EK 307: Electric Circuits

Fall 2017

Lecture 16

Nov 2, 2017

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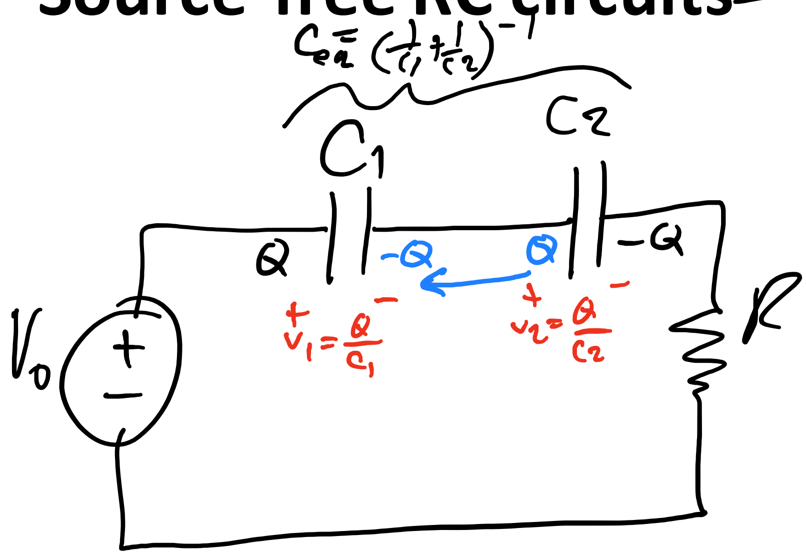
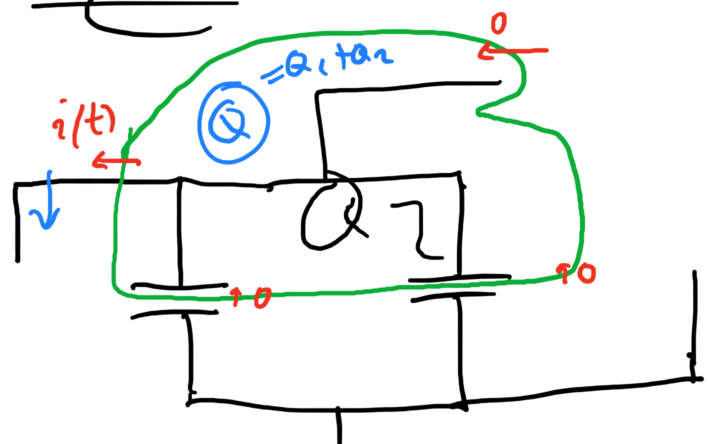
Lecture 16: Chapter 7: 1st order ccts

