Sailing Off the Edge of Tonality: Debussy and Scriabin's Harmonic Adventurism Explained with Harmonic Spectra and Tonal Spaces

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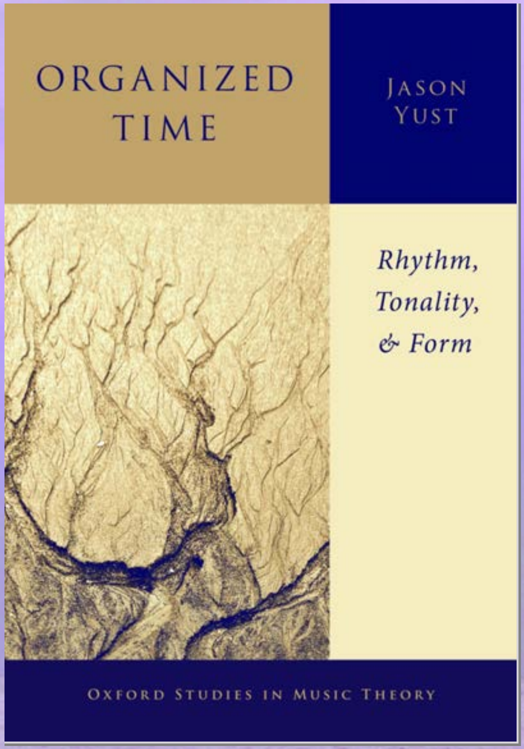
Music Forum, 2/6/2020



Law of Music Theory Concepts*

Importance : Ambiguity = k

See: Phrase

Sonata form

Meter

Key

Function

Tonality

*This is not entirely serious

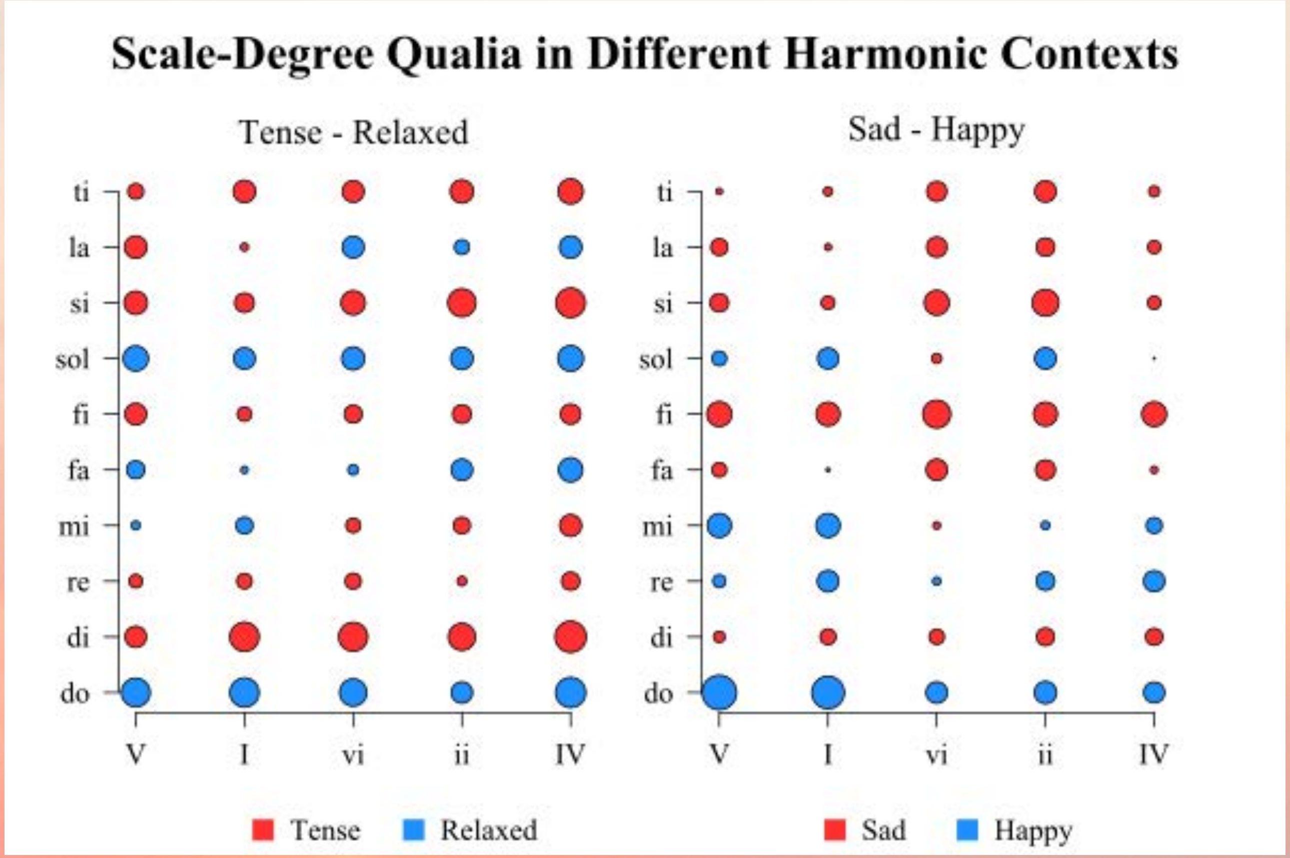
Tonality: What is it?

Some ways that music theorists explain tonality:

- Referential pitch (tonic)
- Harmonic Function / Syntax
- Implication
- Diatonic Scales
- Triads

Tonality: What is it?

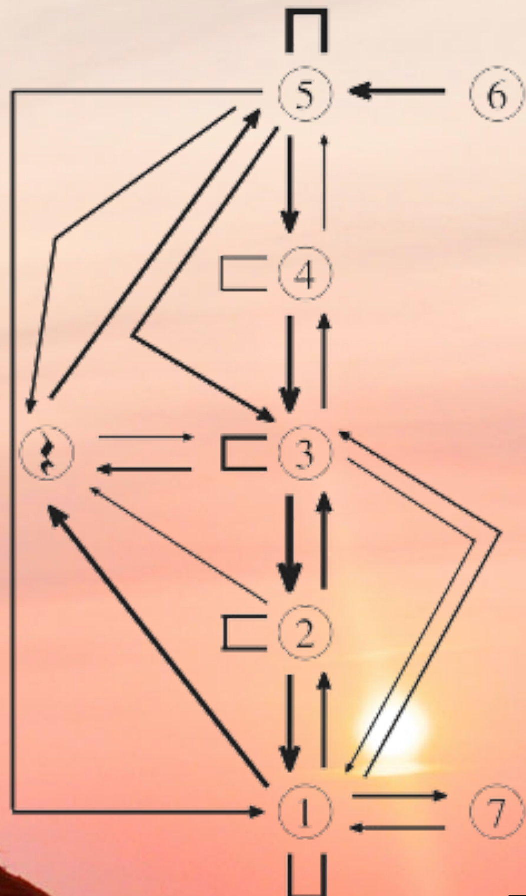
Referential pitch (tonic) → Scale Degree Qualia



—Claire Arthur, 2016. "When the Leading Tone Doesn't Lead: Scale Degree Qualia in Context," PhD Diss., Ohio St. Univ.

Tonality: What is it?

Melodic Syntax → Implication → Scale Degree Qualia



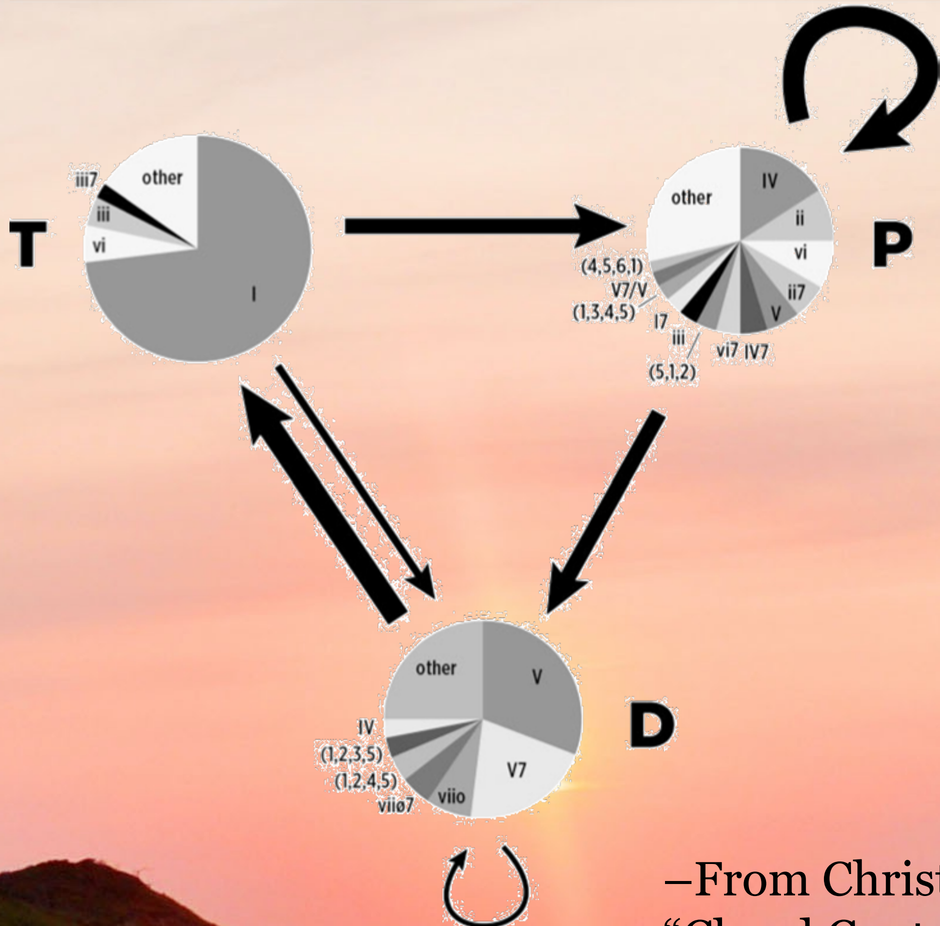
A melodic syntax (represented by **transition probabilities**) creates expectations that lead to qualia.

E.g.: Relaxed tonic, tense leading tone

–From David Huron 2008. *Sweet Anticipation: Music and the Psychology of Expectation* (Bradford Books)

Tonality: What is it?

Harmonic Function / Syntax → Implication



Harmonic function can be understood similarly as a syntax.

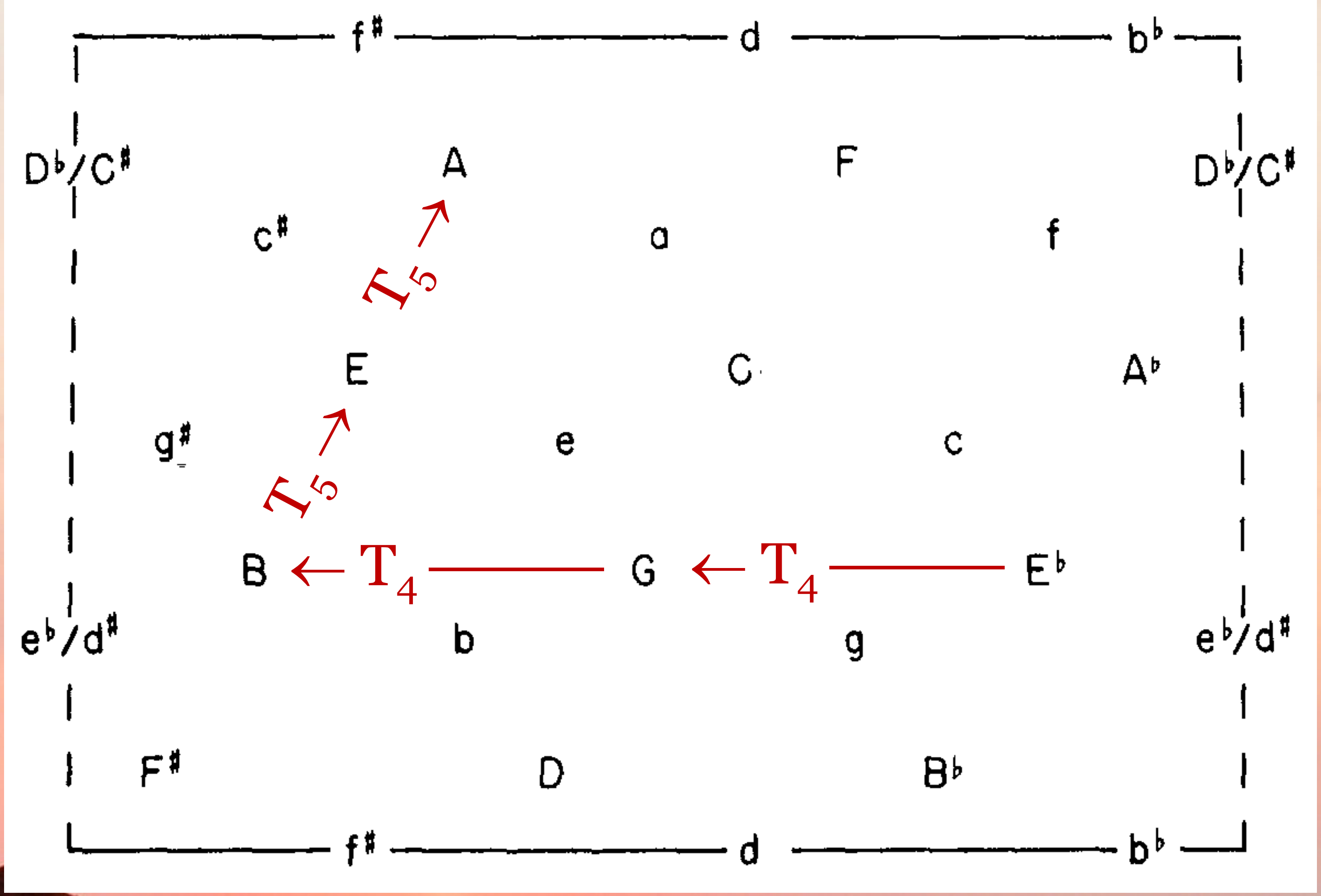
Triads and **diatonic scales** are elements of this syntax.

–From Christopher White and Ian Quinn, 2018. “Chord Context and Harmonic Function in Tonal Music.” *Music Theory Spectrum* 40/2.

Tonal Space

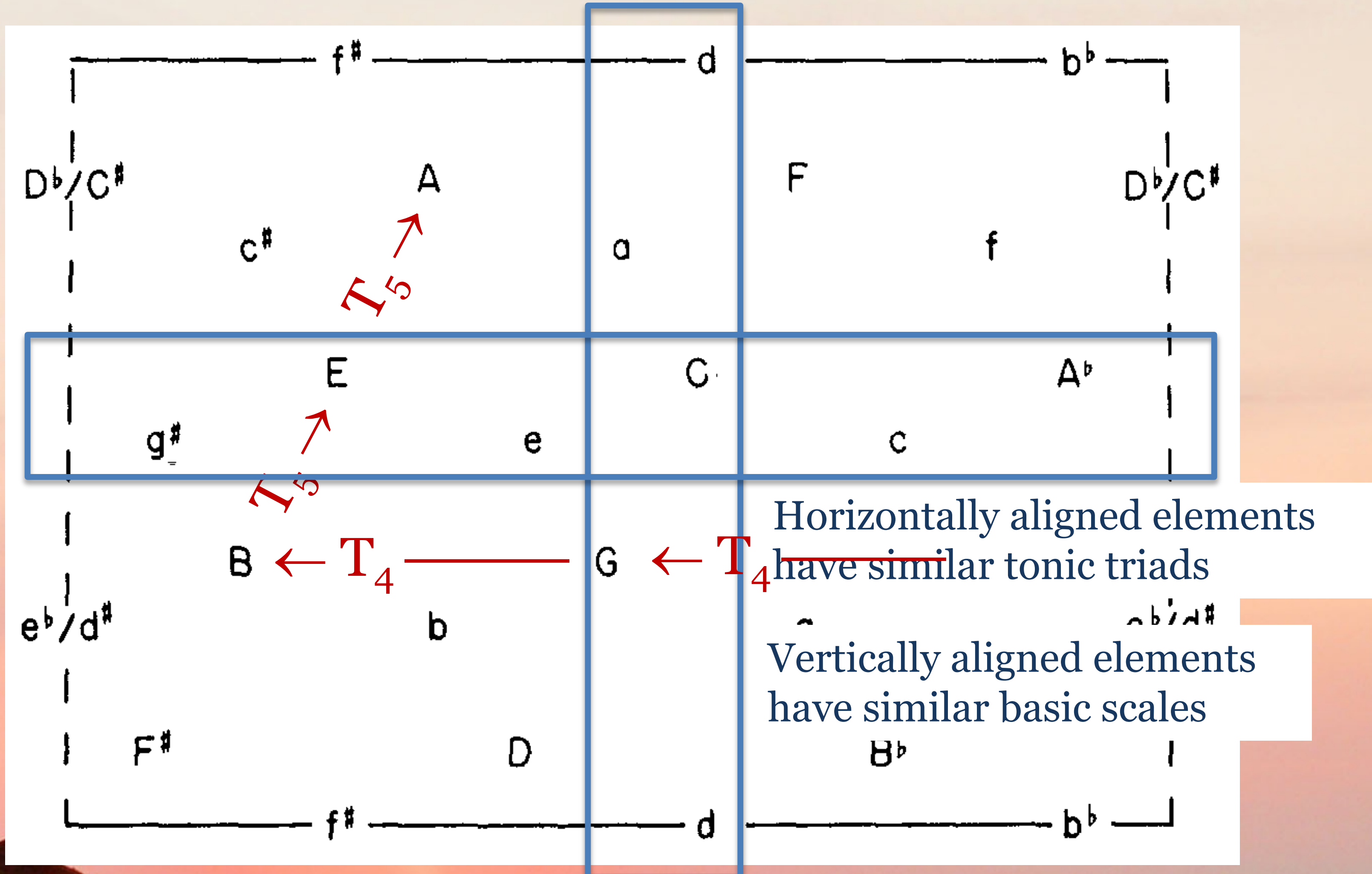
Tonality as a triadic/diatonic syntax
can be represented by a tonal space.

Krumhansl's Tonal Space



Toroidal map of key profiles from Krumhansl and Kessler 1982

Krumhansl's Tonal Space



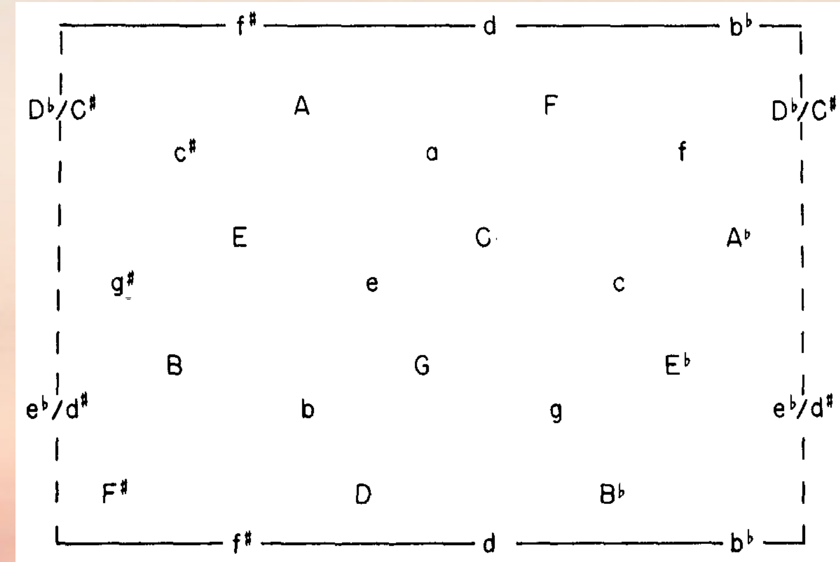
Toroidal map of key profiles from Krumhansl and Kessler 1982

Krumhansl's Tonal Space

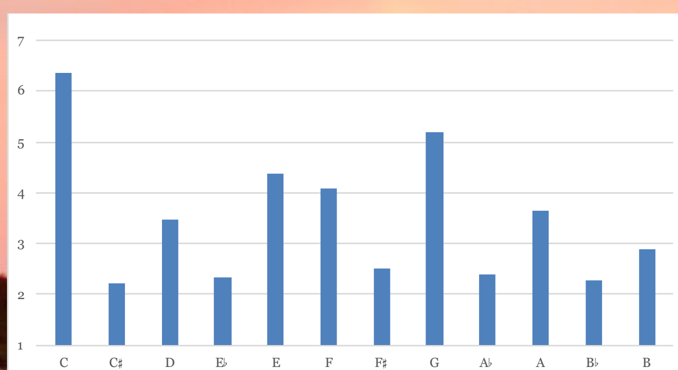
Krumhansl's space is based on **correlations** between ***key profiles.***

Key profiles may be derived from

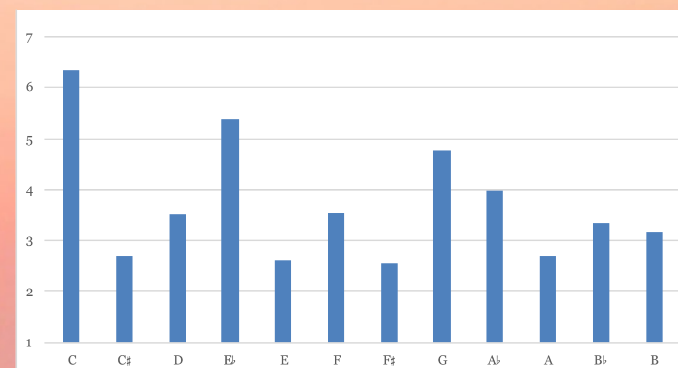
- Listener rankings of stability of tones in a given context,
- Frequency of occurrence of tones in the given key,
- etc.



