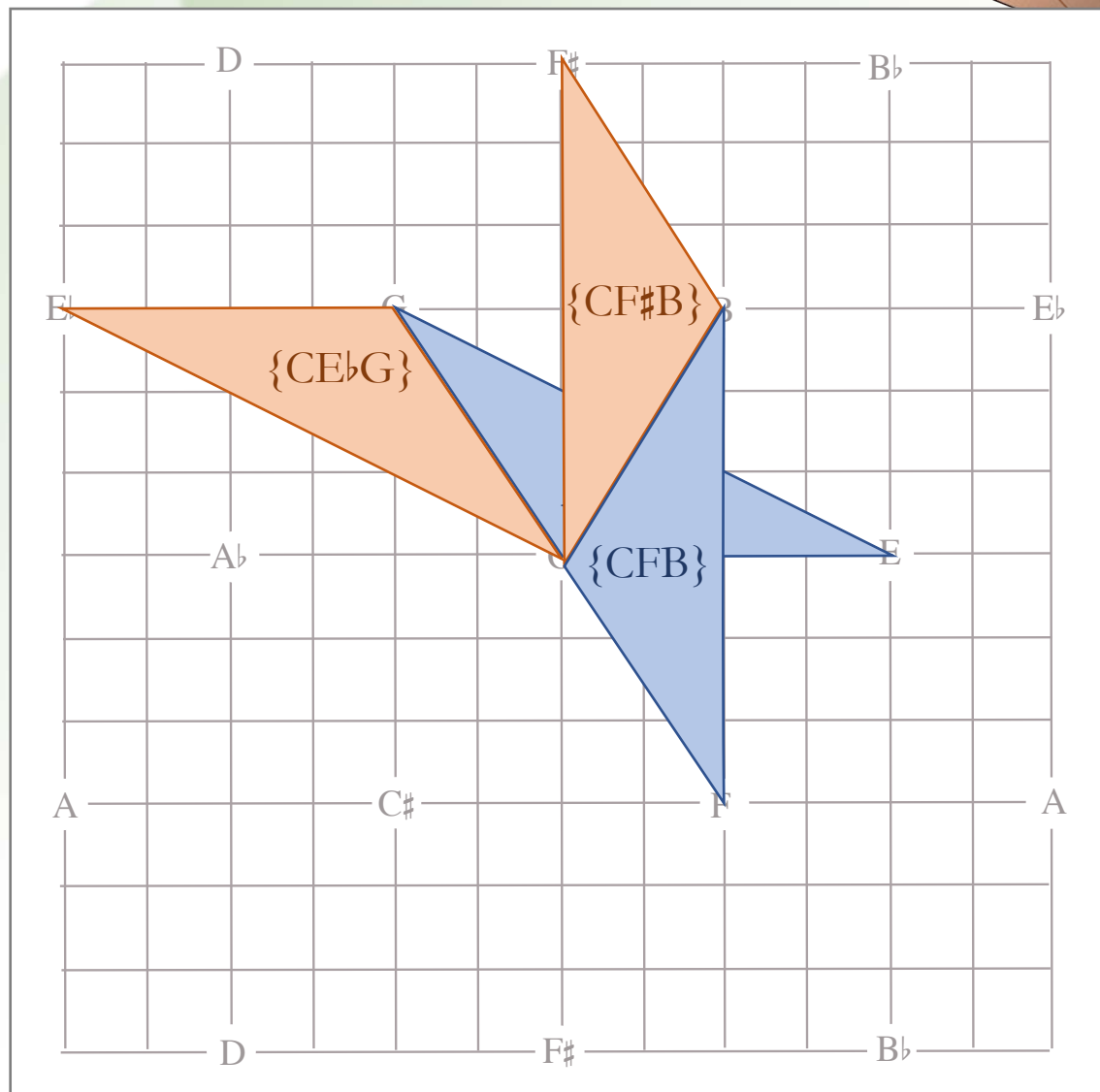
Compositional Application: “Not Marble”

Ph_{2,3}-space

Trichords:

(1) **037**

(2) **016**



Compositional Application: “Not Marble”

Ph_{2,3}-space

Trichords:

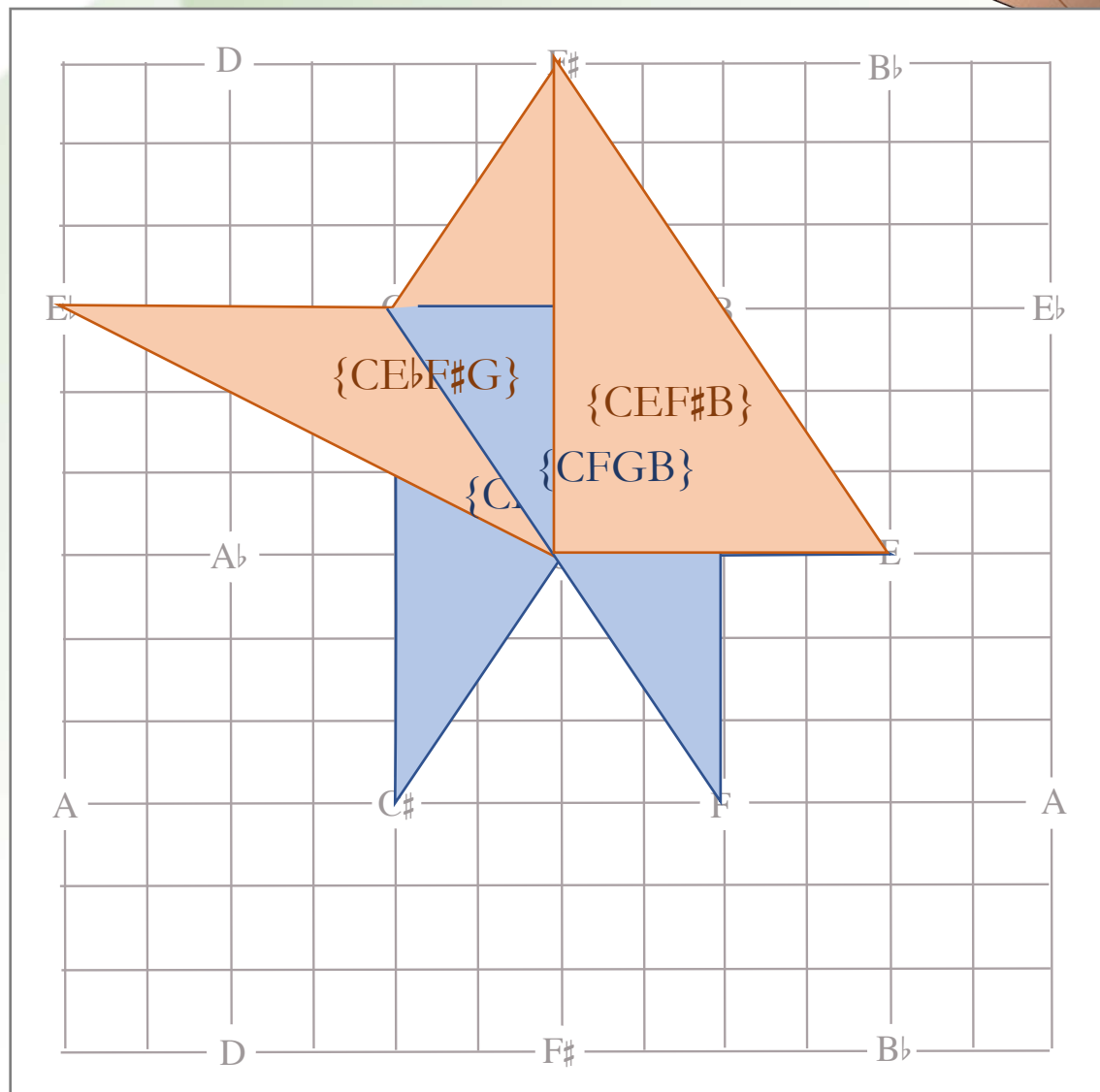
(1) **037**

(2) **016**

Tetrachords:

(3) **0147**

(4) **0157**



Compositional Application: “Not Marble”

Ph_{2,3}-space

Trichords:

(1) **037**

(2) **016**

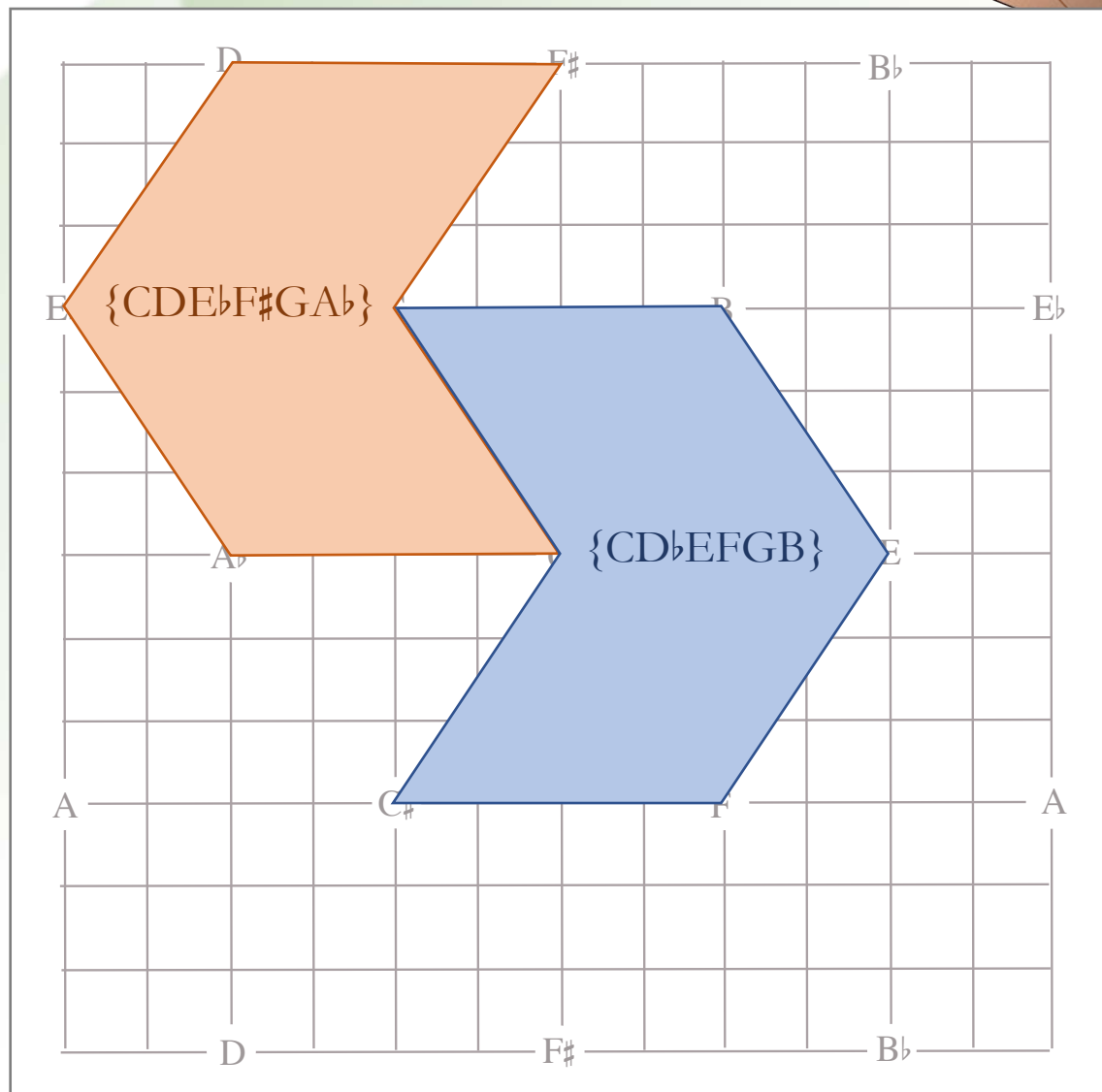
Tetrachords:

(3) **0147**

(4) **0157**

Hexachords:

