Restoring the Structural Status of Keys through DFT Phase Space

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A copy of this talk is available at
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Outline

I. Schenker, Brahms, and Keys: The Problem

1. Schenker's analysis of Brahms's F major Cello Sonata
2. Structure as long-range voice leading
3. An alternative: DFT phase space

II. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

III. Beethoven's *Heiliger Dankgesang*

1. Scalar context and triadic orbits
2. The D–C motive
3. Strength and weakness

I. Schenker, Brahms, and Keys

1. Schenker's analysis of Brahms's F major Cello Sonata
2. Structure as long-range voice leading
3. An alternative: DFT Phase Space

Schenker, Brahms, and Keys

A Music-Analytic Problem with a Mathematical Solution

The Schenkerian Syllogism

False!

~~*Hidden Premise:* Voice leading is always a relationship between individual harmonies.~~

Premise: Long-range structure is contrapuntal (based on voice-leading).

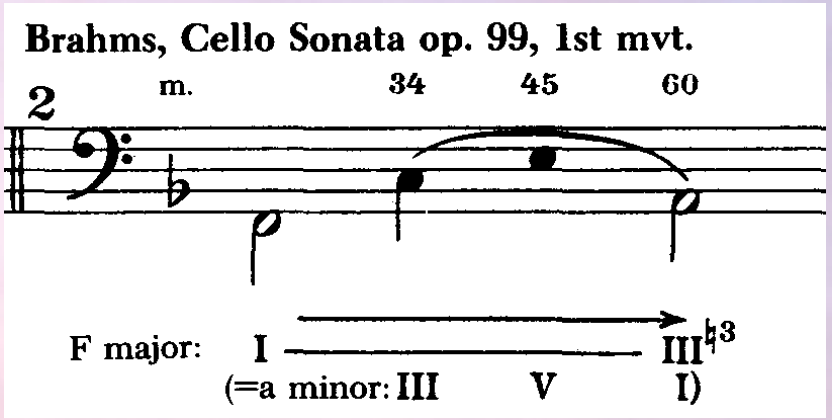
Conclusion: Long-range structure is *reductive*—i.e., it consists of relationships between non-adjacent harmonies.

Problematic

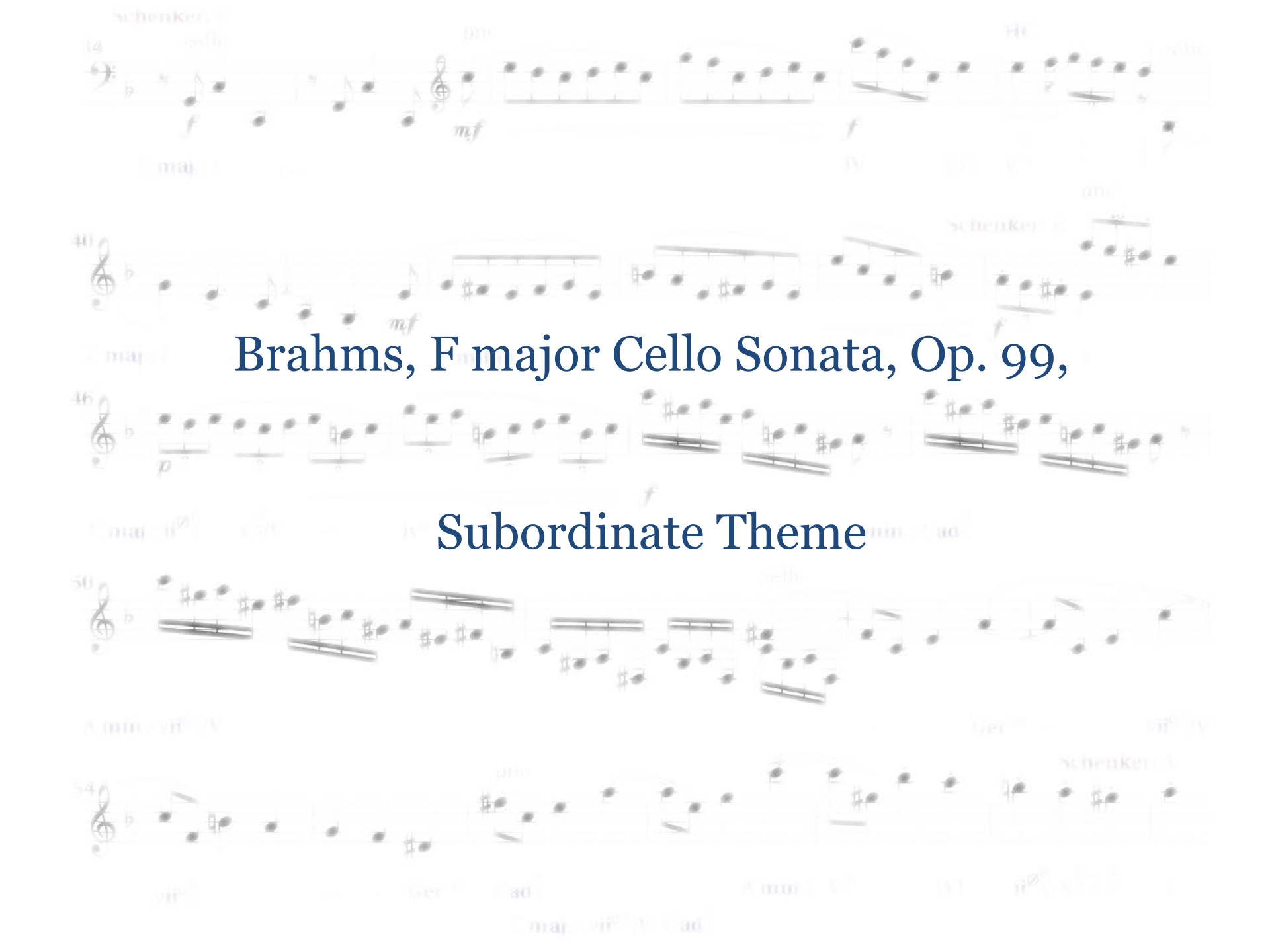
Schenker, Brahms, and Keys

A Music-Analytic Problem with a Mathematical Solution

So what's wrong with reductive analysis?!?

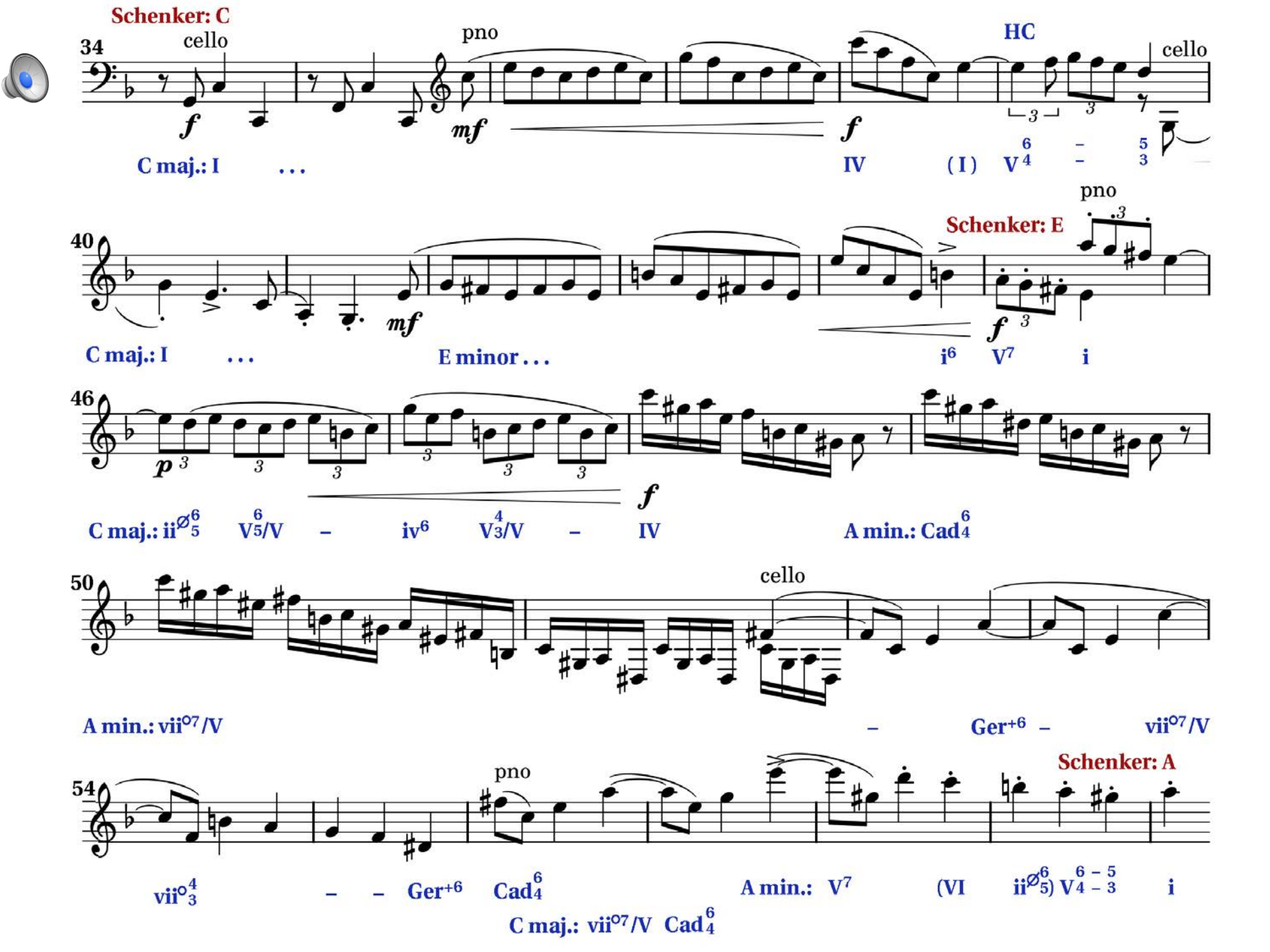


Schenker: *Der Freie Satz* Fig. 110d(2)

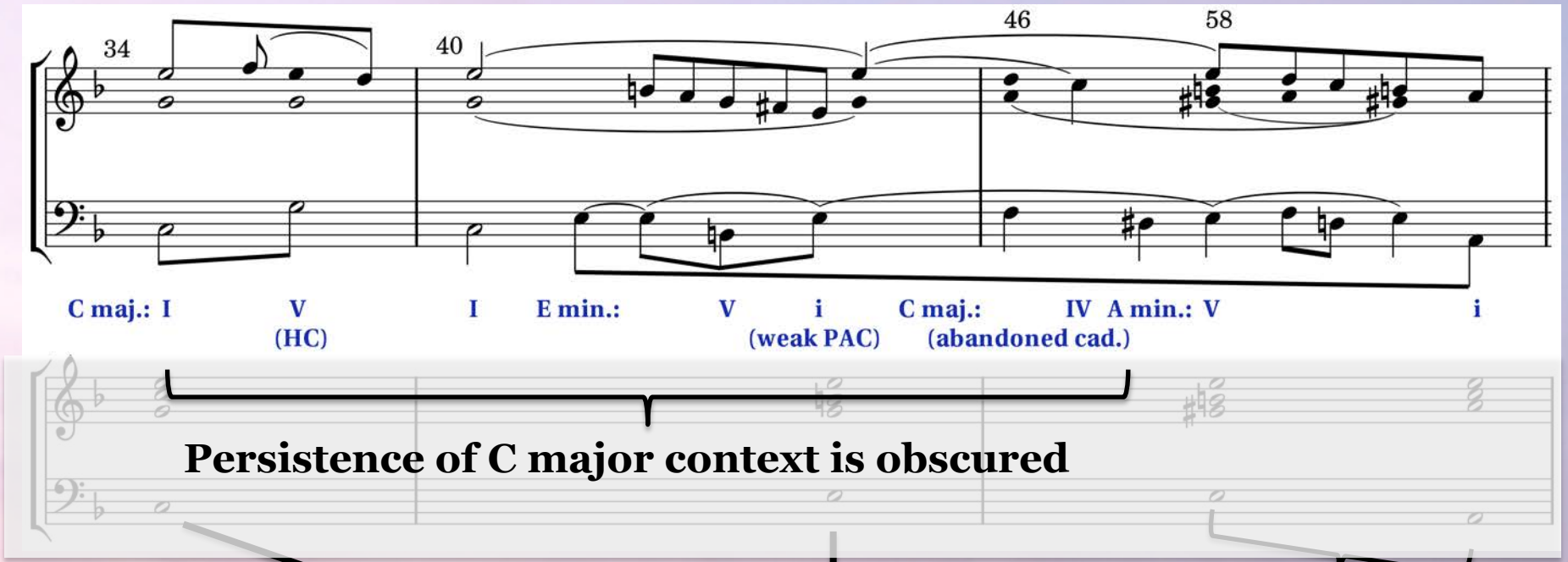


Brahms, F major Cello Sonata, Op. 99,

Subordinate Theme


Schenker: C
34
cello
pno
HC
cello
C maj.: I . . .
IV (I) V 6 4 – 5 3
40
pno
Schenker: E
C maj.: I . . .
E minor . . .
i6 V7 i
46
C maj.: ii ø 6 5 V 6 5/V – iv6 V 4 3/V – IV
A min.: Cad 6 4
50
cello
A min.: vii o7 /V
– Ger+6 – vii o7 /V
54
pno
Schenker: A
vii o 4 3 – – Ger+6 Cad 6 4
A min.: V7 (VI ii ø 6 5) V 6 – 5 4 – 3 i
C maj.: vii o7 /V Cad 6 4

Schenker, Brahms, and Keys



Persistence of C major context is obscured

Structure is based on *large-scale voice leadings*,
which must occur between **distinct musical objects (*chords*).**
This leads to a ***reductive*** approach.
Cadential points must take precedence as structural events

Schenker, Brahms, and Keys

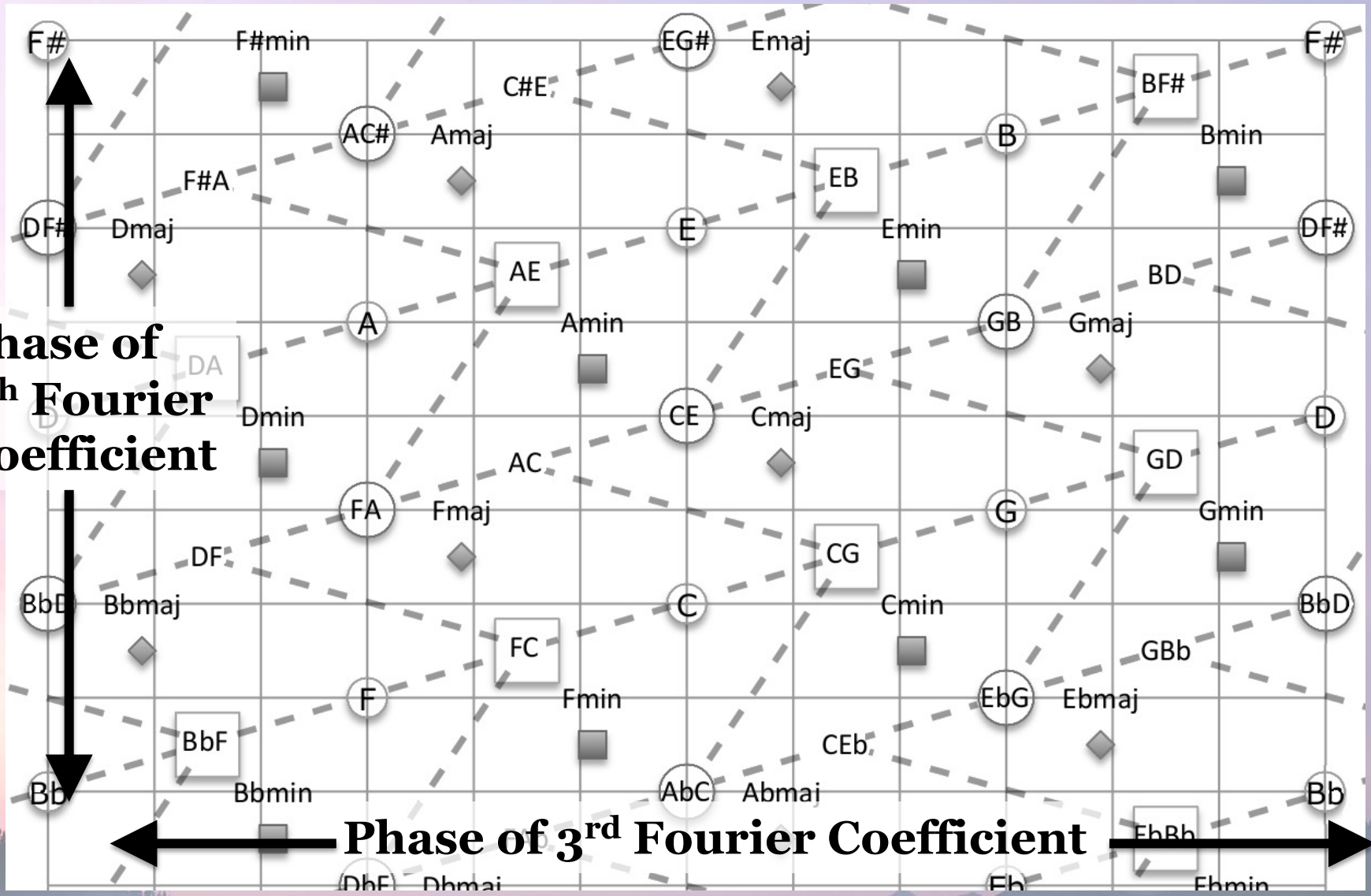
Alternative:
A spatial concept of tonality . . .