Krumhansl and Kessler's key profiles based on listener ratings



C major



C minor

Krumhansl's Tonal Space

Considered mathematically . . .

Constraint: **Transposability** (Arrangement of keys invariant under transposition).

$\Rightarrow$

—The space is **toroidal** (circular in two dimensions)

—Each dimension represents an **interval cycle**

Possible interval cycles:

Interval class:	1	2	3	4	5	6
8^{ve} division:	1	6	4	3	5	2

Fourier Transform on Pitch-Class Sets and Distributions

Using discrete Fourier transform, we can

- Identify harmonic qualities, through **spectra**
- Relate harmonies of similar quality through **phase spaces**

Krumhansl’s tonal space is a phase space.

Fourier Transform on Pitch-Class Vectors: A brief history

Lewin, David (1959). "Re: Intervallic Relations between Two Collections of Notes," *JMT* 3/2.

——— (2001). "Special Cases of the Interval Function between Pitch Class Sets X and Y." *JMT* 45/1.

Quinn, Ian (2006–2007). "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1.

Callender, Cliff (2007). "Continuous Harmonic Spaces," *JMT* 51/2

Amiot, Emmanuel (2007). "David Lewin and Maximally Even Sets." *Journal of Mathematics and Music* 1/3.

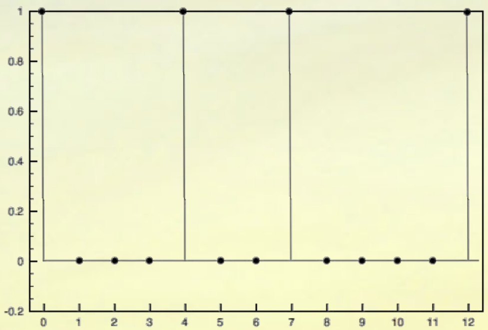
——— (2016). *Music Through Fourier Space: Discrete Fourier Transform in Music Theory*. (Springer)

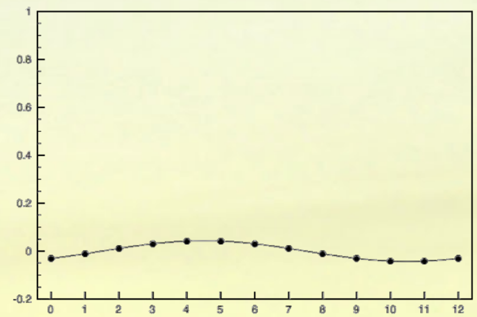
Yust, Jason (2015). "Schubert's Harmonic Language and Fourier Phase Spaces." *JMT* 59/1.

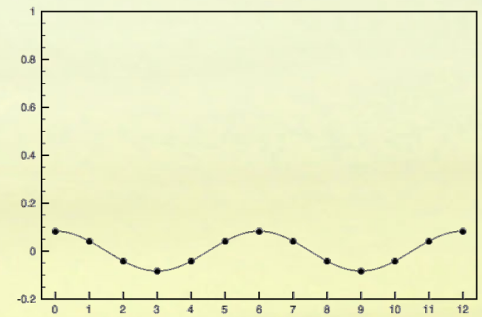
——— (2016). "Special Collections: Renewing Forte's Set Theory." *JMT* 60/2.

Fourier Transform of a Pitch-Class Vector

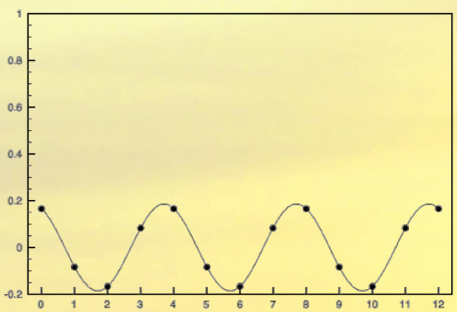
C major triad = f_0 + f_1 + f_2

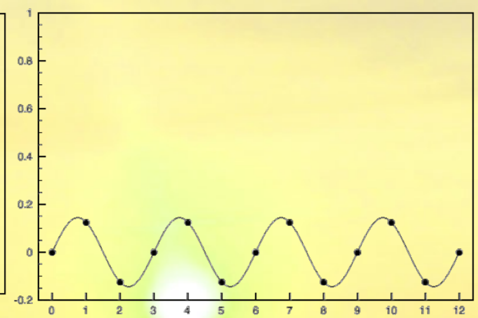


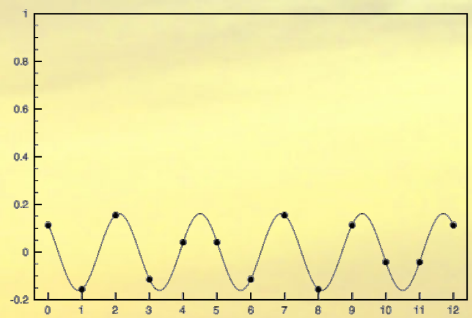


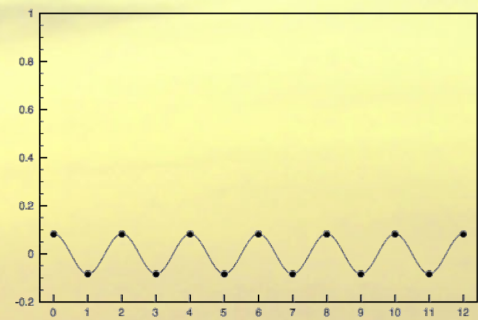


+ f_3 + f_4 + f_5 + f_6 + . . .



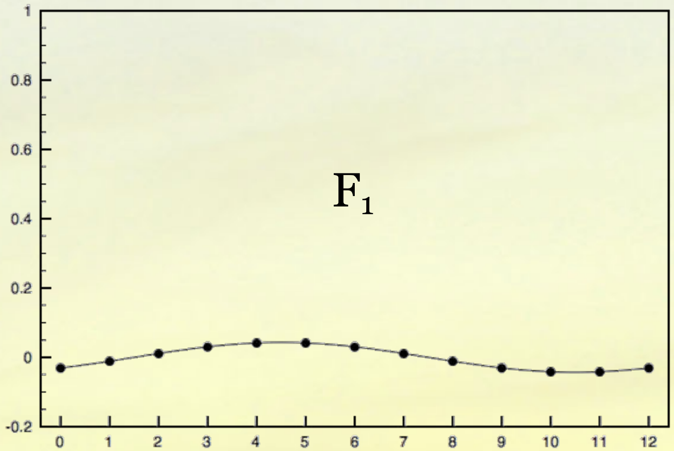


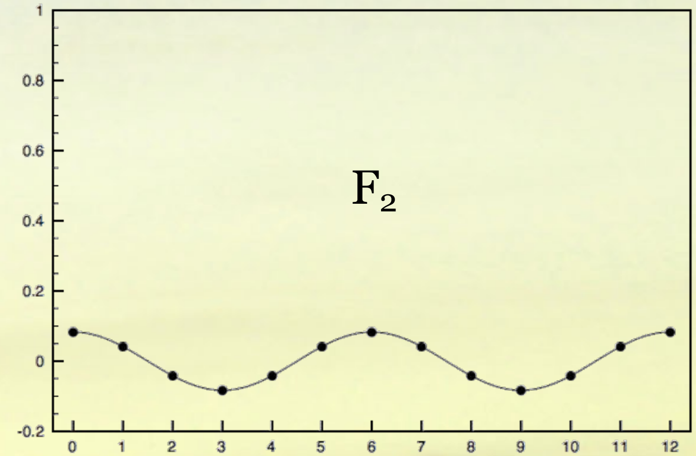




The DFT is a **lossless transformation** from a pitch-class vector to a sum of **periodic functions** dividing the octave into 1–6 parts.

Fourier Qualities

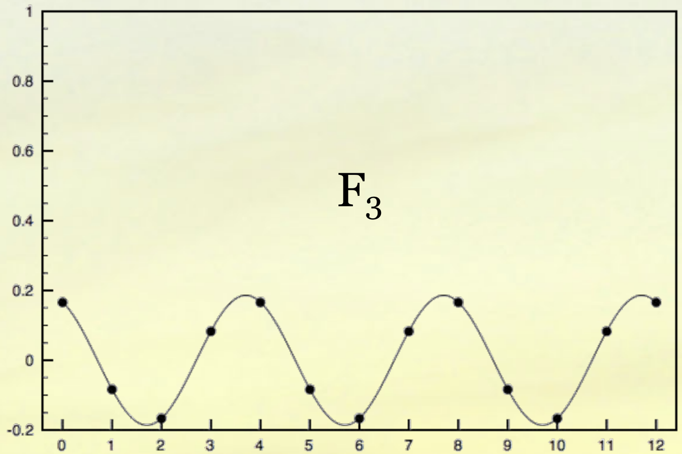




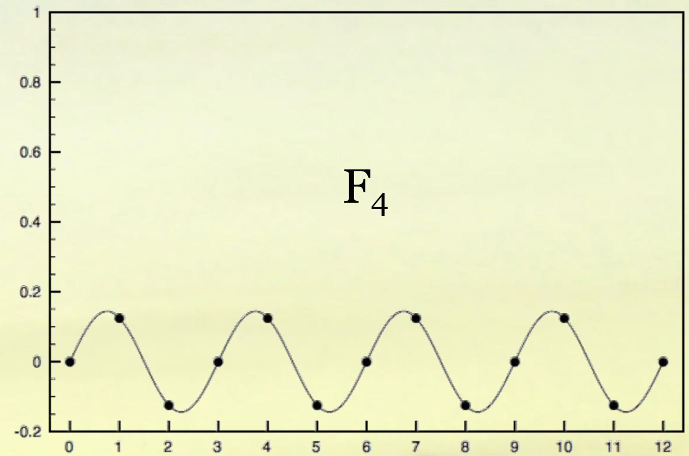
F_1 represents a concentration of pitch-class weight on the full pc circle.

F_2 represents a concentration of pitch-class weight on a half-octave (tritone) cycle.

Fourier Qualities

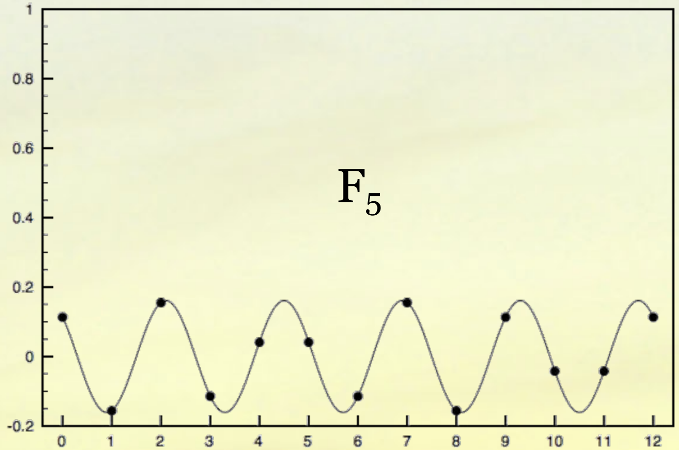


f_3 gives the weighting on the nearest *augmented triad* or ***hexatonic*** *scale*.

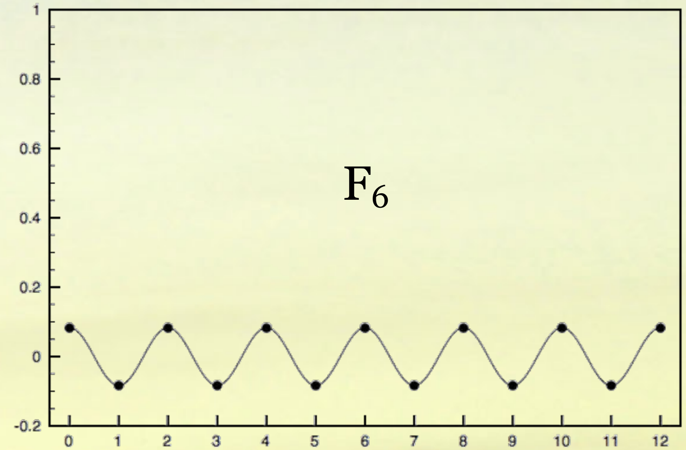


f_4 gives the weighting on the nearest *diminished seventh* or ***octatonic*** *scale*.

Fourier Qualities



f_5 give the balance on the ***circle of fifths***



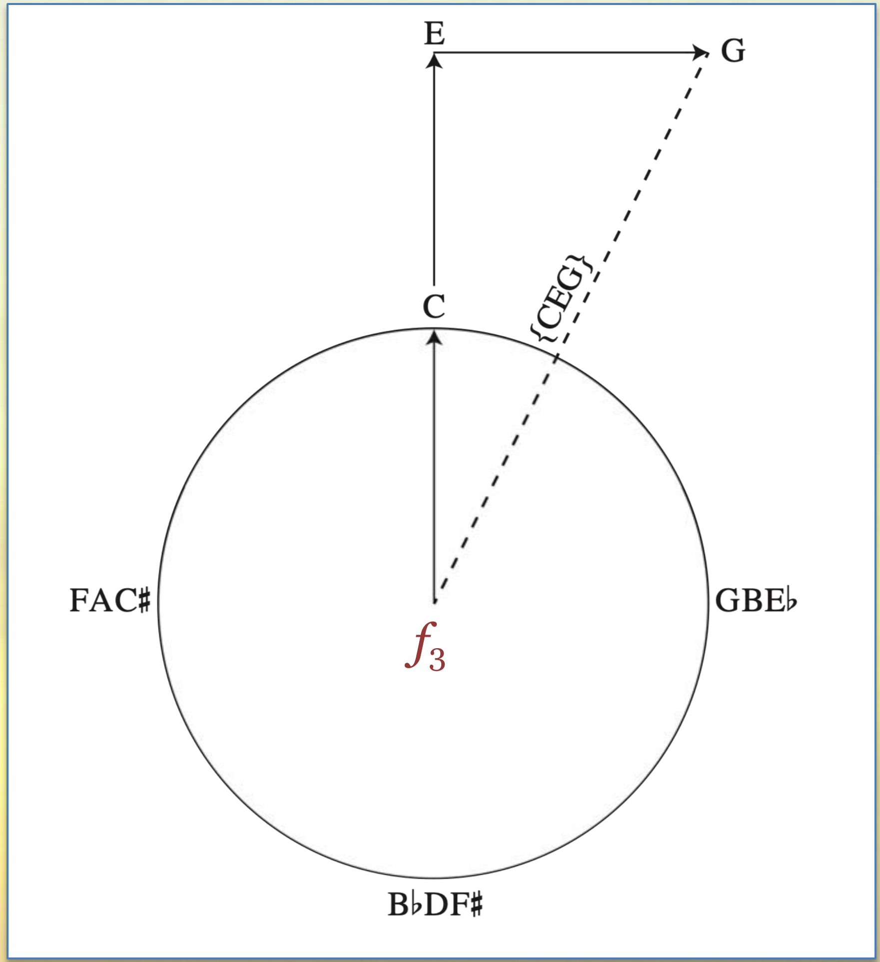
f_6 gives the weighting on one of the two ***whole-tone*** collections.

Fourier Transform as Vector Sums

Fourier component f_k can be derived as a vector sum with each pitch class as a unit vector, where the unit circle is the 8ve/k.

The length of the resulting vector is the **magnitude** of the component, and the angle is its **phase.**

Example: C maj. triad, k = 3

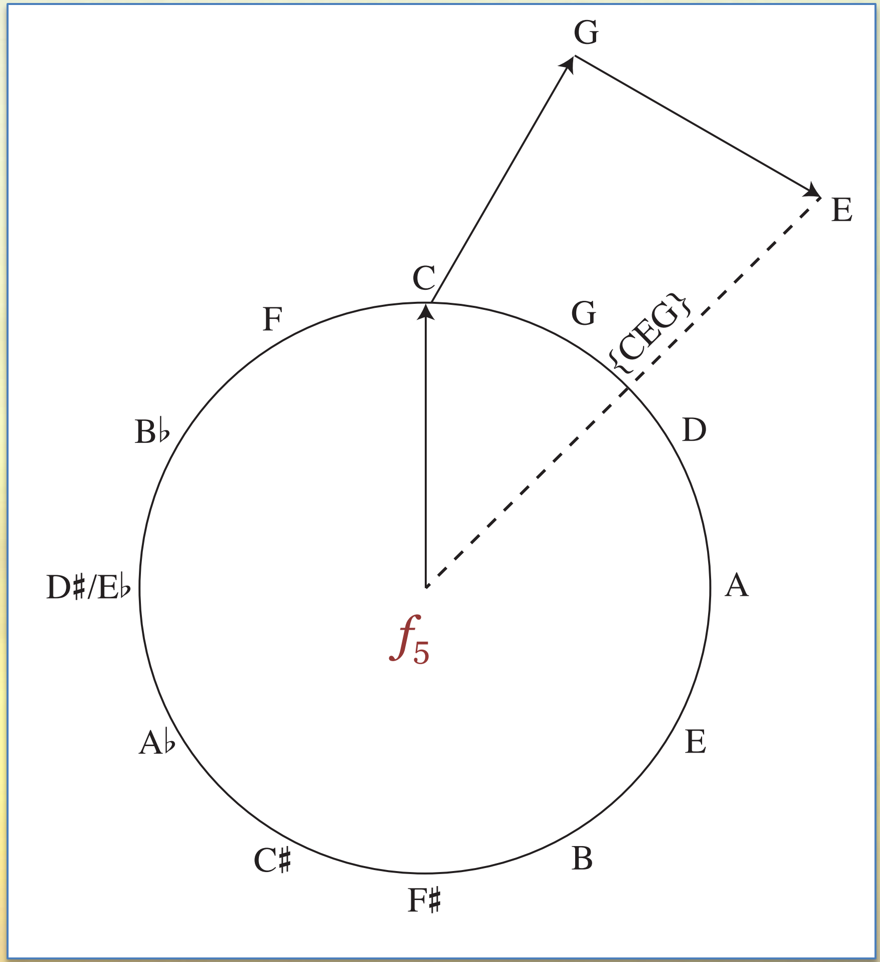


Fourier Transform as Vector Sums

Fourier component f_k can be derived as a vector sum with each pitch class as a unit vector, where the unit circle is the 8ve/k.

