Serial Analysis: A digital library of rows in the repertoire and their properties, with applications for teaching and research

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Digital Libraries for Musicology 2021

Application: DFT on row segments

- Discrete Fourier transform (DFT) on pitch-class vectors and harmonic qualities
- Parsing of rows
- Some results

Fourier Transform on Pitch-Class Vectors: A brief history

Lewin, David (1959). “Re: Intervallic Relations between Two Collections of Notes,” *JMT* 3/2.

——— (2001). “Special Cases of the Interval Function between Pitch Class Sets X and Y.” *JMT* 45/1.

Quinn, Ian (2006–2007). “General Equal-Tempered Harmony,” *Perspectives of New Music* 44/2–45/1.

Callender, Cliff (2007). “Continuous Harmonic Spaces,” *JMT* 51/2

Amiot, Emmanuel (2007). “David Lewin and Maximally Even Sets.” *Journal of Mathematics and Music* 1/3.

——— (2016). *Music Through Fourier Space: Discrete Fourier Transform in Music Theory*. (Springer)

Yust, Jason. (2016). “Special Collections: Renewing Forte’s Set Theory.” *JMT* 60/2.

———. (2019) “Stylistic Information in Pitch-Class Distributions.” *Journal of New Music Research* 48(3).

Pitch-Class Vectors

12-place vector

Each place corresponds to a pitch-class.

1 = present, 0 = absent

Example:

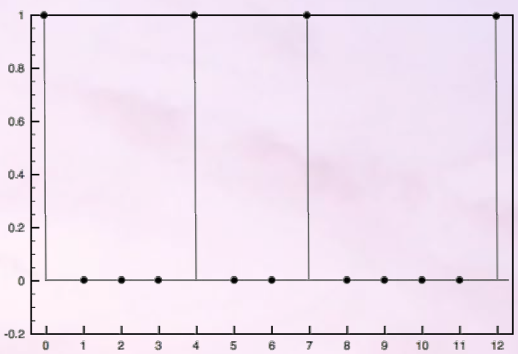
	C	C♯	D	E♭	E	F	F♯	G	A♭	A	B♭	B
C major triad:	(1,	0,	0,	0,	1,	0,	0,	1,	0,	0,	0,	0)

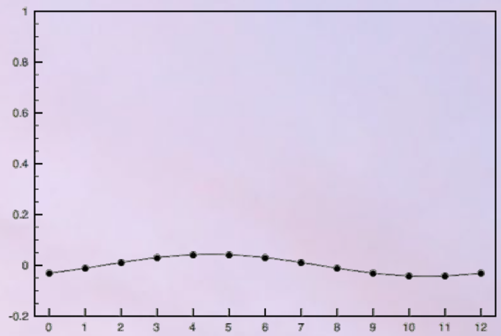
Other names: “Characteristic function,” “pitch-class distribution”

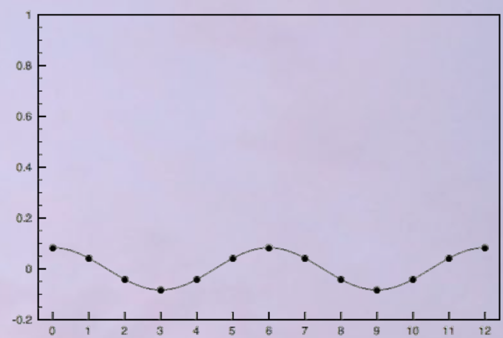
The *discrete Fourier transform* (DFT) of this vector converts it in periodic functions at divisions of the octave into 1–6 parts.

Fourier Transform of a Pitch-Class Vector

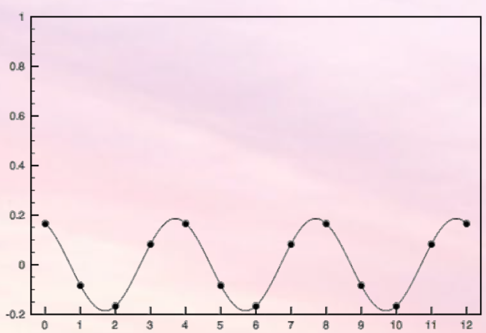
C major triad = F_0 + F_1 + F_2

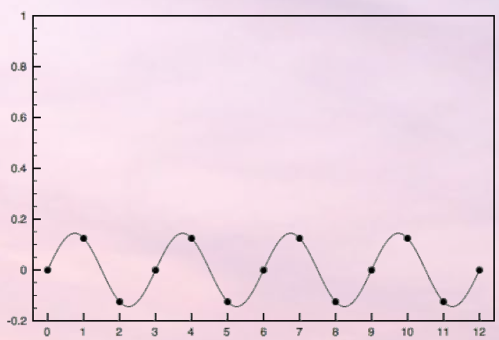


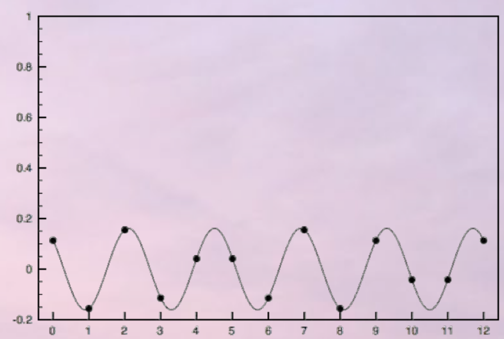


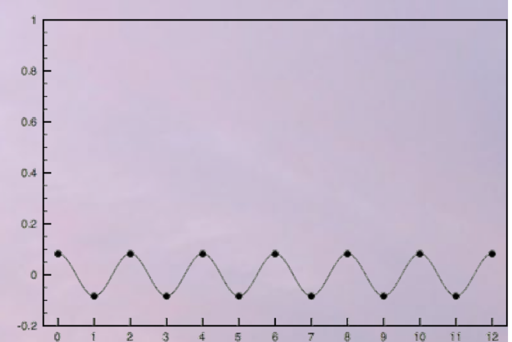


+ F_3 + F_4 + F_5 + F_6 + . . .