DFT phase space

Amiot, Emmanuel. (2013). "The Torii of Phases," *Mathematics and Computation in Music, MCM 2013* (ed. Yust, Wild, & Burgoyne) 1–18.

Yust, "Schubert's Harmonic Language and Fourier Phase Space." *Journal of Music Theory* (forthcoming). Available at http:/people.bu.edu/jyust/SchubertDFT.pdf

Schenker, Brahms, and Keys



Schenker, Brahms, and Keys

Properties of DFT Phase Space

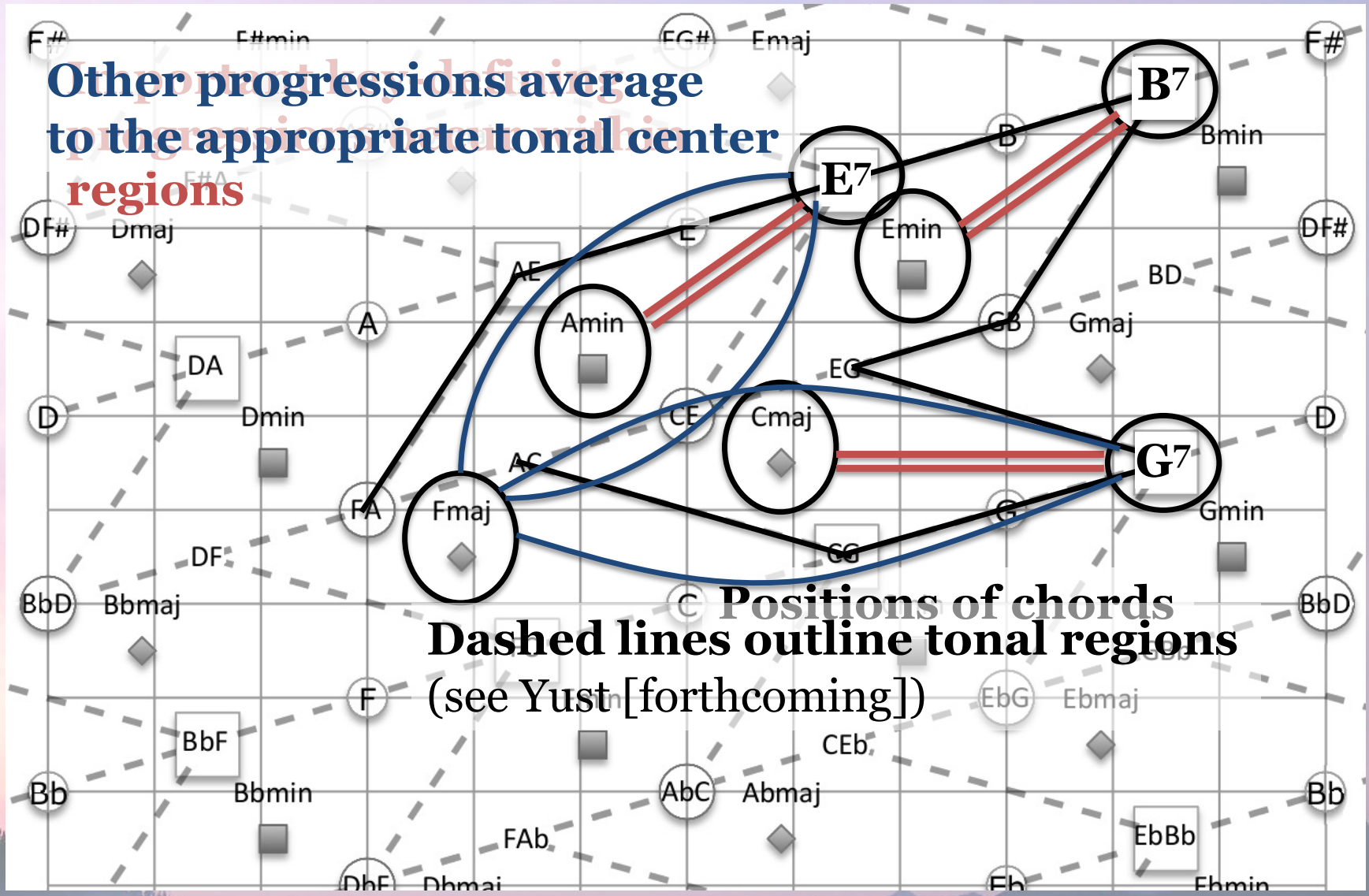
- Objects are pcsets, multisets, or statistical pc distributions
- Toroidal geometry
- Vertical axis indicates circle-of-fifths position
- Horizontal axis captures triadic voice-leading properties
- Many kinds of harmonic objects exist in the space: single pcs, harmonies, scales, etc.

Schenker, Brahms, and Keys

Properties of DFT Phase Space

- Space is *continuous*—paths connect points via a potential infinite series of intermediate states (pc distributions)
- Spatial conceptualization of DFT is inspired by Mazzola's concept of **gesture.**
- Nearness in the space (its ***topology***) is based on *common pc content*.

Schenker, Brahms, and Keys



Schenker, Brahms, and Keys

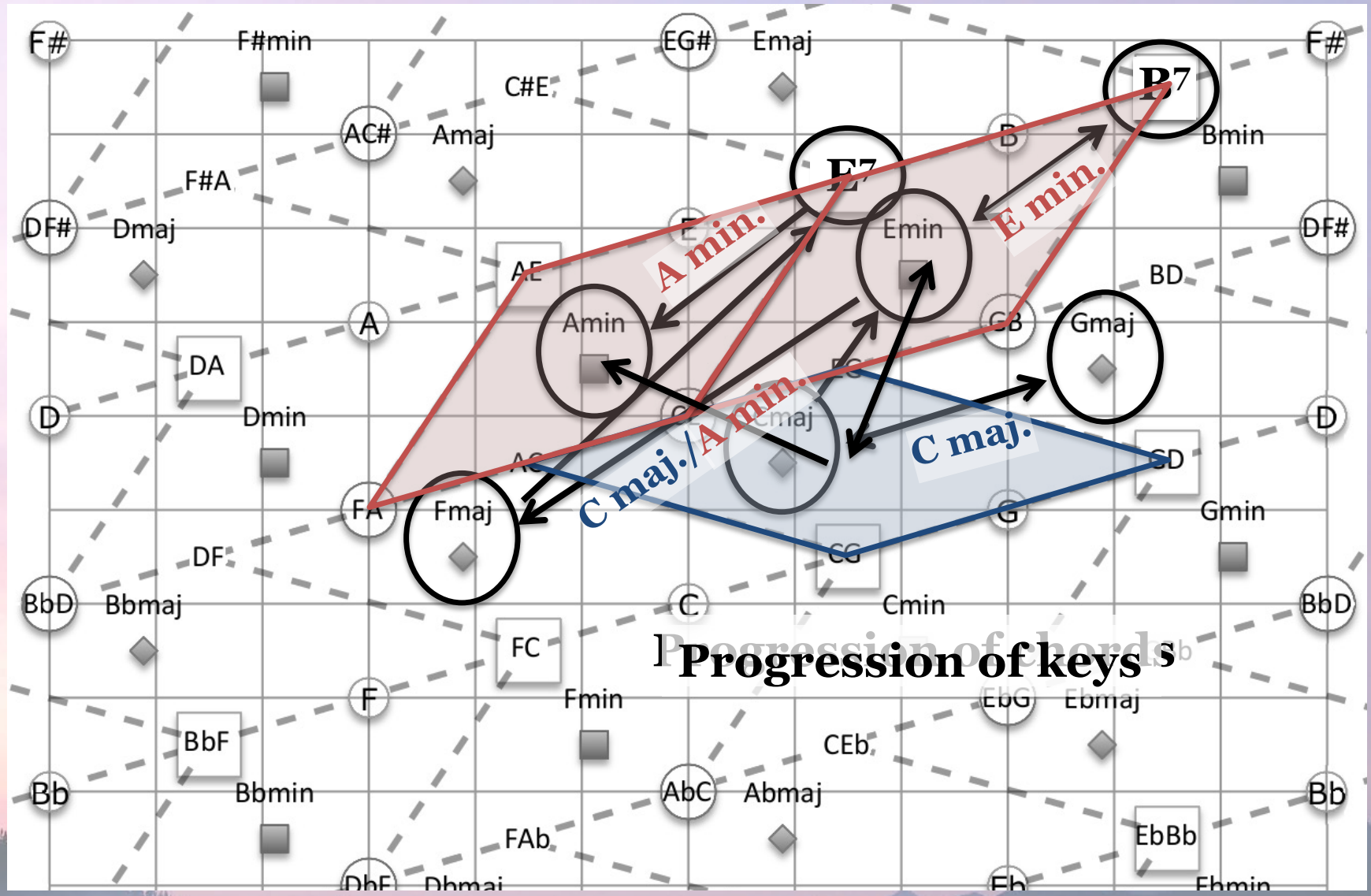
Properties of DFT Phase Space

A ***path*** can represent a motion from A to B, but it can also represent "B in the context of A."

Combination of pcsets is highly tractable: the position of $A + B$ is easily predictable from the path $A \rightarrow B$.

Averaging over *more objects* (pcs, triads) restricts the range of activity.

Schenker, Brahms, and Keys



2. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

Discrete Fourier Transform on Pcsets

Lewin, David (1959). "Re: Intervallic Relations between Two Collections of Notes," *JMT* 3/2.

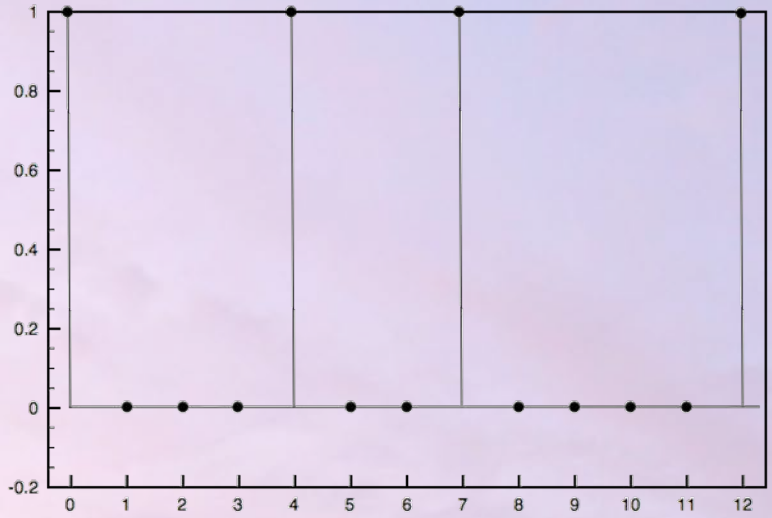
——— (2001). "Special Cases of the Interval Function between Pitch Class Sets X and Y." *JMT* 45/1.

Quinn, Ian (2006–2007). "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1.

Amiot, Emmanuel (2013). "The Torii of Phases." *Proceedings of the International Conference for Mathematics and Computation in Music, Montreal, 2013* (Springer).

DFT components

C major triad



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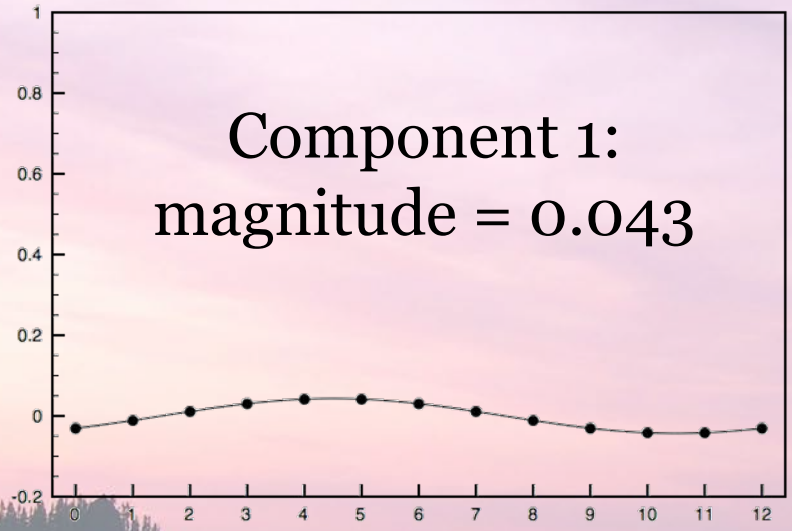

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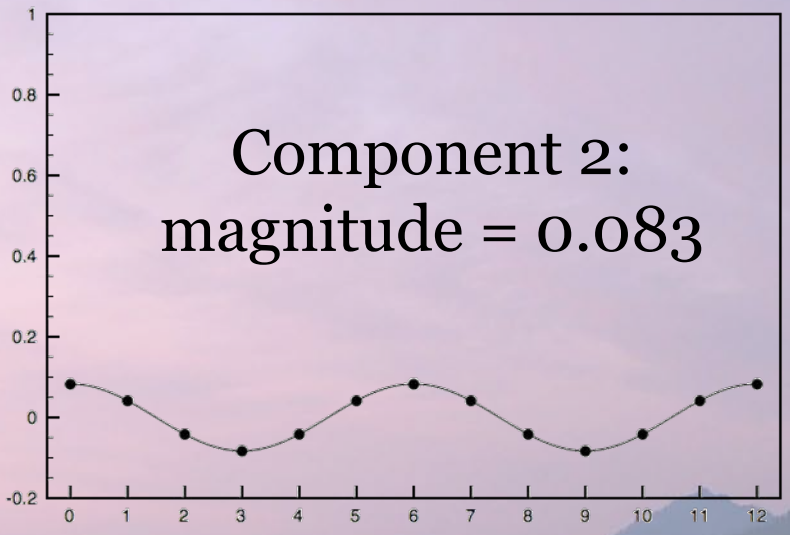

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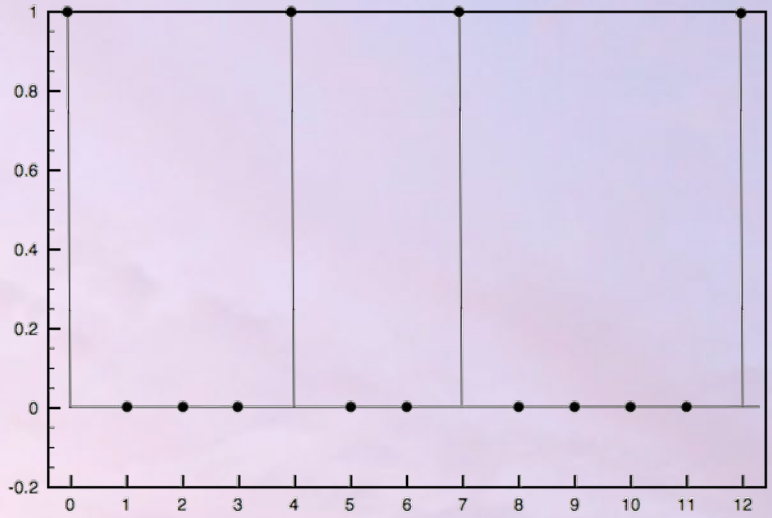
DFT components

C major triad



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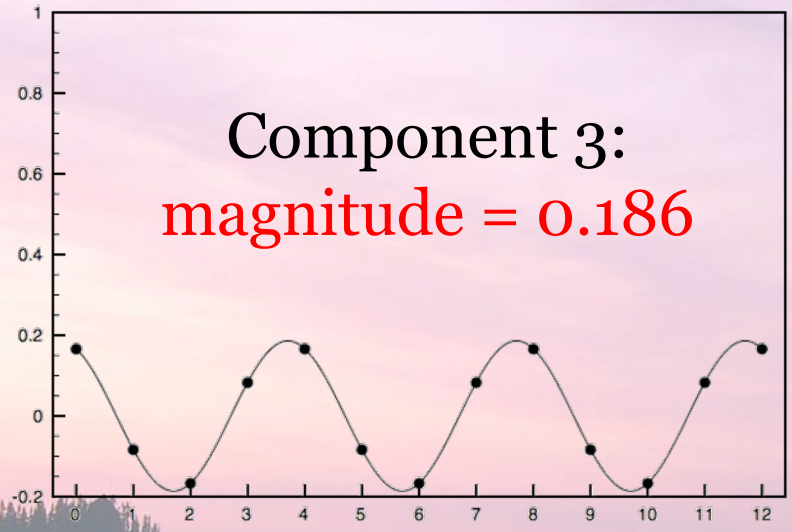