1. **First order circuits**
2. **Source-free RC circuits**



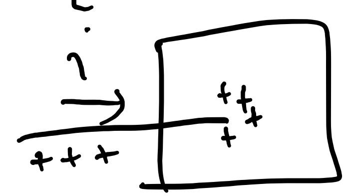


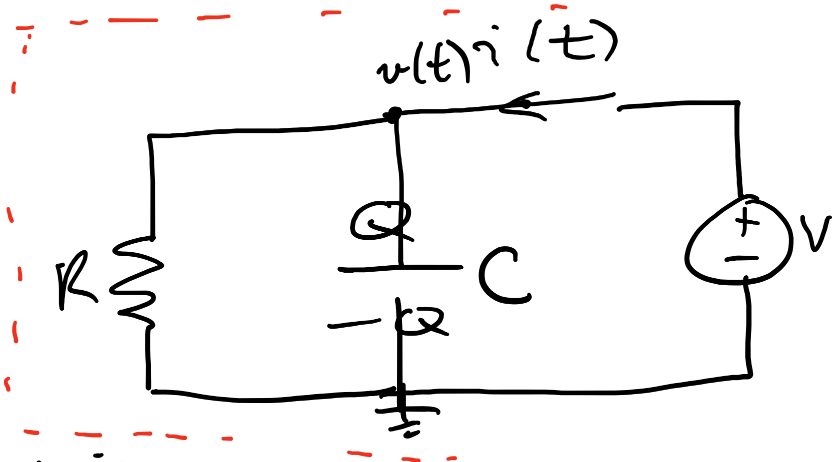
$Q = C_{eq} V_0$

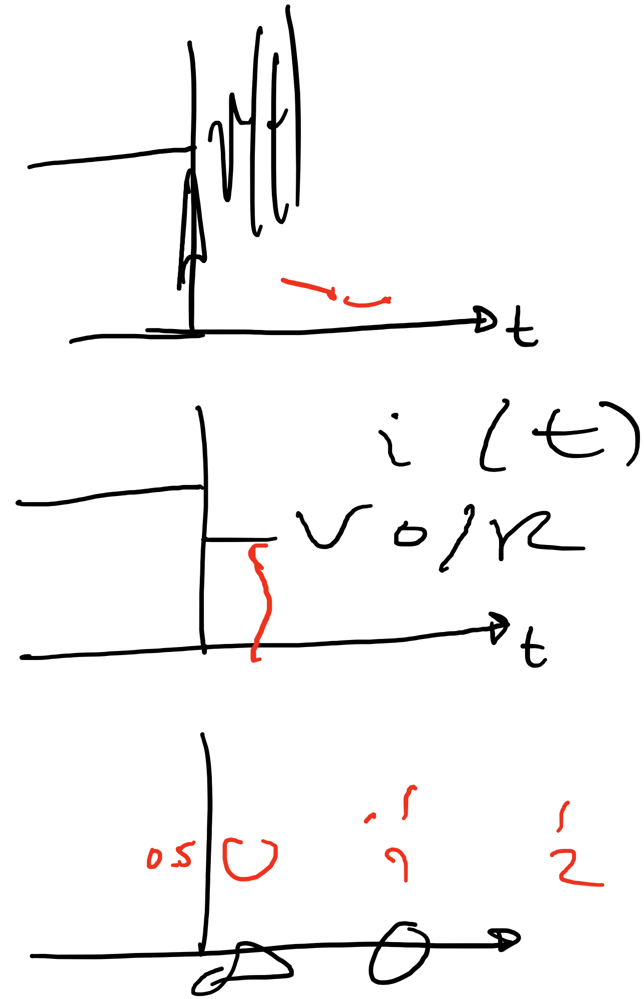
$i = \frac{v(t)}{R} = \frac{V_0}{R}$ at $t=0$

$-i = \frac{d}{dt} Q$

$v(t) = \frac{Q_1 + Q_2}{C_1 + C_2}$





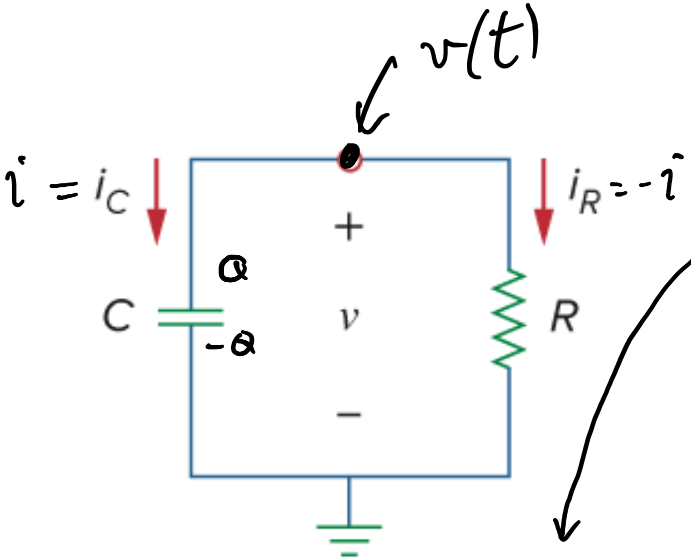


At $t=0^-$:

$$i(t) = \frac{V_0}{R}$$

$$-i(t) = \frac{dQ(t)}{dt}$$

$$v(t) = \frac{Q(t)}{C}$$



$v(0) = V_o$

$$\boxed{v = v_R = v_c}$$

$$i_c = -i_R$$

$$i = i_c = C\frac{dv_c}{dt} = C\frac{dv}{dt}, \quad v = v_R = i_R R = -iR$$

$$\boxed{\frac{dv(t)}{dt} = -\frac{1}{RC}v(t)}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)$$

$$\frac{1}{v(t)}\frac{d}{dt}v(t) = -\frac{1}{RC}$$

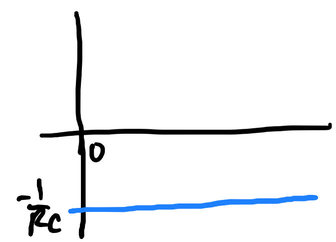
$$\frac{d}{dt}\left[\ln v(t)\right] = -\frac{1}{RC}$$