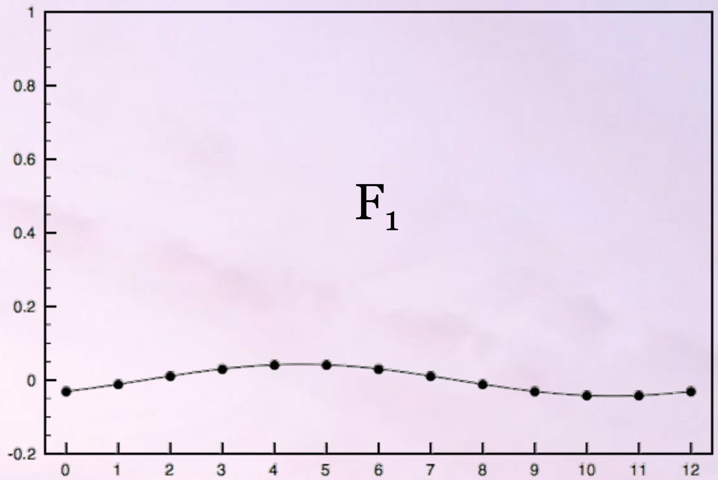


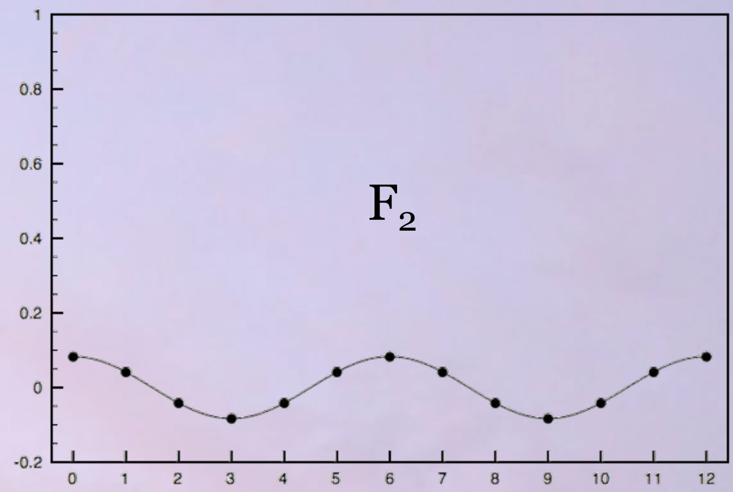




The DFT is a **lossless transformation** from a pitch-class vector to a sum of **periodic functions** dividing the octave into 1–6 parts.

Fourier Qualities

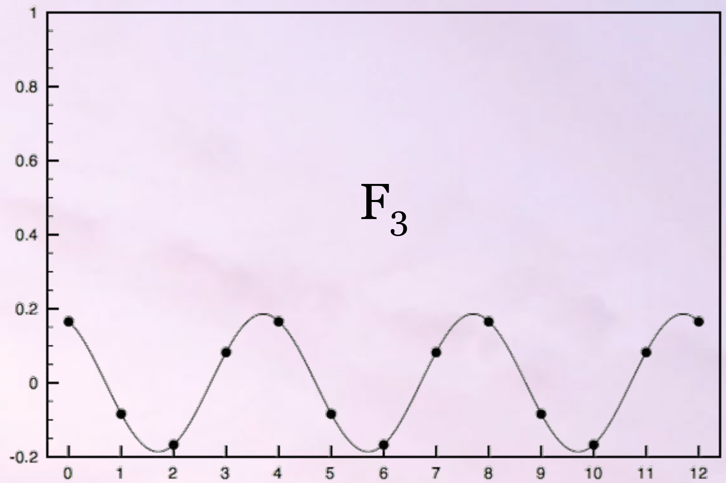


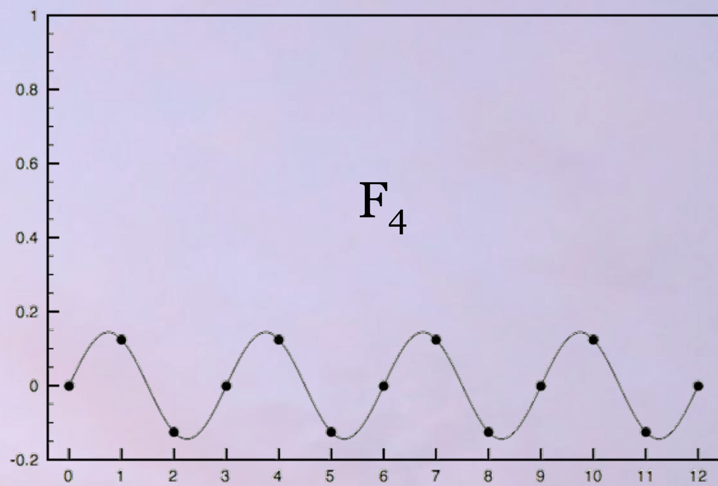


F_1 represents a concentration of pitch-class weight on the full pc circle.

F_2 represents a concentration of pitch-class weight on a half-octave (tritone) cycle.

Fourier Qualities

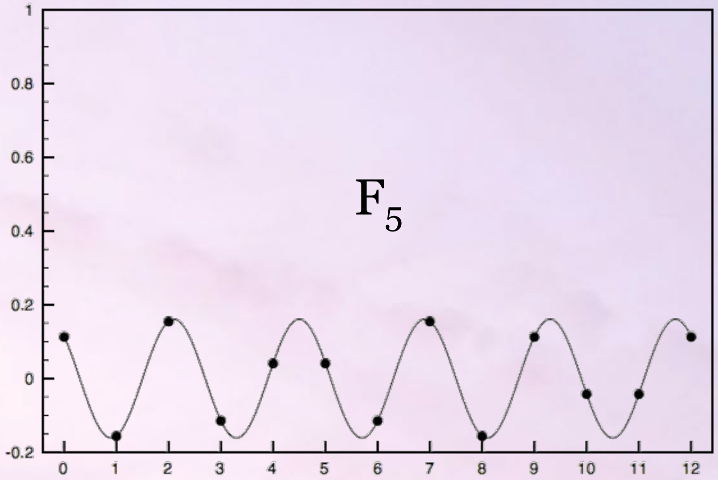


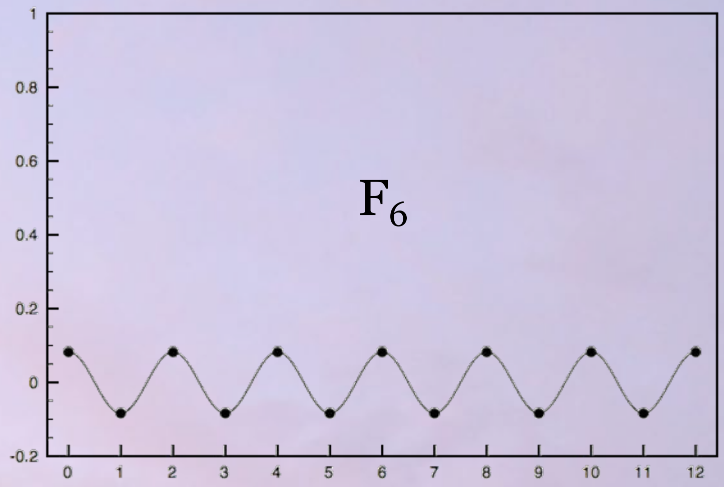


F_3 gives the weighting on the nearest *augmented triad* or ***hexatonic*** *scale*.