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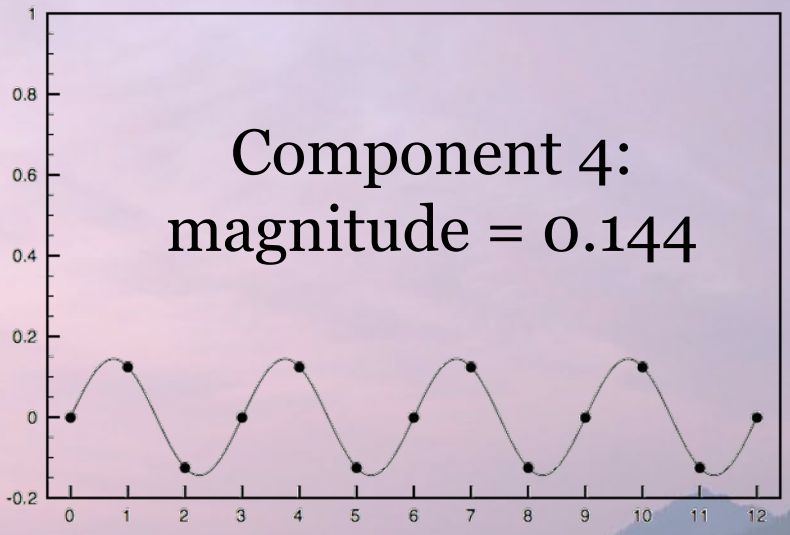

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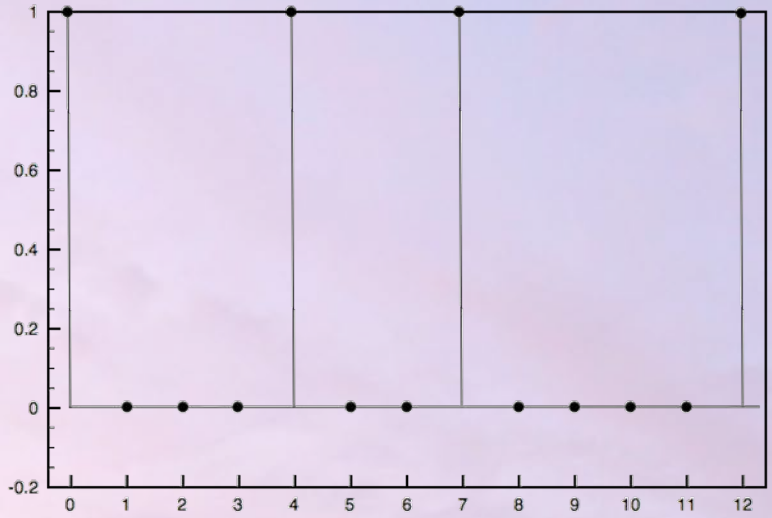
DFT components

C major triad



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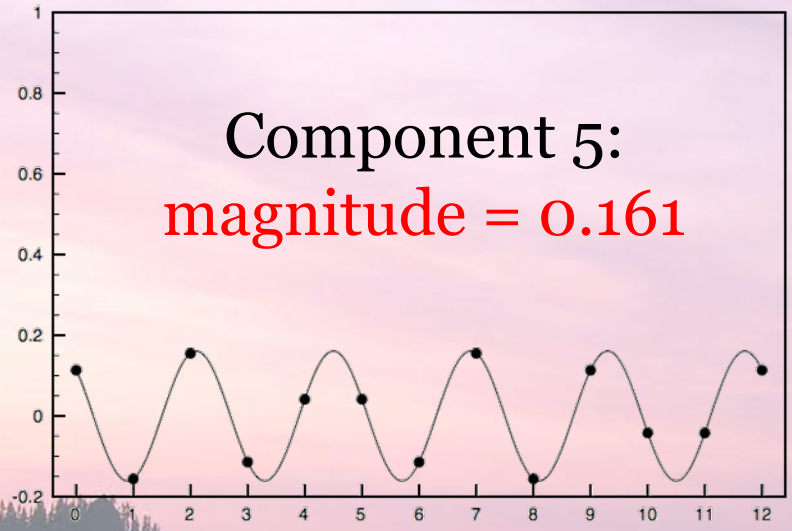
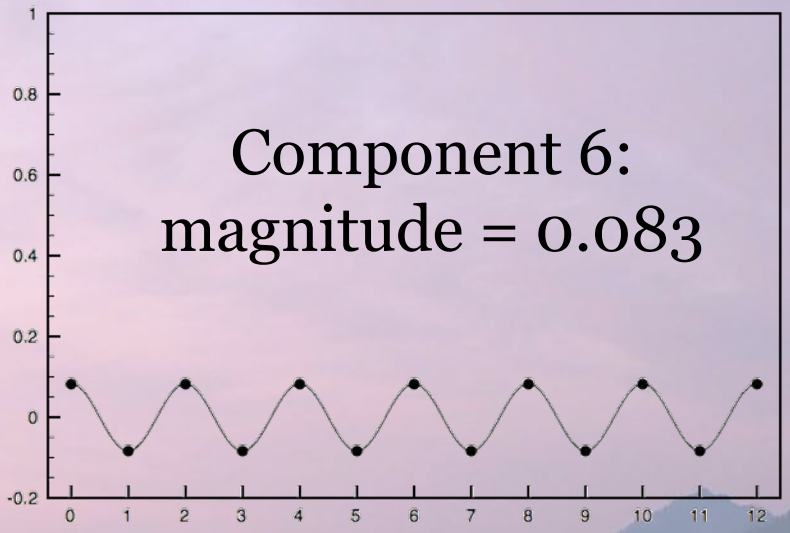


Component 5:
magnitude = 0.161

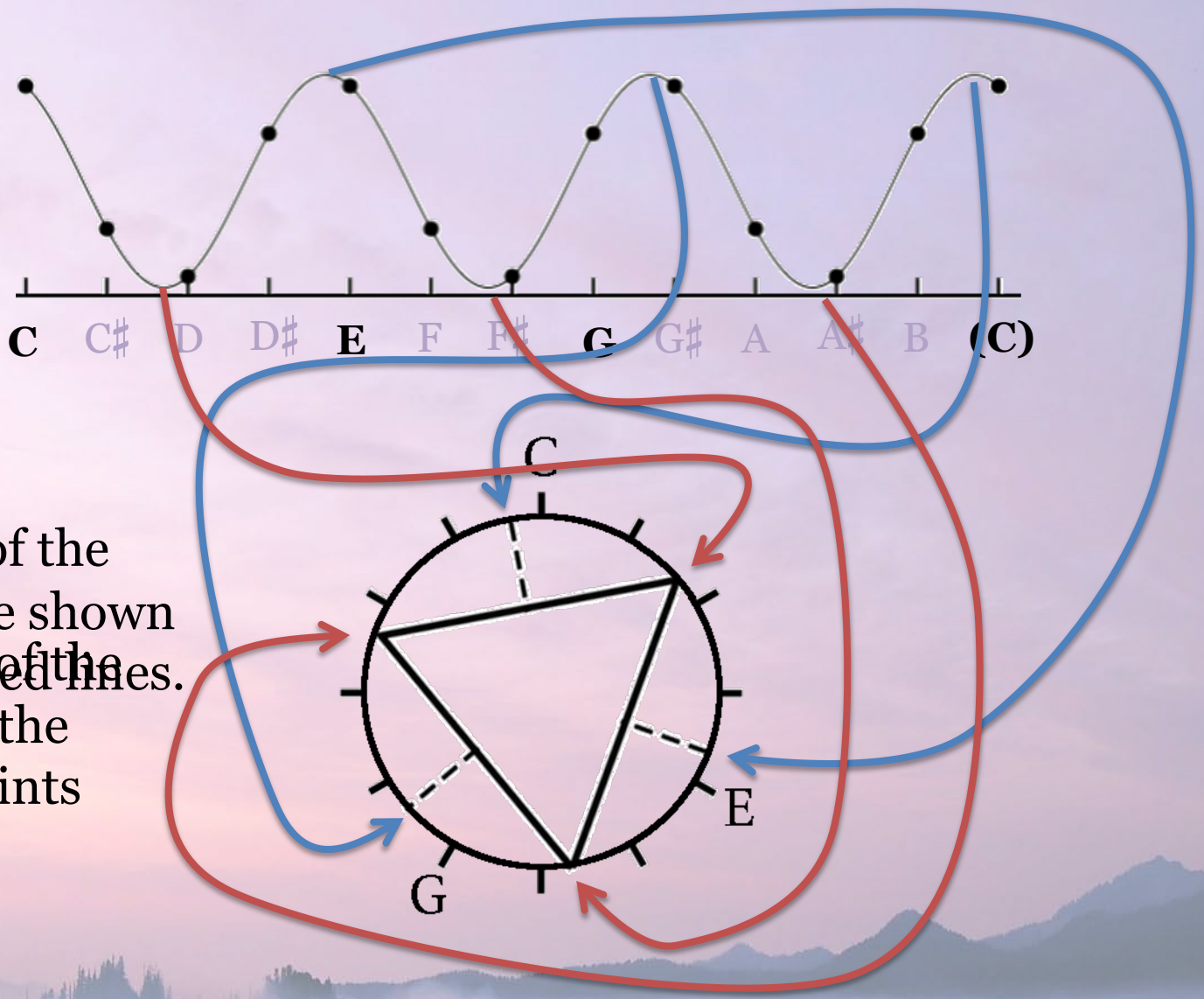
+



Component 6:
magnitude = 0.083

+ . . .

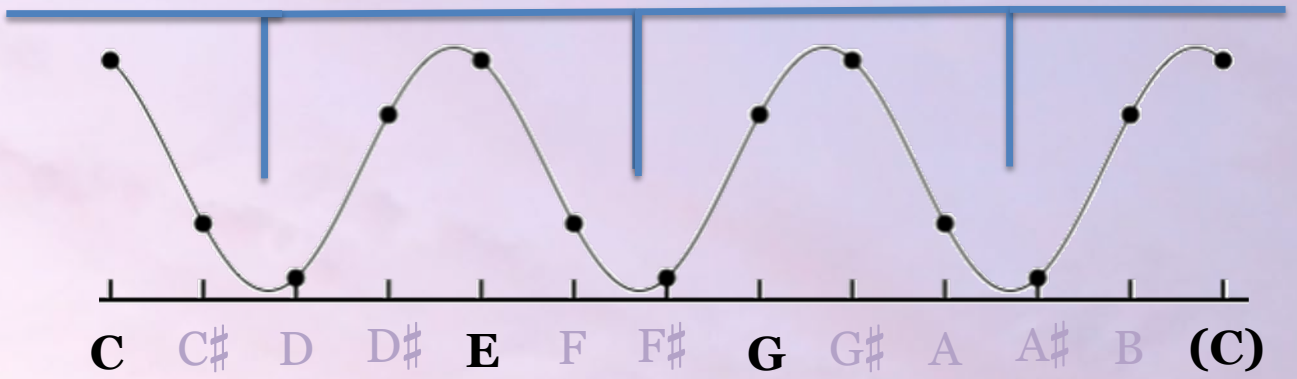
Triadic Orbits



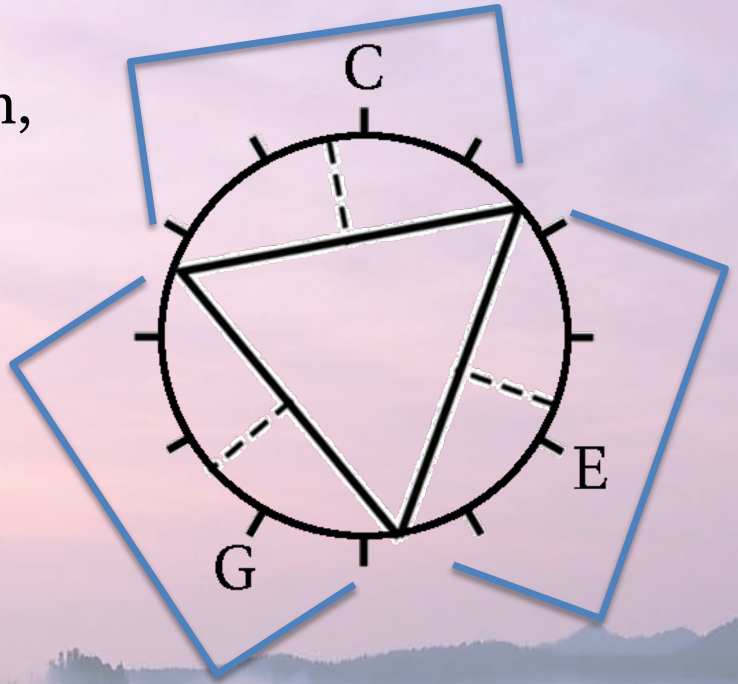
The peaks of the sinusoid are shown by the dashed lines.

The troughs of the sinusoid are the boundary points

Triadic Orbits

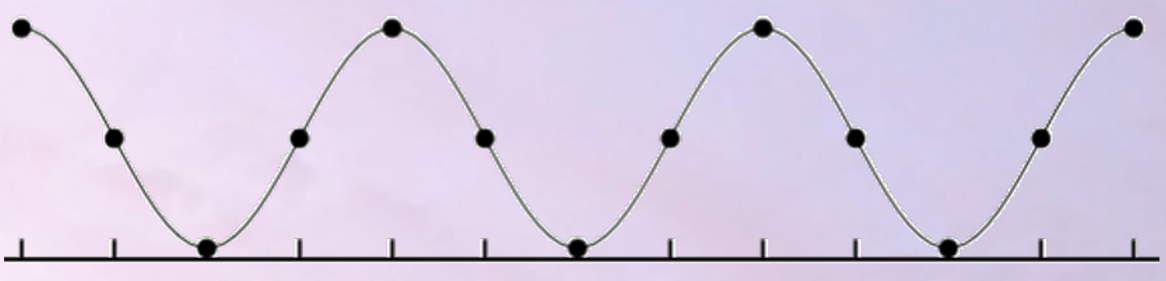


The triadic orbits go from trough to trough, and group pcs that may be considered displacements of those in the triad



Triadic Orbits

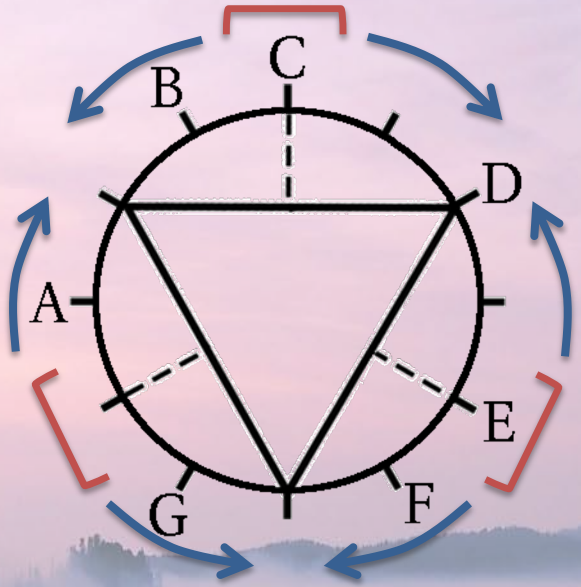
Any pc-distribution can define a set of triadic orbits, including **scales.**



C diatonic: C C♯ D D♯ E F F♯ G G♯ A A♯ B (C)

Regions of stability

Periphery: Unstable upper and lower neighbors



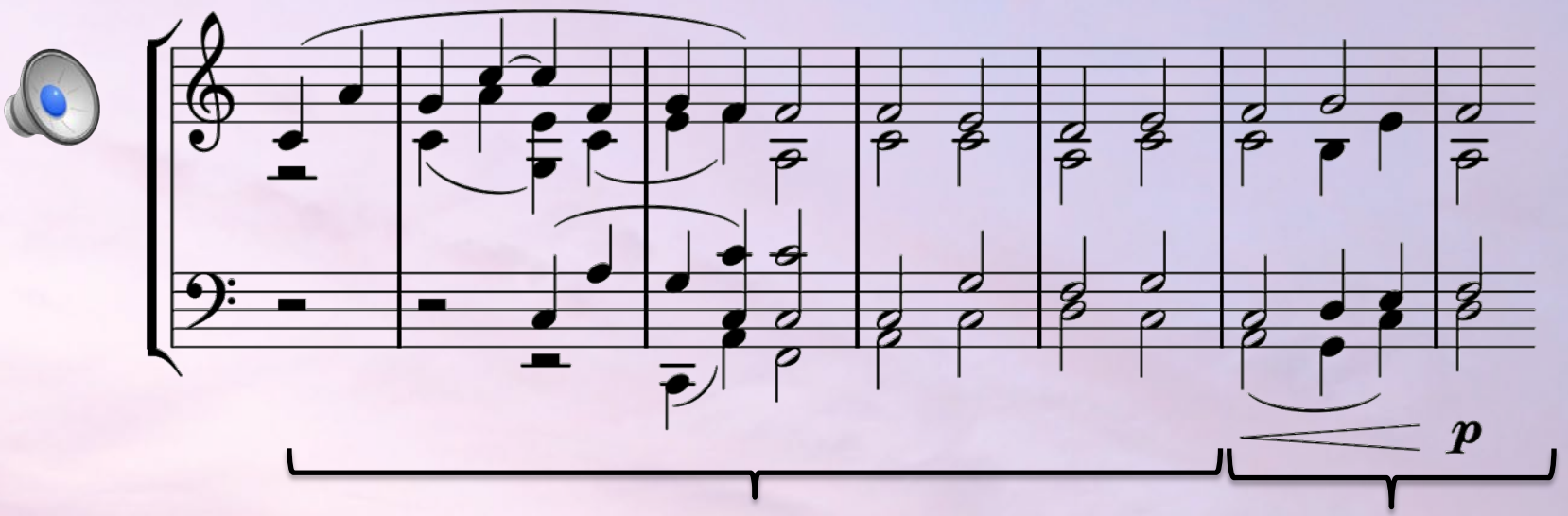
Pitch classes at the center of a region are more stable displacements