(5) **012568**



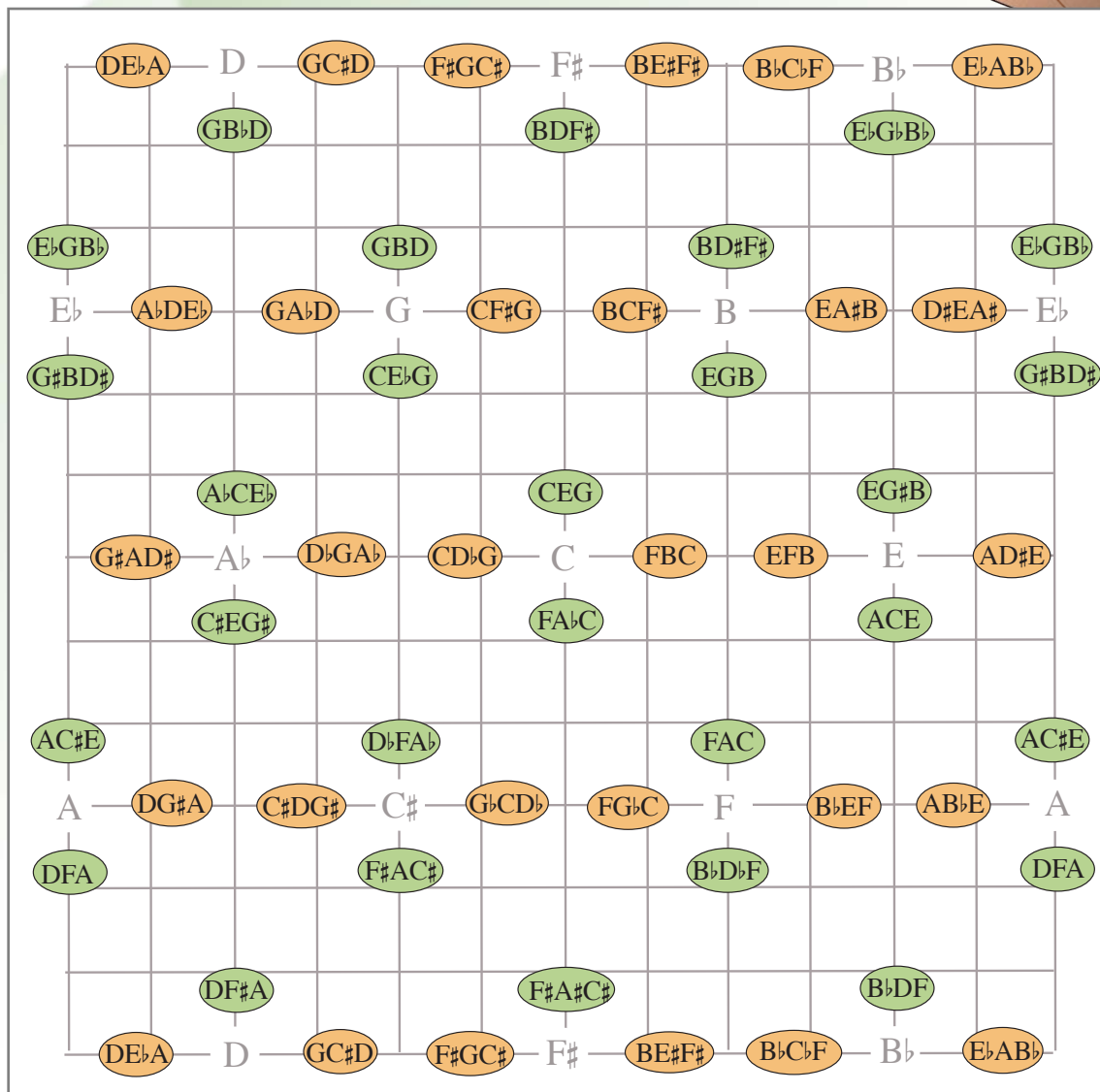
Compositional Application: "Not Marble"

Ph_{2,3}-space

All trichords:

037

016



Compositional Application: "Not Marble"

Ph_{2,3}-space

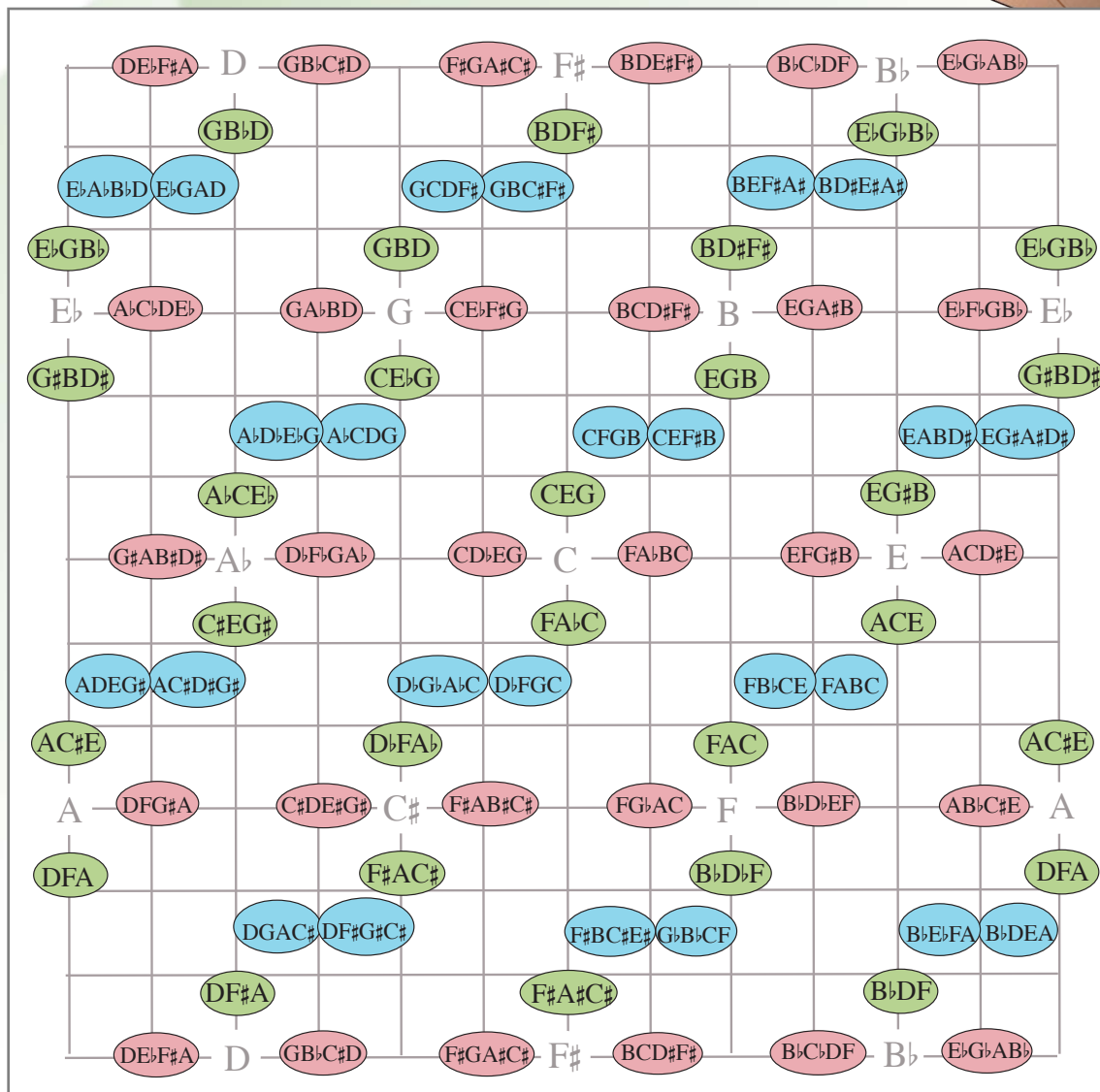
Trichords:

037

Tetrachords:

0147

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Compositional Application: "Not Marble"

Ph_{2,3}-space

Trichords:

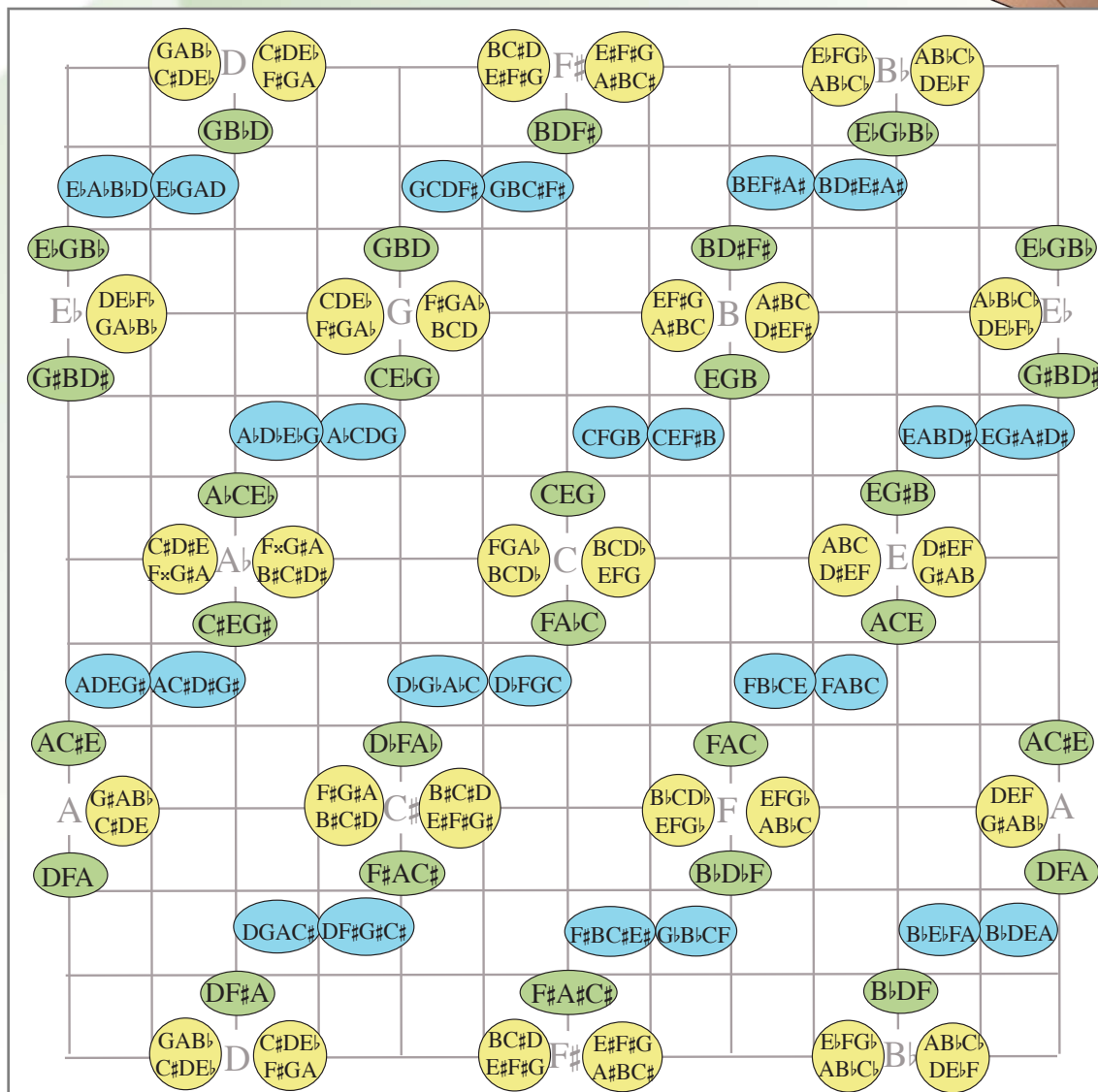
037

Tetrachords:

0157

Hexachords:

012568



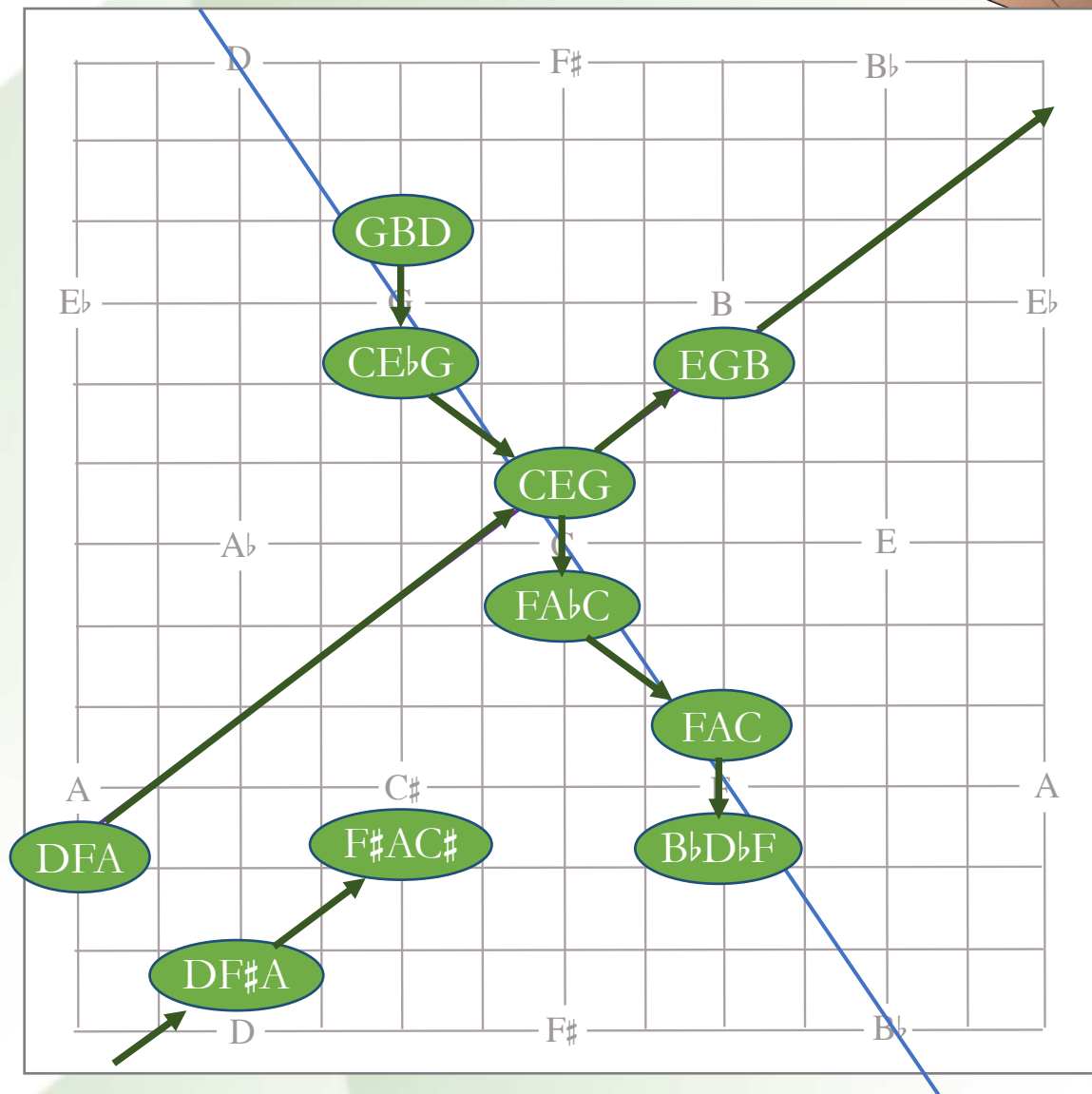
Compositional Application: "Not Marble"