F_4 gives the weighting on the nearest *diminished seventh* or ***octatonic*** *scale*.

Fourier Qualities





F_5 give the balance on the ***circle of fifths***

F_6 gives the relative weighting on the two ***whole-tone*** collections.

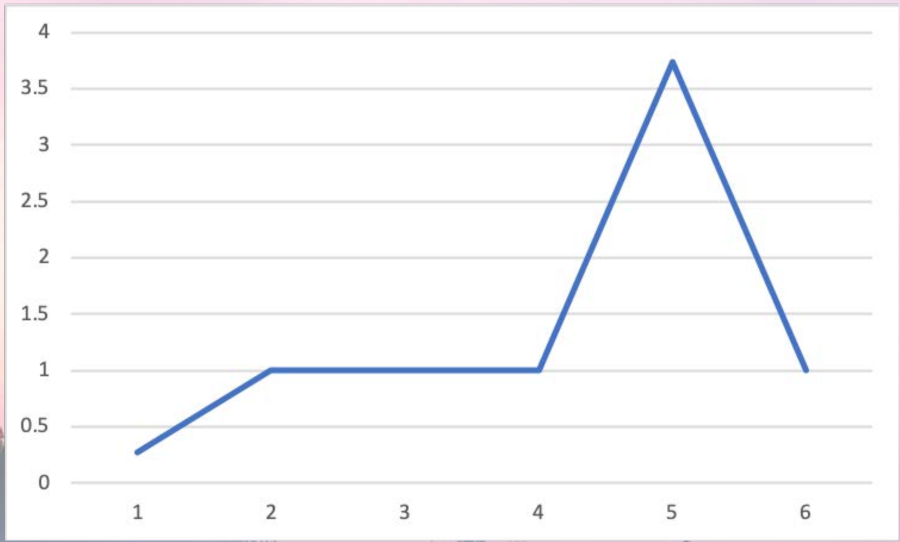
Fourier Spectra

The ***spectrum*** of a pitch-class vector shows the magnitudes of all its Fourier coefficients (ignoring phases)

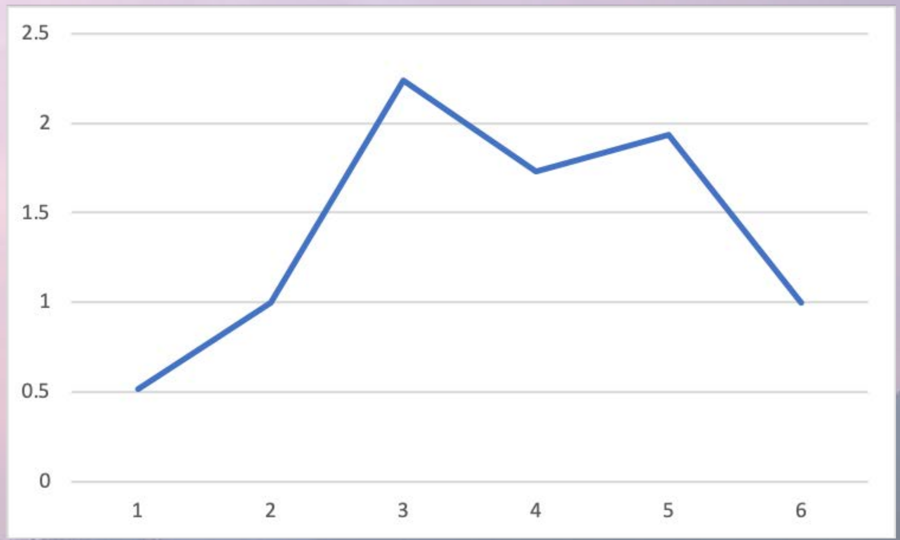
The spectrum is **invariant with respect to transposition and inversion** (i.e. it is a *set class* property) and also **complementation** and **Z-relation**

Examples:

Diatonic scale



Major/minor triad



Parsing of rows

Parsing of rows

Example: Webern Op. 27

0 8 7 e t 9 3 1 4 2 6 5 Note: t = 10, e = 11

- Initial and final hexachord: {7 8 9 t e 0} {1 2 3 4 5 6}

Set class: (012345) (012345)

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- Inner and outer hexachord: {9 e t 1 3 4} {0 2 5 6 7 8}

  Set class: (012467) (012368)

Parsing of rows

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- Inner and outer hexachord: {9 e t 1 3 4} {0 2 5 6 7 8}

  Set class: (012467) (012368)
- Middle tetrachord: {9 t 1 3} {e 0 2 4 5 6 7 8}

  Set class: (0146) (01234689)

Parsing of rows

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DFT of row segments

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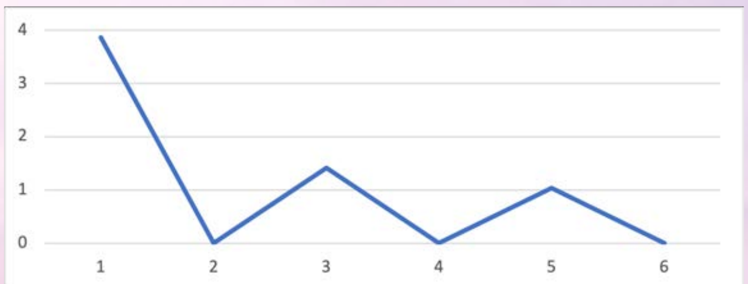
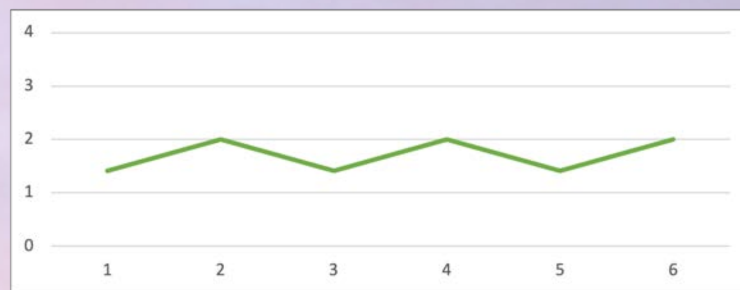
Initial/final hex.: (012345)

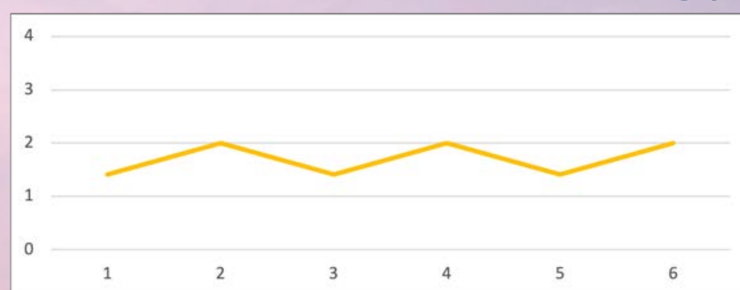


Middle tetrachord: (0146)