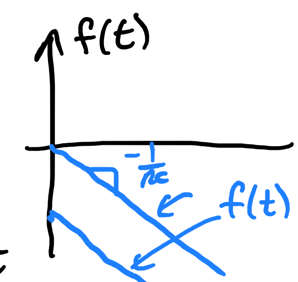
$$f(t) = \ln v(t)$$

$$\int_0^t dt'\frac{d}{dt'}f(t') = \int_0^t -\frac{1}{RC}\,dt'$$

$$f(t) - f(0) = -\frac{1}{RC}t'\Big|_0^t$$

$$= -\frac{1}{RC}(t-0) = \frac{-1}{RC}t$$





$$f(t) - f(0) = -\frac{1}{RC} t$$

$$\underbrace{\ln v(t) - \ln v(0)}_{\ln\left(\frac{v(t)}{v(0)}\right)} = -\frac{t}{RC}$$

$$e^{(\cdot)} \quad \ln\left(\frac{v(t)}{v(0)}\right) = \frac{-t}{RC}$$

$$\boxed{\frac{v(t)}{v(0)} = e^{-t/RC}}$$

$$\boxed{v(t) = v(0)\, e^{-t/RC}}$$

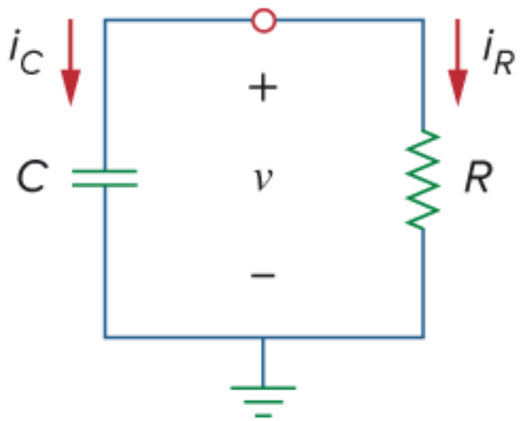
$$f(t) = \int_0^t t'^2 \, dt' = \frac{t^3}{3}$$