The length of the resulting vector is the **magnitude** of the component, and the angle is its **phase.**

Example: C maj. triad, k = 5

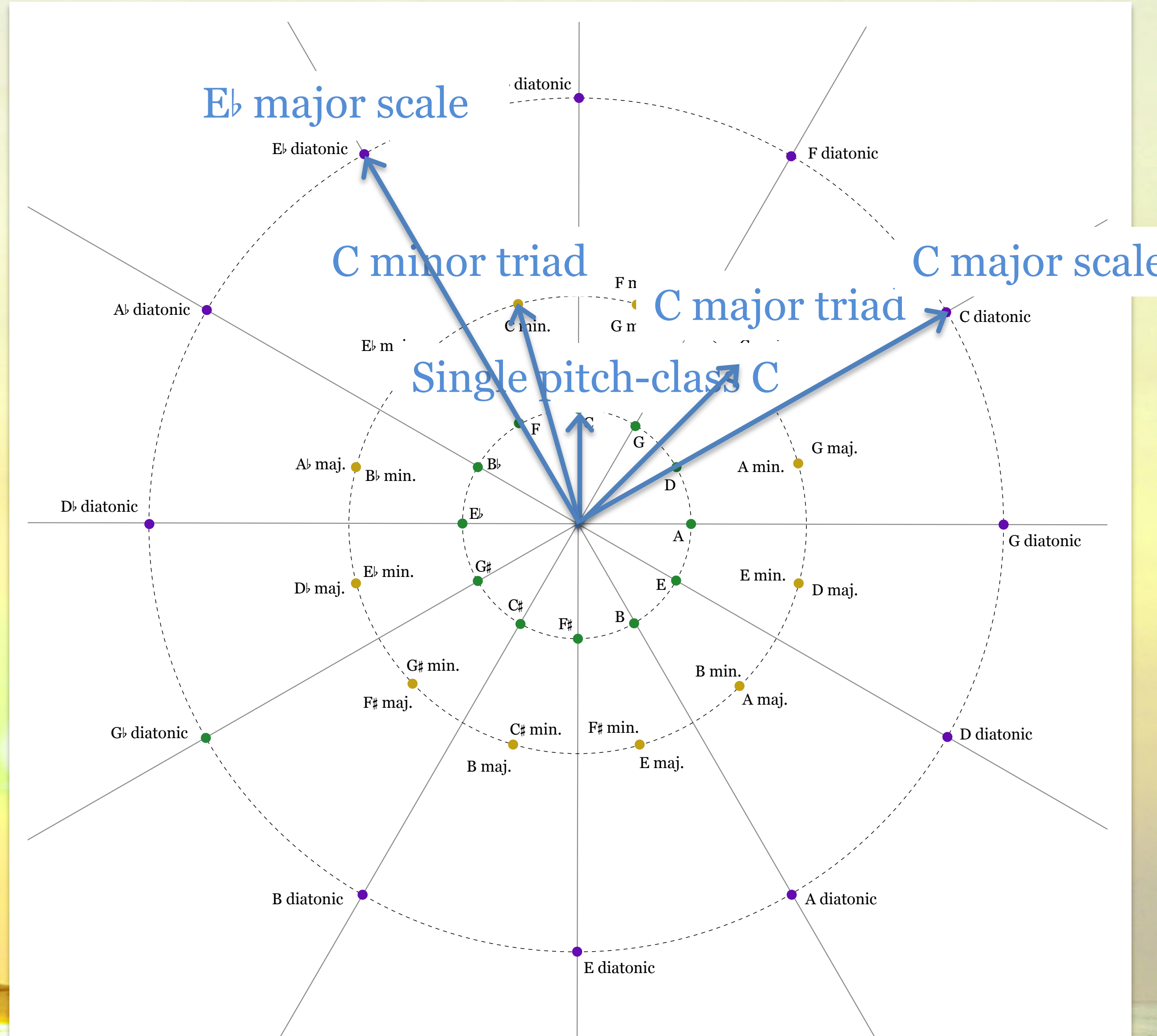


Single component spaces (complex plane)

Example: F_5 space

Distance from the center is the ***magnitude*** of f_5

Angle is the ***phase*** of f_5



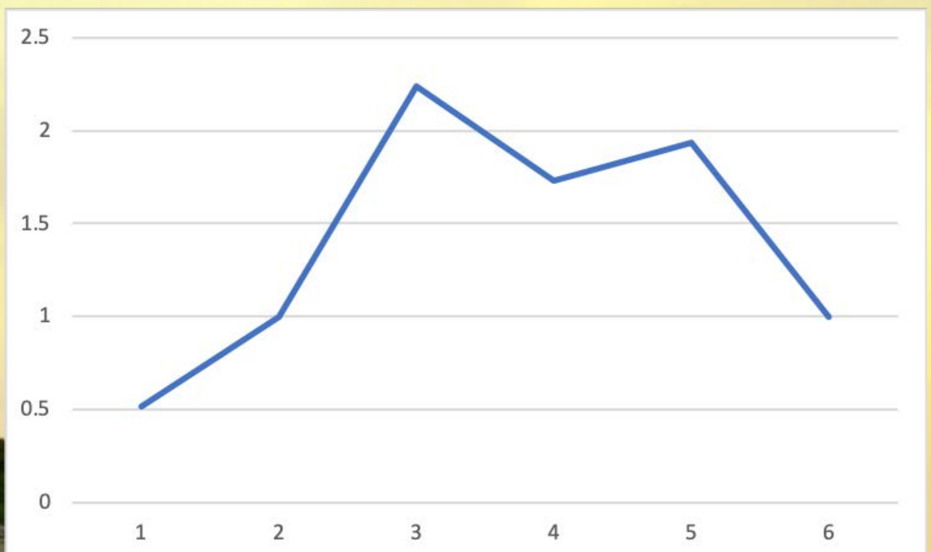
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

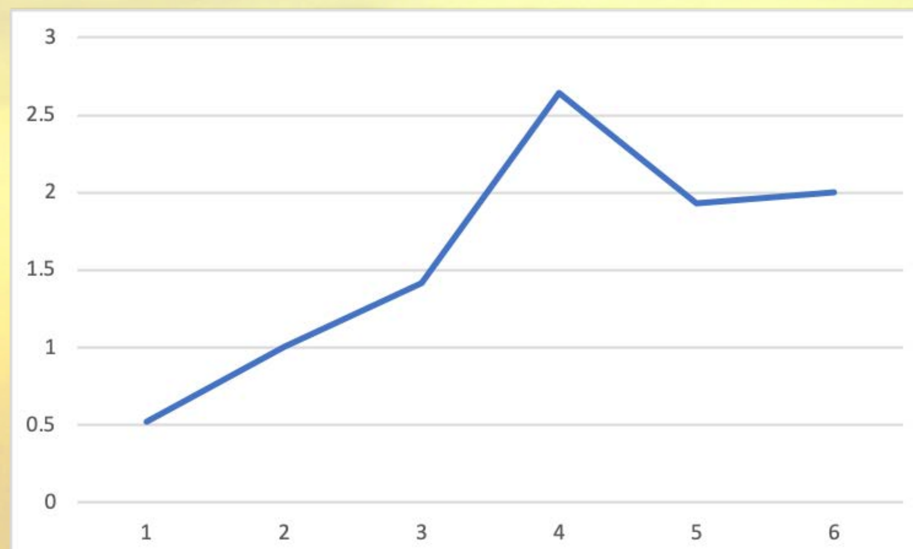
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Major/minor triad



Dominant 7th



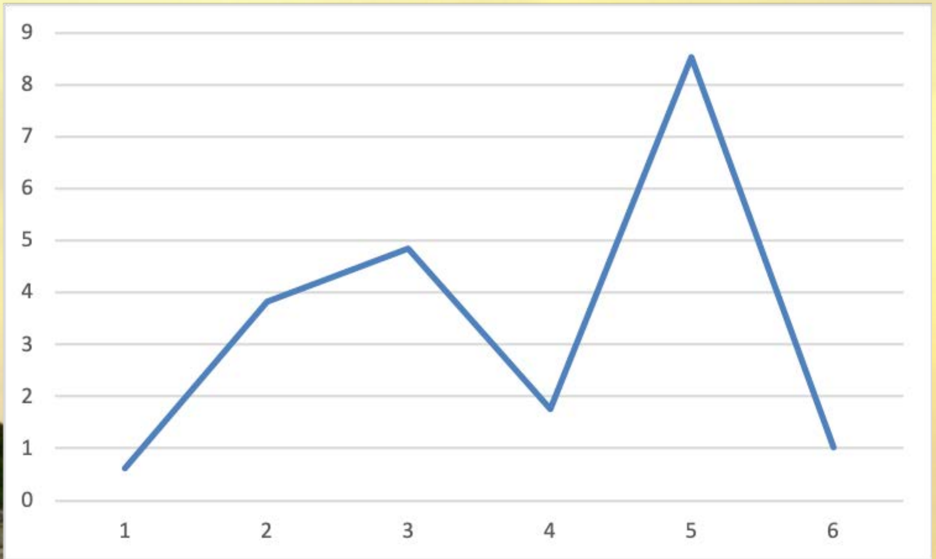
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

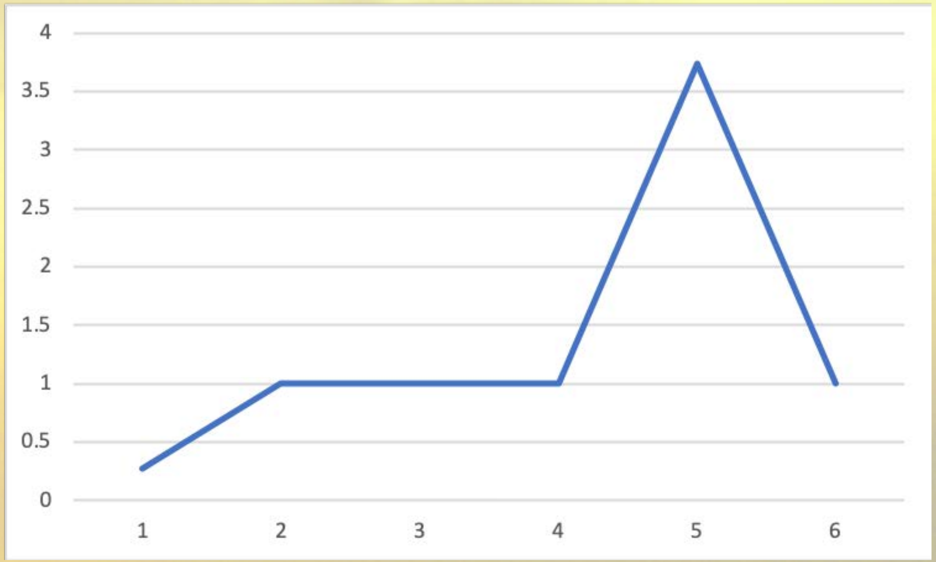
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property)

Examples:

Krumhansl-Kessler major key

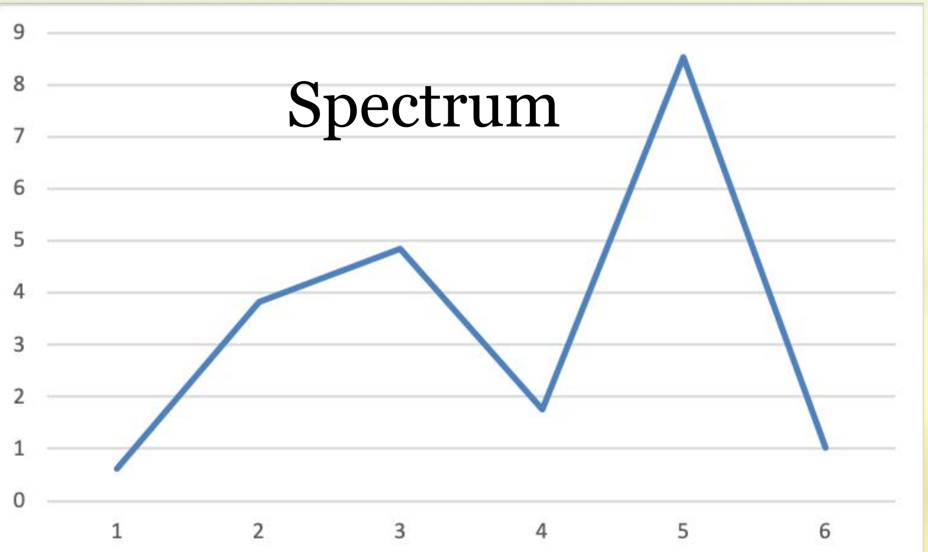


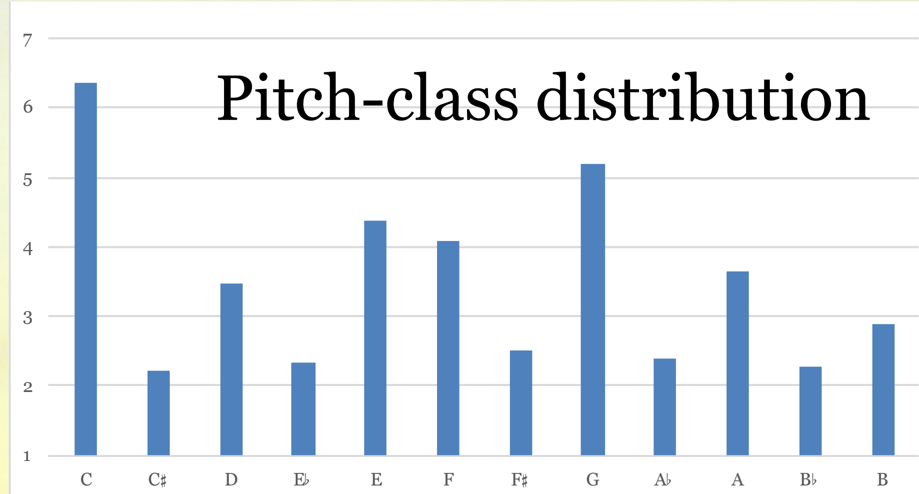
Diatonic scale



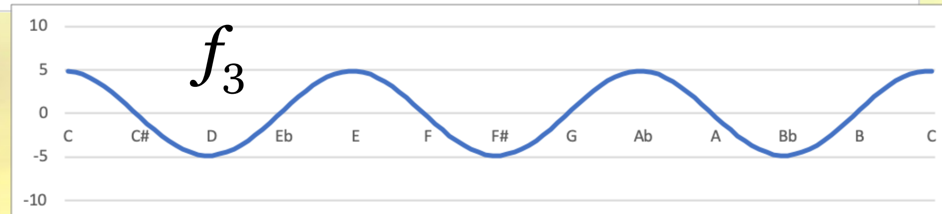
From Spectra to Phase Spaces

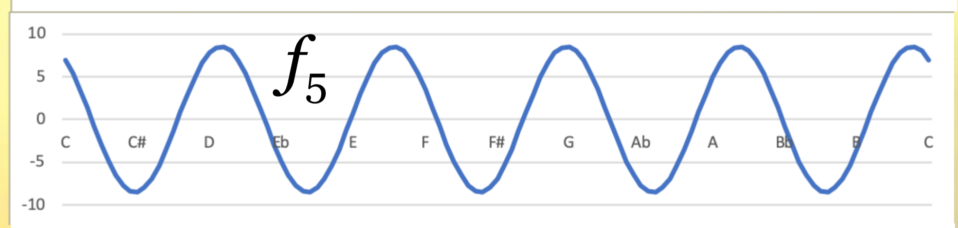
Krumhansl-Kessler major key



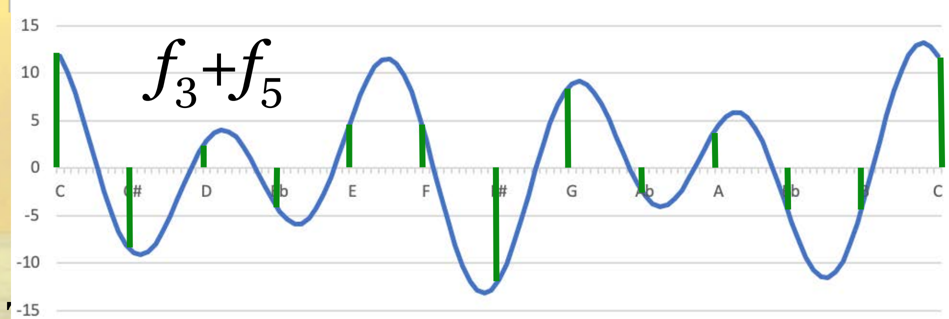


Principal components:



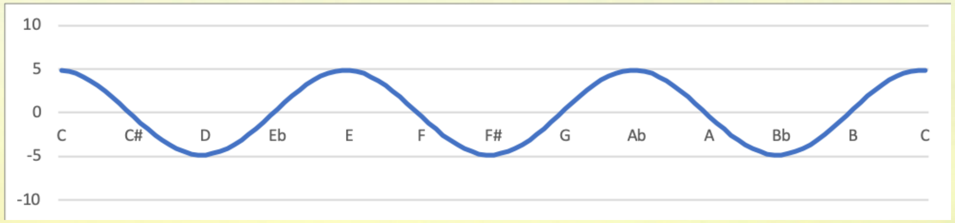


The sum of these gives a good approximation:

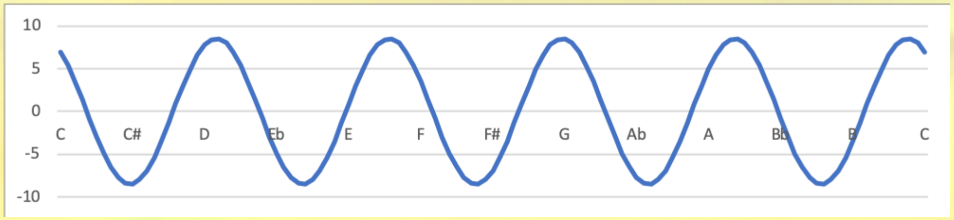


From Spectra to Phase Spaces

Transpositions of the key correspond to phase shifts of the components

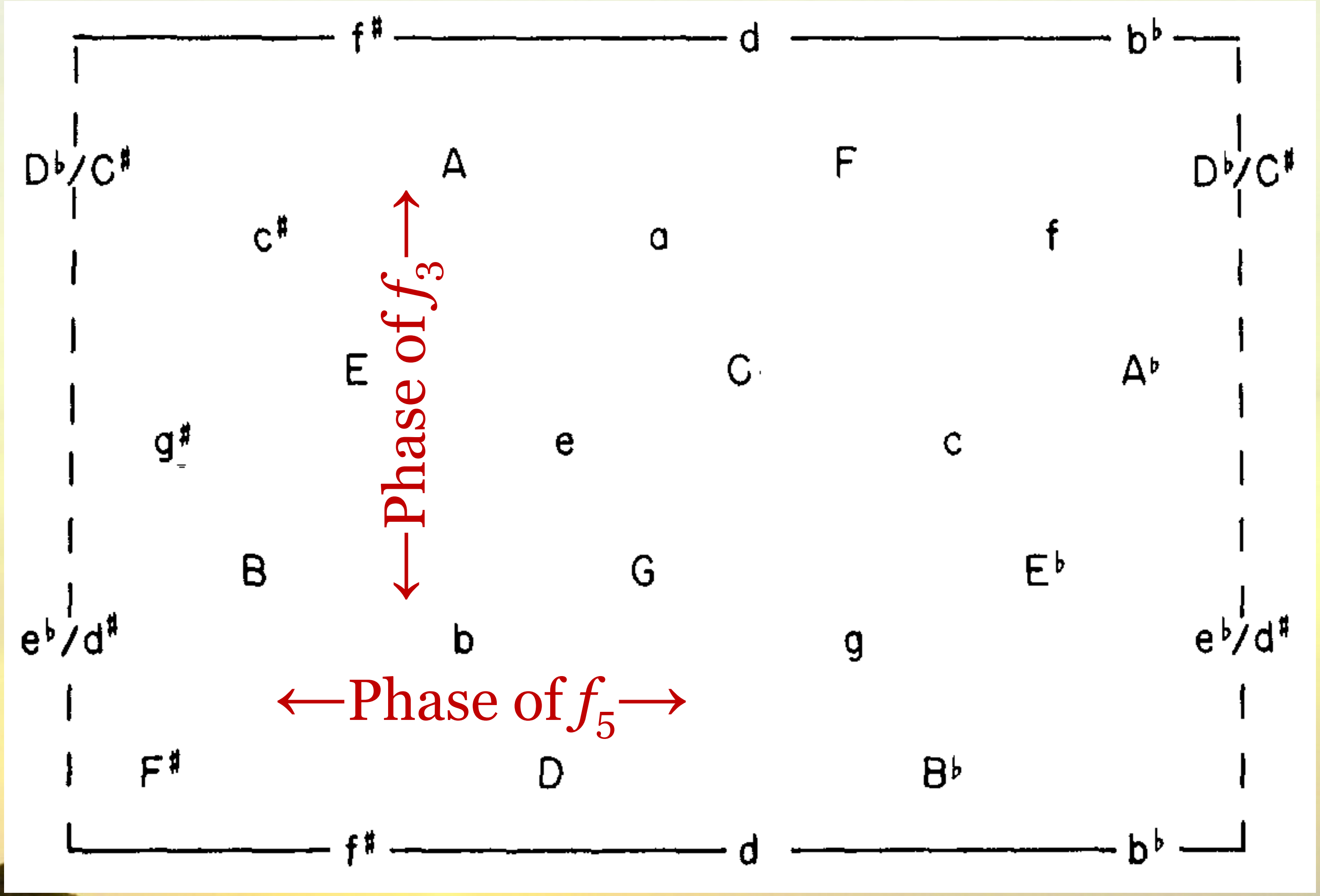


$\longleftarrow$Phase of $f_3$$\longrightarrow$



$\longleftarrow$Phase of $f_5$$\longrightarrow$

From Spectra to Phase Spaces

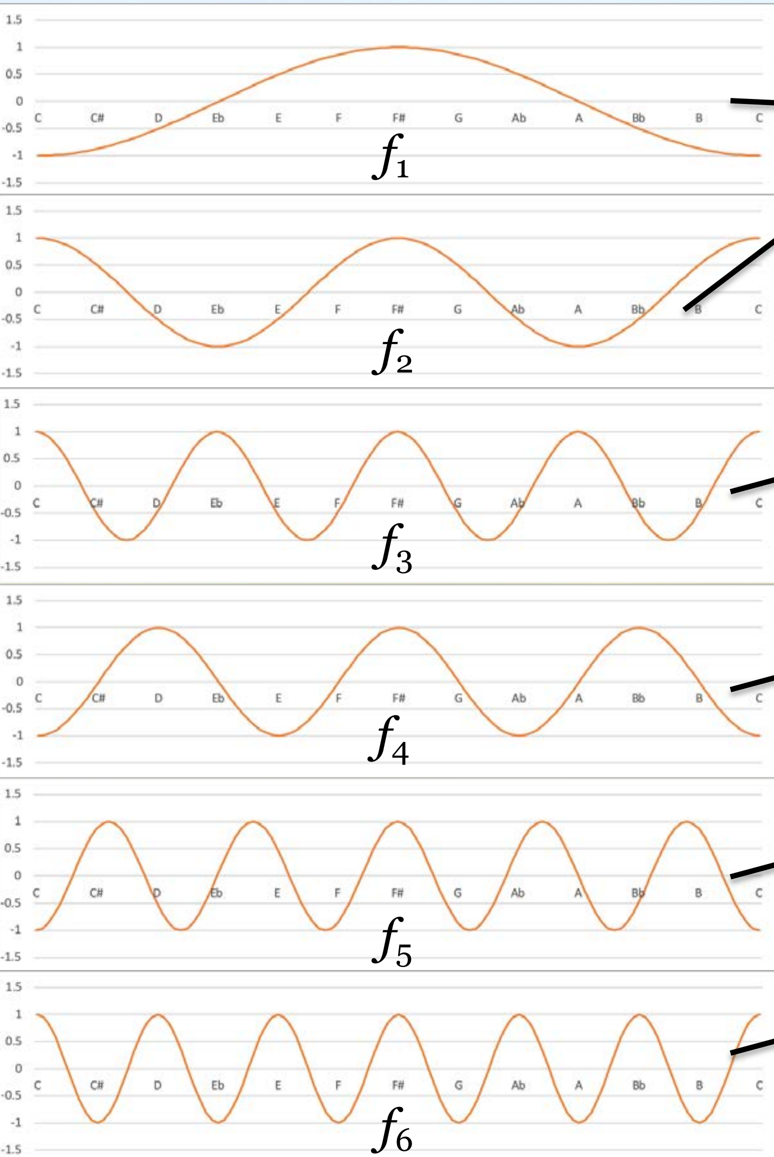


Krumhansl and Kessler Tonal Space

Alternate Tonalities

Twentieth-century composers extend tonality by redefining the basic space

The Six Harmonic Qualities



Use of f_1 and f_2 requires dissonant harmony and abandonment of traditional voice-leading principles

f_3 represents the **triadic/ functional** element of tonality.

f_4 represents **octatonicism**, a potential extension of tonality.

f_5 represents the **diatonic** element of tonality.