Inner/outer hex.: (012467)

Middle octachord: (0137)



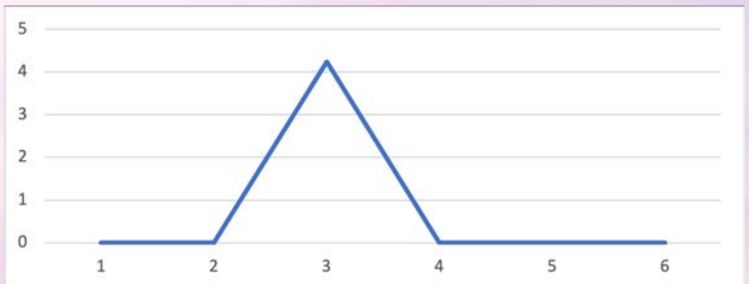
DFT of row segments

Example: Babbitt, *Vision and Prayer*

0 e 8 3 7 4 t 9 2 6 1 5

Note: t = 10, e = 11

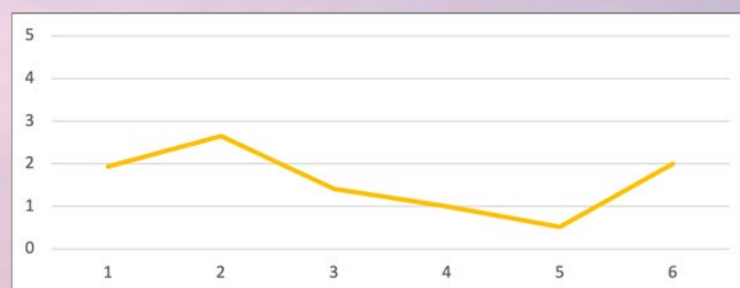
Initial/final hex.: (013478)



Middle tetrachord: (0136)

Inner/outer hex.: (012578)

Middle octachord: (0126)



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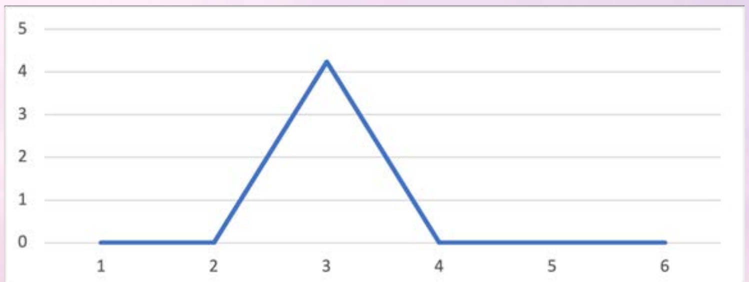

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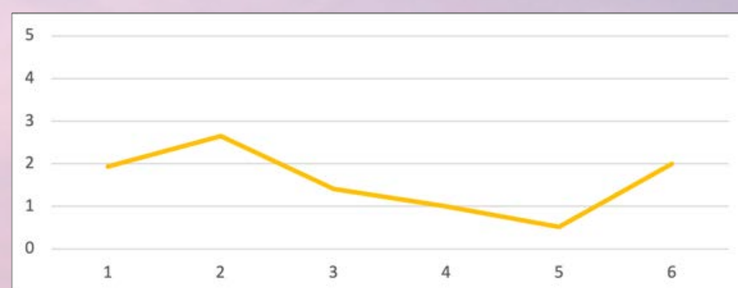

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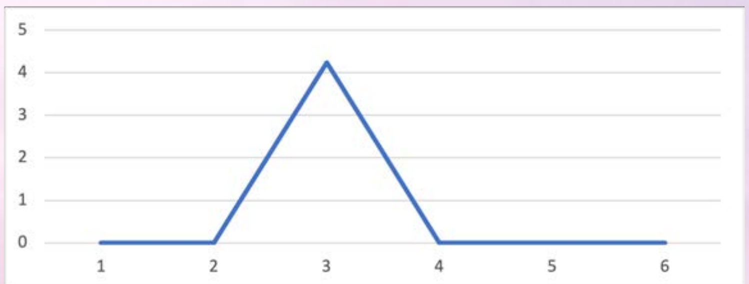

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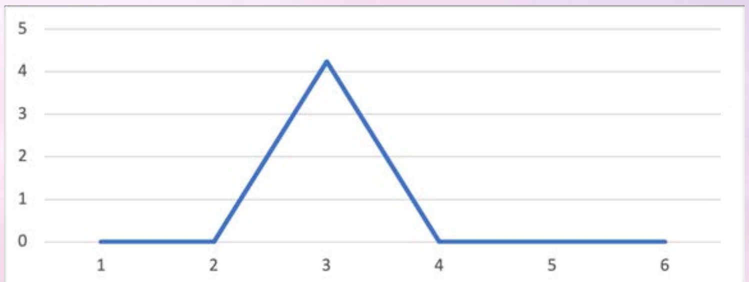