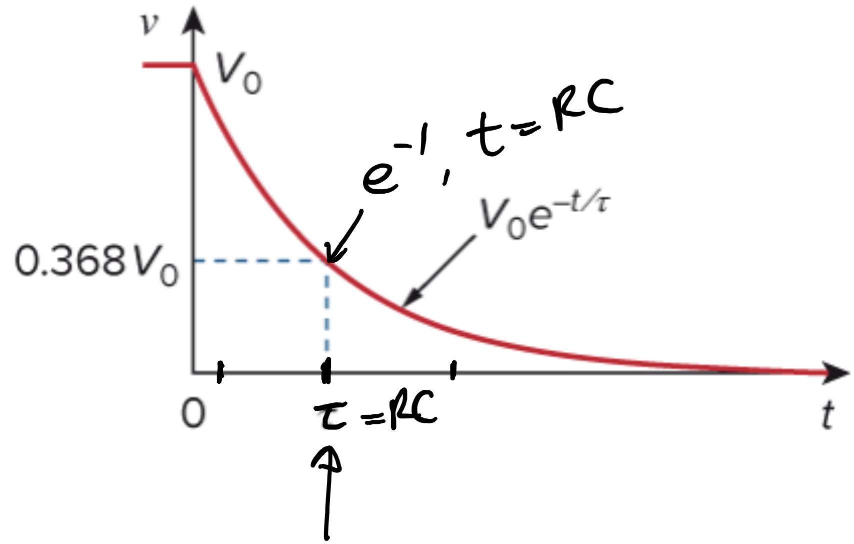
$$f(t) = \int_0^t t^2 \, dt$$

$$f(5) = \int_0^5 5^2 \, d5$$

$$f(2) = \int_0^2 t'^2 \, dt'$$

$v(0) = V_o$

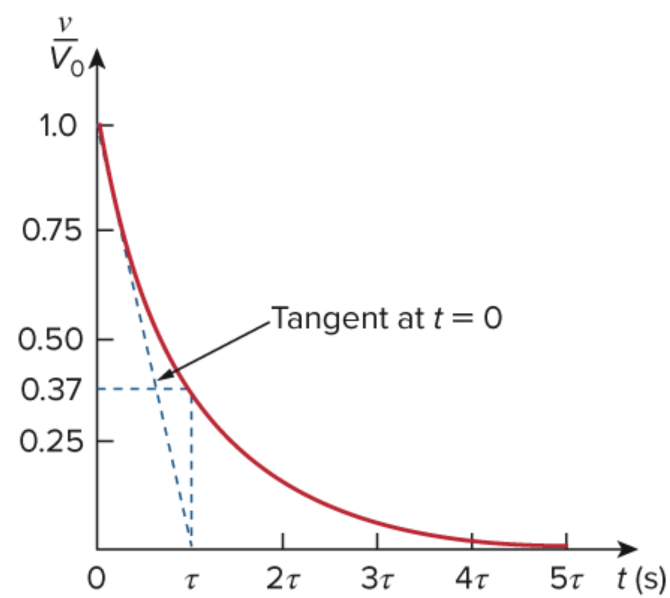


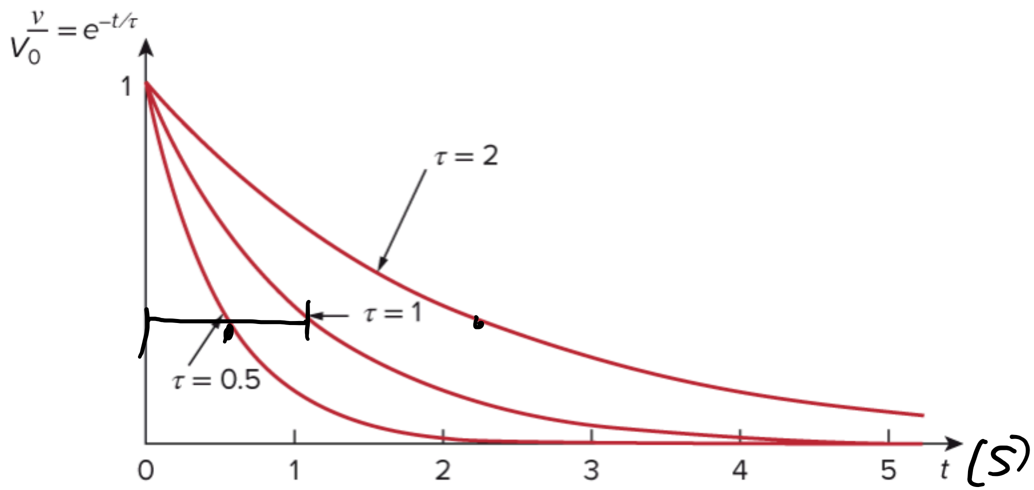


Natural response

$$v(t) = v(0)\, e^{-t/RC}$$

$$= v(0)\, e^{-t/\tau}$$

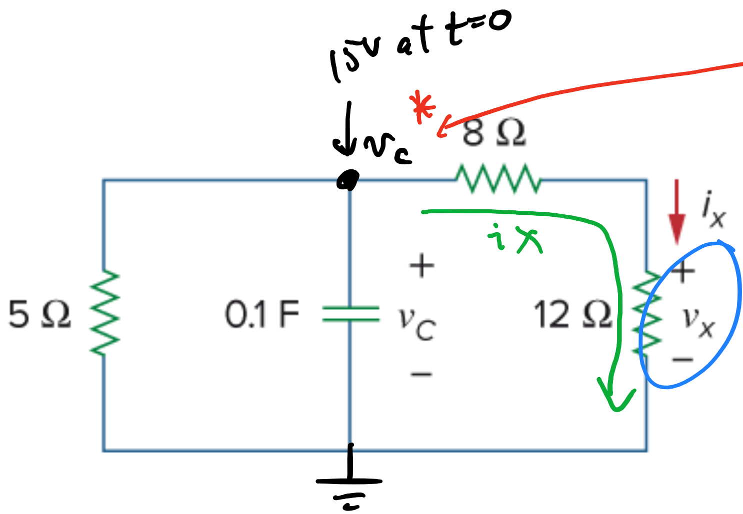


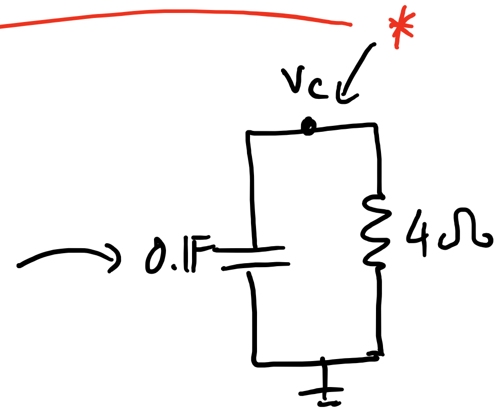


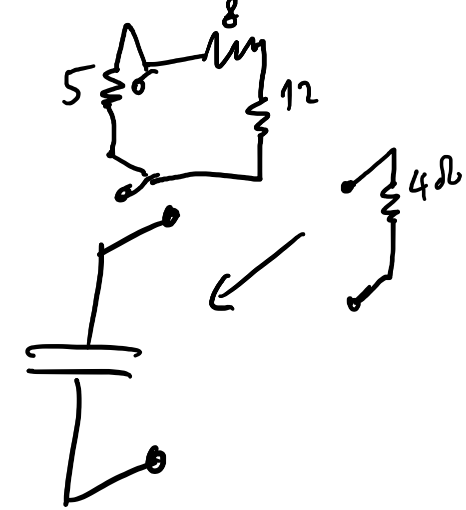
$\frac{v}{V_0} = e^{-t/\tau}$
1
$\tau = 2$
$\tau = 1$