Pitch classes at the periphery of regions are unstable

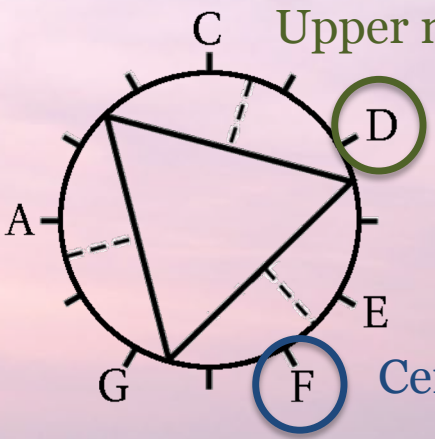
3. *Heiliger Dankgesang*

1. Scalar contexts and triadic orbits
2. The D–C motive
3. Strength and weakness

Heiliger Dankgesang: Scalar Contexts

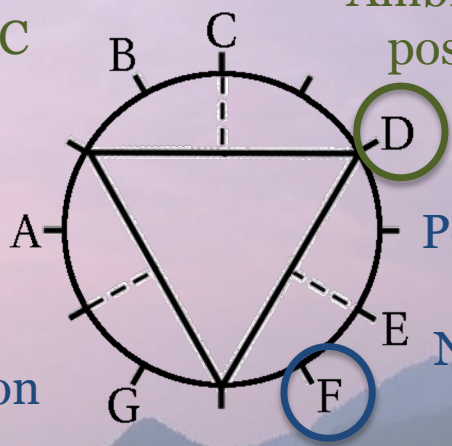


First intonation and chorale



Upper neighbor to C

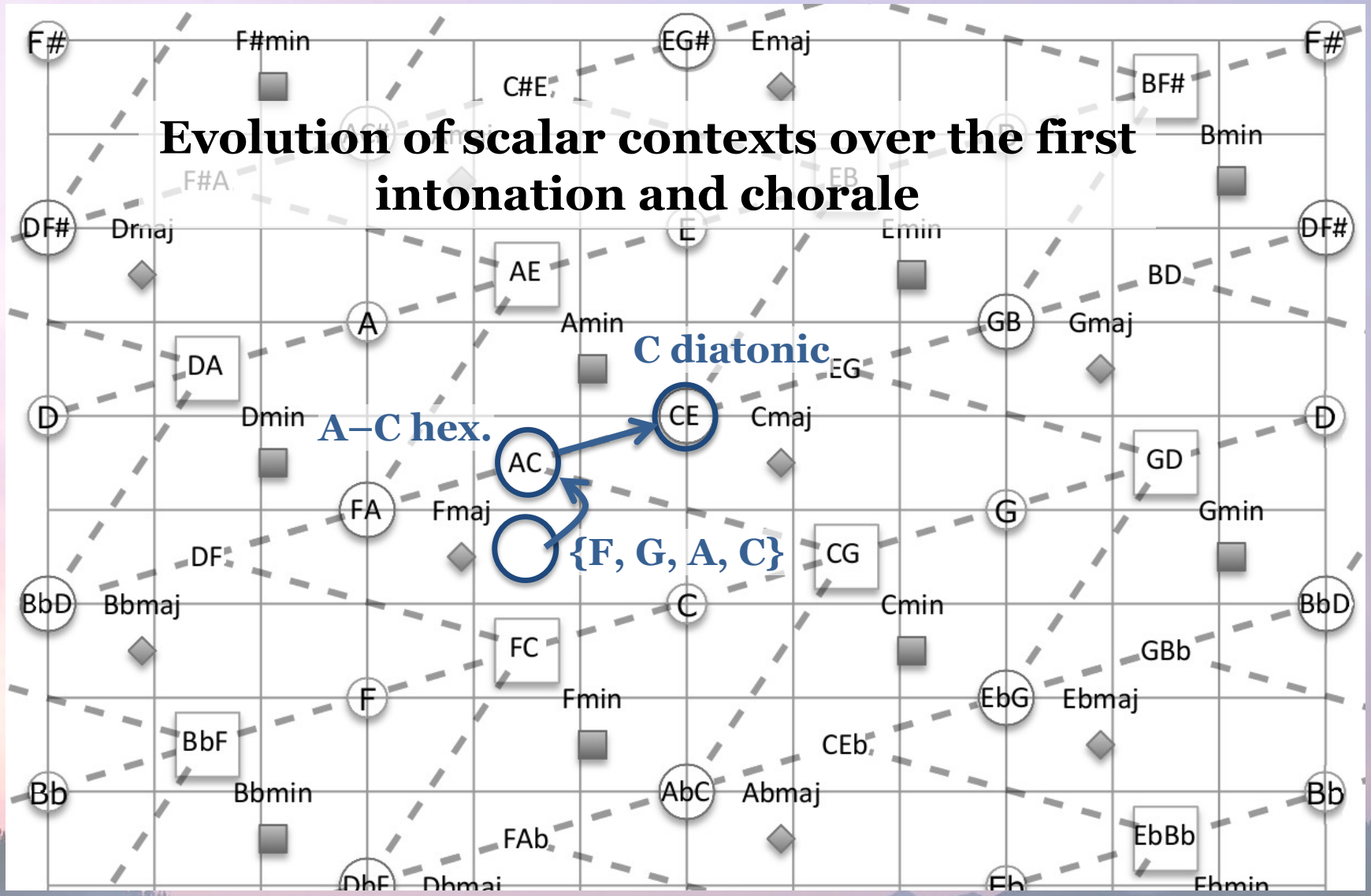
Central position



Ambiguous position

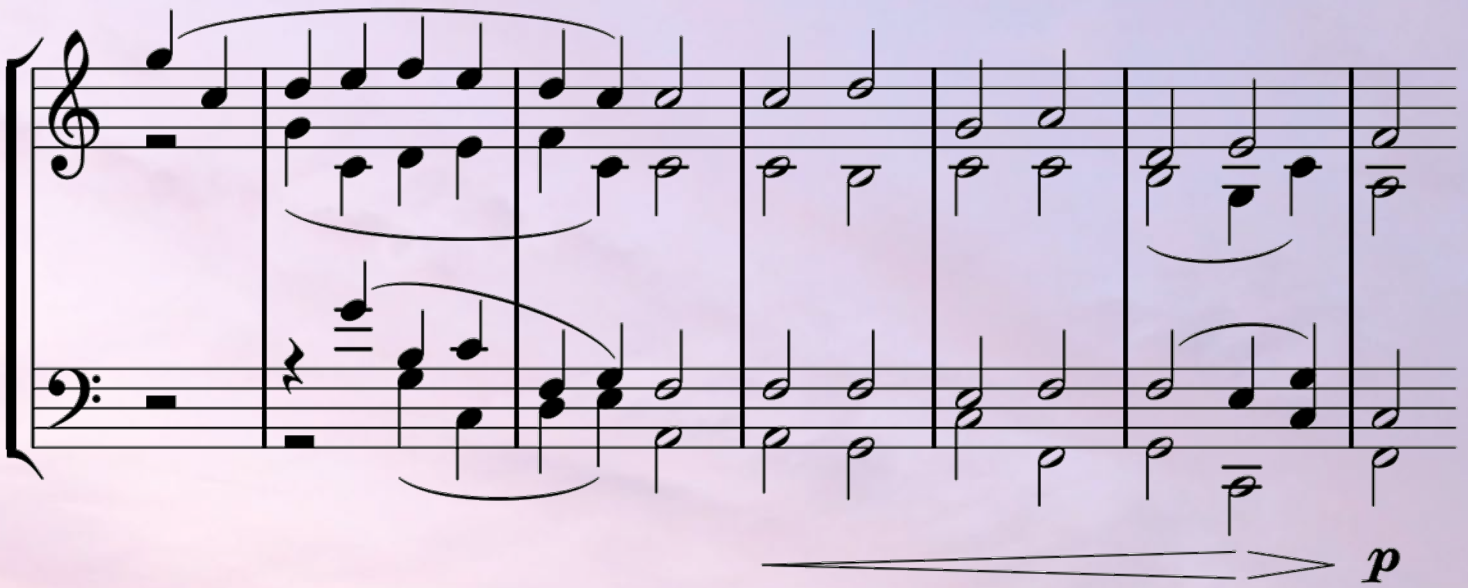
Peripheral Upper Neighbor

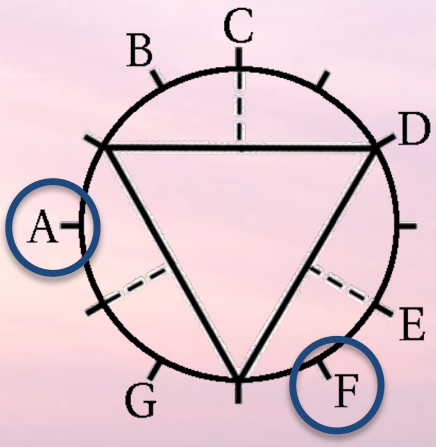
Heiliger Dankgesang: Scalar Contexts



Heiliger Dankgesang: Scalar Contexts

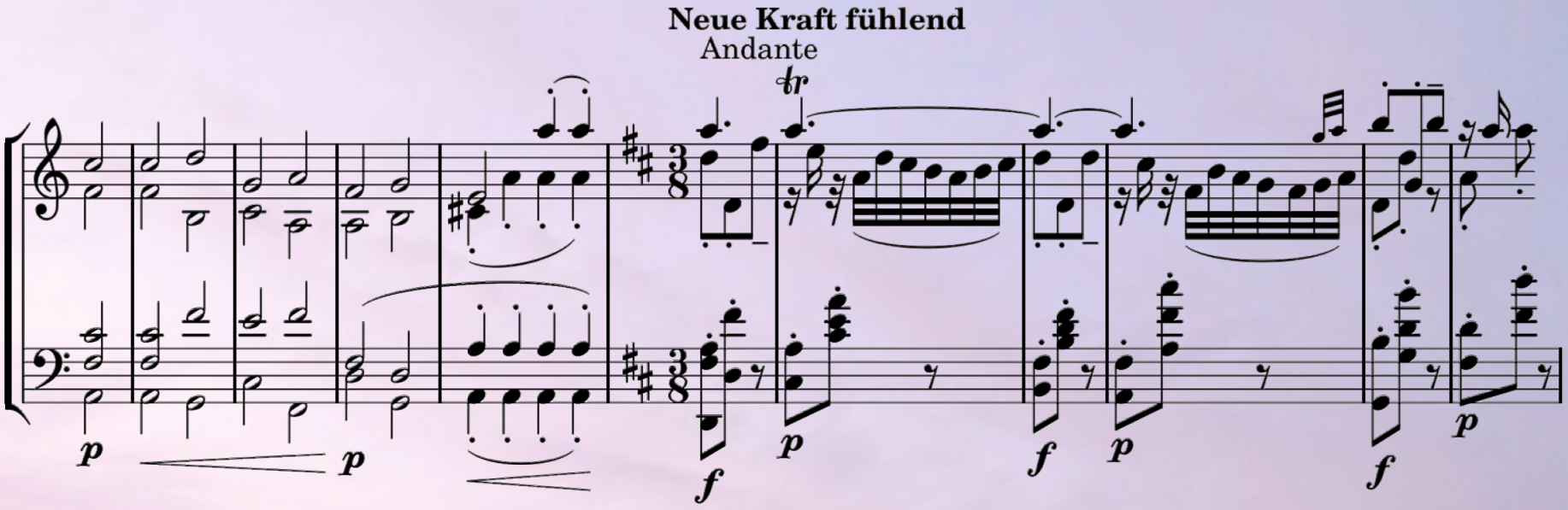
Chorale phrase 4:

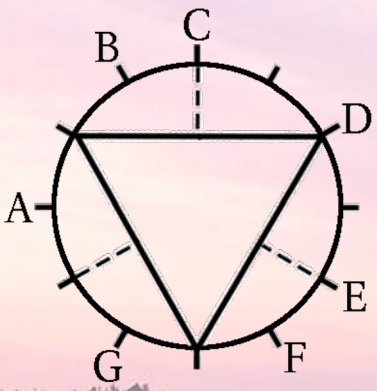




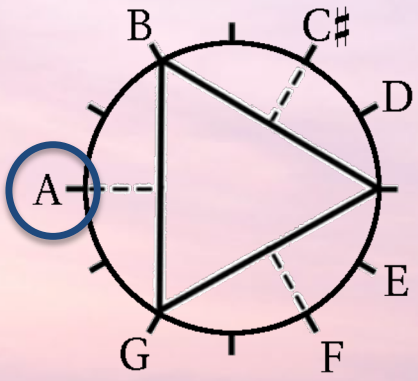
Ending high in the orbit gives the effect of suspension