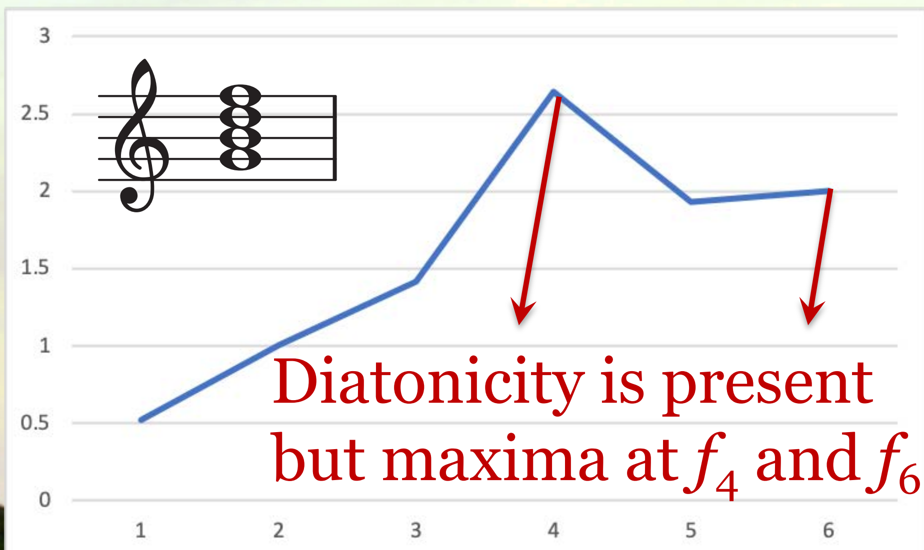
f_6 represents the balance between **whole tone scales.**

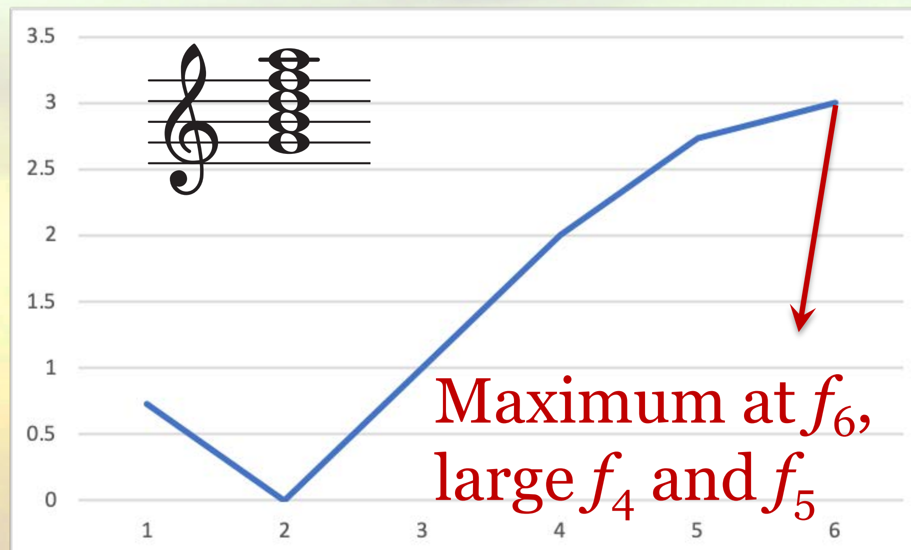
Alternate Basic Sonorities

Debussy:

Acoustic scale/Dominant 9th

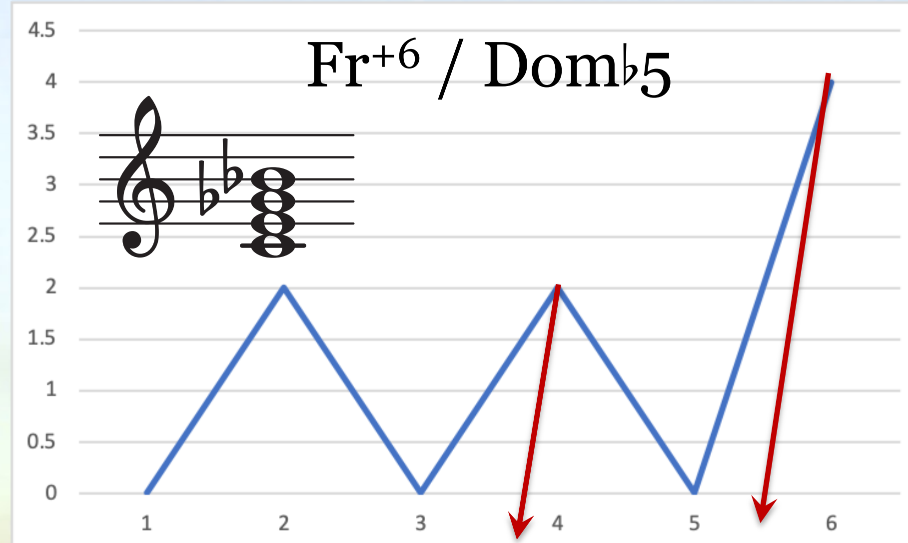
Dominant/half-diminished 7th



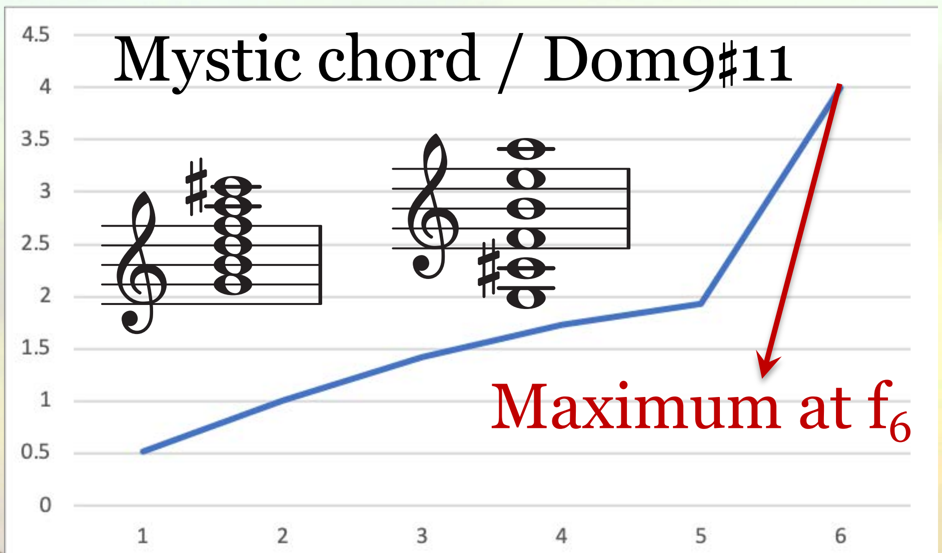


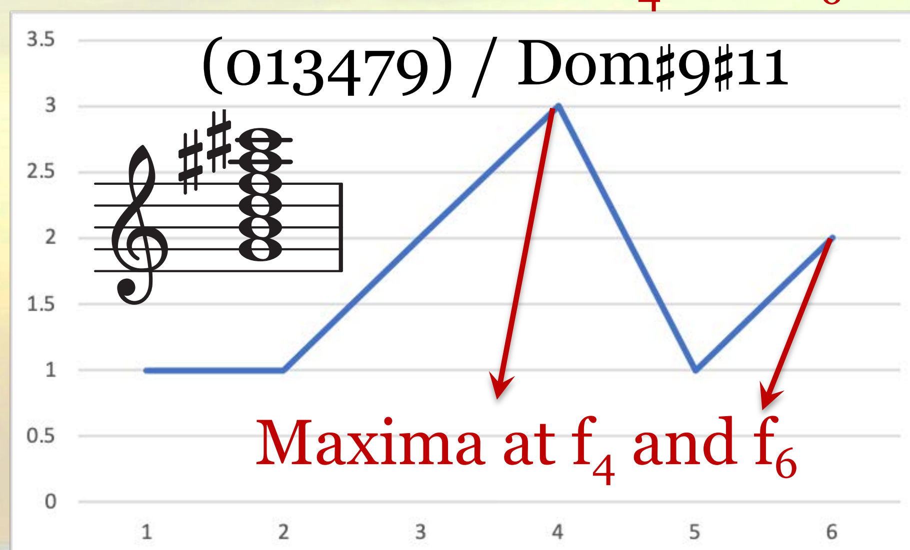
Alternate Basic Sonorities

Scriabin:

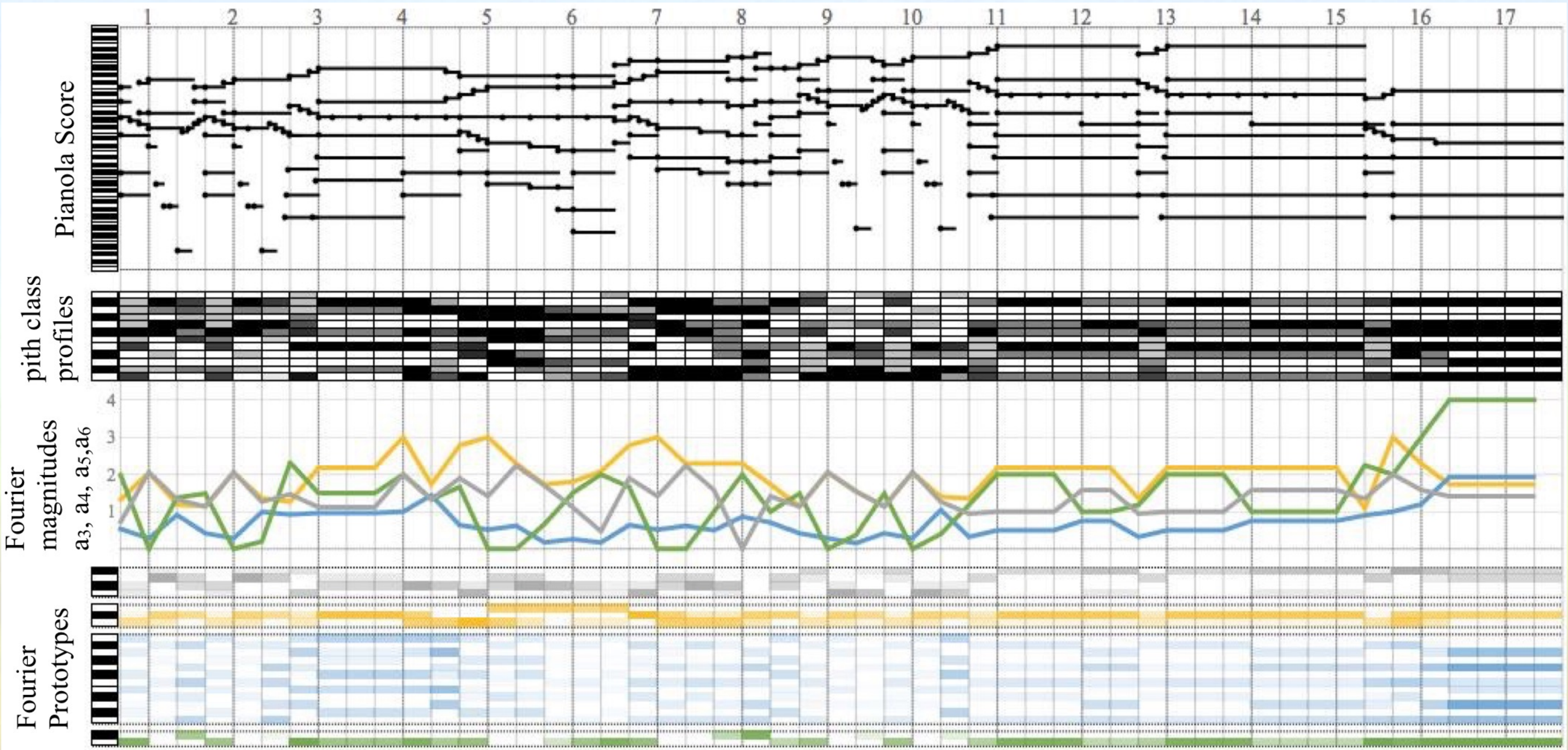


Maxima at f_4 and f_6





Automated DFT Analysis of Op. 74/1



By Thomas Noll

Analysis: Debussy
“D’un Cahier des Esquisses”

Debussy relates sets via **diatonic (f_5)** and **whole-tone (f_6)** qualities

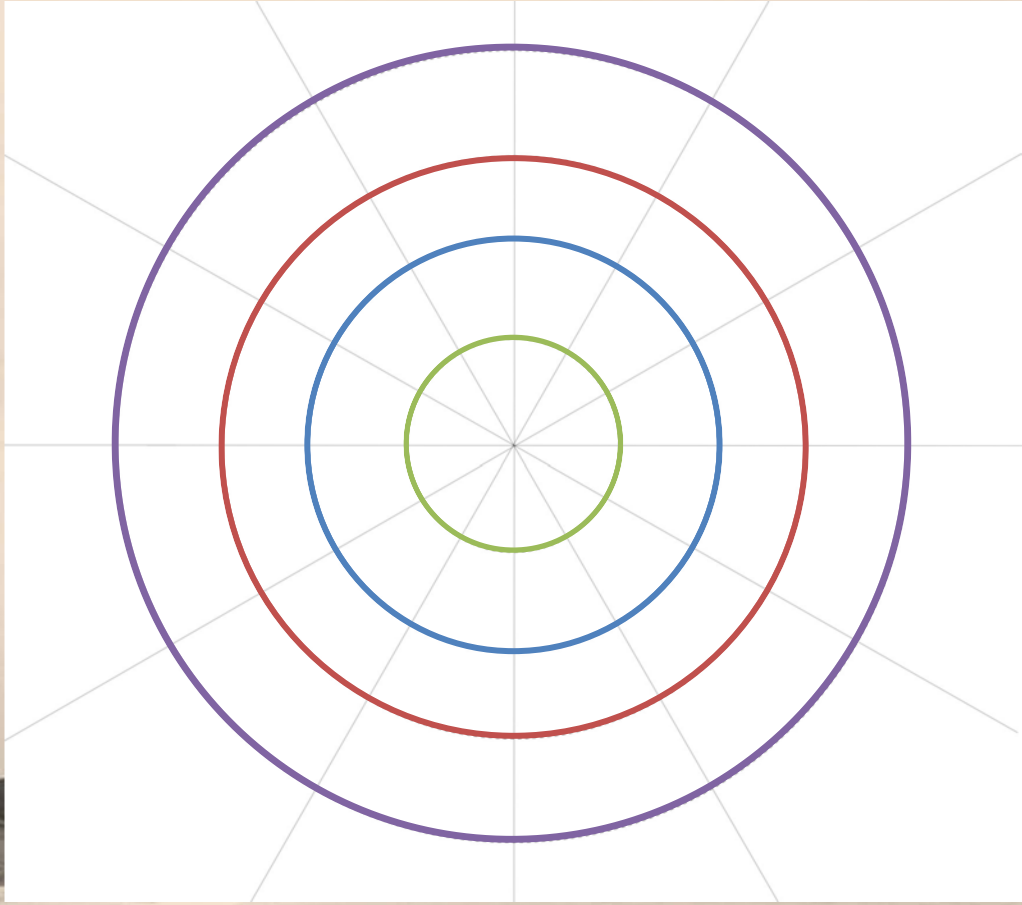
Diatonic Space (F_5)

Diatonic and **pentatonic scales** have the largest $|f_5|$

Acoustic scales and **dominant ninths** also have large $|f_5|$

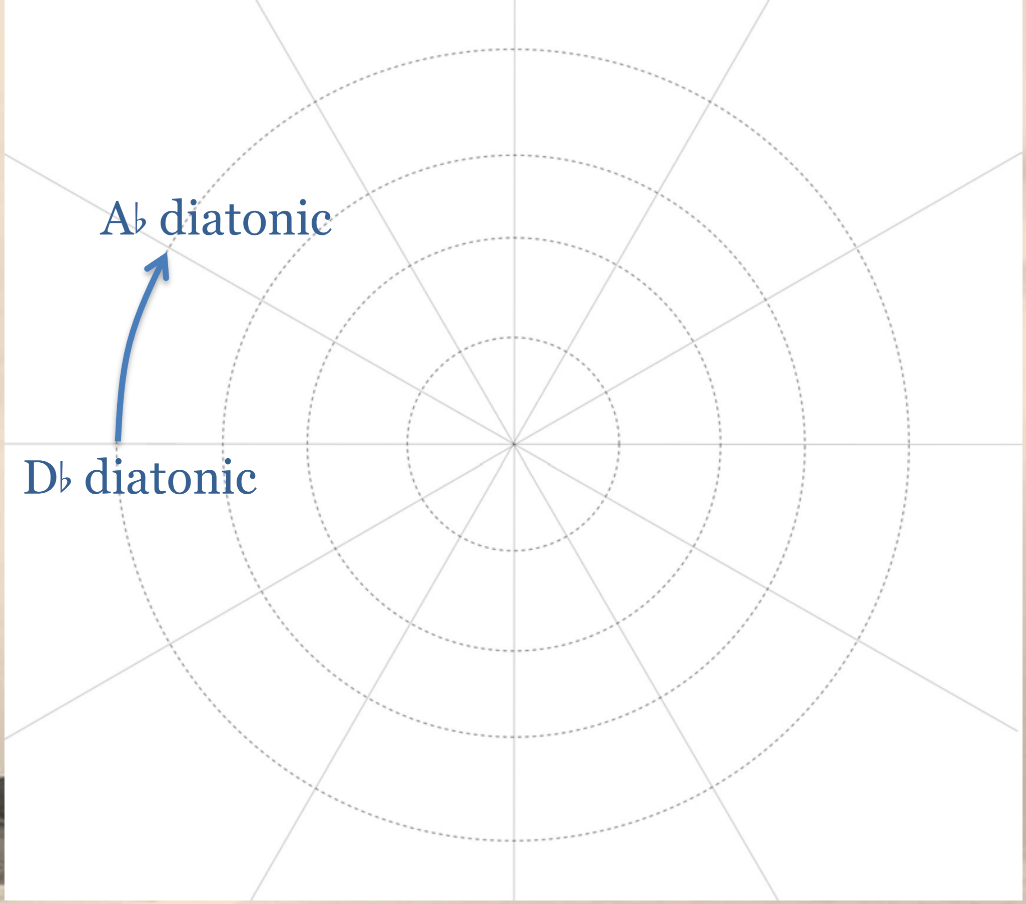
Major and minor triads and **dominant 7ths** are closer to the center