$\tau = 0.5$
0
1
2
3
4
5
t
(s)

Example 7.1

In **Fig. 7.5**, let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.

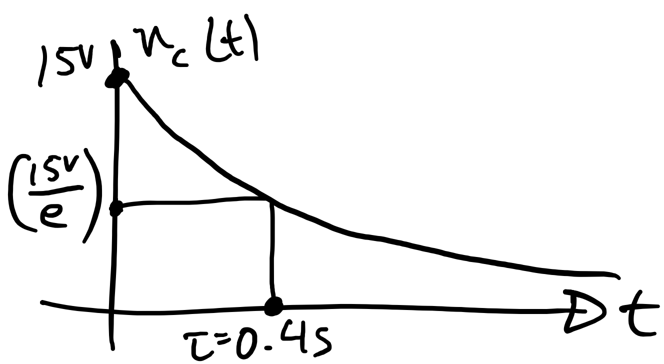






$\tau = RC = 0.1F \cdot 4\Omega = 0.4S$

* $v_c(t) = v_c(0)\, e^{-t/0.4s} = v_c(0)\, e^{-(2.5s^{-1})t}$



$$i_x(t) = \frac{v_c(t) - 0}{8\Omega + 12\Omega}$$

$$= \frac{v_c(t)}{20\Omega}$$

$$= \frac{15V}{20\Omega} e^{-t/0.4s}$$

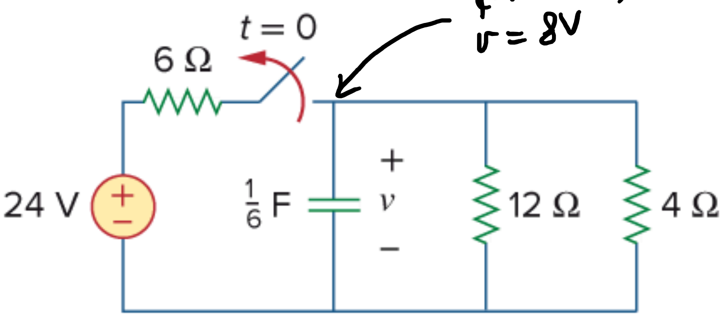
$$\boxed{i_x(t) = 0.75A\, e^{-t/0.4S}}$$