Heiliger Dankgesang: Scalar Contexts

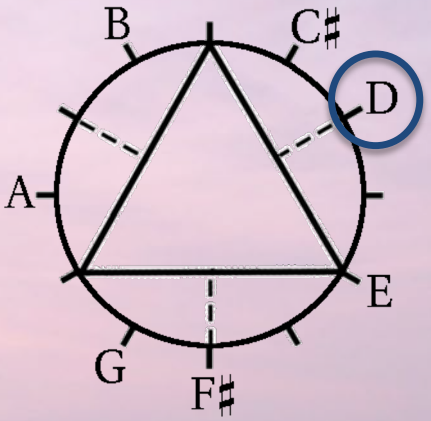




C diatonic



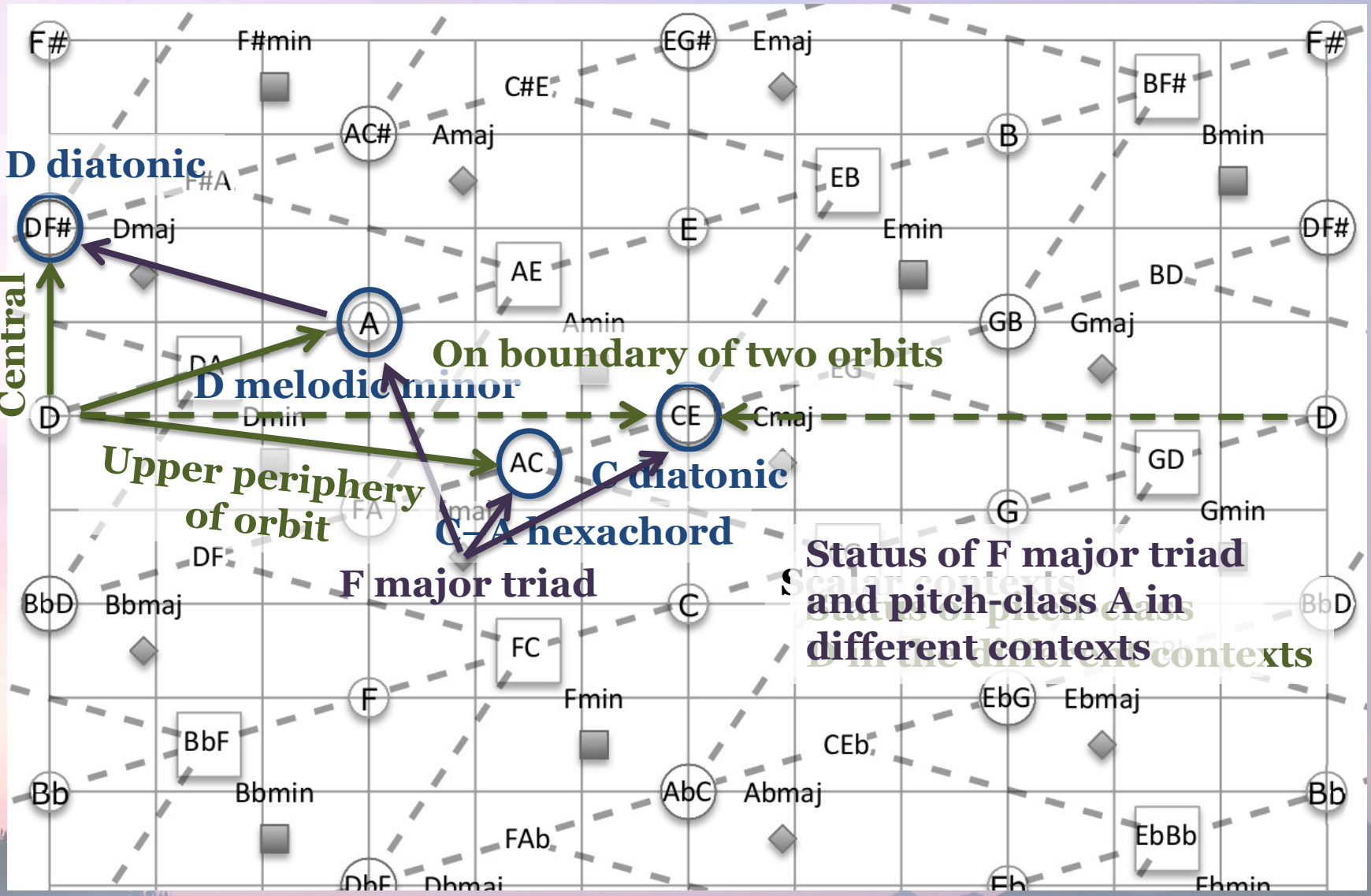
A is central



D diatonic

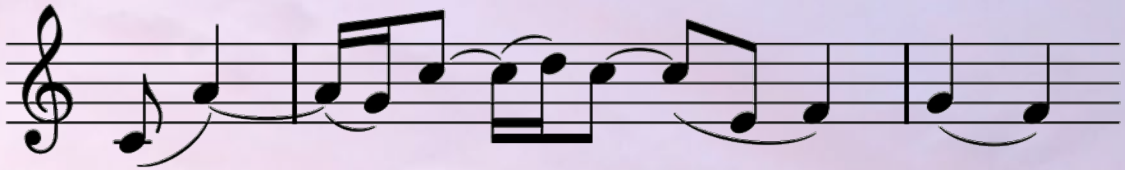
D is central

Heiliger Dankgesang: Scalar Contexts



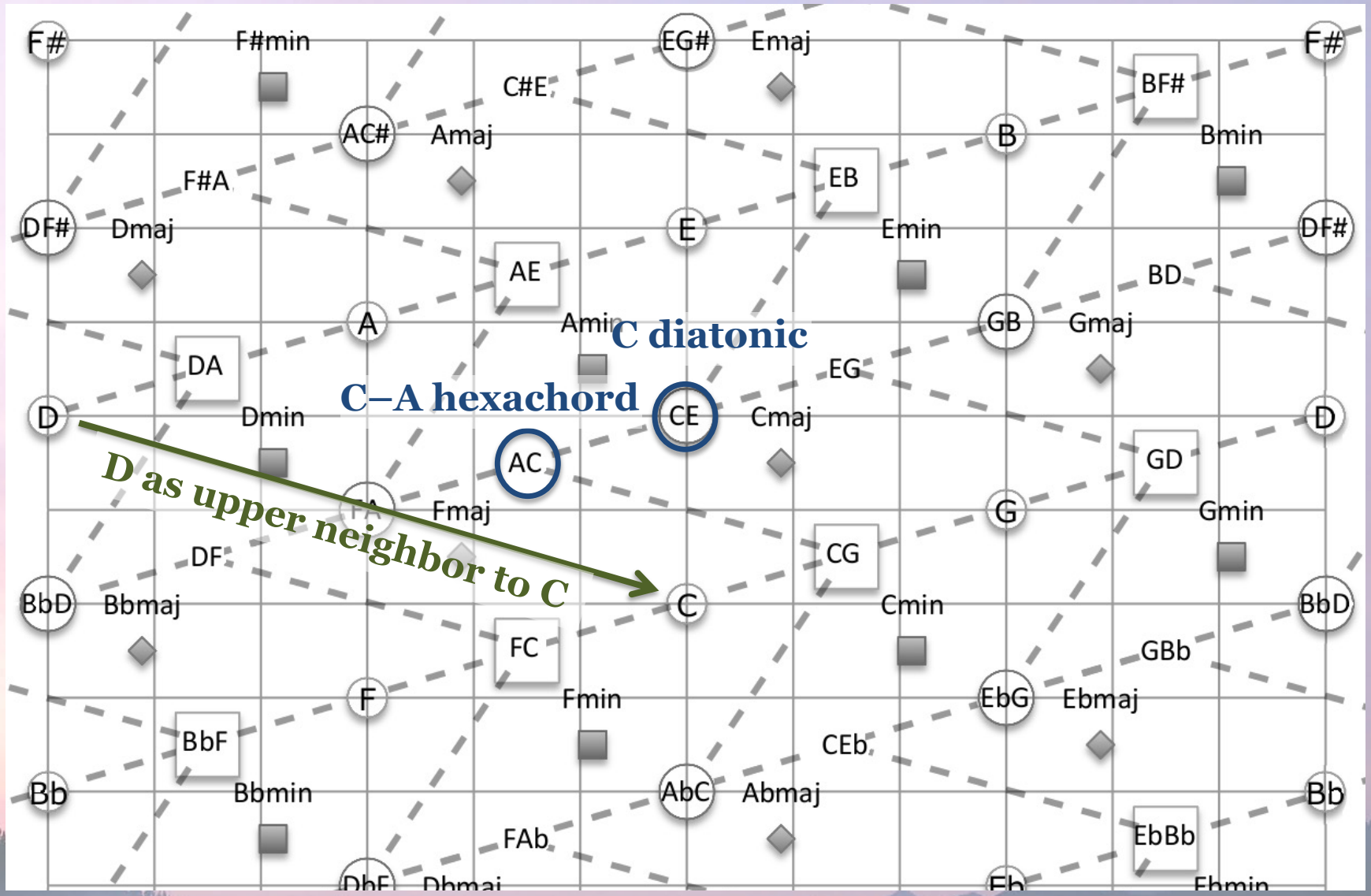
Heiliger Dankgesang: Motivic D–C

Final form of the intonation:

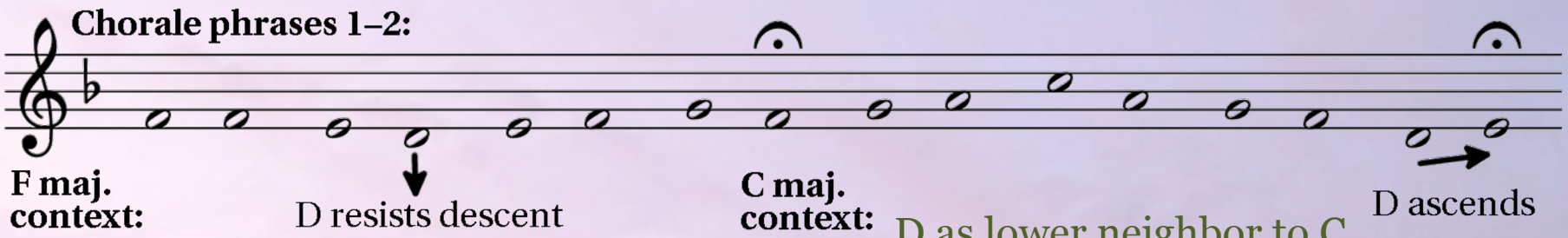


D–C

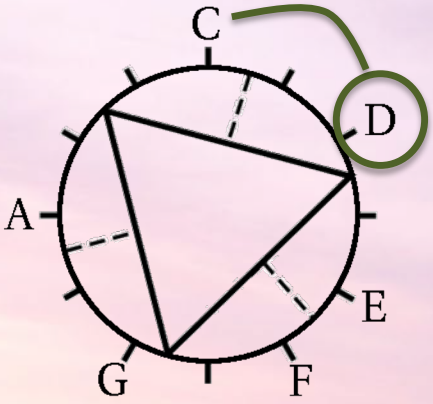
Heiliger Dankgesang: Motivic D–C



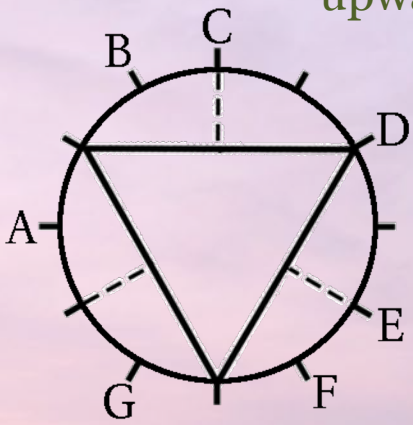
Heiliger Dankgesang: Motivic D–C



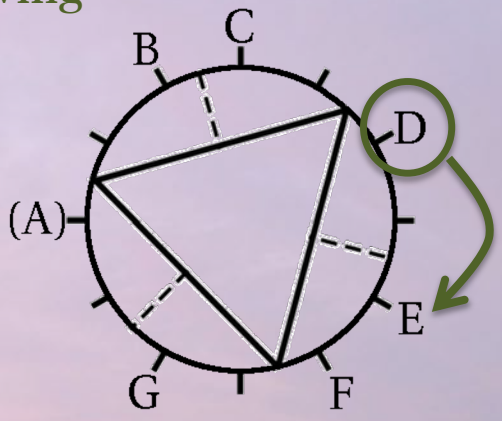
D as lower neighbor to C, upward striving



C–A hexachord



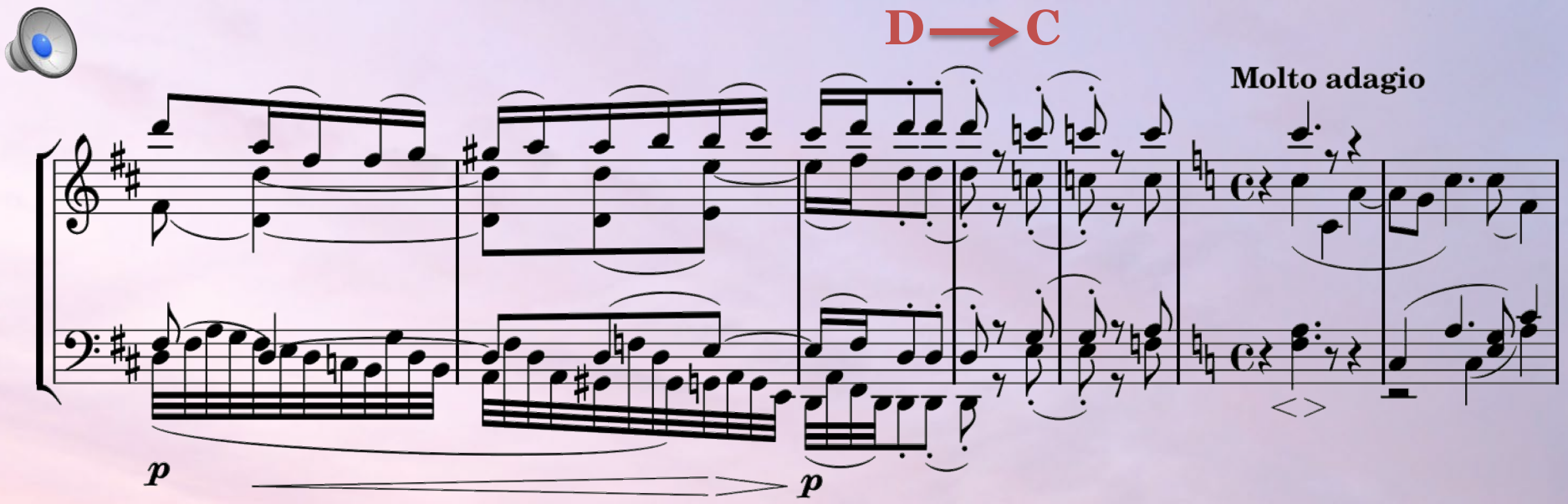
C diatonic

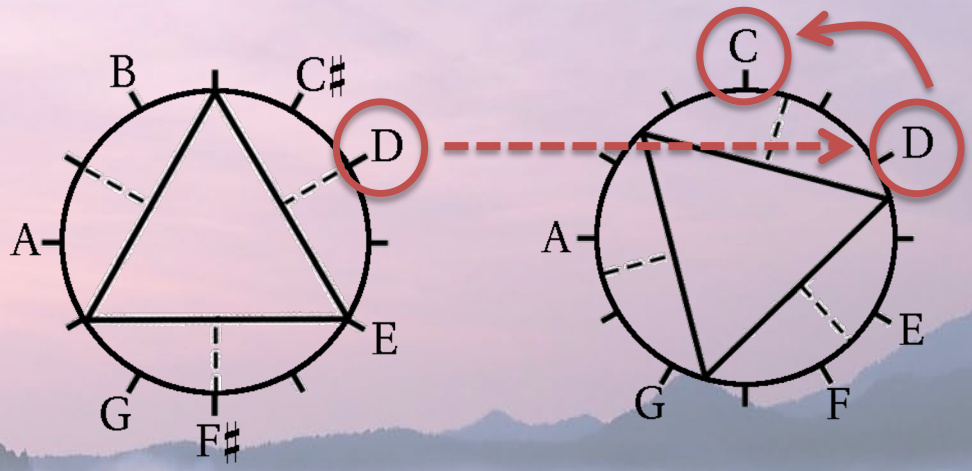


C maj. tonality

Heiliger Dankgesang: Motivic D–C

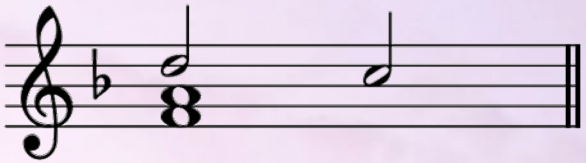
End of *Neue Kraft* section



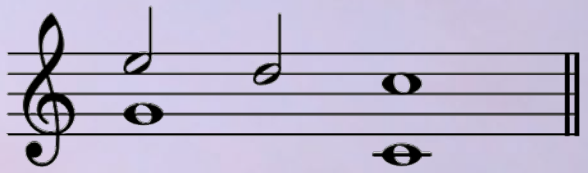


Weakness and Strength

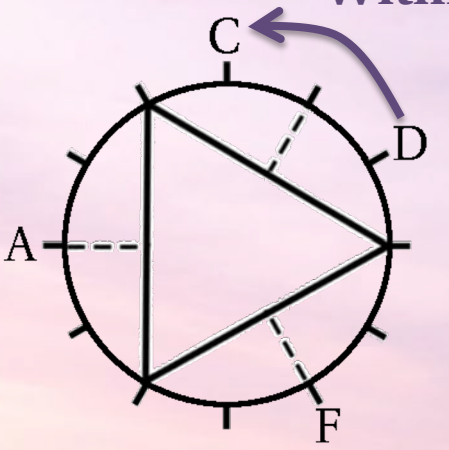
In an F major context, the step C–D is a weak neighbor motion



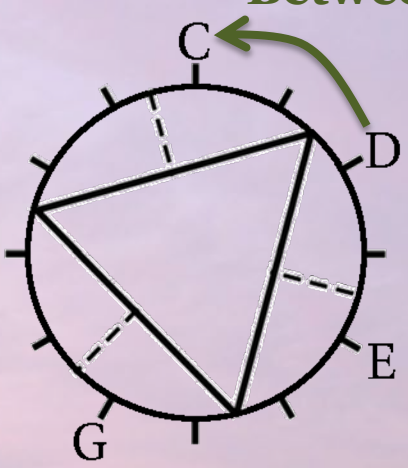
In a C major context, it is a strong completion of passing motion