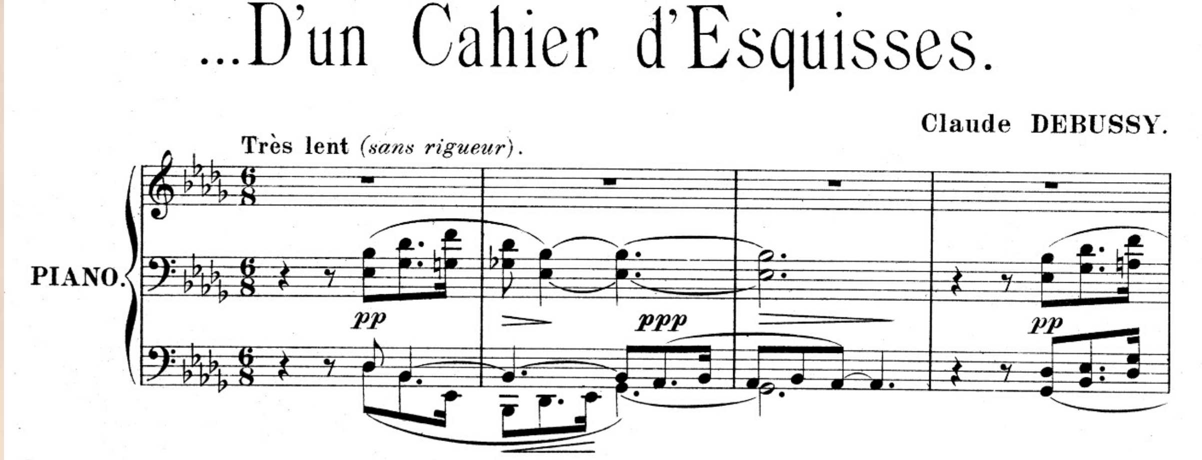
Individual pitch classes are on the unit circle



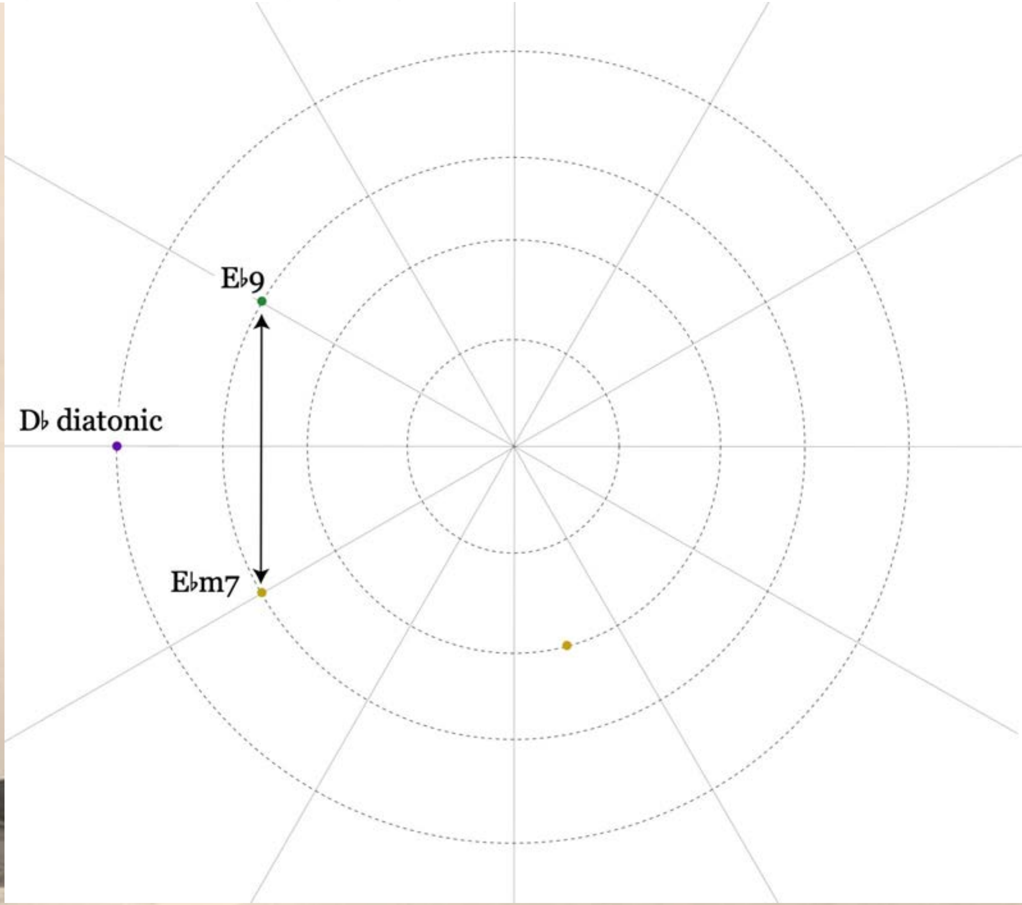
Diatonic Space (F_5)

Transposition by fifth circles the space 30°

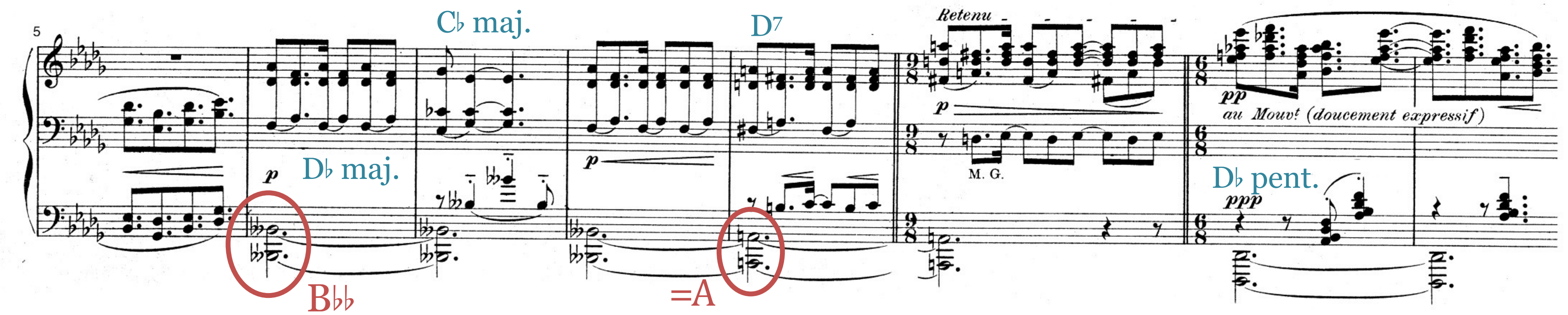




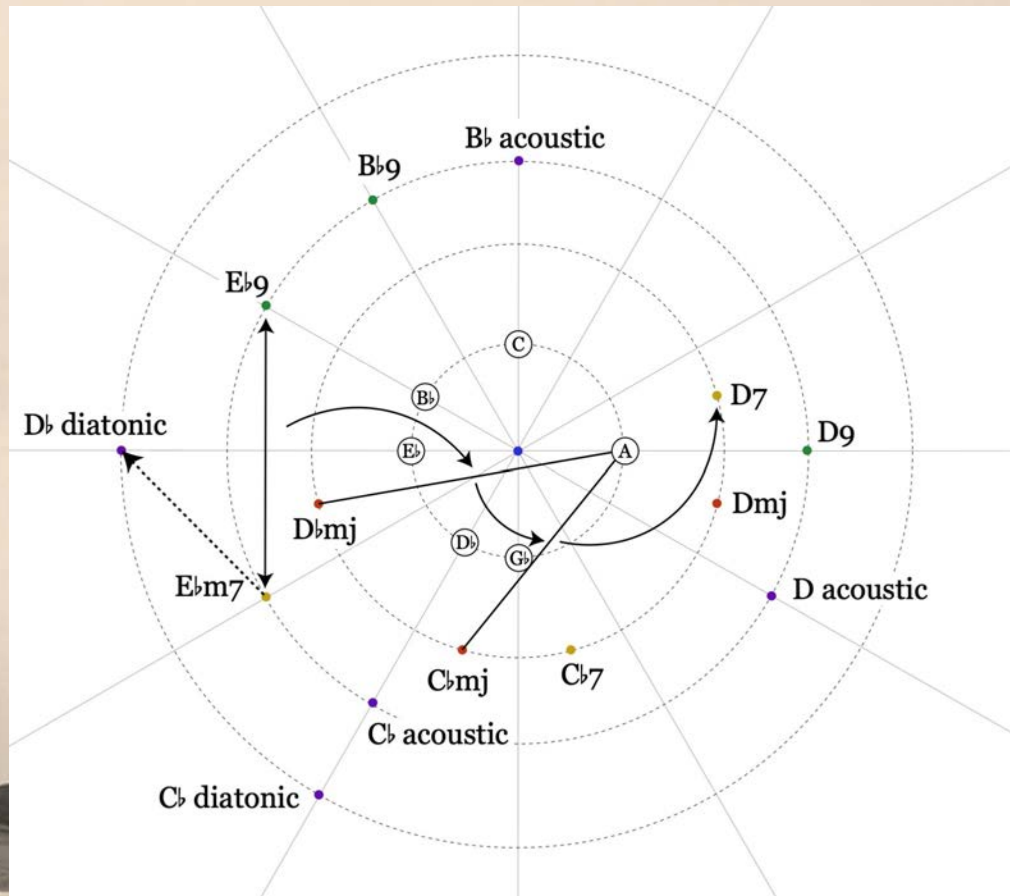
...D'un Cahier d'Esquisses.
Claude DEBUSSY.
Très lent (sans rigueur).
PIANO.
pp
ppp
pp



E♭9
D♭ diatonic
E♭m7

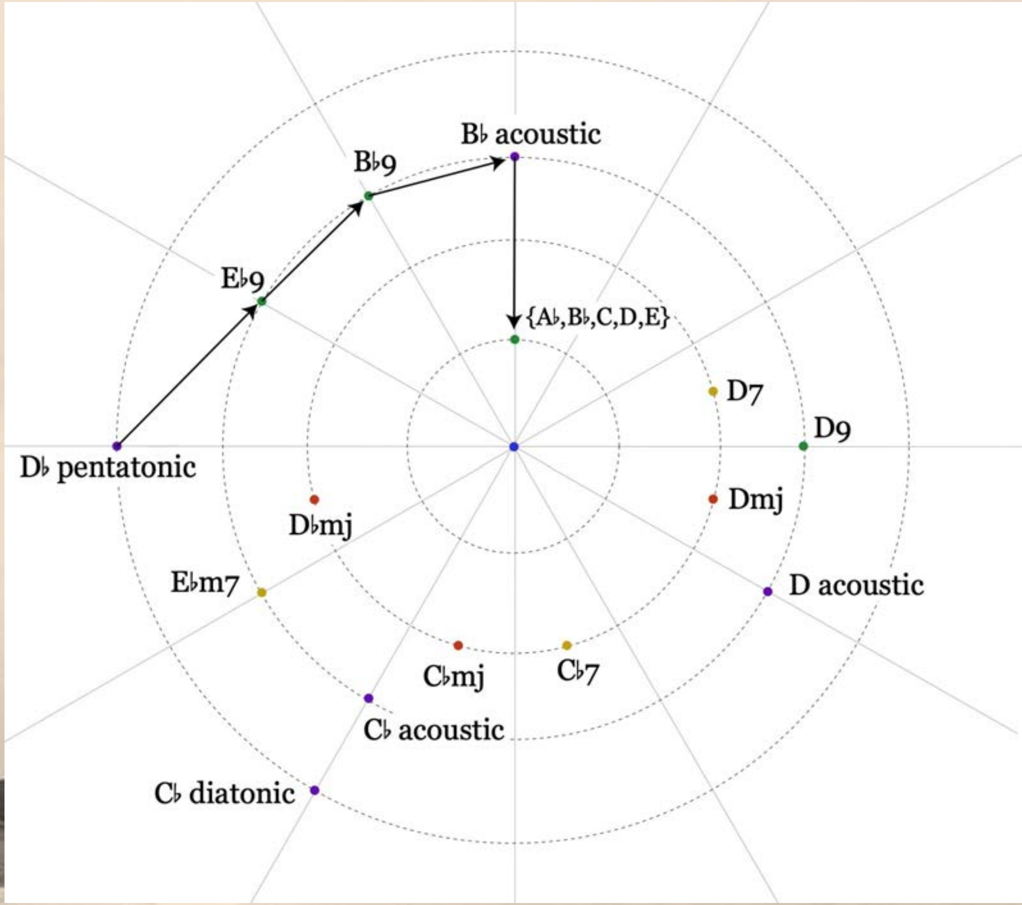

5
C♭ maj.
D7
Retenu
D♭ maj.
au Mouvt (doucement expressif)
M. G.
D♭ pent.
B♭♭
=A

Meas. 5–11


B♭ acoustic
B♭9
E♭9
C
B♭
D7
D9
D♭ diatonic
E♭
A
Dmj
D♭mj
D♭
G♭
D acoustic
E♭m7
C♭mj
C♭7
C♭ acoustic
C♭ diatonic

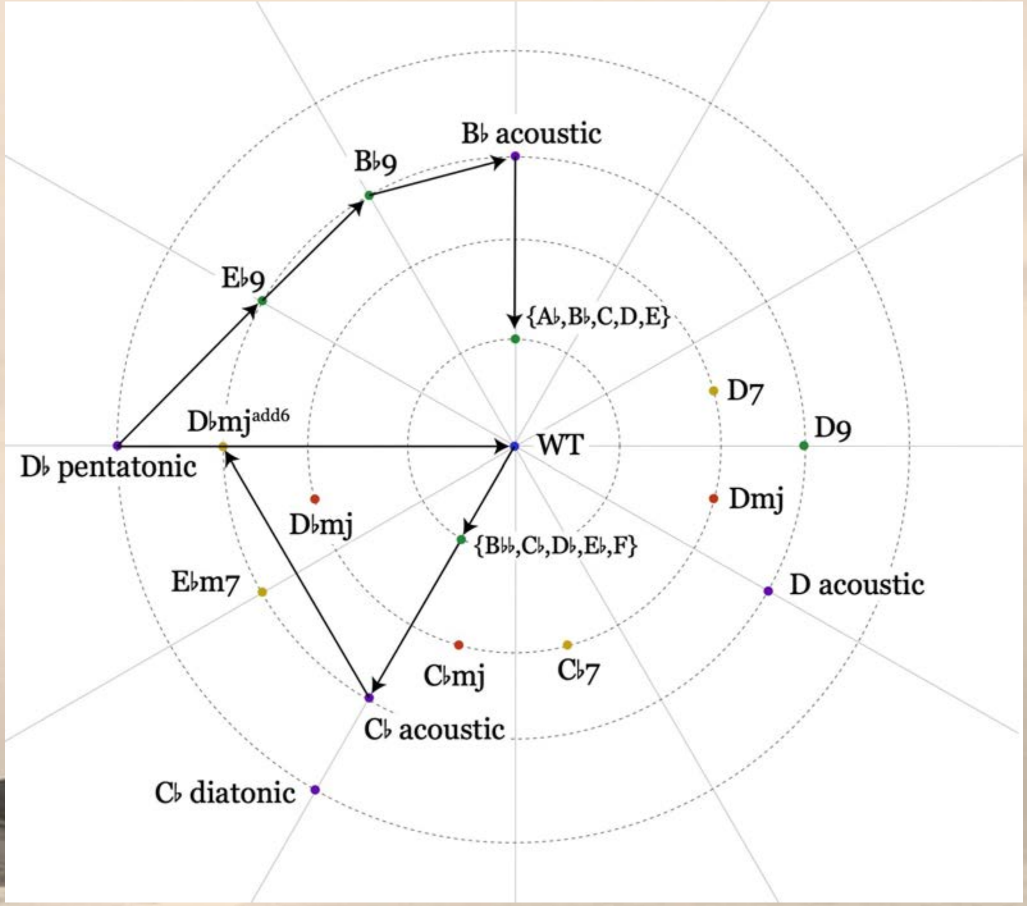

En animant peu à peu
18
poco a poco cresc.
E♭9
B♭9
B♭ acoustic
{A♭,B♭,C,D,E}

Meas. 11–28


B♭ acoustic
B♭9
E♭9
{A♭,B♭,C,D,E}
D7
D9
D♭ pentatonic
Dmj
D♭mj
E♭m7
D acoustic
C♭mj
C♭7
C♭ acoustic
C♭ diatonic

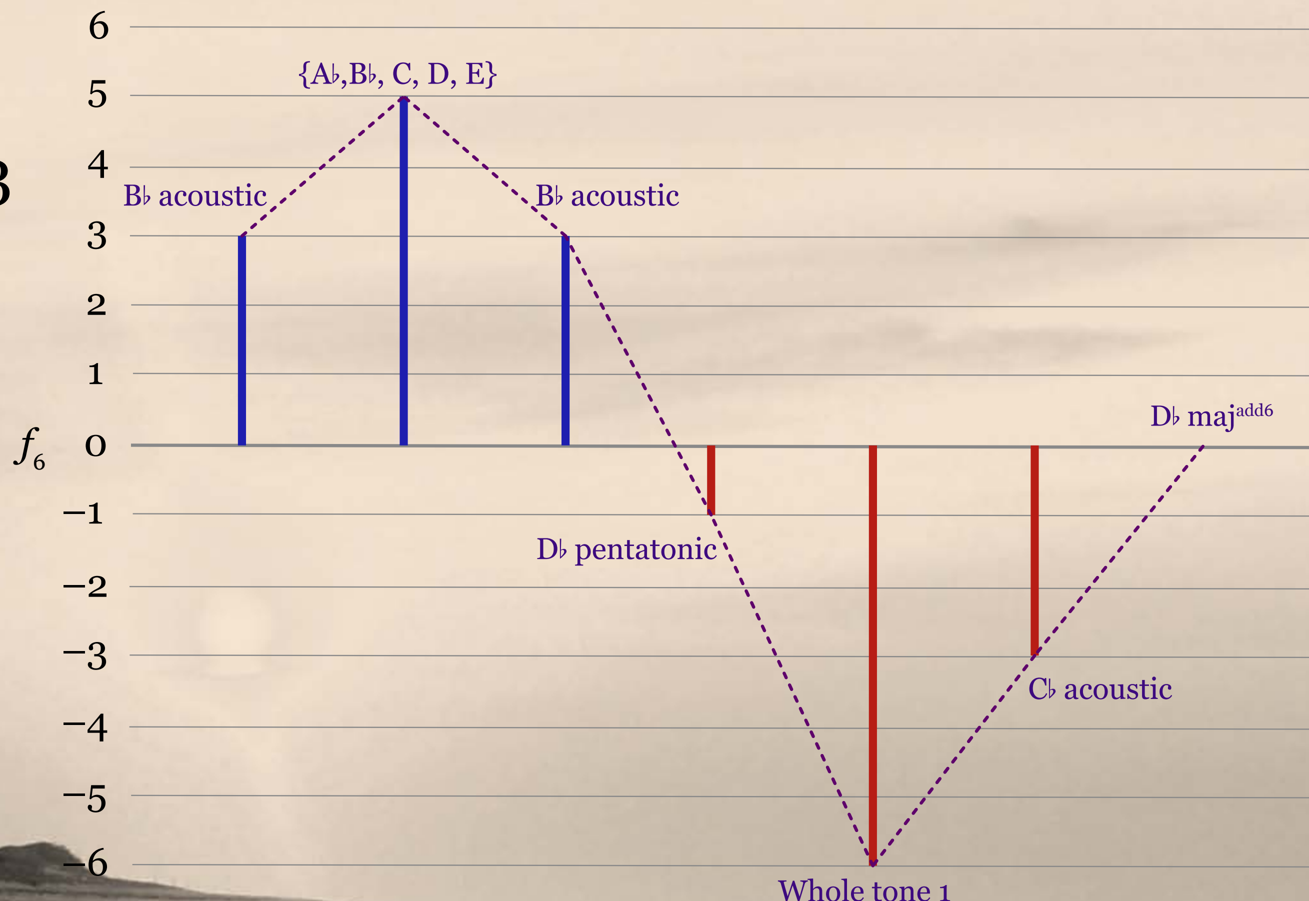

Retenu
au Mouvt
En animant
28
p
pp
mf
f
più p
dim.
M. G.
p dim.
B♭ acoustic
D♭ pentatonic
WT1
C♭ acoustic
D♭ maj^{add6}

Meas. 15–35


B♭ acoustic
B♭9
E♭9
{A♭,B♭,C,D,E}
D7
D9
D♭mj^{add6}
D♭ pentatonic
WT
D♭mj
Dmj
{B♭♭,C♭,D♭,E♭,F}
E♭m7
D acoustic
C♭mj
C♭7
C♭ acoustic
C♭ diatonic