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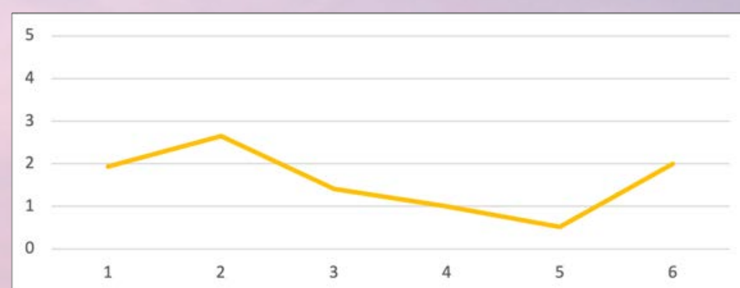
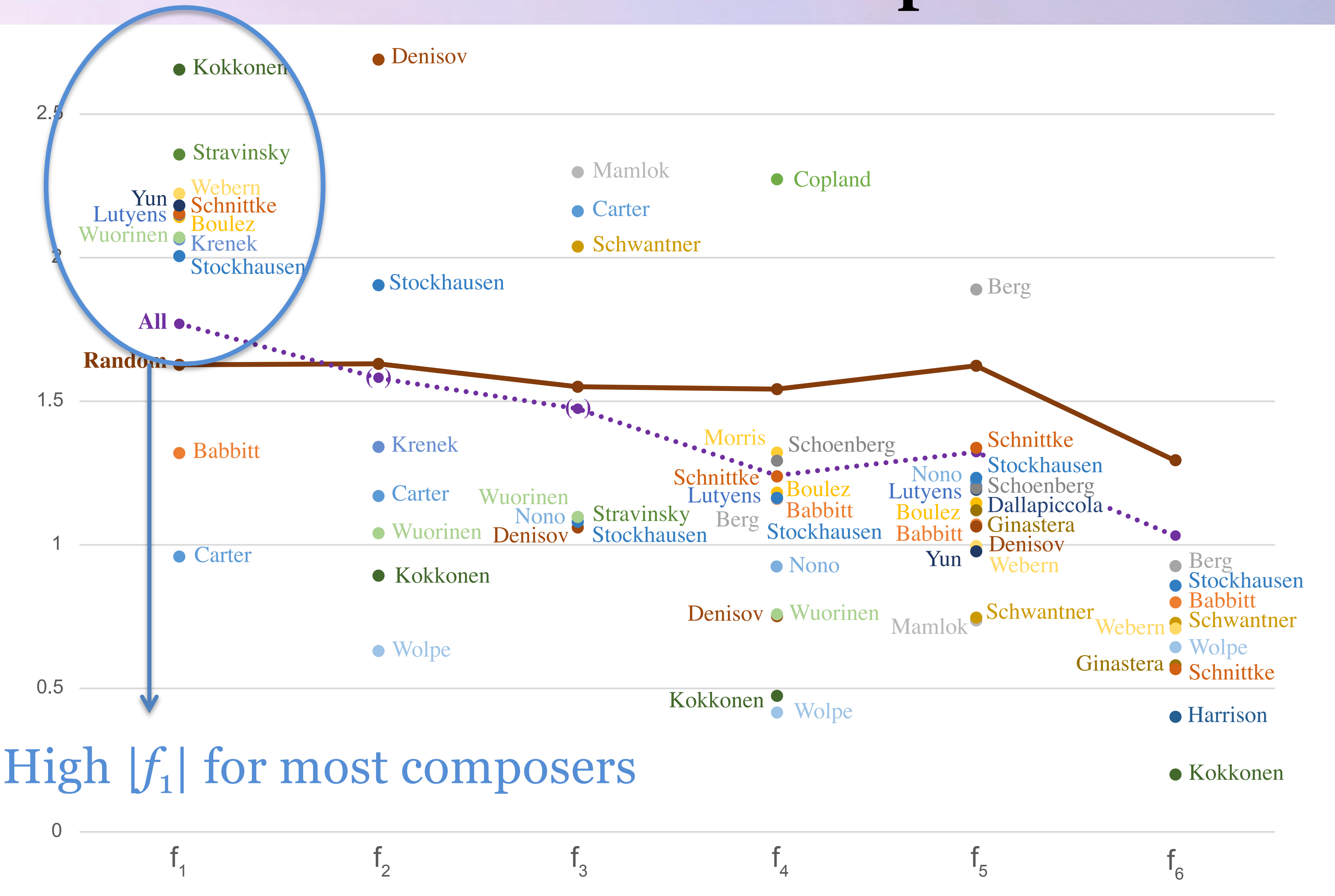


Results

Results: Both hexachord partitions



High $|f_1|$ for most composers

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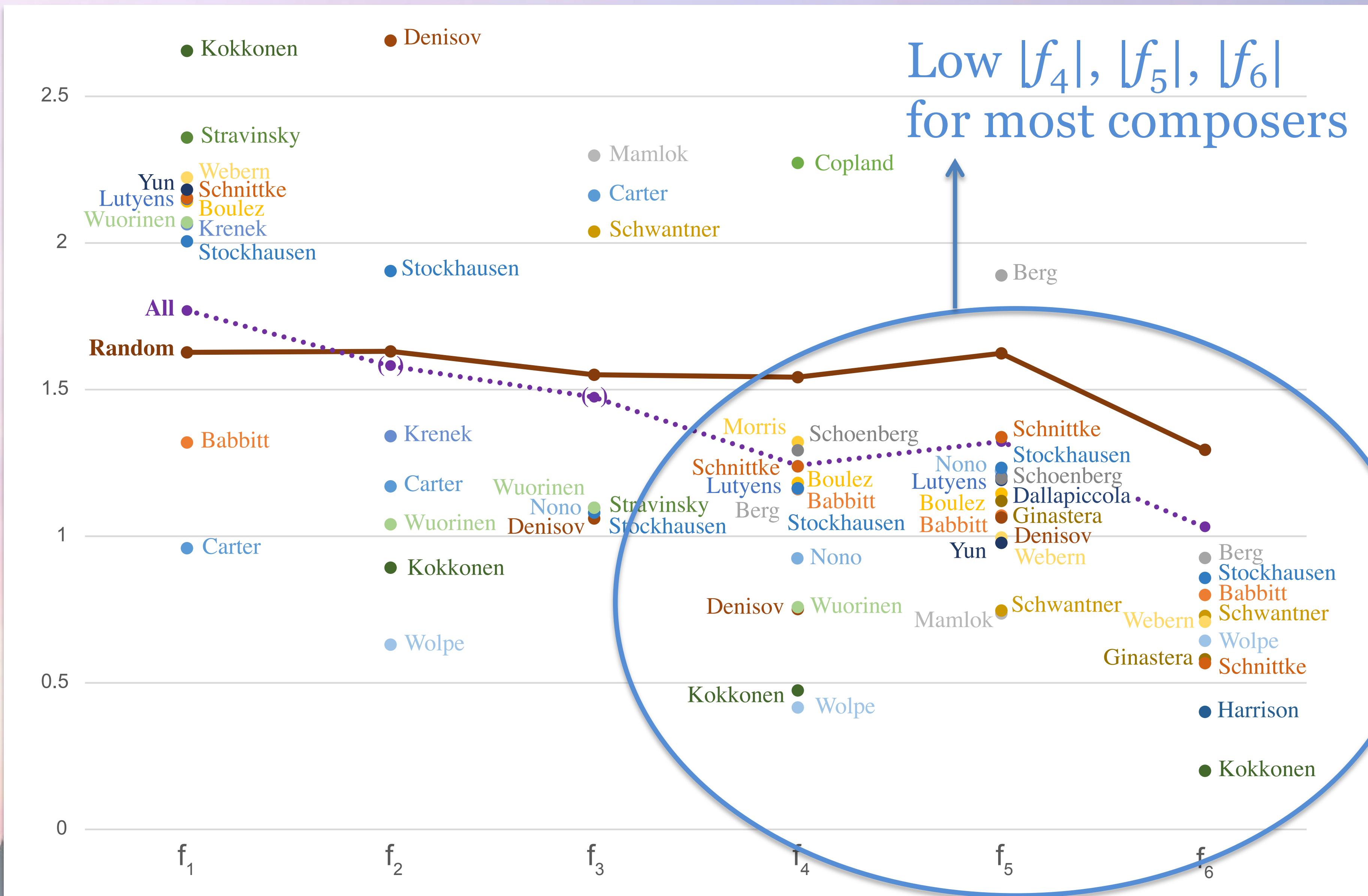