Ph_{2,3}-space

(037) Trichords

Fifths axis

Whole-tone axis



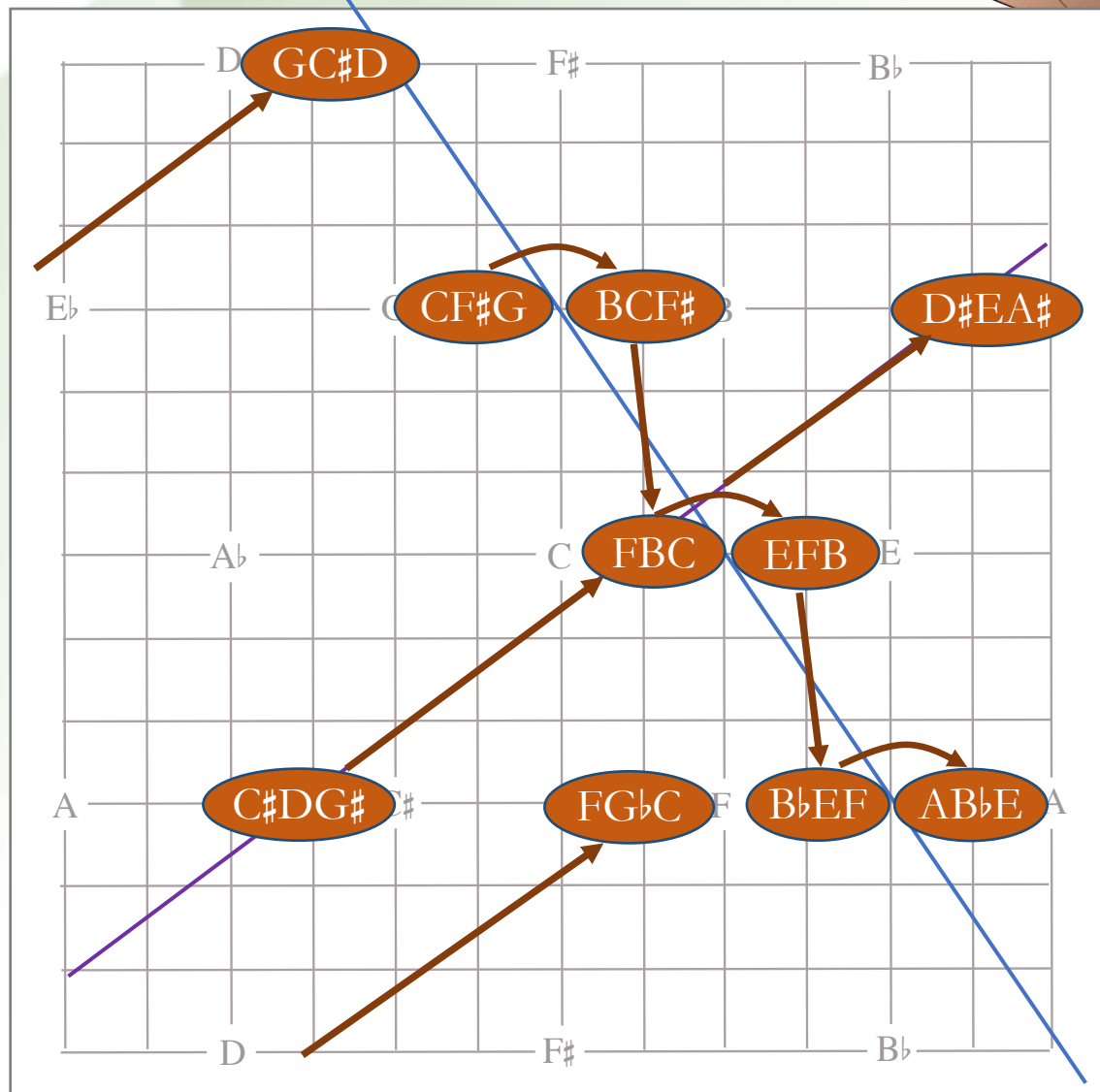
Compositional Application: “Not Marble”