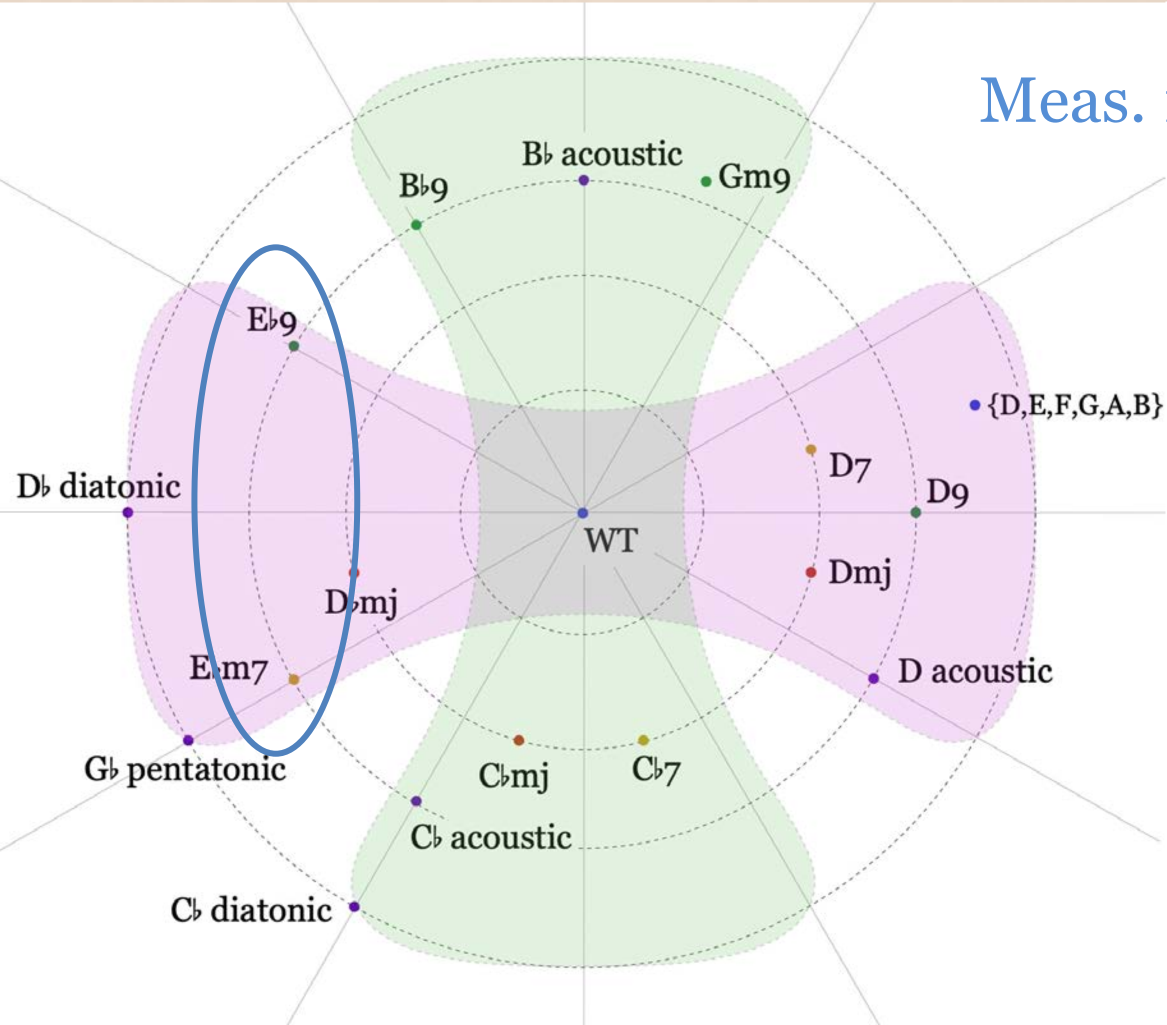
28
Retenu
au Mouvt
En animant
p
pp
mf
f
più p
dim.
p dim.
M. G.
B♭ acoustic
D♭ pentatonic
WT1
C♭ acoustic
D♭ maj^add6

Meas. 28–33



f_6
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
{A♭,B♭, C, D, E}
B♭ acoustic
B♭ acoustic
D♭ pentatonic
Whole tone 1
C♭ acoustic
D♭ maj^add6

F_5 oppositions — F_6 oppositions



Meas. 1

6 ⟶ Whole tone 0

5 ⟶ WT_0−1: {A♭,B♭, C, D, E}, {B♭, C, D, E, F♯}

4 ⟶ C Fr^{+6}

3 ⟶ B♭ acoustic, D acoustic, $B♭^9$

2 ⟶ D^7

1 ⟶ G♭ pentatonic, D♭ diatonic

0 ⟶ $E♭^{m7}$

−1 ⟶ D♭ pentatonic

−2 ⟶ B♭♭+D♭mj, $C♭^7$, {D,E,F,G,A,B}

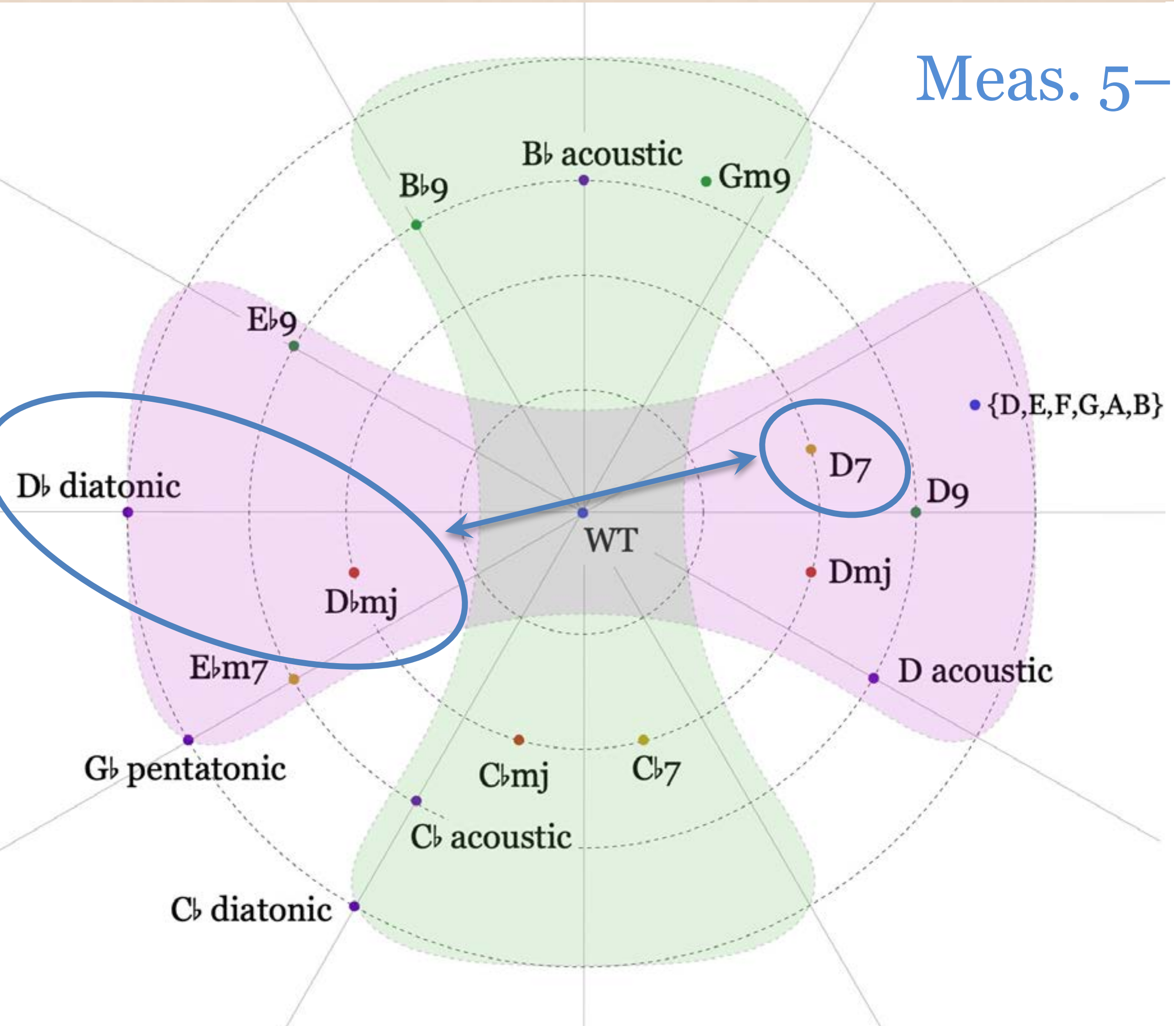
−3 ⟶ C♭ acoustic, $E♭^9$

−4

−5 ⟶ WT_1−1: {B♭♭,C♭,D♭,E♭,F}

−6 ⟶ Whole tone 1

F_5 oppositions — F_6 oppositions



Meas. 5–10

6 ⟶ Whole tone 0

5 ⟶ WT_0–1: {A♭,B♭, C, D, E}, {B♭, C, D, E, F♯}

4 ⟶ C Fr^{+6}

3 ⟶ B♭ acoustic, D acoustic, $B♭^9$

2 ⟶ D^7

1 ⟶ G♭ pentatonic, D♭ diatonic

0 ⟶ $E♭^{m7}$

−1 ⟶ D♭ pentatonic

−2 ⟶ B♭♭+D♭mj, $C♭^7$, {D,E,F,G,A,B}

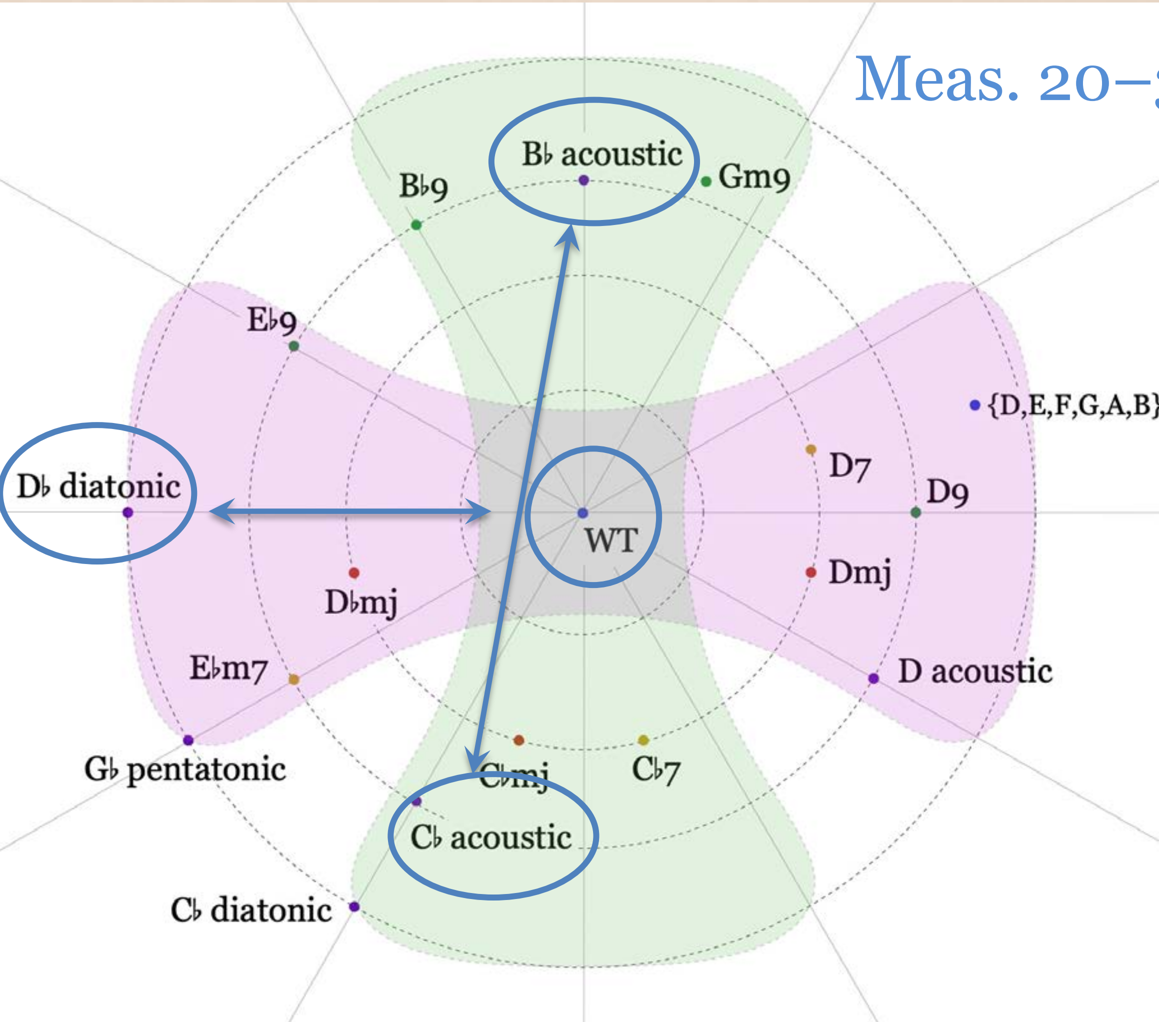
−3 ⟶ C♭ acoustic, $E♭^9$

−4

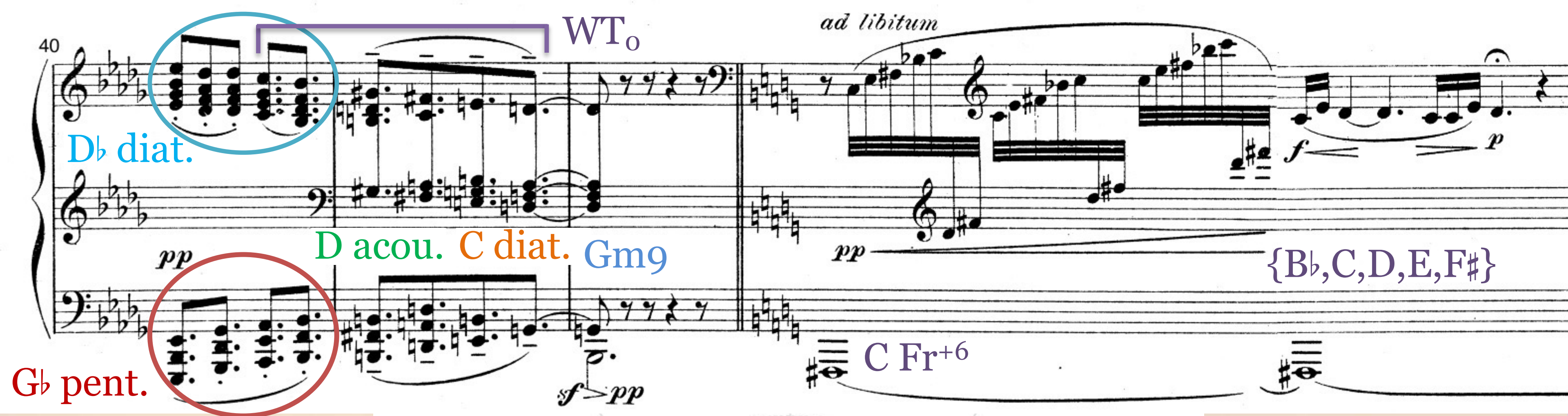
−5 ⟶ WT_1–1: {B♭♭,C♭,D♭,E♭,F}

−6 ⟶ Whole tone 1

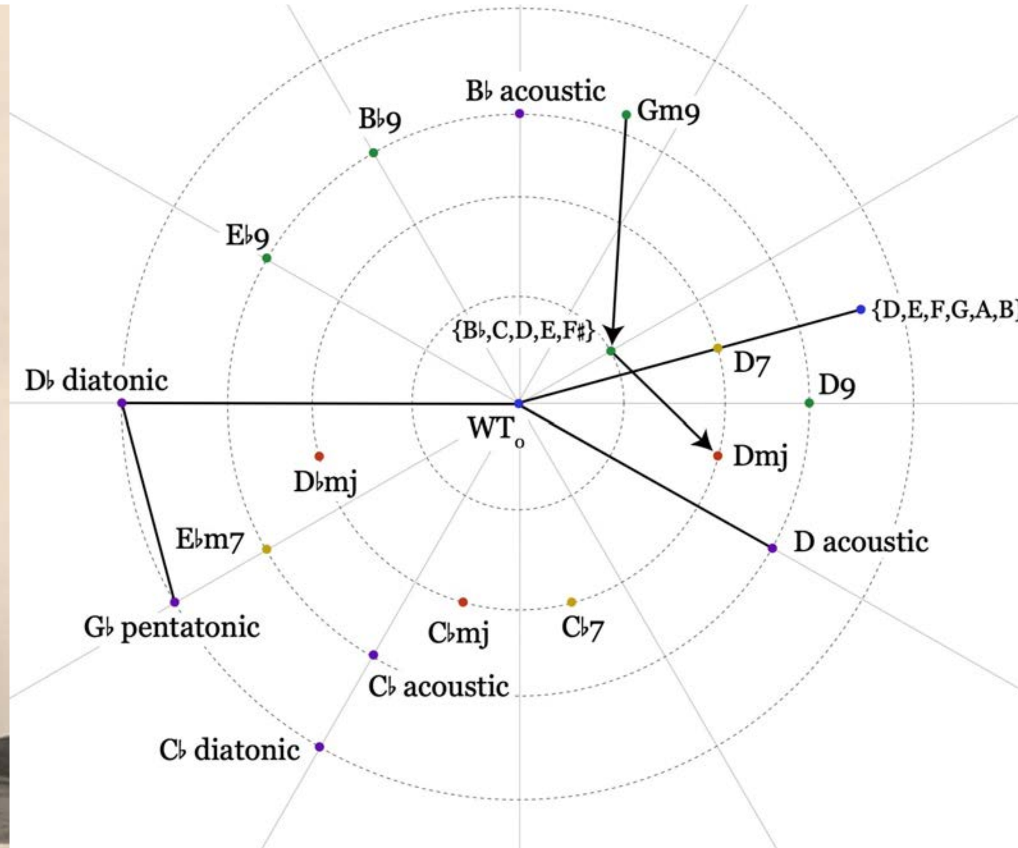
F_5 oppositions — F_6 oppositions



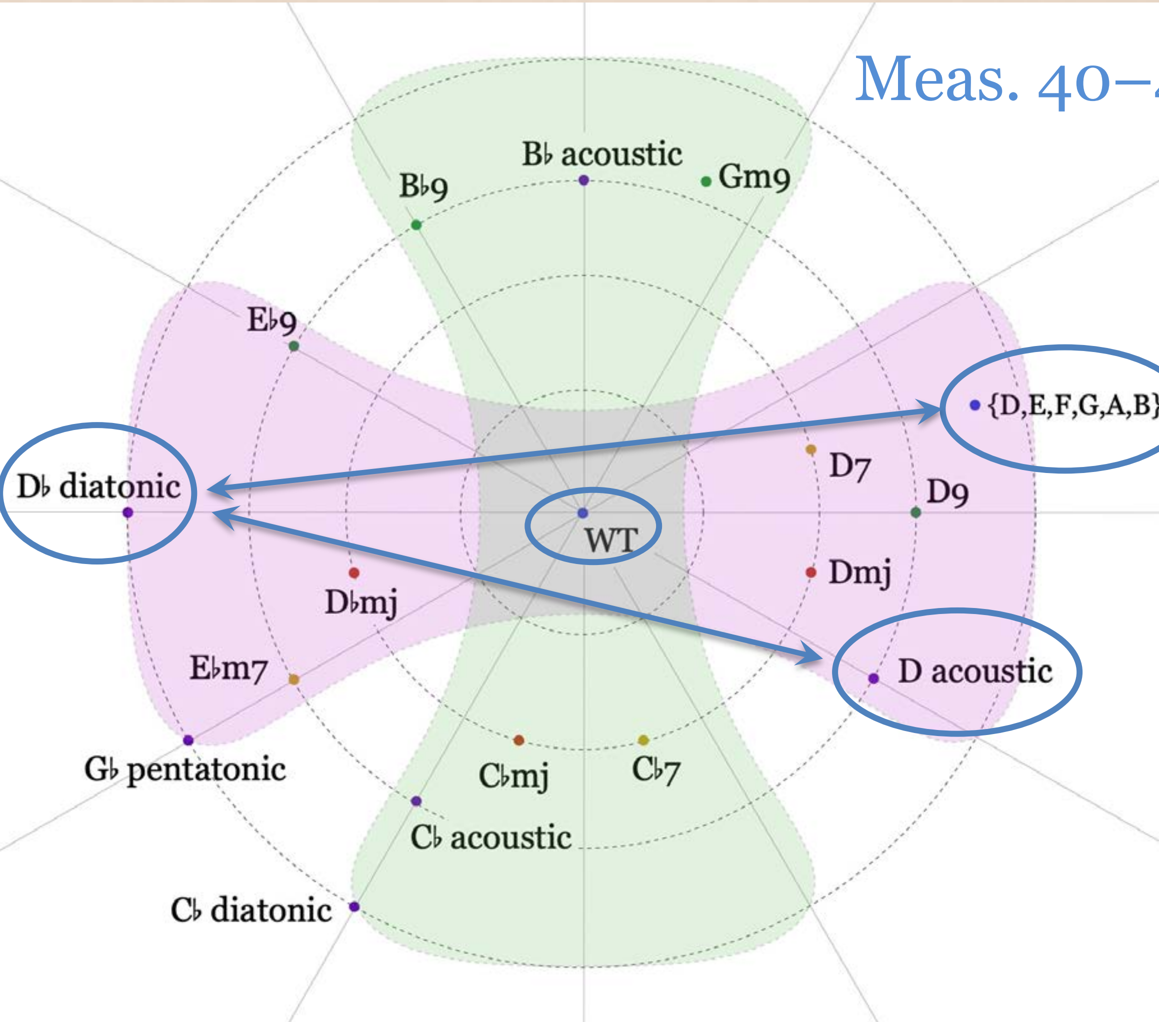
- 6 ⟶ Whole tone 0
- 5 ⟶ WT_0–1: {A♭,B♭, C, D, E}, {B♭, C, D, E, F♯}
- 4 ⟶ C Fr^{+6}
- 3 ⟶ B♭ acoustic, D acoustic, $B♭^9$
- 2 ⟶ D^7
- 1 ⟶ G♭ pentatonic, D♭ diatonic
- 0 ⟶ $E♭^{m7}$
- −1 ⟶ D♭ pentatonic
- −2 ⟶ B♭♭+D♭mj, $C♭^7$, {D,E,F,G,A,B}
- −3 ⟶ C♭ acoustic, $E♭^9$
- −4
- −5 ⟶ WT_1–1: {B♭♭,C♭,D♭,E♭,F}
- −6 ⟶ Whole tone 1


