Hyperbolic Discounting and the Standard Model: Eliciting Discount Functions*

Jawwad Noor

April 24, 2009

Abstract

Experiments on static intertemporal choice find evidence of particularly extreme impatience toward immediate rewards. While this is often taken as support for hyperbolic discounting, it could also arise because the most likely participants in experiments are also those with the most immediate need for money. We conduct a calibration exercise and find that the extreme impatience observed in experiments can be accommodated by a standard exponential discounting model with no discounting and expectation of a 'small' increase in the base consumption level. The calibration uses existing estimates of curvature of utility.

Keywords: Experiments, Hyperbolic Discounting, Exponential Discounting, Preference Reversals, Decreasing Impatience, DU anomalies.

JEL classification number: D11

---

*Dept of Economics, Boston University, 270 Bay State Road, Boston MA 02215. Email: jnoor@bu.edu. I would like to thank Attila Ambrus, Alberto Bisin, Larry Epstein, David Laibson, Bart Lipman, Alessandro Lizzeri, Klaus Nehring, Efe Ok, Parag Pathak, Larry Samuelson and the Associate Editor for useful comments. Financial support from the NSF (grant SES-0822927) is gratefully acknowledged. The usual disclaimer applies.

1 Introduction

The standard model of intertemporal choice, the *exponential discounting model*, posits that agents discount the utility of a reward by an exponential discount function: $D(t) = \delta^t$ for $\delta \in (0, 1)$. Most experiments on intertemporal choice investigate discount functions by observing the manner in which subjects trade-off money and time, as revealed in subjects' choices between *dated rewards* (money-time pairs (m, t)) at a given point in time. A sizeable experimental literature provides evidence that appears to reject the standard model (see [6] for a review of the evidence). A finding from this literature that has received much attention in economics is that subjects' discount functions possess the *decreasing impatience* property: subjects tend to be more impatient in their money-time trade-offs when the trade-offs involve more immediate dated rewards. This motivates the *hyperbolic discounting model*, which adopts the discount function $D(t) = \frac{1}{1+t}$ (Ainslie [1]).[1] The popular psychological explanation for decreasing impatience is the presence of a desire for immediate gratification and the inability to exert self-control. The evidence has been used to suggest the existence of extreme self-control problems.

The evidence for decreasing impatience is based on direct estimates of discount functions and also on peculiar properties of behavior. Thaler [13] asks subjects to specify the amount of money they would require in [one month/one year/ten years] to make them indifferent to receiving $15 now. From the median responses [$20/$50/$100] it is inferred that subjects use an average annual discount rate of 345% over a one-month horizon, 120% over a one-year horizon and 19% over a ten-year horizon, thereby suggesting decreasing impatience and extreme desire for immediate gratification. Ainslie and Haslam [2] report that "a majority of subjects say they would prefer to have a prize of a $100 certified check available immediately over a $200 certified check that could not be cashed before 2 years; the same people do not prefer a $100 certified check that could be cashed in 6 years to a $200 certified check that could be cashed in 8 years." Such behavior, known as a *preference reversal*, suggests decreasing impatience in the increased willingness to wait an extra 2 yrs for an extra $100 when the trade-off is farther in the future.

[1] The general *decreasing impatience model* is defined by a discount function D that satisfies the property that $\frac{D(t+1)}{D(t)}$ increases with t. A *generalized hyperbolic discounting model* is introduced by Lowenstein and Prelec [11] and the more tractable *quasi-hyperbolic discounting model* is studied by Laibson [10] and subsequent literature.