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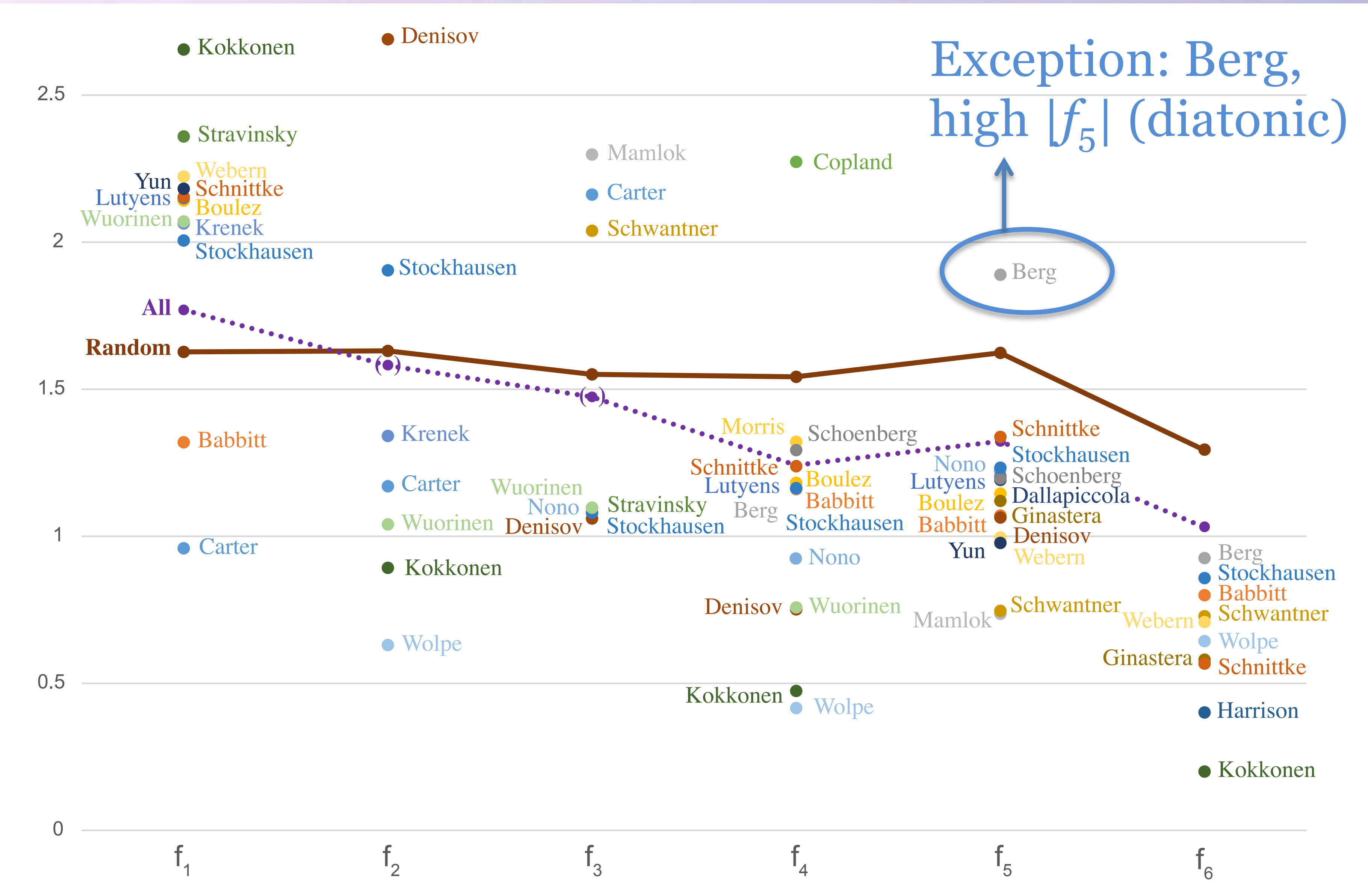