40
WT₀
ad libitum
D♭ diat.
pp
D acou. C diat. Gm9
{B♭,C,D,E,F♯}
pp
G♭ pent.
sf pp
C Fr+6
f
p

Meas. 40–44


B♭ acoustic
Gm9
B♭9
E♭9
{D,E,F,G,A,B}
{B♭,C,D,E,F♯}
D7
D♭ diatonic
D9
WT₀
Dmj
D♭mj
E♭m7
D acoustic
G♭ pentatonic
C♭mj
C♭7
C♭ acoustic
C♭ diatonic

F_5 oppositions — F_6 oppositions



6 ⟶ Whole tone 0

5 ⟶ WT_0–1: {A♭,B♭, C, D, E}, {B♭, C, D, E, F♯}

4 ⟶ C Fr^{+6}

3 ⟶ B♭ acoustic, D acoustic, B♭^{9}

2 ⟶ D^7

1 ⟶ G♭ pentatonic, D♭ diatonic

0 ⟶ E♭^{m7}

−1 ⟶ D♭ pentatonic

−2 ⟶ B♭♭+D♭mj, C♭^{7}, {D,E,F,G,A,B}

−3 ⟶ C♭ acoustic, E♭^{9}

−4

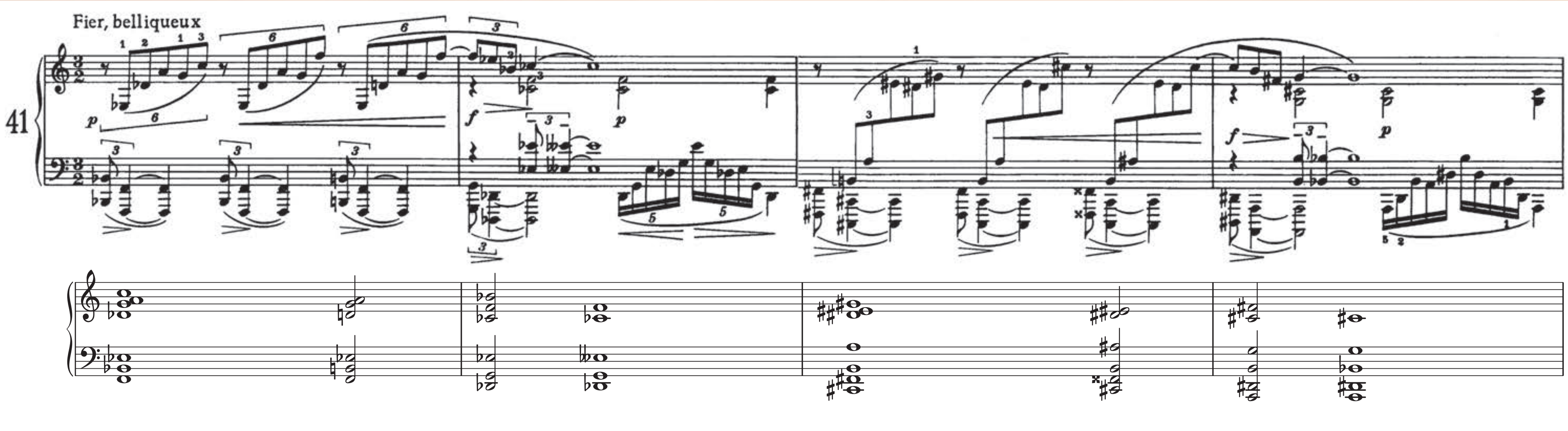
−5 ⟶ WT_1–1: {B♭♭,C♭,D♭,E♭,F}

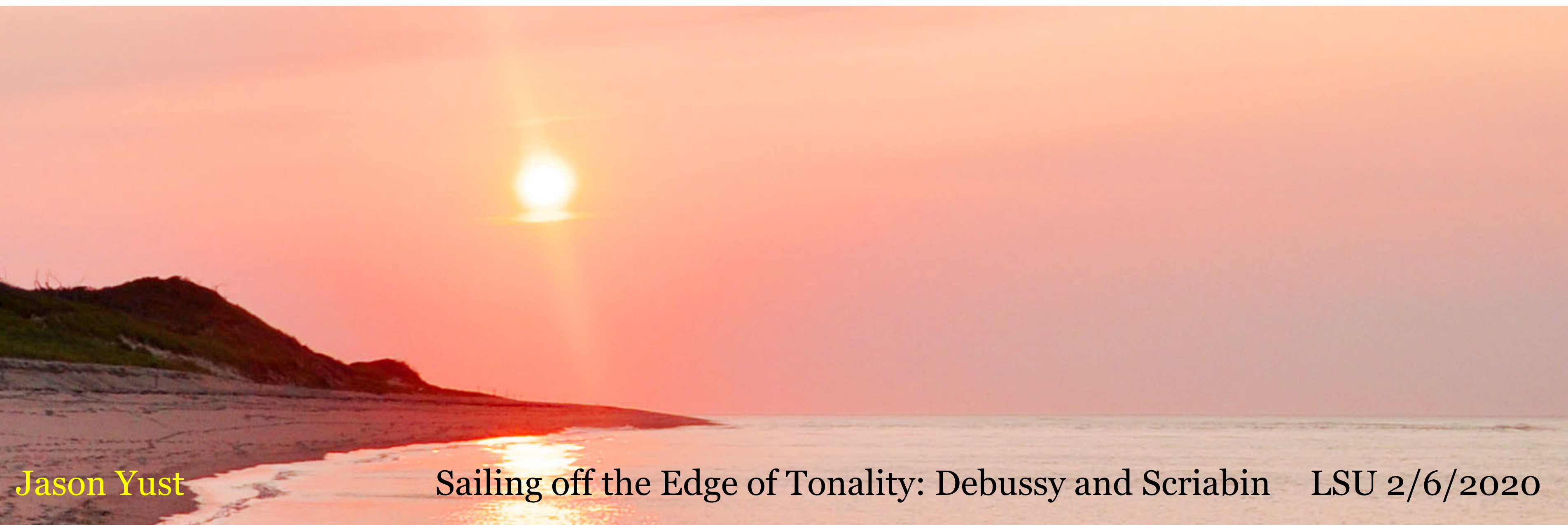
−6 ⟶ Whole tone 1

Analysis: Scriabin Prelude, Op. 74/5

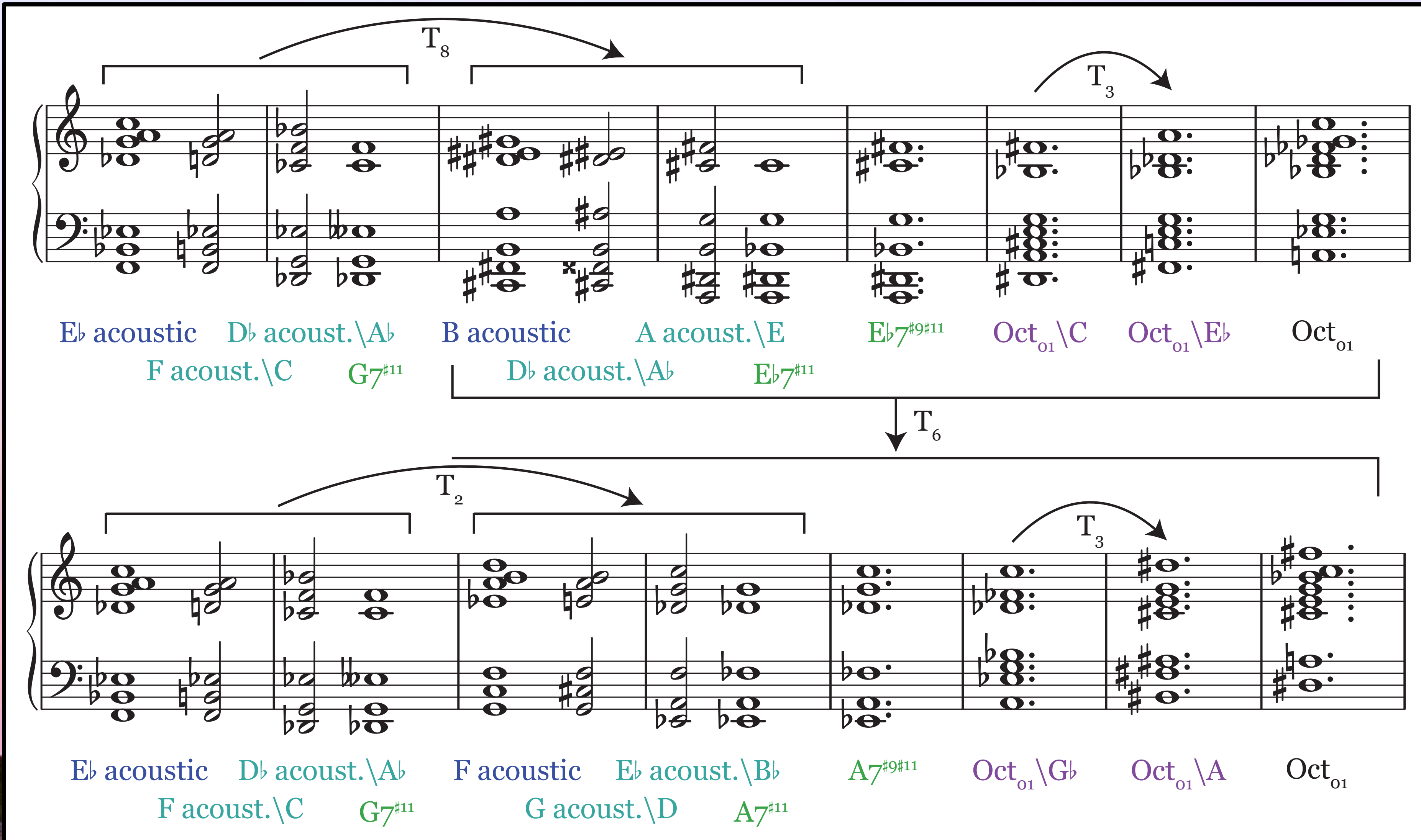
Scriabin relates sets via **diatonic (f_5)** and **octatonic (f_4)** qualities

Harmonic reduction mm. 1–4

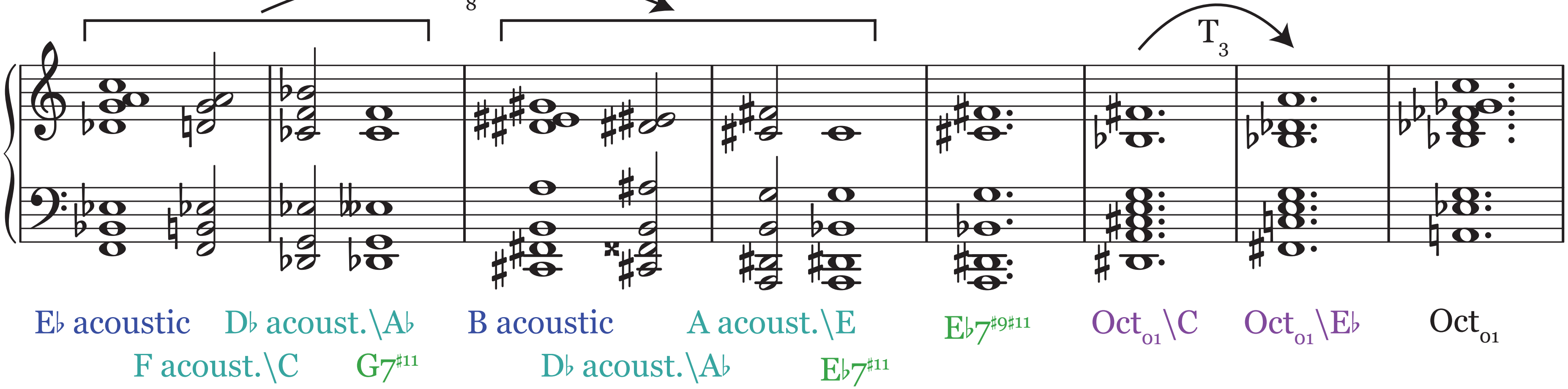




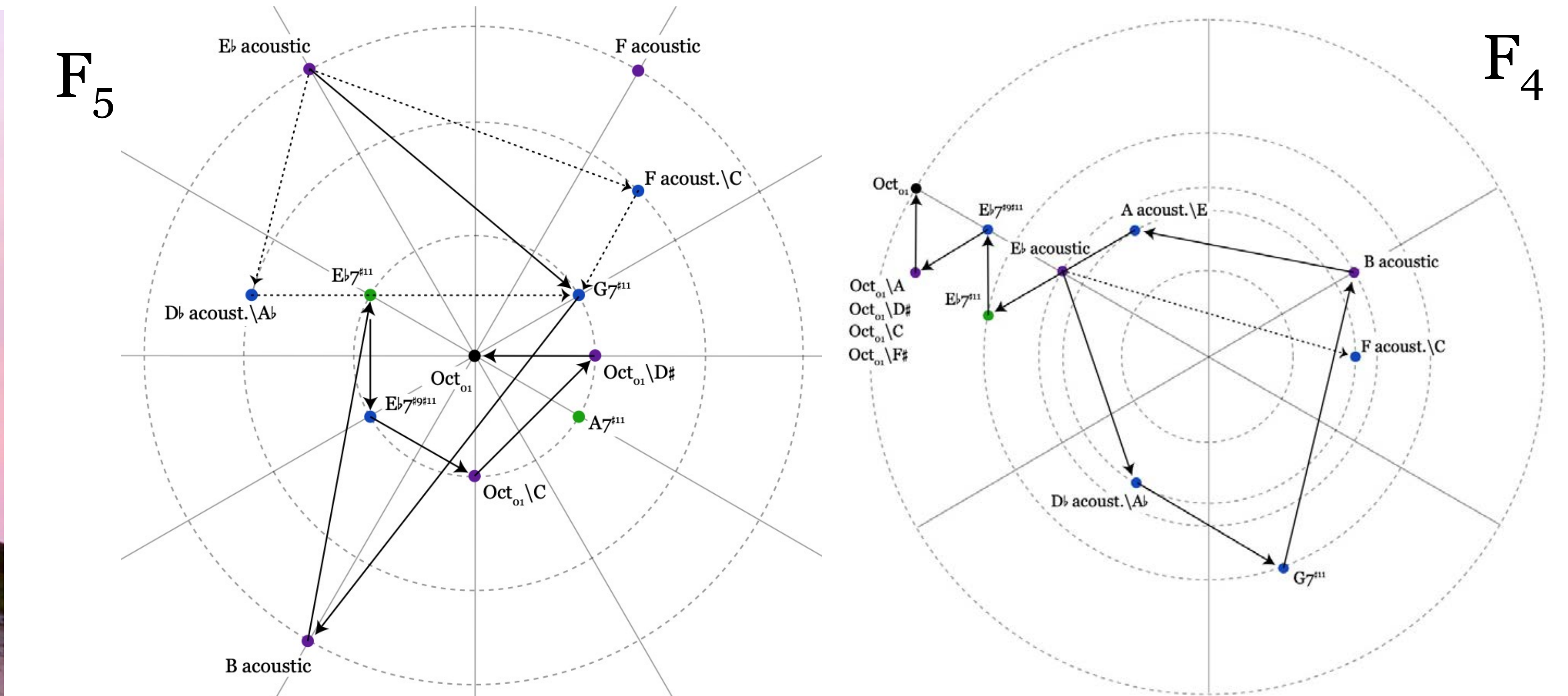
Harmonic reduction



Meas. 1–8

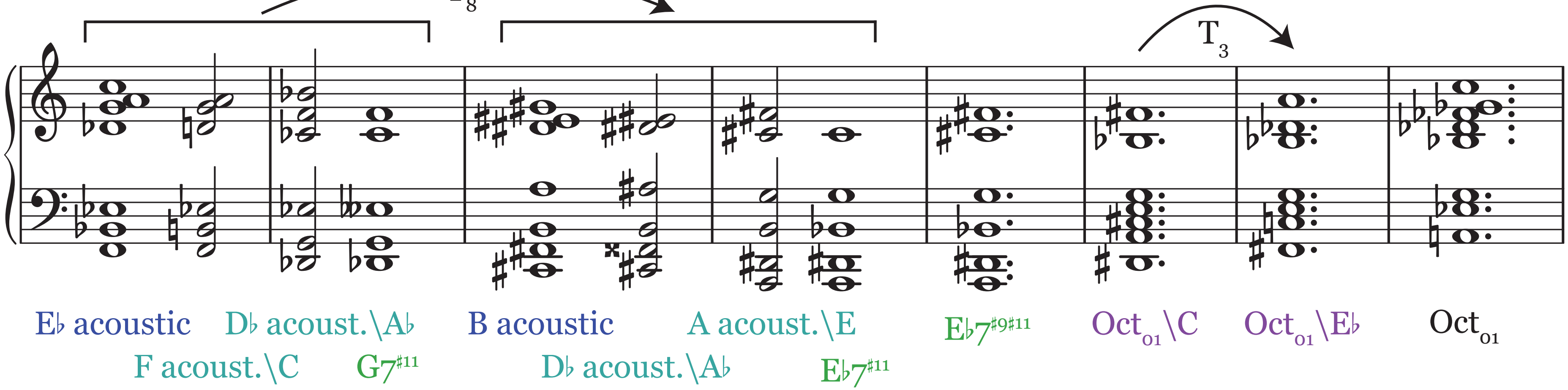


T8
T3
E♭ acoustic
D♭ acoust.\A♭
B acoustic
A acoust.\E
E♭7♯9♯11
Oct01\C
Oct01\E♭
Oct01
F acoust.\C
G7♯11
D♭ acoust.\A♭
E♭7♯11

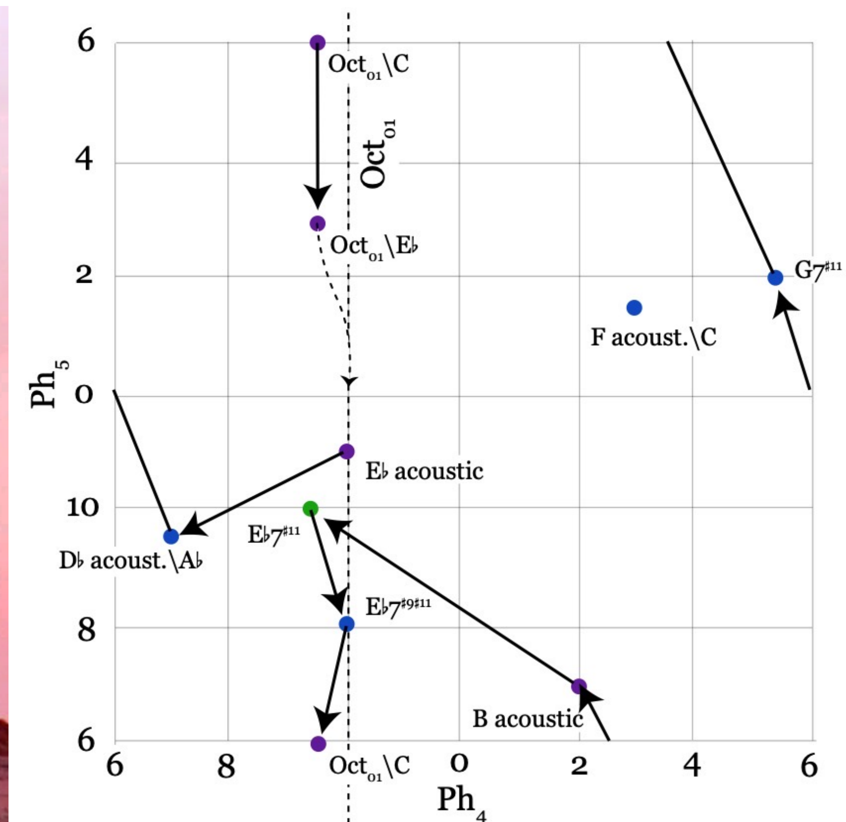


F5
E♭ acoustic
F acoustic
F acoust.\C
E♭7♯11
D♭ acoust.\A♭
G7♯11
Oct01
Oct01\D♯
E♭7♯9♯11
A7♯11
Oct01\C
B acoustic
F4
Oct01
E♭7♯9♯11
A acoust.\E
E♭ acoustic
B acoustic
Oct01\A
Oct01\D♯
Oct01\C
Oct01\F♯
E♭7♯11
F acoust.\C
D♭ acoust.\A♭
G7♯11