However, in order to infer the underlying discount function from preferences over dated rewards, it is necessary that the worth to the subject of an extra dollar (the marginal utility for money) be the same across time. It is clear that a subject who is relatively cash constrained today will exhibit the behavior patterns in the evidence even if his underlying discount function is exponential. A potential concern is that experiments (typical subjects of which are undergraduates) may systematically find such subjects because those with the most need for cash are also the most likely participants in experiments.[2]

There is little evidence available to evaluate the extent and significance of such selection bias. The following findings in Zauberman and Lynch [14] are suggestive. These authors directly elicit subjects' expectations regarding the future "slackness" in their budgets. In [14, experiment 1], for instance, 76 undergraduates responded on a 10-point scale ranging from 1 (= much more slack today) to 10 (= much more slack a month from today). Although the time-horizon was only one month, the mean response was 7.0 with a notably large standard deviation of 2.9. See also [14, experiment 7].[3]

Without the assumption of constant marginal utility for money across time (the 'constant marginal utility' assumption for short) the evaluation of money depends on time, and consequently the standard model ceases to have any peculiar testable implications for preferences over dated rewards other than basic restrictions such as monotonicity and transitivity. Indeed, qualitative findings such as preference reversals no longer reject the model. We conduct a calibration exercise to get a sense of the model's ability to fit quantitative findings, specifically estimated discount functions. We show that the extreme self-control problem suggested by the well-known findings in Thaler [13] is in fact also consistent with a standard exponential discounting model (CARA utility) with *no discounting* and a very slight relaxation of the constant marginal utility assumption (as reflected in close-to-zero changes in anticipated base consumption levels). The calibration uses estimates of CARA utility based on aggregate consumption data and also some recent data on choice under risk.

The results suggest that an accounting of expectations may be desirable

[2] We owe this observation to Alessandro Lizzeri.

[3] In the psychology literature, Zauberman and Lynch [14] show that the anticipation of "slack" in a relevant resource (specifically, time or money) is related to the pattern of discounting. This work comes closest in spirit to ours given that resource slack is interpretable in terms of marginal utility.

in experimental studies that seek to elicit discount functions.

2 Eliciting Discount Functions

This section outlines how experiments elicit discount functions and derives the relationship between estimates and the marginal utility of money. Calibration results follow in Section 3.

2.1 DU-Rationalizability

For the nondegenerate set of rewards $\mathcal{M} = [0, M]$ and the set of dates $\mathcal{T} = \mathbb{N}_+$, denote by $\succsim$ a preference relation over the set of dated rewards $X = \mathcal{M} \times \mathcal{T}$. Indifference is denoted by $\sim$ and strict preference by $\succ$.

If the subject evaluates consumption streams according to a *discounted utility model*:[4]

$$V(c_0, .., c_T) = \sum_{t=0}^{T} D(t)u(c_t),$$

and integrates any dated reward (m, t) into his anticipated *base-consumption stream* $(b_t)_{t\in\mathcal{T}}$, then for all $(m, t), (m', t') \in X$, we have $(m, t) \succsim (m', t') \iff$

$$D(t)u(b_t+m)+D(t')u(b_{t'})+\sum_{\tau\neq t,t'} D(\tau)u(b_\tau) \geq D(t)u(b_t)+D(t')u(b_{t'}+m')+\sum_{\tau\neq t,t'} D(\tau)u(b_\tau)$$

$$\iff D(t)[u(b_t + m) - u(b_t)] \geq D(t')[u(b_{t'} + m') - u(b_{t'})],$$

and thus the representation induced on $\succsim$ is

$$U(m, t) = D(t)[u(b_t + m) - u(b_t)]. \quad (1)$$

That is, a dated reward (m, t) is evaluated according to the discounted increase in utility due to (m, t). Say that $\succsim$ is *DU-rationalizable* if it admits

[4] We assume that D is a discount function in the sense that it satisfies $D(0) = 1$ and $D(t)$ is strictly decreasing and tends to 0 as $t \to \infty$. Moreover u is strictly increasing and continuous.

such a representation U for some discount function D, strictly increasing continuous utility u and $(b_t)_{t\in\mathcal{T}}$.[5]

Observe that the receipt of a reward (m,t) matters only to the extent that it adds to the anticipated base-consumption level b_t at t, and in particular, that the marginal utility of the reward is determined by b_t. Thus, in the discounted utility model's abstraction, differences in base-consumption over time is the only vehicle for talking about the marginal utility of a reward depending on t. Consequently, it must be viewed as a *stand-in* for all the factors that might influence marginal utility.