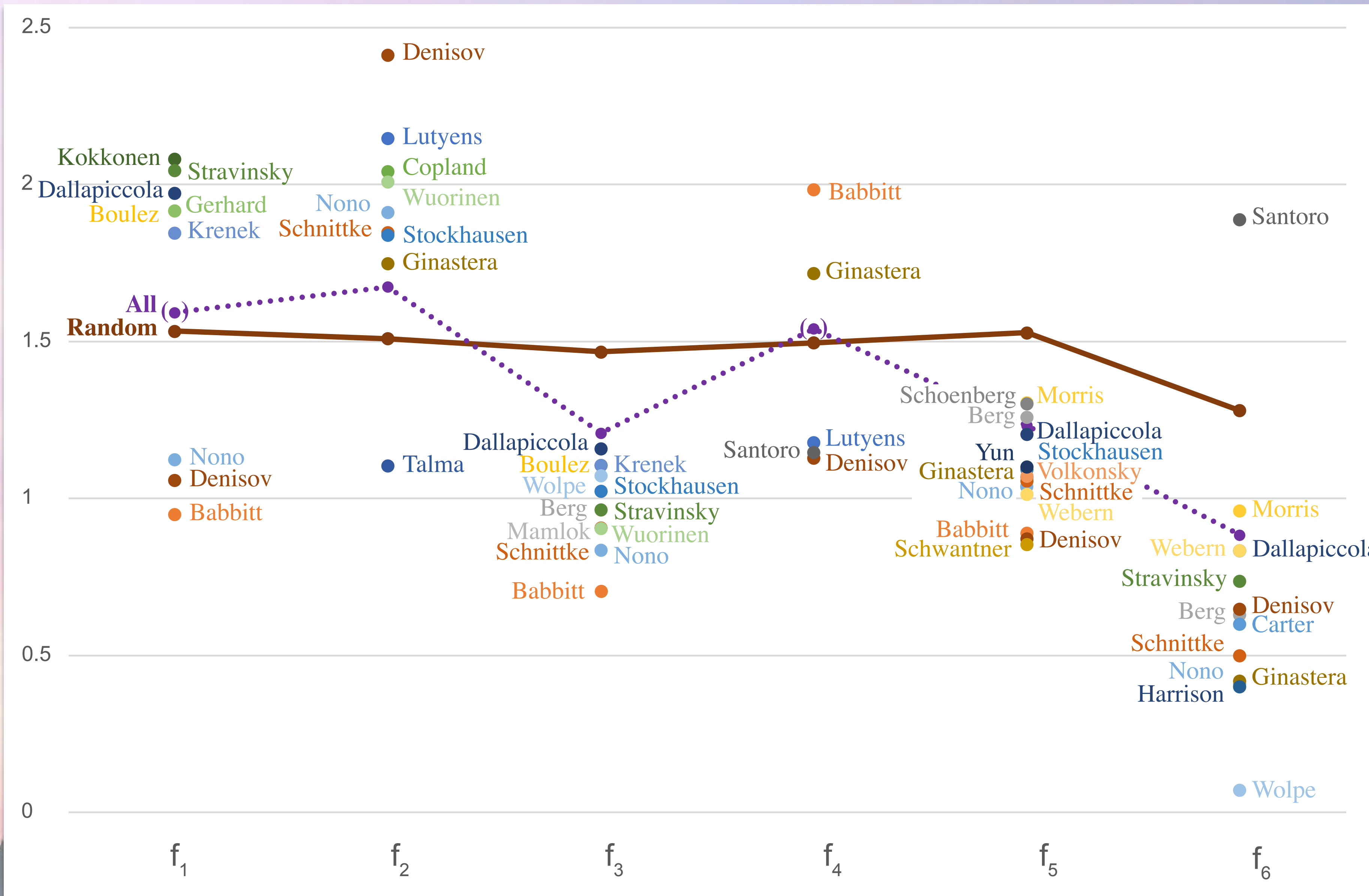
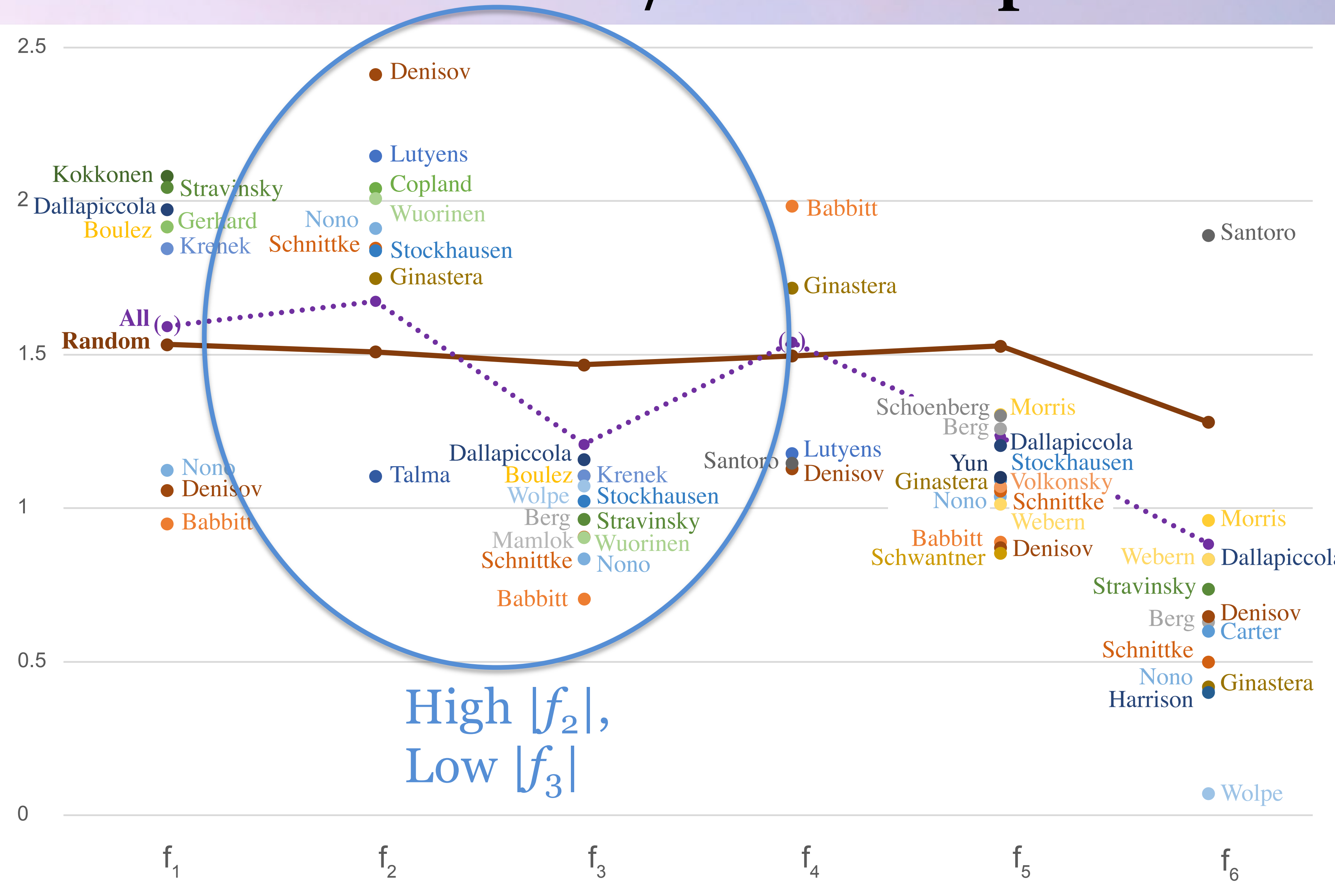