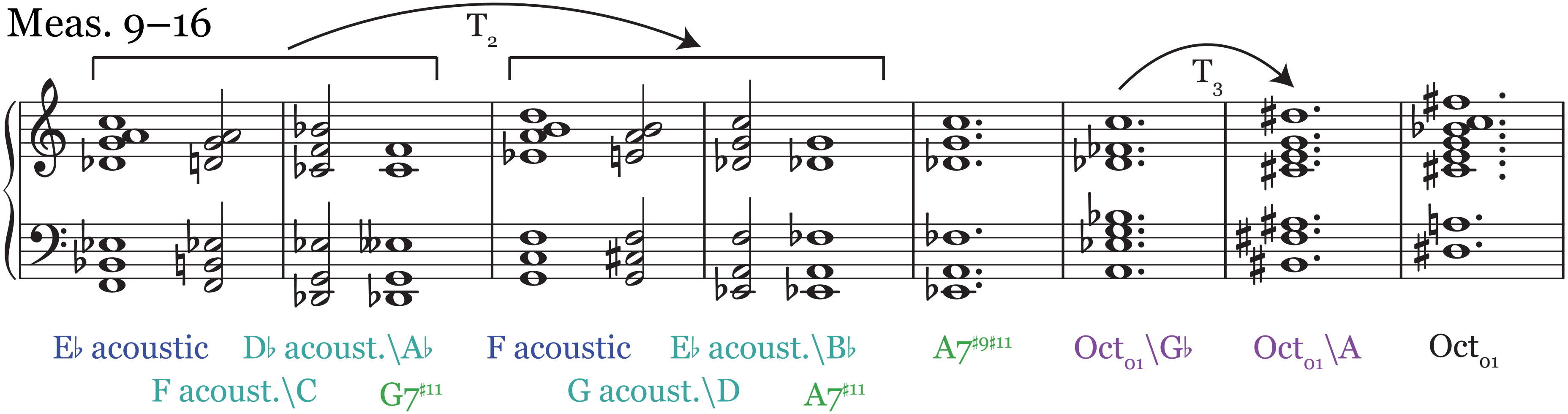
Meas. 1–8

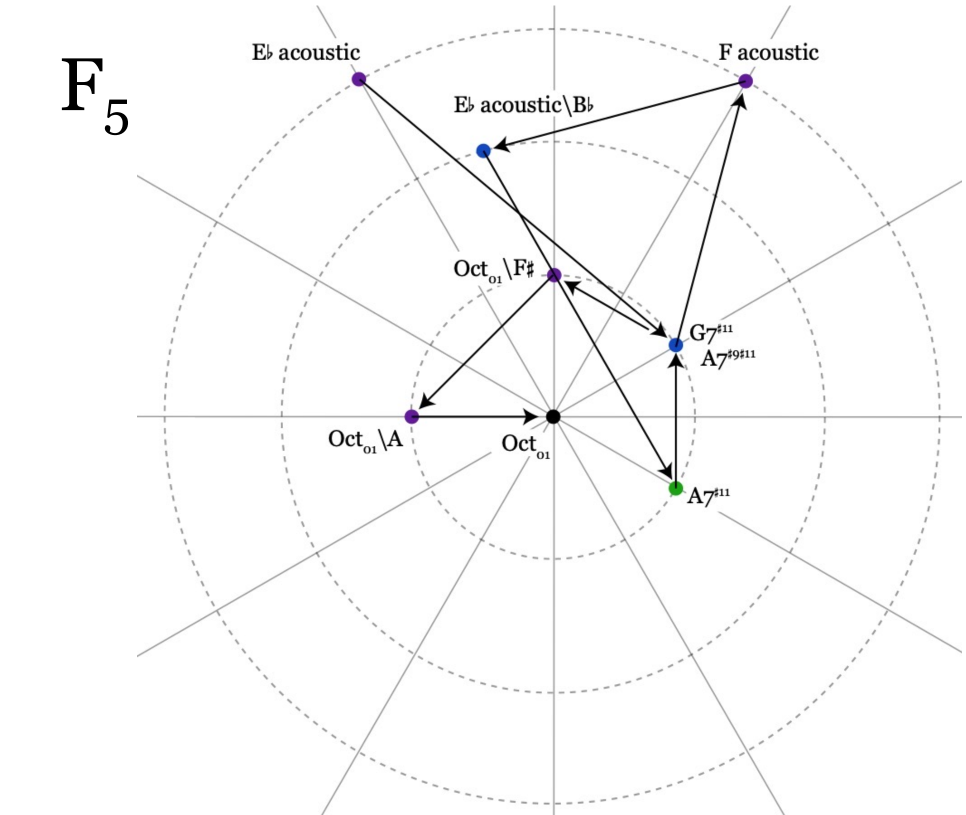


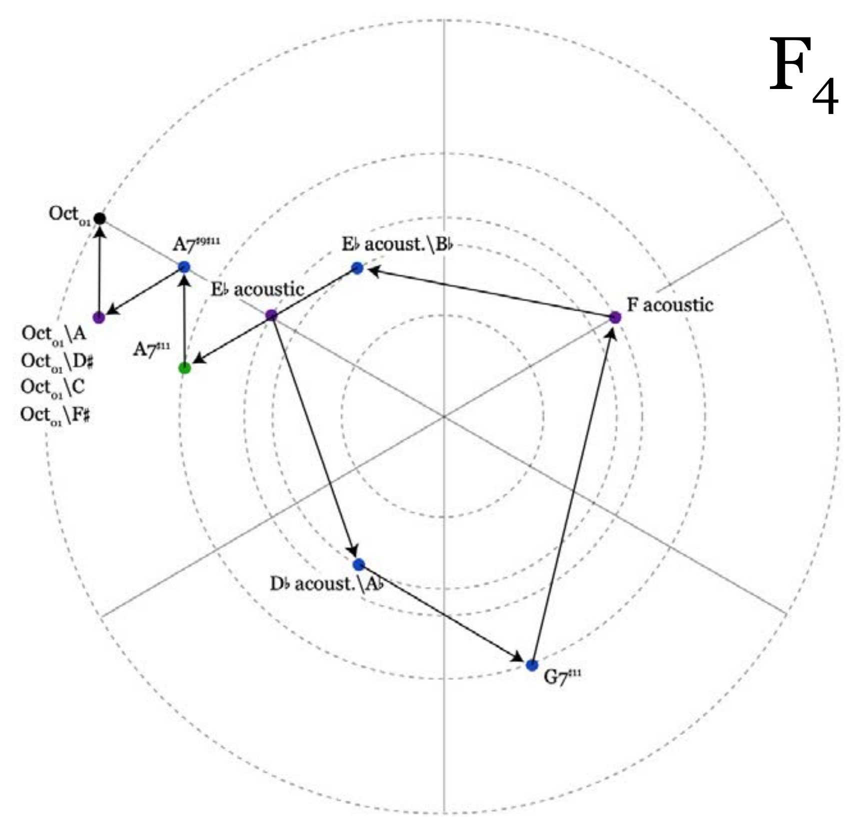
F_4/F_5 phase space

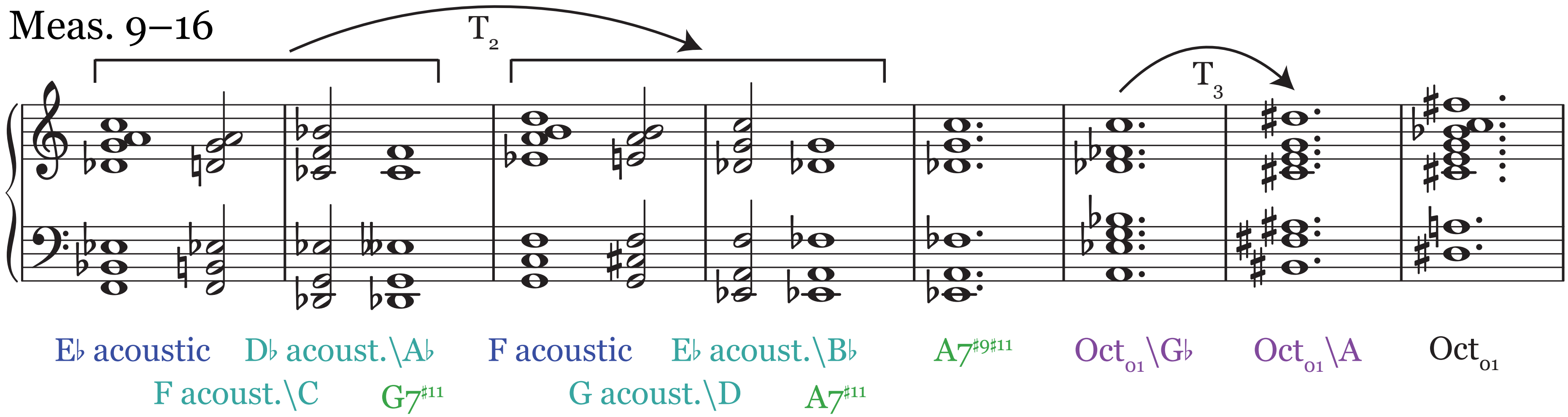


The antecedent begins with a double cycle returning to the E♭ region, followed by a consolidation to the "tonic" Ph_4 value.

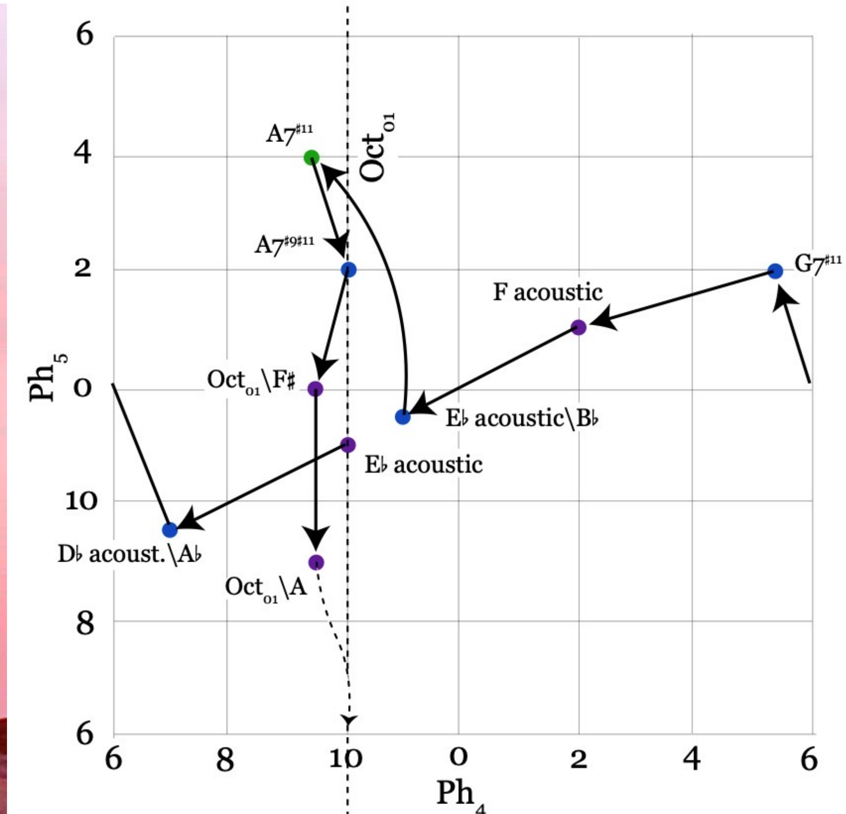

Meas. 9–16
T_2
T_3
E♭ acoustic
D♭ acoust.\A♭
F acoustic
E♭ acoust.\B♭
$A7^{\sharp9\sharp11}$
Oct_{01}\G♭
Oct_{01}\A
Oct_{01}
F acoust.\C
$G7^{\sharp11}$
G acoust.\D
$A7^{\sharp11}$


F_5
E♭ acoustic
F acoustic
E♭ acoustic\B♭
Oct_{01}\F♯
$G7^{\sharp11}$
$A7^{\sharp9\sharp11}$
Oct_{01}\A
Oct_{01}
$A7^{\sharp11}$


F_4
Oct_{01}
$A7^{\sharp9\sharp11}$
E♭ acoust.\B♭
E♭ acoustic
F acoustic
Oct_{01}\A
Oct_{01}\D♯
Oct_{01}\C
Oct_{01}\F♯
$A7^{\sharp11}$
D♭ acoust.\A♭
$G7^{\sharp11}$



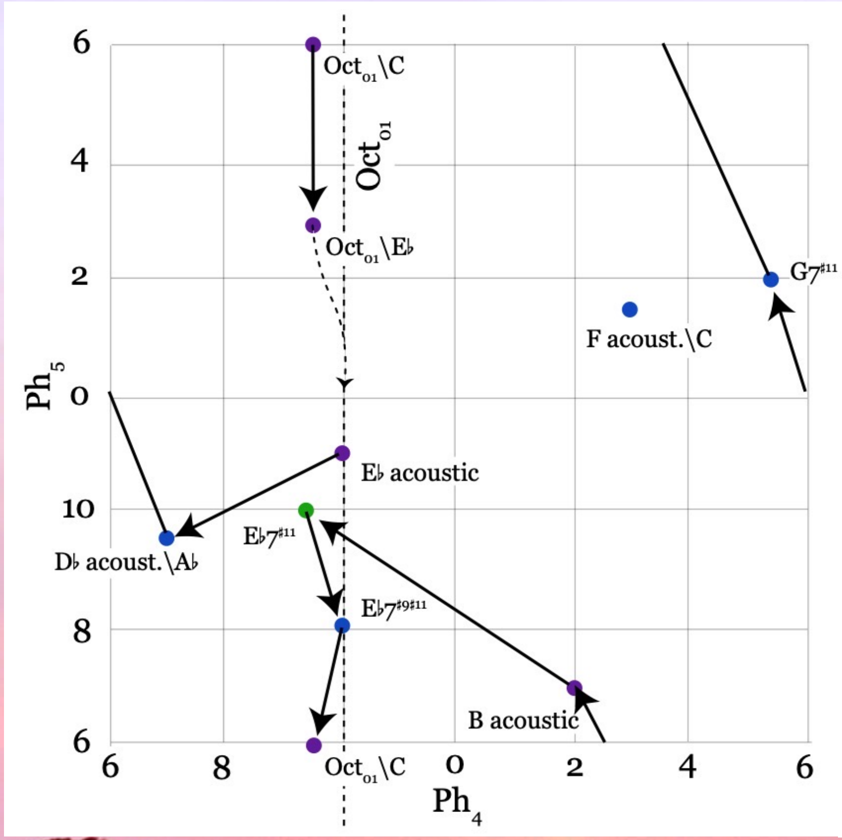
F_4/F_5 phase space



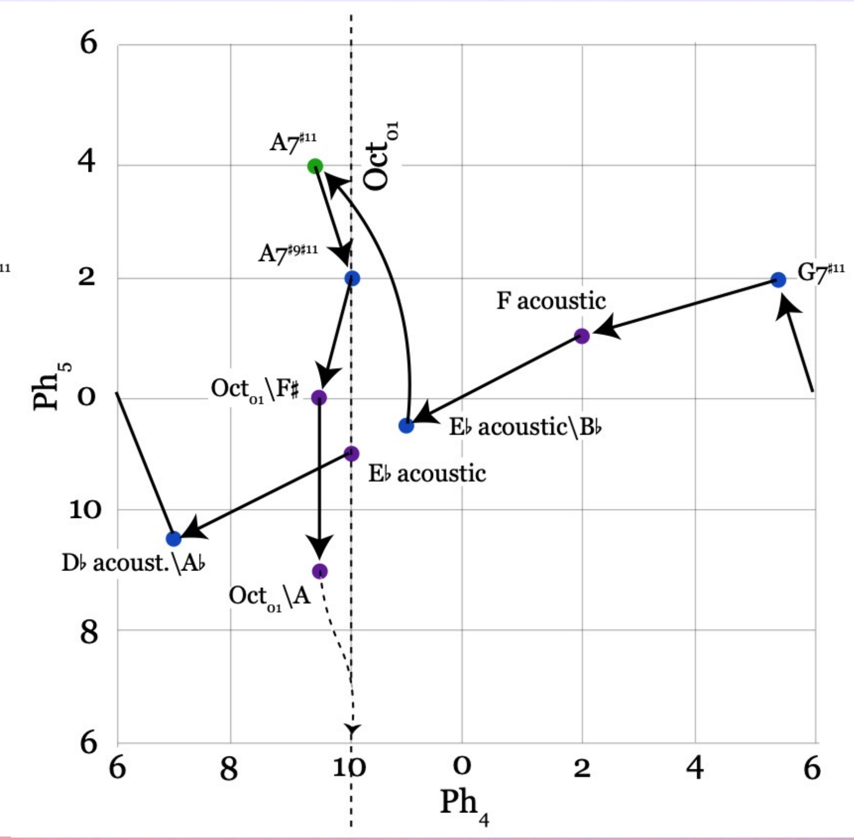
The consequent begins with the same Ph_4 cycle eliminating the enharmonic cycle. The f_4 consolidation now occurs in the diatonic vicinity to E♭ acoustic.

Phase space plots

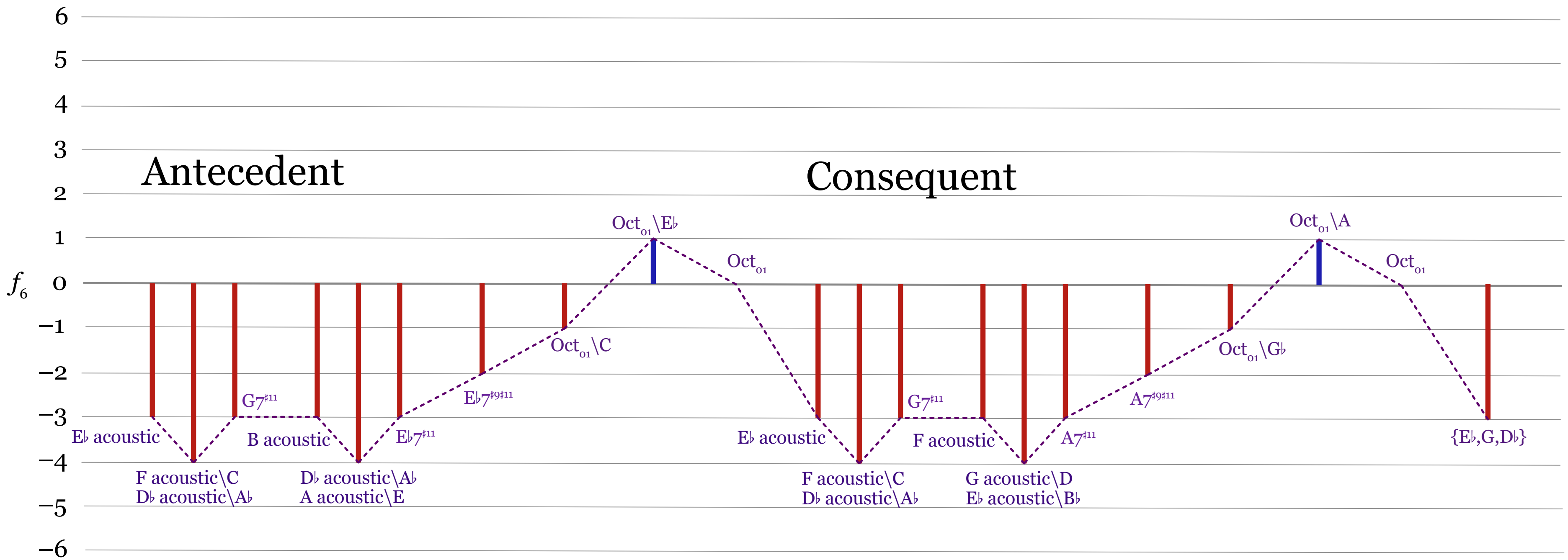
Meas. 1–8



Meas. 9–16



F^6 (whole-tone) plot



Some conclusions

- Conventional tonality is based on f_3 and f_5. A phase space on these shows relationships between keys and harmonies.
- Other Fourier components were therefore a resource for expanded tonality in the early 1900s. The most attractive alternatives were f_4 and f_6.
- The organized use of f_4 and f_6 (and manipulation of f_5) explains the sonorities and progressions that characterize the harmonic languages of Debussy and Scriabin.

Sailing Off the Edge of Tonality: Debussy and Scriabin's Harmonic Adventurism Explained with Harmonic Spectra and Tonal Spaces

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You can download my slides at http://people.bu.edu/jyust

Thanks!