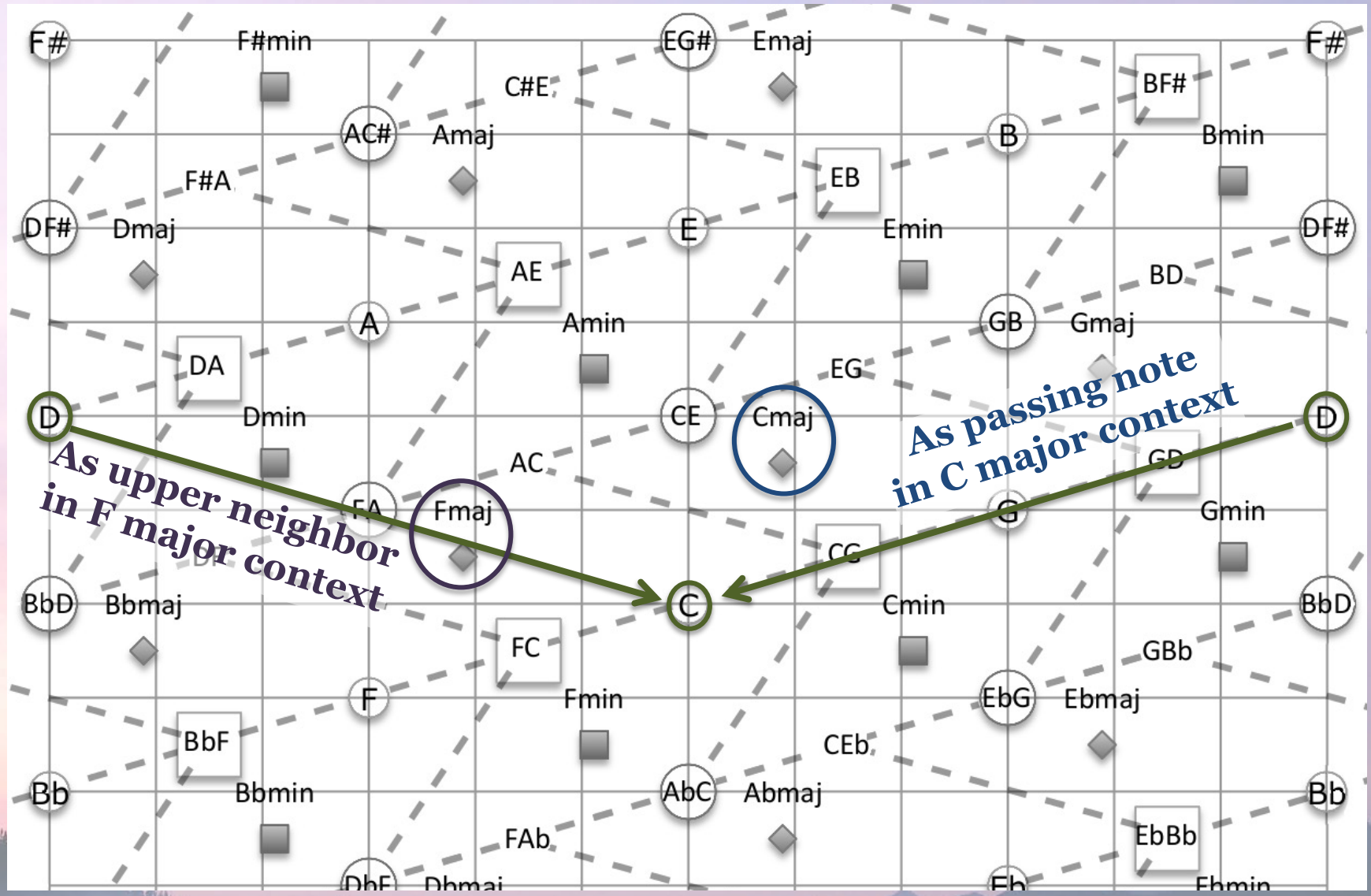
Within orbit



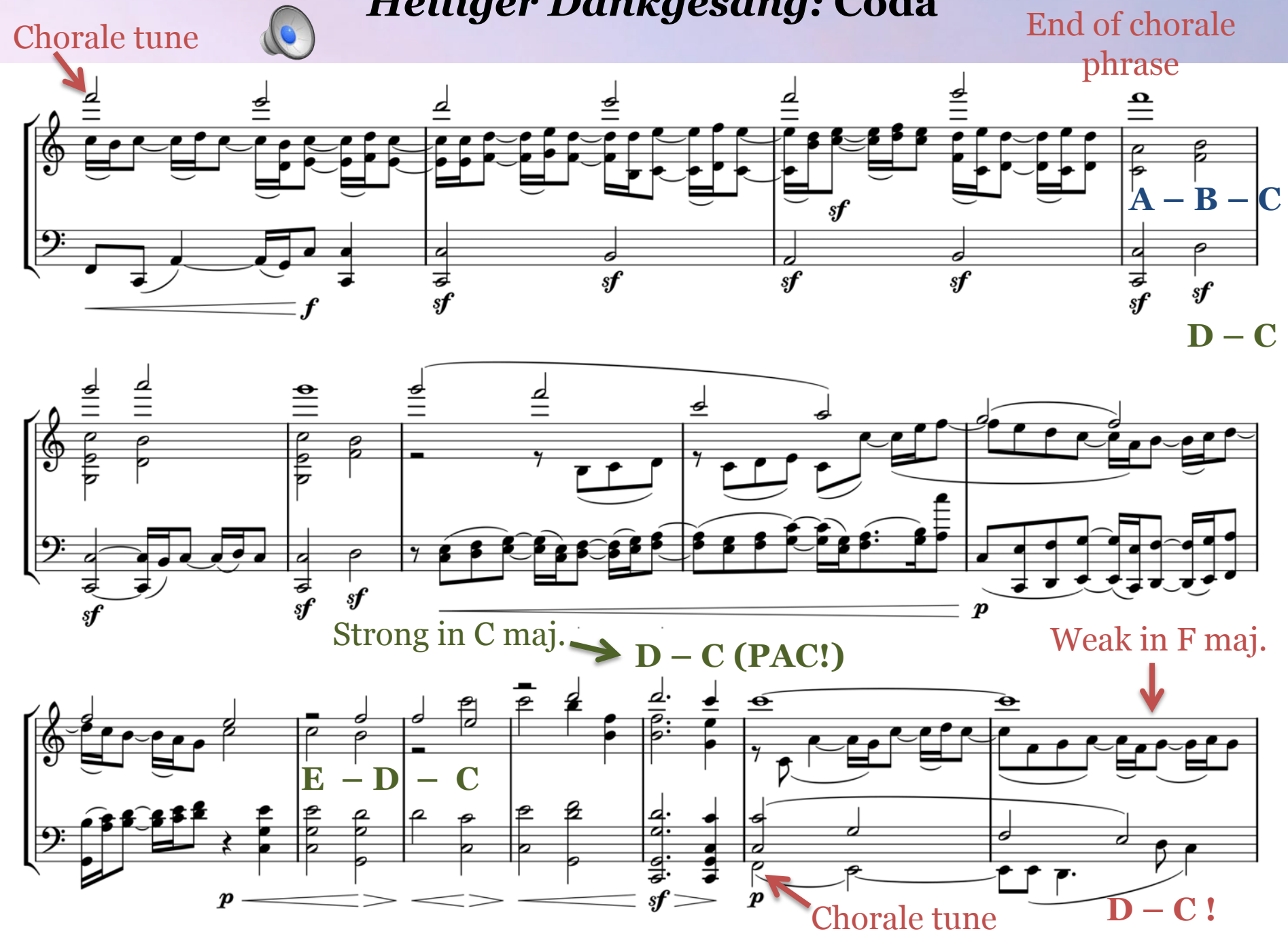
Between orbits



Weakness and Strength



Heiliger Dankgesang: Coda



Conclusions

- DFT phase space effectively reflects tonal process at multiple **levels of structure.**
- It does so through processes of **combination** rather than **reduction**.
- Relating levels through combination better reflects the **traditional notion of keys**.
- Motions in DFT phase space can be construed as a kind of **voice leading** through the idea of **triadic orbits**.
- Triadic orbits also have hermeneutic value in showing the **gravitational forces** that color tones and distinguishing **strong** and **weak** melodic motions.

Restoring the Structural Status of Keys through DFT Phase Space

Jason Yust, Boston University

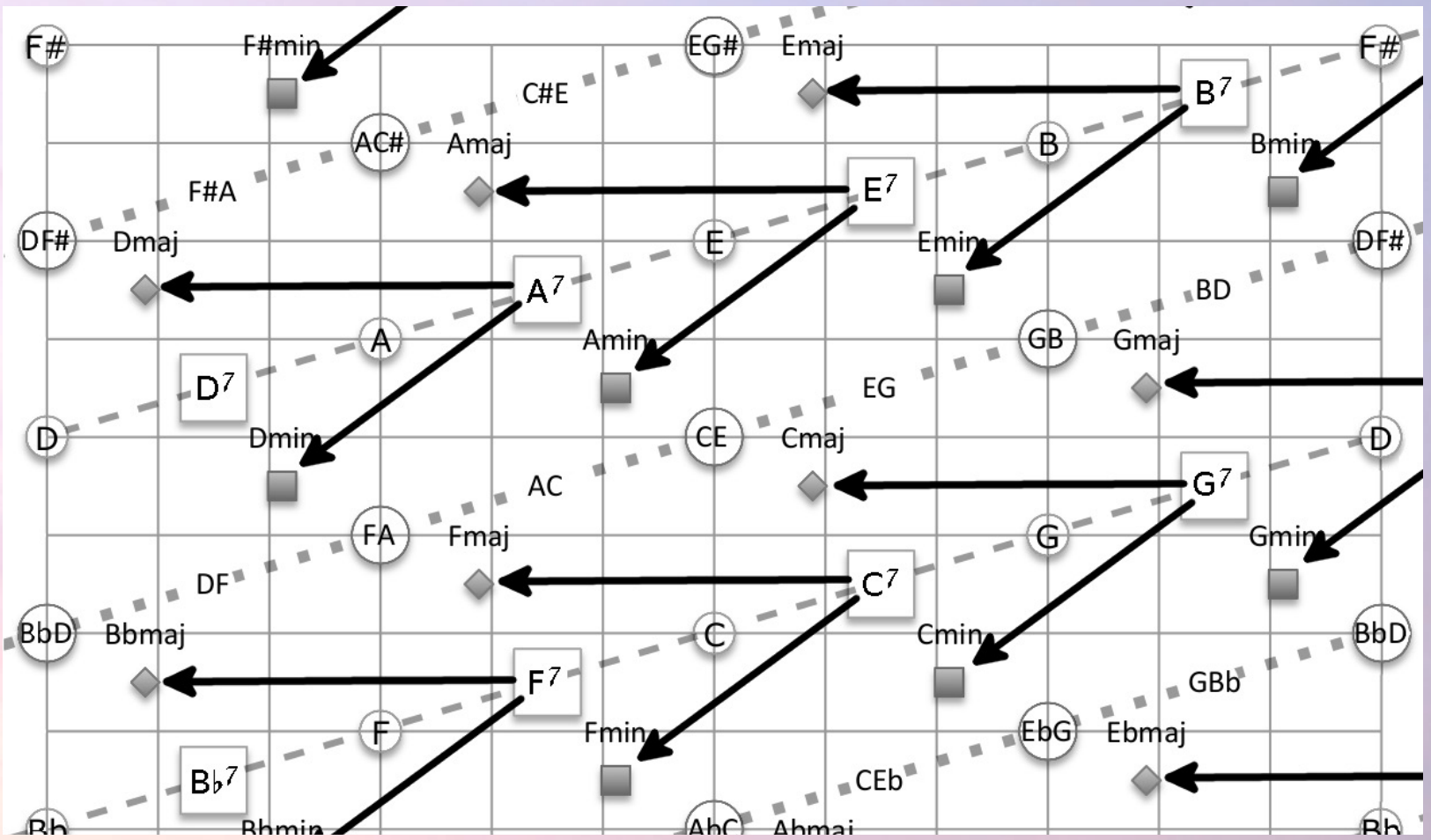
Presentation to the
International Congress on Music and Mathematics
Nov. 26–29, 2014

A copy of this talk is available at
people.bu.edu/jyust/

Appendices:

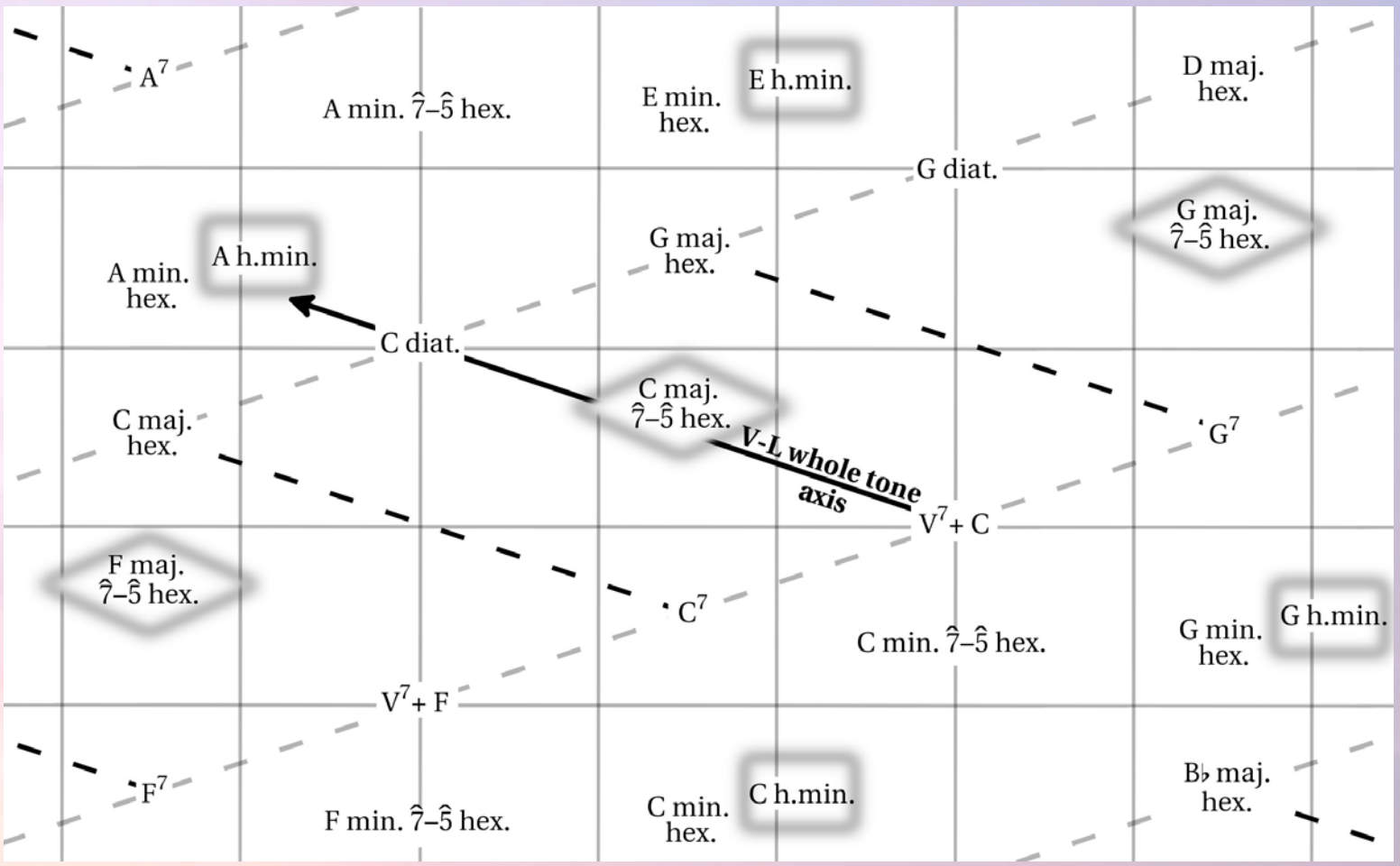
A1: Derivation of tonal regions

Derivation of Tonal Regions



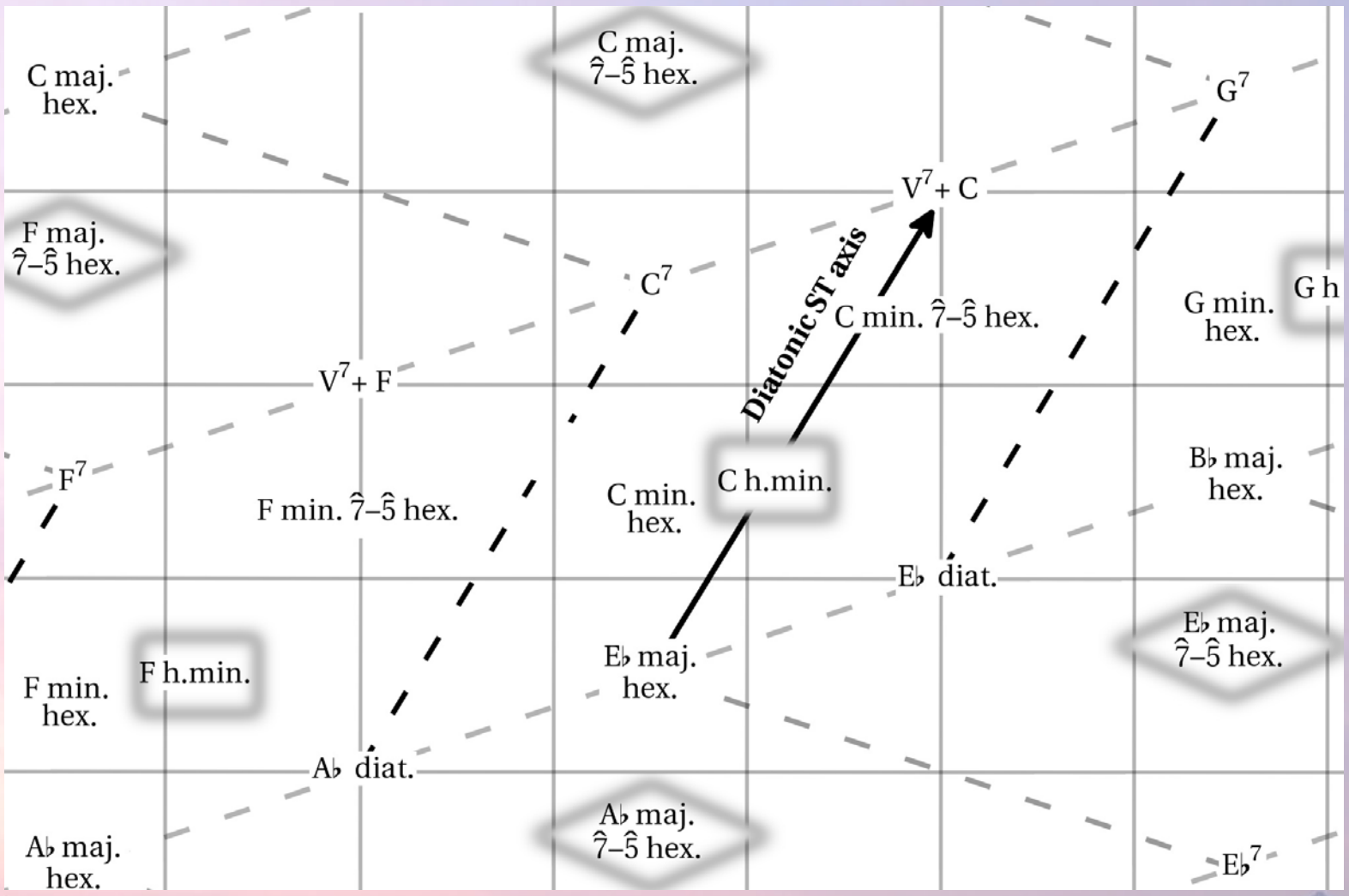
Boundaries between major and minor follow the circle of fifths through diatonic scales and dominant sevenths / individual pcs.