Ph_{2,3}-space

(016) Trichords

Fifths axis

Whole-tone axis



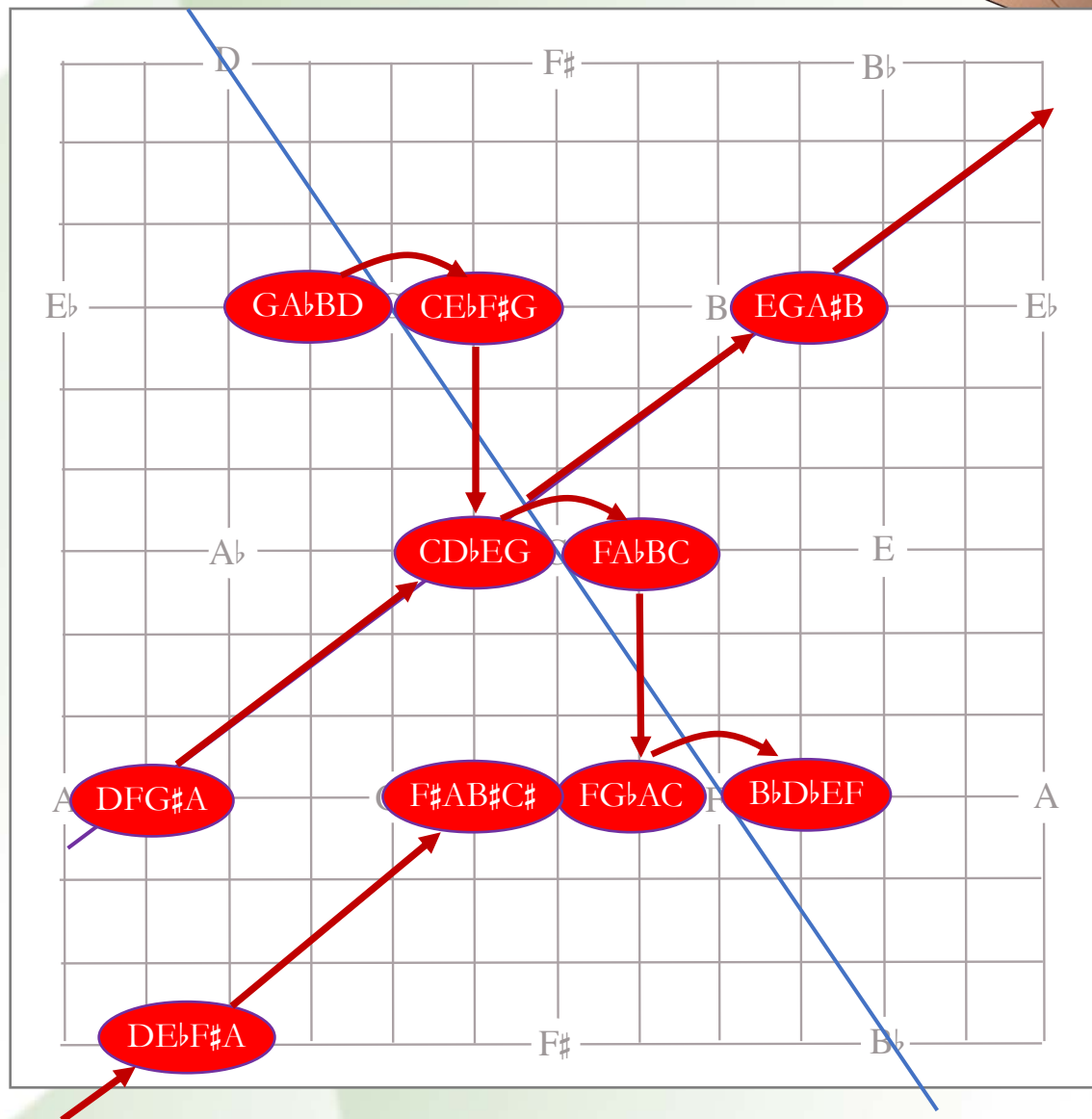
Compositional Application: “Not Marble”

Ph_{2,3}-space

(0147) Tetrachords

Fifths axis

Whole-tone axis



Compositional Application: "Not Marble"

Ph_{2,3}-space

(0157) Tetrachords

Fifths axis

Whole-tone axis