$v_x(t) = i_x(t)\, 12\Omega$

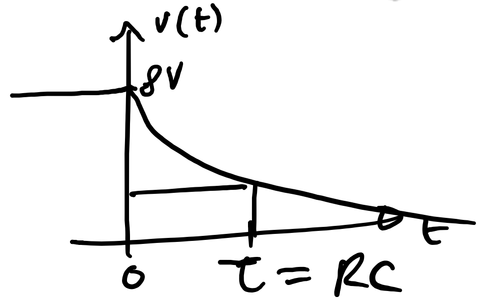
$$\boxed{v_x(t) = 9V\, e^{-t/0.4S}}$$

Practice Problem 7.2

If the switch in **Fig. 7.10** opens at $t = 0$, find $v(t)$ for $t \geq 0$ and $w_C(0)$.



for $t<0$, $v = 8V$

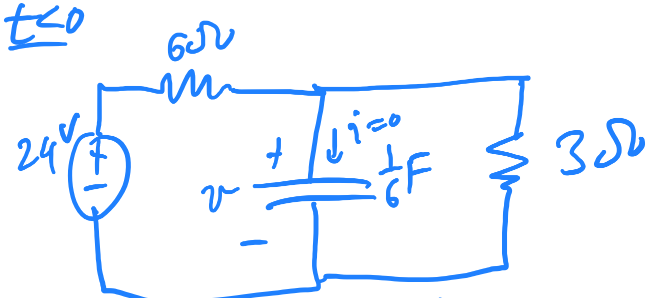


$\tau = RC$

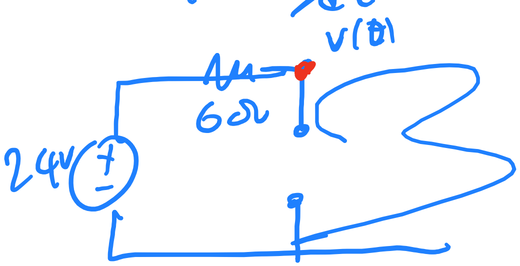
$= \frac{1}{6}F \cdot 3\Omega$

$= 0.5s$

$t<0$

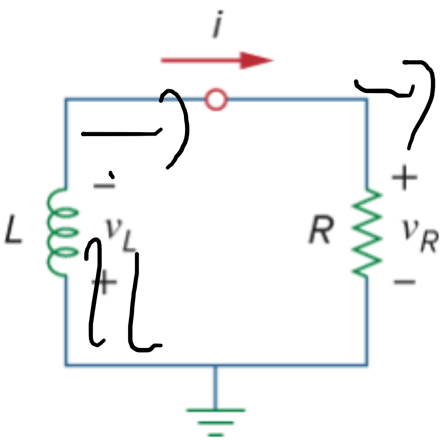


$i = C\cancel{\frac{dv}{dt}}^{\,0} = 0$



$V = \frac{3}{3+6} 24 = 8V$

Source-free RL circuits



$i =$ (KCL)

$v = v_L = -v_R$ (KVL)