Derivation of Tonal Regions



A characteristic hexachord is at the center of the major regions. Boundaries between fifth-related major regions are parallel to an axis that approximately passes through this hexachord.

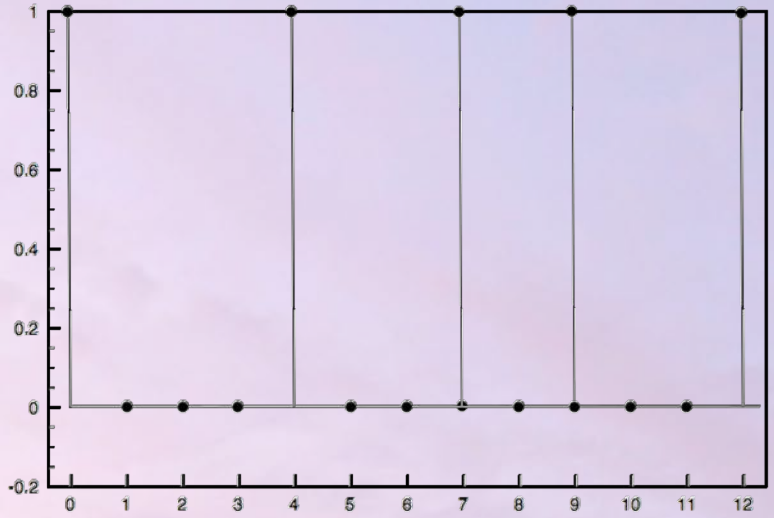
Derivation of Tonal Regions



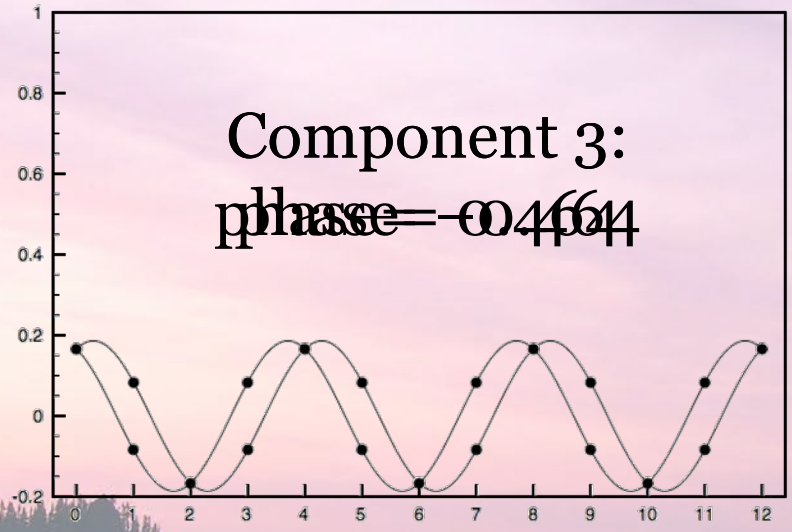
Minor region boundaries are parallel to an axis that approximately passes through the harmonic minor scale

Discrete Fourier Transform: Periodicity

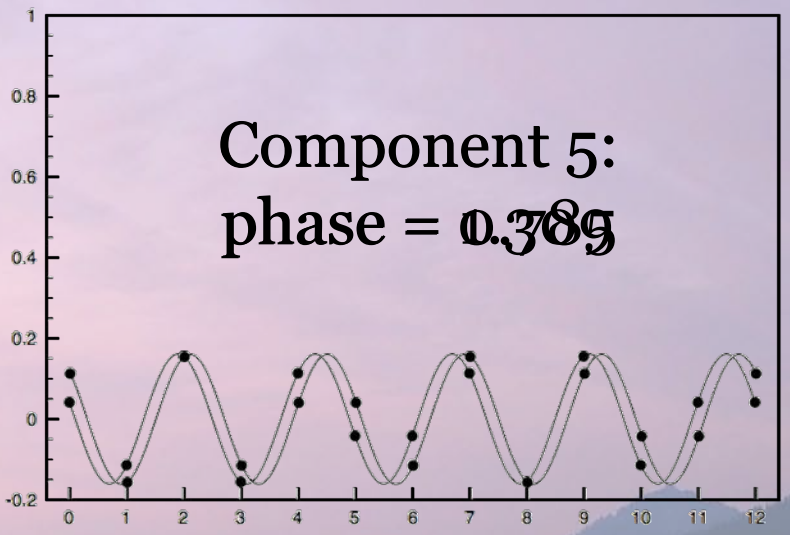
A major triad



Component 3:
phase = 0.464

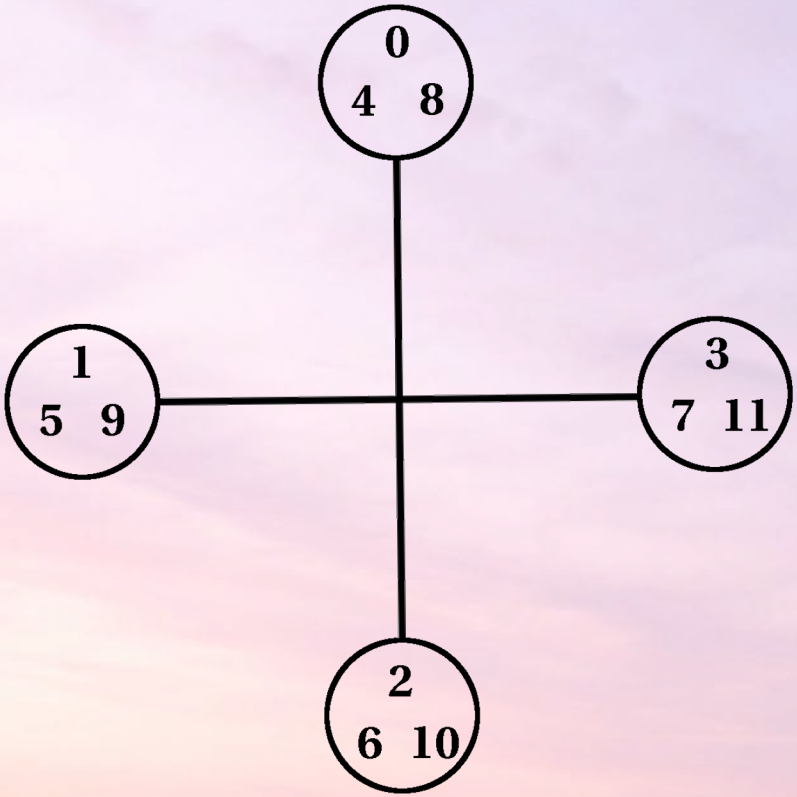


Component 5:
phase = 0.785

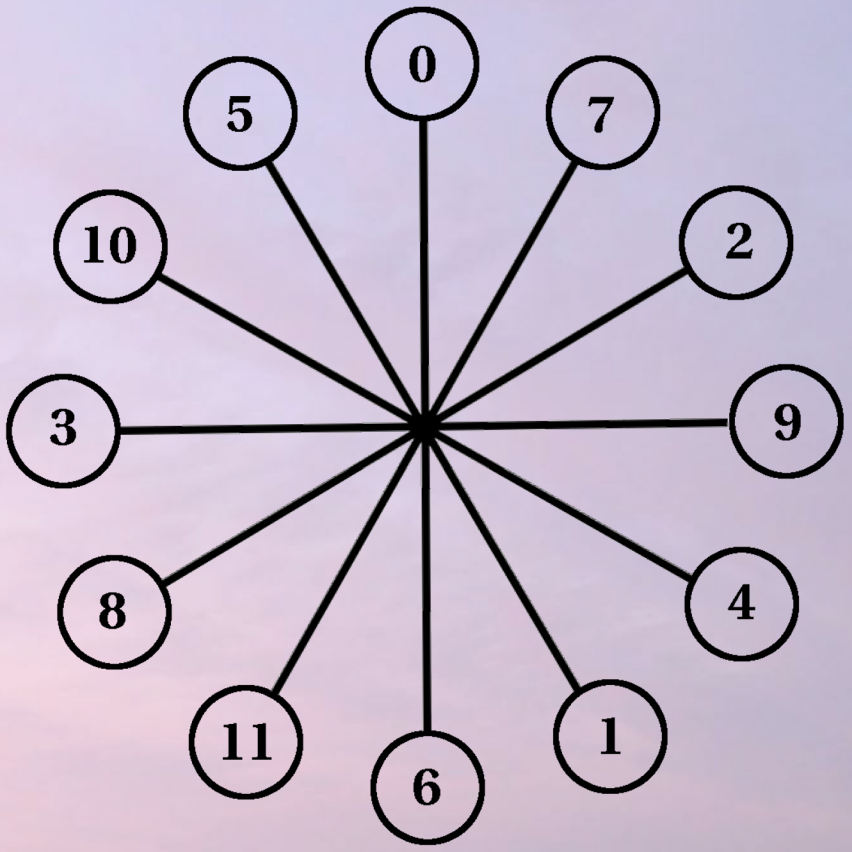


Discrete Fourier Transform: Balances

3-component balance



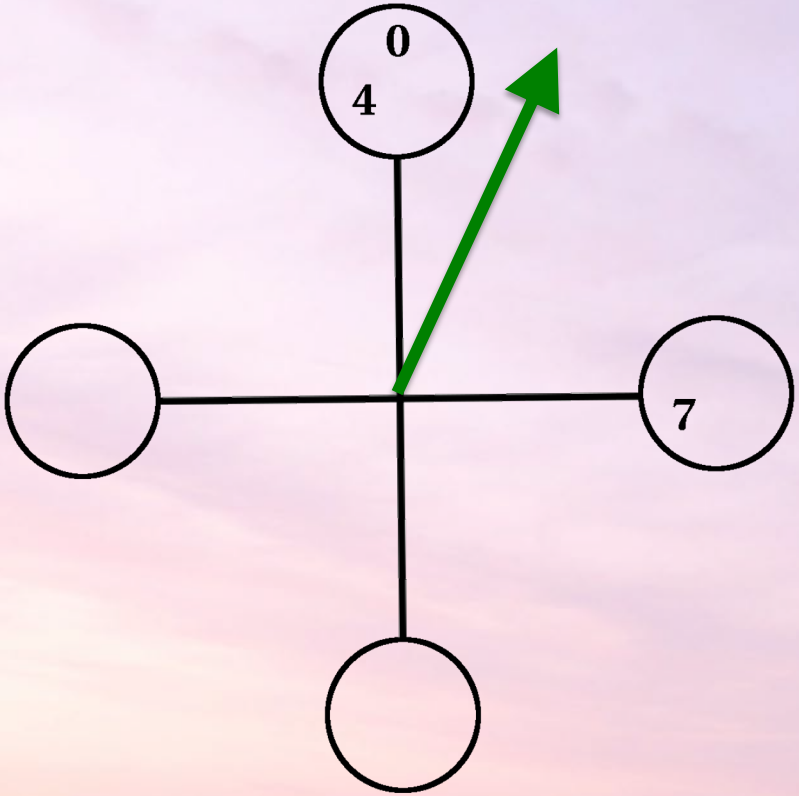
5-component balance



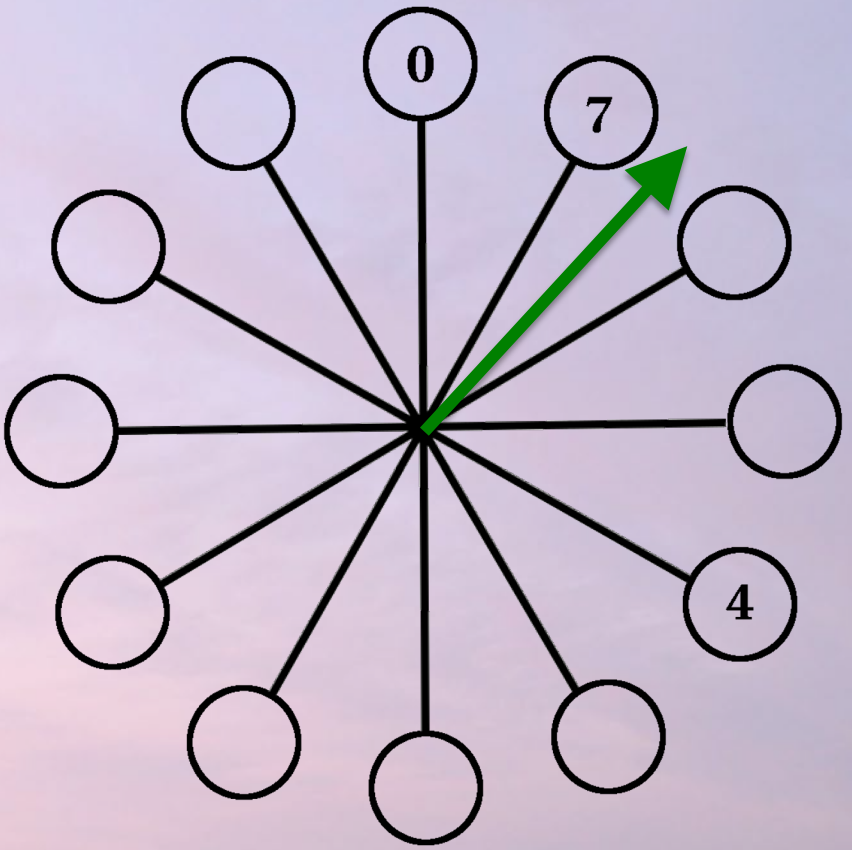
See Quinn "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1 (2006–2007).