2.2 Elicitation

Most experimental findings (such as decreasing impatience, the magnitude effect, the sign effect, etc.) are properties of estimated discounted functions. The estimation is based on the following procedure.

By having the subject face a sequence of choices, or by asking him directly, the experiments elicit the *present equivalent* $\psi(m,t)$ of any dated reward (m,t):

$$(\psi(m,t),0)\sim(m,t).$$

This information is used to construct a discount function by setting, for all $m>0$ and t,

$$\phi^m(t)\equiv\frac{\psi(m,t)}{m}. \tag{2}$$

For instance, if \$80 is the present equivalent of $(\$100,t)$ then the subject is said to discount the \$100 reward by a factor of $\phi^{100}(t)=0.8$. We refer to $\phi^m(t)$ as the subject's *money-discount function*.

Money-discount functions are viewed as estimates of the subjects' underlying discount function when the size of rewards used are small. The justification for the claim comes from the presumption that utility over small intervals is approximately linear, but also crucially depends on the constant marginal

[5] This definition rests on an implicit *commitment assumption*: for any (m,t), the money m is consumed entirely at time t without any other changes in the consumption stream. Note that perfect consumption smoothing implies that money-discount functions (defined by (2) below) must be exponential: a $\$m$ reward at t is only good as its present value $\frac{m}{(1+r)^t}$, and thus for all m, $\phi^m(t)=\frac{1}{(1+r)^t}$. Since nonexponential money-discount functions are a robust finding in the experiments, restricting attention to imperfect consumption smoothing is justified, and the commitment assumption serves as a useful benchmark in this case.

utility assumpion. Note first that by the assumption of DU-rationalizability (1) and the definition of present equivalents, for any (m, t),

$$u(b_0 + \psi(m,t)) - u(b_0) = D(t)[u(b_t + m) - u(b_t)]. \tag{3}$$

Assuming differentiability where needed, differentiating with respect to m and evaluating at $m = 0$ yields $\frac{\partial\psi(m,t)}{\partial m}\mid_{m=0}= \frac{D(t)u'(b_t)}{u'(b_0)}$. Given $\psi(0,t) = 0$ and definition (2), l'Hopital's rule yields that $\lim_{m\to 0}\phi^m(t) = \frac{\partial\psi(m,t)}{\partial m}\mid_{m=0}$. Hence,

$$\lim_{m\to 0}\phi^m(t) = \frac{D(t)u'(b_t)}{u'(b_0)}. \tag{4}$$

But then, for concave u, the claim that $\phi^m(t)$ approximates $D(t)$ for small m is justified only when b_t is close to b_0.

3 Calibration

Consider the evidence in Thaler [13] where, on average, agents exhibit:

$$(\$15,\ \text{now}) \sim (\$20, 1\ \text{month}),\ (\$50,\ 1\ \text{yr}),\ (\$100, 10\ \text{yrs}). \tag{5}$$

These preferences imply that – under the constant marginal utility assumption and under the assumption that rewards of \$100 or less are sufficiently "small" rewards – on average the subjects use an annual discount rate of 345%, 120% and 19% over a one-month, one-year horizon and ten-year horizon respectively. This suggests an extreme desire for immediate gratification and corresponding self-control problem, and motivates the hyperbolic discounting model.

We determine how much change in base-consumption over time is required in order to rationalize these preference with a standard exponential discounting model

$$V(c_0, .., c_T) = \sum_{t=0}^{T} \delta^t u(c_t),$$

with CARA utility $u(c) = \frac{1-e^{-ac}}{a}$, $a > 0$. Use (3) to determine that for any (m, t),

$$b_t - b_0 = \frac{1}{a}\ln\left(\delta^t \frac{1-e^{-am}}{1-e^{-a\psi(m,t)}}\right). \tag{6}$$

In all that follows we assume *no discounting*, $\delta = 1$; observe that this biases results upward. Taking values for (m, t) from (5) delivers values for $b_t - b_0$ that depend only on the coefficient a that controls the curvature of utility. The unit of time is a month.