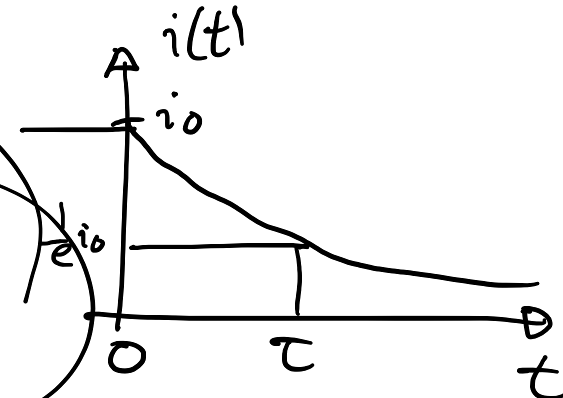
$v_R = i_R R \longrightarrow -v = iR$

$v_L = L\frac{di_L}{dt} \longrightarrow v = L\frac{di}{dt}$

$$L\frac{di}{dt} = -iR$$

$$\boxed{\frac{d}{dt}i(t) = -\frac{R}{L}i(t)}$$

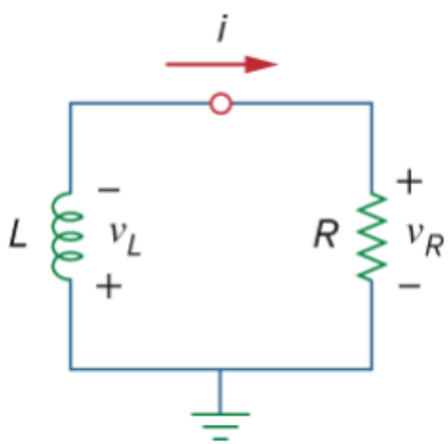
$$\tau = \frac{L}{R}$$

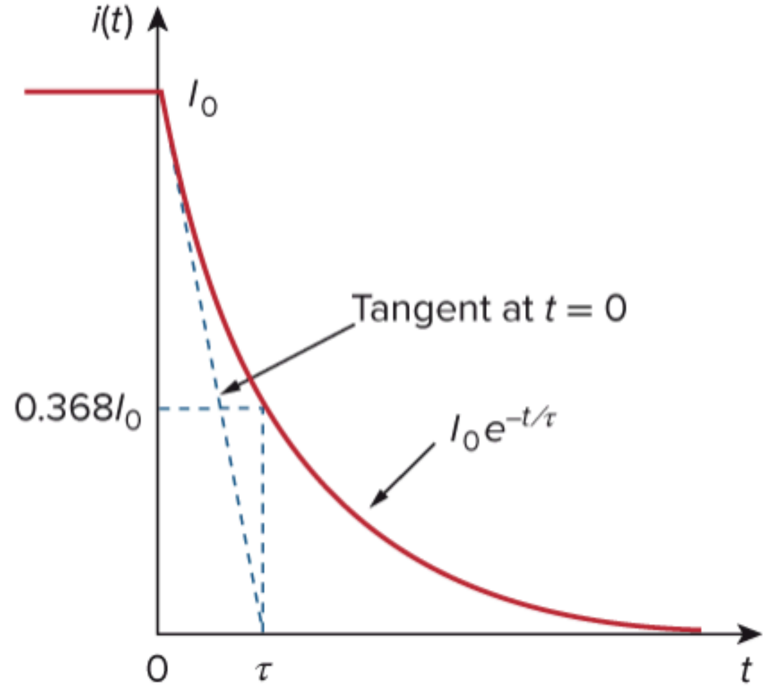


$$\boxed{\frac{d}{dt}v(t) = \frac{-1}{RC}v(t)}$$

$$\tau = RC$$

Source-free RL circuits



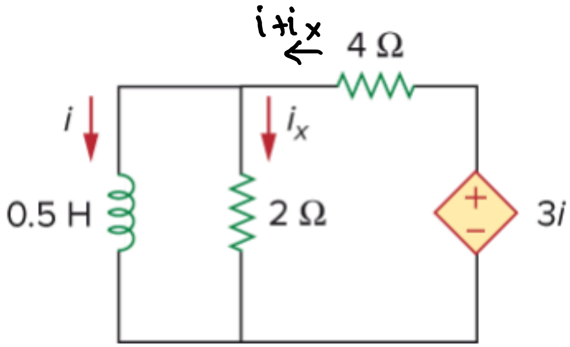


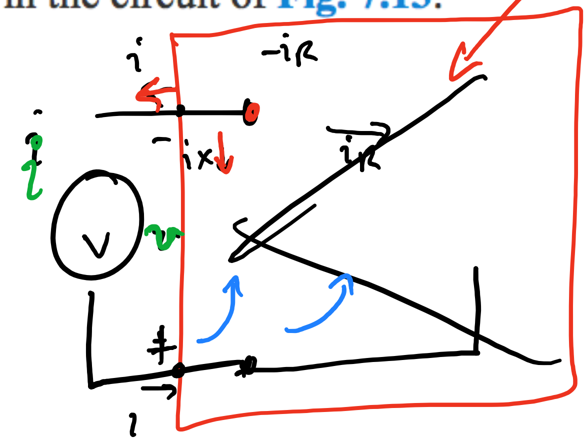
Example 7.3

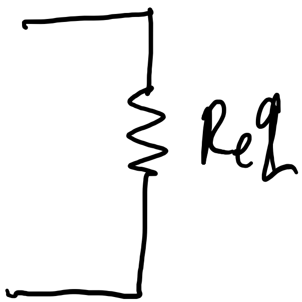
Assuming that $i(0) = 10$ A, calculate $i(t)$ and $i_x(t)$ in the circuit of **Fig. 7.13**.