Discrete Fourier Transform: Balances

3-component balance



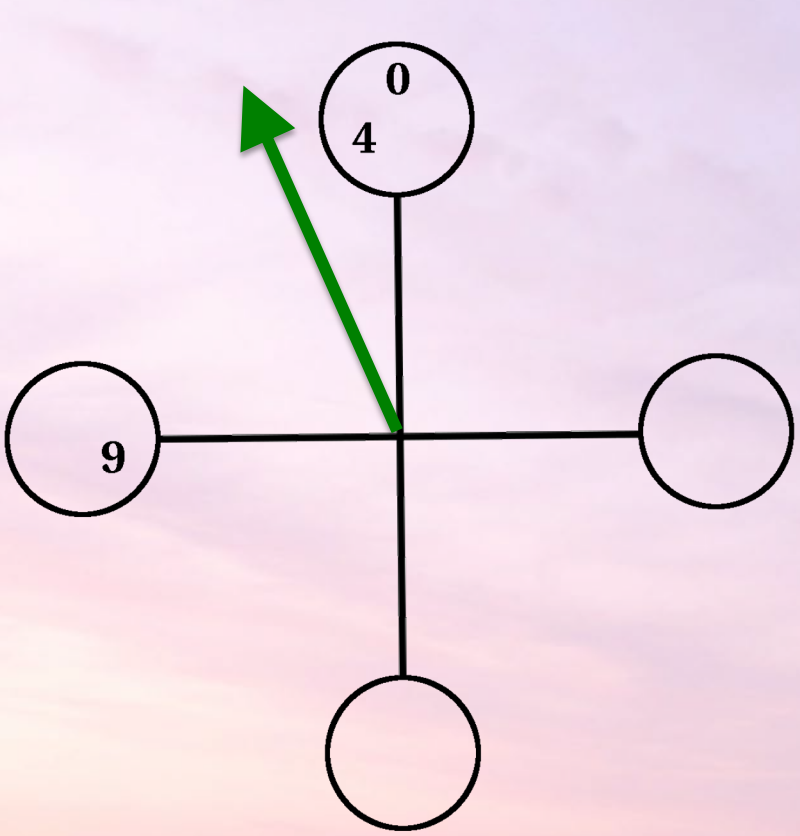
5-component balance



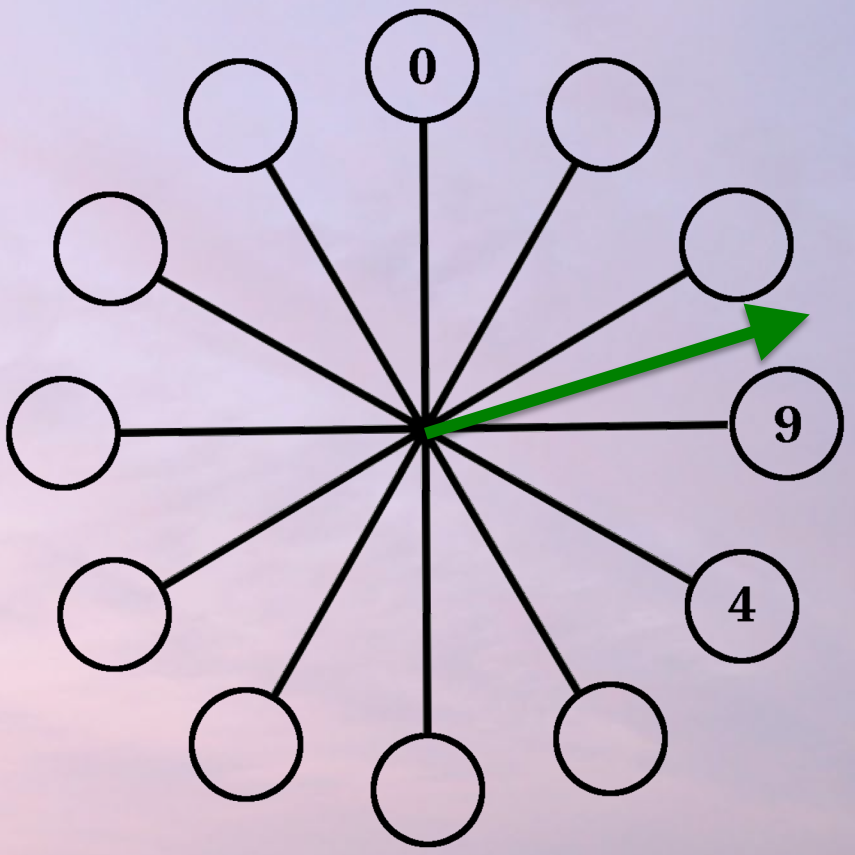
C major triad

Discrete Fourier Transform: Balances

3-component balance

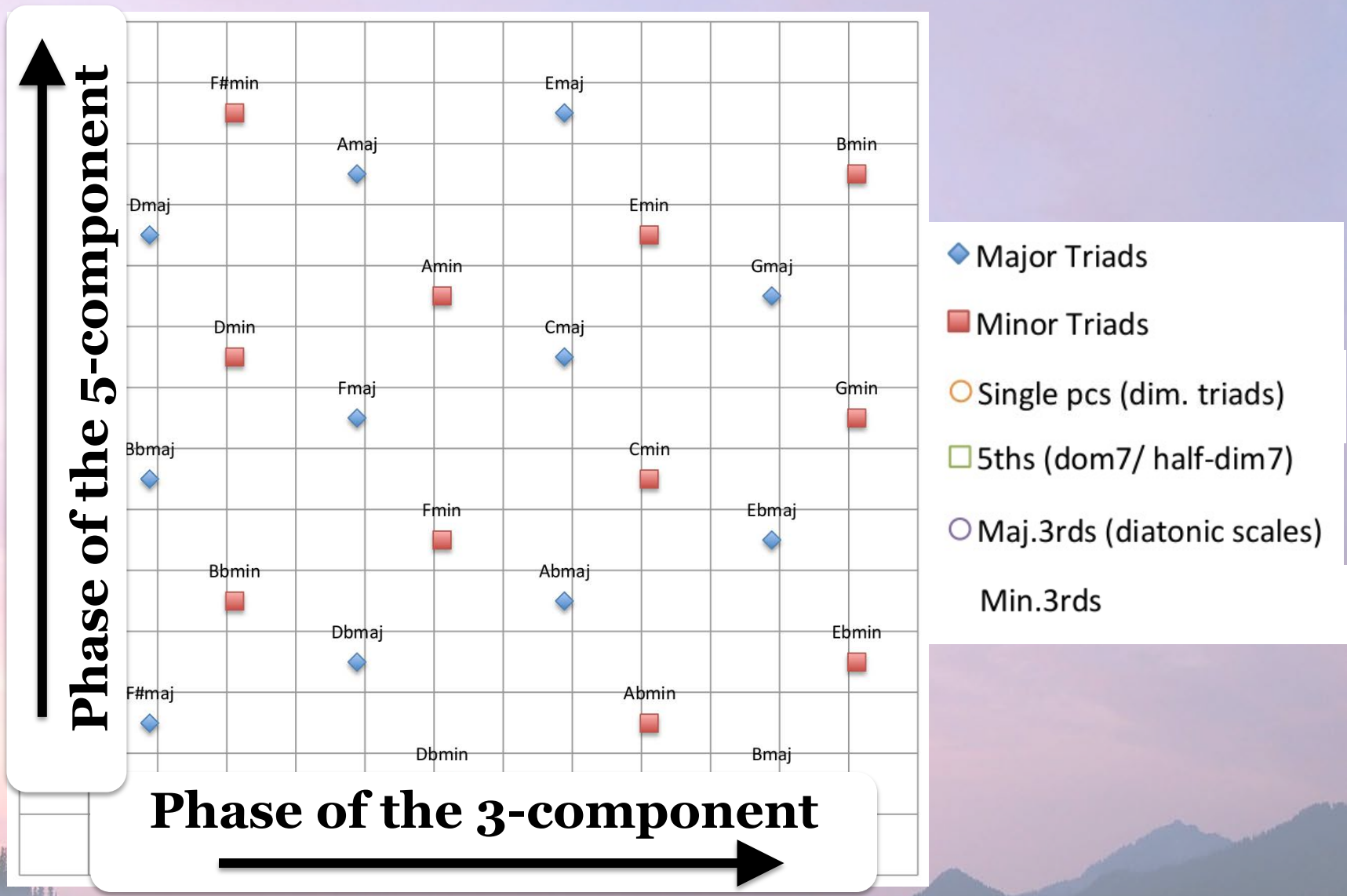


5-component balance

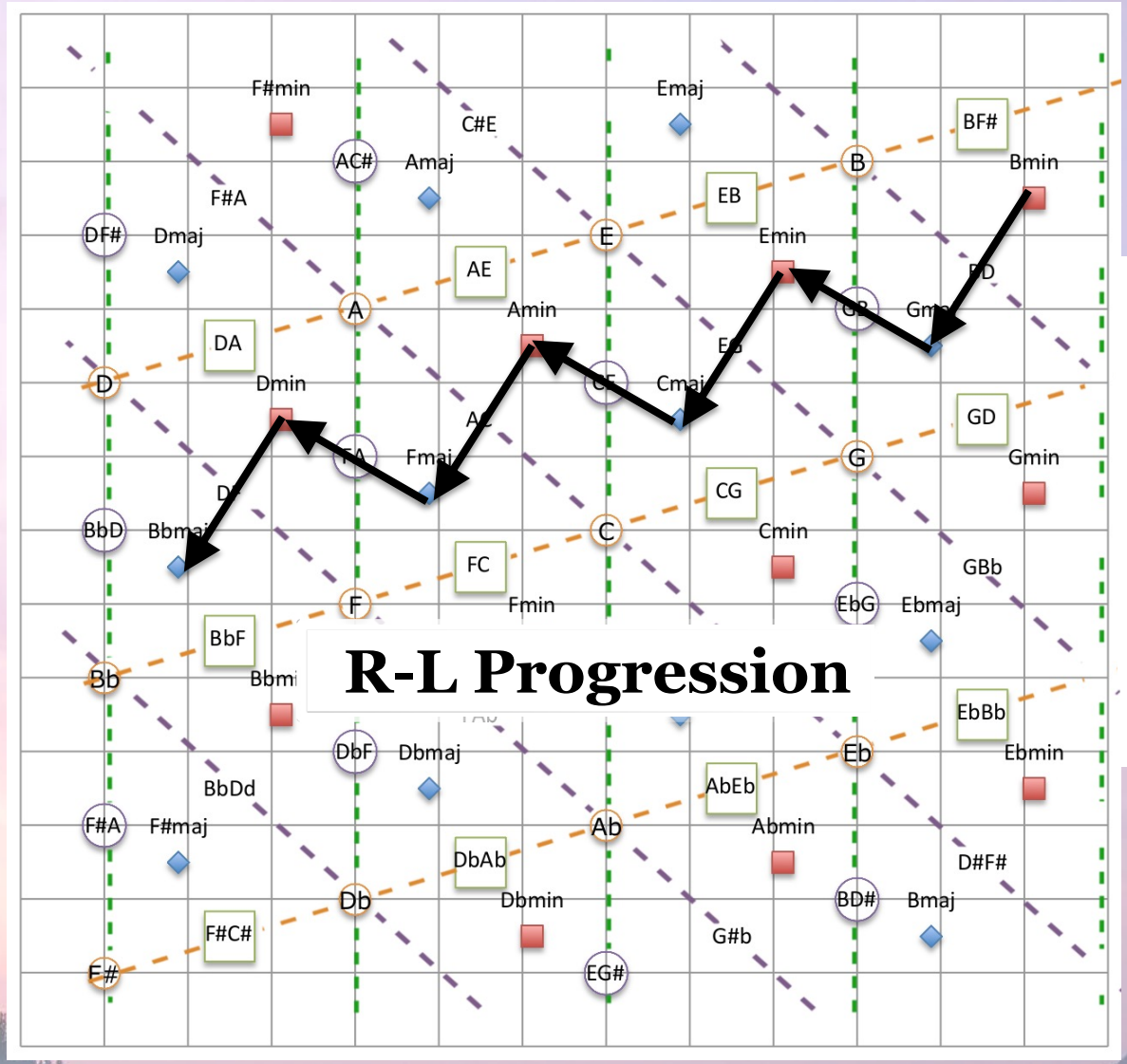


A minor triad

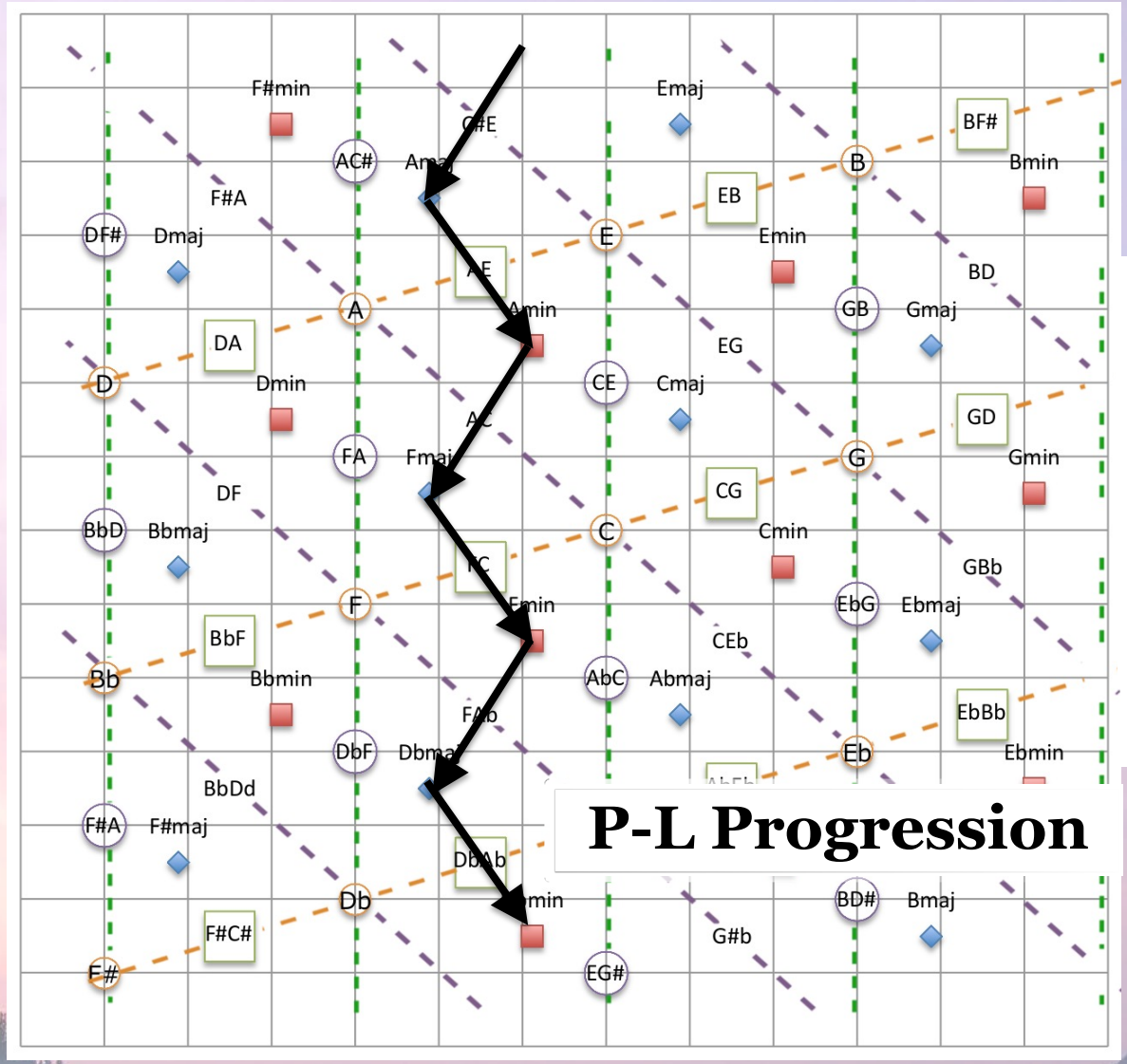
Discrete Fourier Transform: The *Tonnetz*



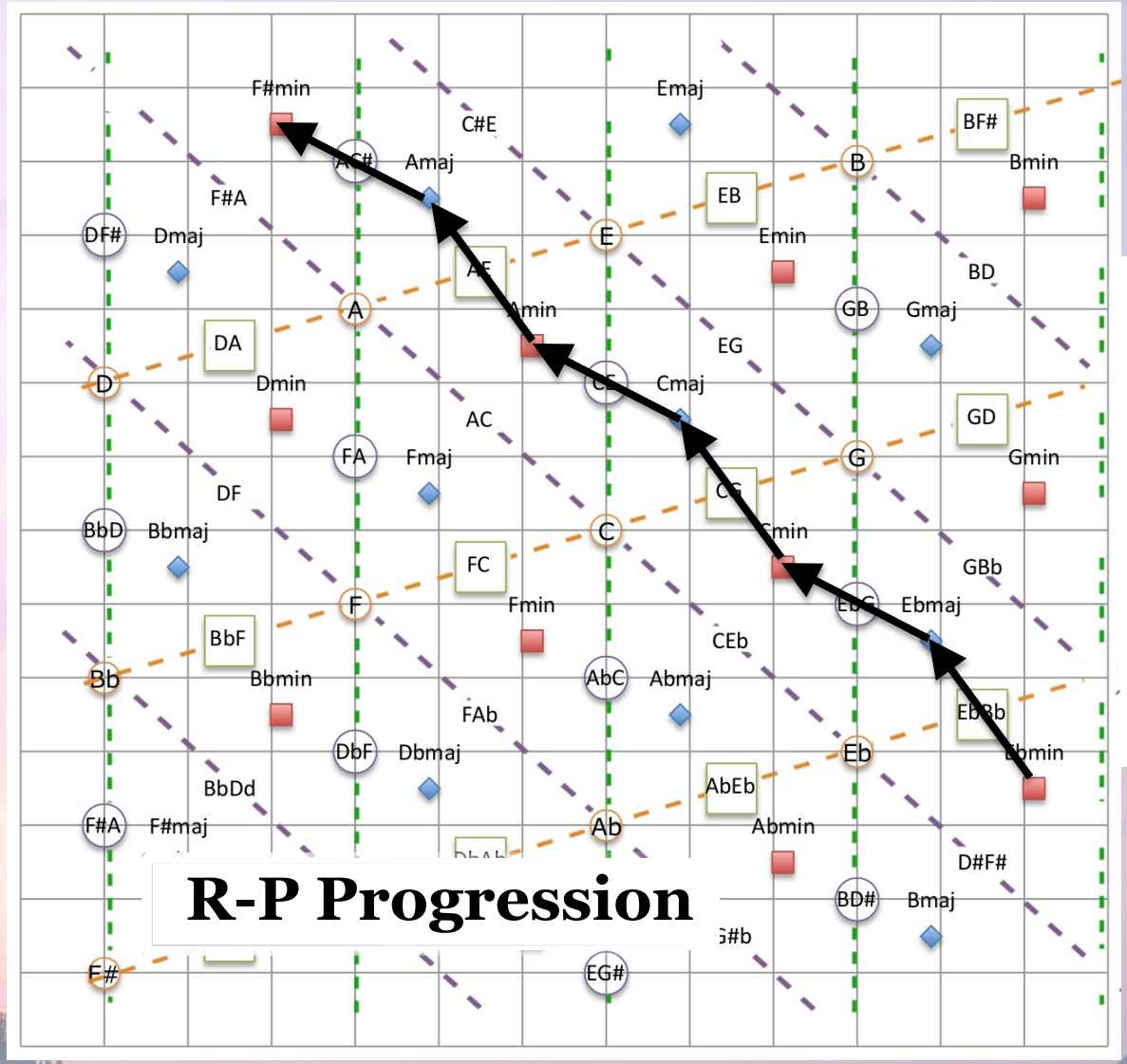
Discrete Fourier Transform: The *Tonnetz*



Discrete Fourier Transform: The *Tonnetz*



Discrete Fourier Transform: The *Tonnetz*



Restoring the Structural Status of Keys through DFT Phase Space

Jason Yust, Boston University

Presentation to the
International Congress on Music and Mathematics
Nov. 26–29, 2014

A copy of this talk is available at
people.bu.edu/jyust/

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