The appropriate data for judging a theory of intertemporal choice is intertemporal substitution data. We consider estimates of a from the macroeconomics literature. Gregory, Lamarche and Smith [7] use US aggregate consumption data on nondurables and services for the period 1959-1998 and estimate an Euler equation with CARA utility. They provide a range of estimates, the smallest being $a = 0.124$.[6] This is in line with various macroeconomic studies that find that estimates of the elasticity of intertemporal substitution based on aggregate consumption data or consumption data of the average consumer (non-stockholders) is typically close to 0 (Hall [8], Attansio et al [3]). Table 1 below shows that for $a = 0.124$, and in fact for values of a as small as 0.01, the data (5) can be accommodated with a practically constant base-consumption stream.

Estimates of a based on choice under risk are not directly relevant for judging a theory of intertemporal choice. Nevertheless, we note that some recent estimates of a based on choice under risk yield values higher than 0.01. Choi, Fisman, Gale and Kariv [4] find that a disappointment aversion model fits better then expected utility; their specification with a CARA utility index yields a mean estimate of $a = 0.086$.[7] The value of a based on the 'large-stakes' treatment data in Holt and Laury [9] is $a = 0.032$.[8] The corresponding calibrated values of $b_t - b_0$ are given in Table 1. For perspective, we also report calibrated values corresponding to the mean estimate $a = 0.0031$ obtained in Cohen and Einav [5] by estimating a structural econometric model using

[6] The values reported in [7, Table 4] are based on annual consumption measured in units of $10,000. The smallest value (103.7) translates to a value of 0.12444 when consumption is measured in monthly terms and units of $1, as required for our purposes. We thank Gregor Smith for clarifications and the translation into our setting.

[7] The values reported in [4, Table 3] are based on consumption in units of experimental currency. We thank Syngjoo Choi for providing us with the estimate that takes $1 as the unit of consumption.

[8] Holt and Laury [9] do not estimate a CARA utility. The figure is taken from [5, Table 3] who estimate a CARA utility for the "×90" treatment in [9].

data on deductible choices in auto insurance contracts.

a	$b_1 - b_0$	$b_{12} - b_0$	$b_{120} - b_0$
0.124	\$0.7	\$1.4	\$1.4
0.100	\$1.1	\$2.5	\$2.5
0.086	\$1.5	\$3.6	\$3.7
0.032	\$6.7	\$23.1	\$28.8
0.010	\$26.3	\$103.8	\$151.3
0.0031	\$90.3	\$371.2	\$570.7

Table 1: Approximate change in monthly base-consumption over t months, $t \in \{1, 12, 120\}$, for different values of a.

The approximation in (4) for 'small' m reveals clearly that even after restricting attention to an exponential discount function $D(t) = \delta^t$, by varying δ and u one can find a large range of possible base-consumption streams that can accommodate any given money-discount function $\phi^m(t)$. For instance, given the data (5), the calibrated values for $b_{120} - b_0$ above could be increased by requiring u to exhibit weakly decreasing absolute risk aversion via a reduction in curvature beyond a particular point. Moreover, all calibrated values are lower if we take $\delta < 1$, and with a suitable δ the calibrated base-consumption stream can even be decreasing.

While we have shown that there exist estimates for CARA u that allow the data (5) to be accommodated without requiring drastic changes in base-consumption, it should be acknowledged that there are also studies that suggest substantially lower estimates for a, and that such estimates would require drastic changes in base-consumption. Macroeconomic studies based on the consumption data of stockmarket investors and those obtained from calibrated macroeconomic models designed to match growth and business cycle facts typically suggest that the elasticity of intertemporal substitution is close to 1, and this corresponds to values of a that are closer to 0. Studies on choice under risk based on data from television game shows also find values closer to 0.