Results: Tetrachord/Octachord partitions



Results: Tetrachord/Octachord partitions



Results: Hexachord partition differences

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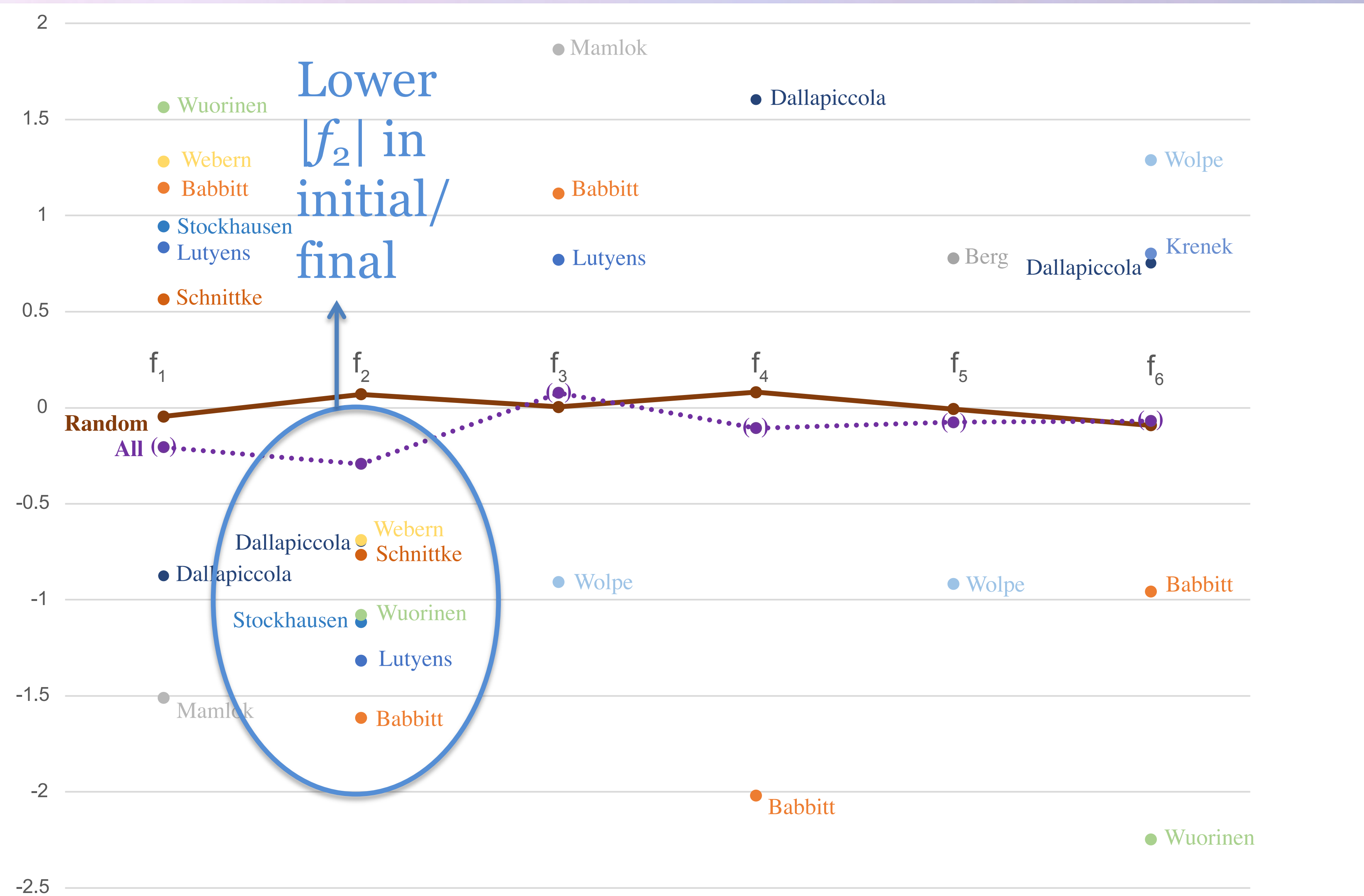

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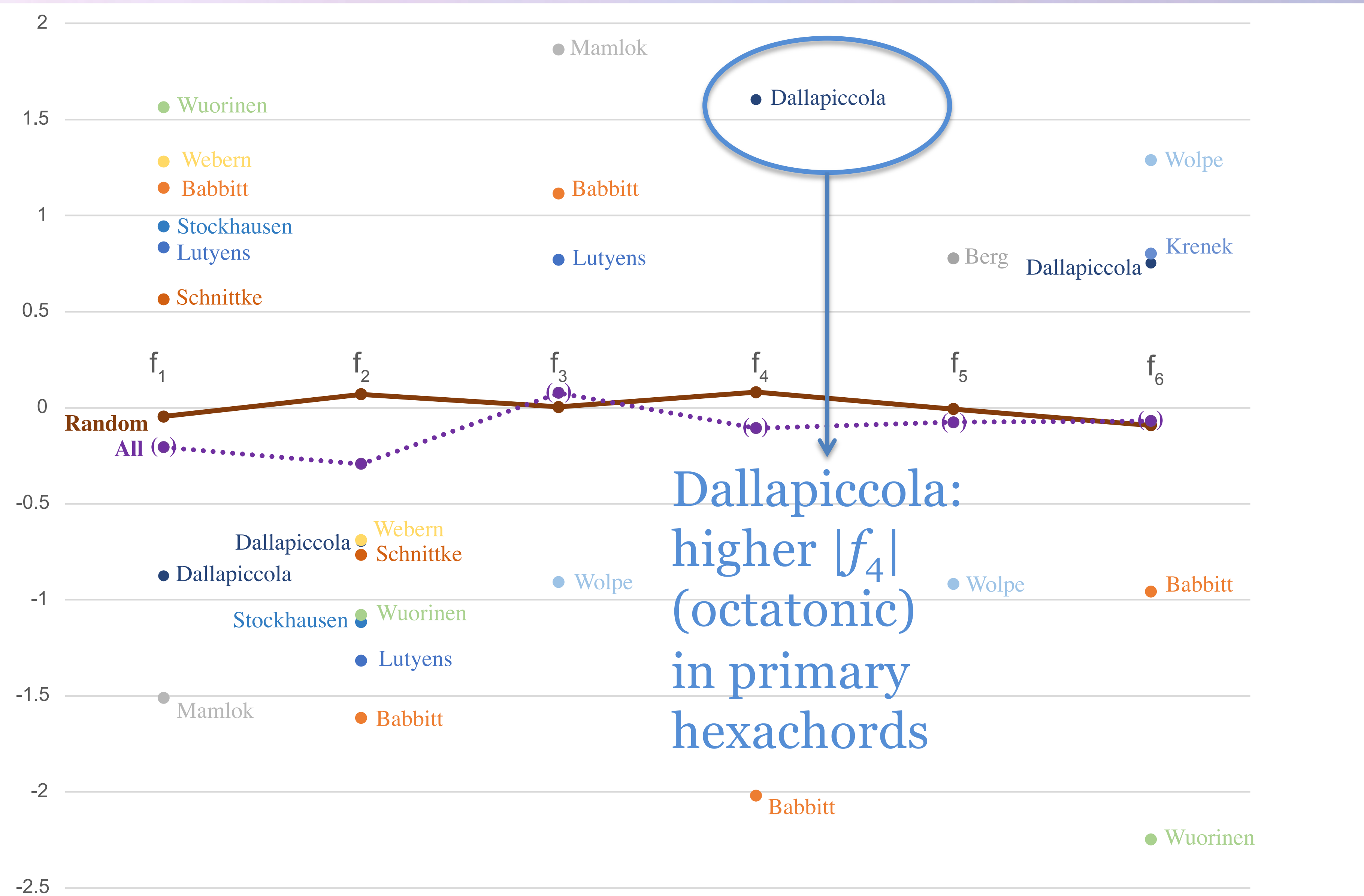

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