4 Concluding Remarks

Without constant marginal utility for money over time, the exponential discounting model has no peculiar testable implications for preferences over dated rewards, other than basic properties like completeness, transitivity and monotonicity. It is readily seen that if a preference $\succsim$ over dated rewards admits some 'regular' (continuous, increasing with respect to money, decreasing with respect to time) representation $U(m,t)$, then that representation can be rewritten in the form (1), even if D is chosen to be exponential. Thus any nonstandard utility model for $\succsim$ that captures the findings in experiments can be replicated behaviorally by an exponential discounting model. Indeed, qualitative findings such as preference reversals do not necessarily reject exponential discounting without the assumption of constant marginal utility over time.[9] Verification of the constant marginal utility assumption or some accounting for possibly non-constant marginal utility is therefore desirable in experimental work on intertemporal choice.

In the literature, various experiments reject exponential discounting on the basis of dynamic choice data by finding violations of *dynamic consistency*. However, expectations are relevant here as well. A robust finding in psychology experiments is the prevalence of positive illusions [12]. If subjects have an optimistic bias in their expectations about future marginal utility for money, then experiments will systematically find a proportion of subjects that exhibit patience in distant trade-offs but appear to become relatively less patient in their re-evaluations over time, in line with what is observed in experiments [6].

References

[1] Ainslie, G. (1992), *Picoeconomics*, Cambridge University Press.

[2] Ainslie, G. and N. Haslam (1992): 'Hyperbolic Discounting' in G. Loewenstein and J. Elster, eds., *Choice over Time*, New York: Russel Sage Foundation.

[9] It is straightforward to show that if a standard agent with concave utility anticipates a one-time increase in base-consumption over the horizon of the experiment, then he exhibits preference reversals. If the commitment assumption (see footnote 5) is relaxed, it is also possible to construct examples whereby preference reversals arise even if a one-time increase occurs *beyond* the horizon of the experiment. Intuitively, if the agent cannot borrow but is free to save, earlier rewards may be relatively more attractive than later rewards as they permit consumption smoothing over more periods (from the period of receipt of reward upto the period of the one-time increase), and this relative attractiveness may decay with delay.

[3] Attanasio, O., J. Banks and S. Tanner (2002): 'Asset Holding and Consumption Volatility', *Journal of Political Economy* 110, pp 771-93.

[4] Choi, S., R. Fisman, D. Gale and S. Kariv (2007): 'Consistency and Heterogeniety of Individual Behavior under Uncertainty', *American Economic Review* 97(5), pp 1921-1938.

[5] Cohen, A., and L. Einav (2007): 'Estimating Risk Preferences from Deductible Choice', *American Economic Review* 97(3), pp 745-788.

[6] Frederick, S., G. Loewenstein and T. O'Donoghue (2002): 'Time Discounting and Time Preference: A Critical Review', *Journal of Economic Literature* 40(2), pp 351-401.

[7] Gregory, A., J. Lamarche and G. Smith (2002): 'Information-Theoretic Estimation of Preference Parameters: Macroeconomic Applications and Simulation Evidence', *Journal of Econometrics* 107, pp 213-233.

[8] Hall, R.(1988): 'Intertemporal Substitution in Consumption', *Journal of Political Economy* 96(2), pp 339-356.

[9] Holt, C. and S. Laury (2002): 'Risk Aversion and Incentive Effects', *American Economic Review* 92(5), pp 1644-1655

[10] Laibson, D. (1997): 'Golden Eggs and Hyperbolic Discounting', *Quarterly Journal of Economics* 112, pp 443-77.

[11] Loewenstein, G. and D. Prelec (1992): 'Anomalies in Intertemporal Choice: Evidence and an Interpretation', *The Quarterly Journal of Economics* 107(2), pp 573-597.

[12] Taylor, S. and J. Brown (1988): 'Illusion and Well-being: A Social Psychological Perspective on Mental Health', *Psychological Bulletin* 103, pp 193-210.

[13] Thaler, R. (1981): 'Some Empirical Evidence on Dynamic Inconsistency', *Economic Letters* 8, pp 201-207.

[14] Zauberman, G. and J. Lynch (2005): 'Resource Slack and Propensity to Discount Delayed Investments of Time Versus Money', *Journal of Experimental Psychology: General* 134(1), pp 23-37.