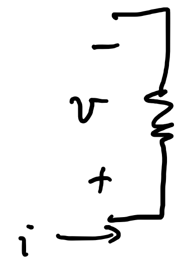
$i_R + i_x + i = 0$

$i_R = -(i_x + i)$









$R = \frac{v}{i}$

$v = -i_x \cdot 2 =$

$v = -(3i + i_R \cdot 4\Omega)$

$-(i_x + i)$

$v = -(3i - 4(i_x + i))$

$\boxed{v = i + 4i_x}$

$v = -2i_x$

$-2i_x = i + 4i_x \qquad i = -6i_x$

$\boxed{\frac{i_x}{i} = -\frac{1}{6}}$

$v = -7i + 4i_x = -2i_x$

$R = \frac{v}{i} = \frac{-2i_x}{i}$

$R = -2\left(-\frac{1}{6}\right) = \frac{1}{3}\,\Omega$

$i(t) = i_0 e^{-t/\tau}$

$= 10A\, e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{0.5H}{\frac{1}{3}\Omega} = 1.5s$

Practice Problem 7.5

Determine i, i_o, and v_o for all t in the circuit shown in Fig. 7.22. Assume that the switch was closed for a long time. It should be noted that opening a switch in series with an ideal current source creates an infinite voltage at the current source terminals. Clearly this is impossible. For the purposes of problem solving, we can place a shunt resistor in parallel with the source (which now makes it a voltage source in series with a resistor). In more practical circuits, devices that act like current sources are, for the most part, electronic circuits. These circuits will allow the source to act like an ideal current source over its operating range but voltage-limit it when the load resistor becomes too large (as in an open circuit).

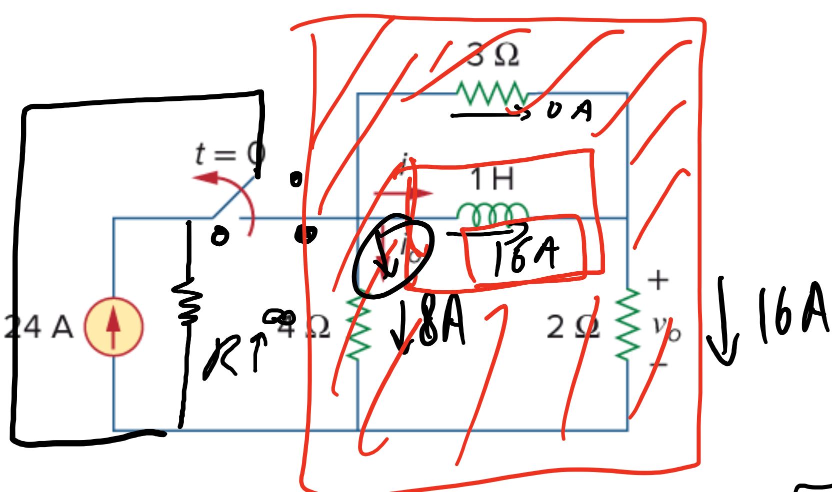
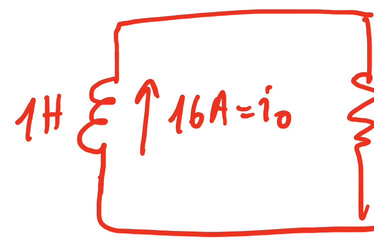
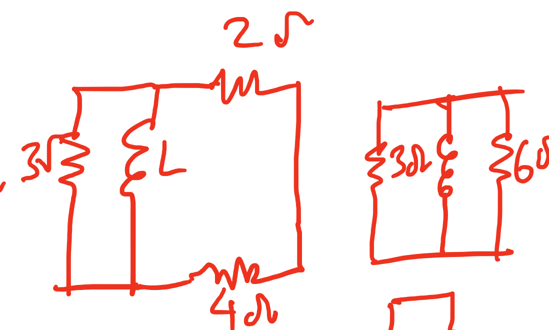
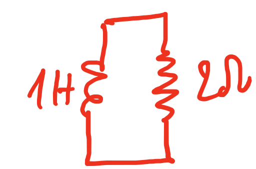


Figure 7.22 For Practice Prob. 7.5.

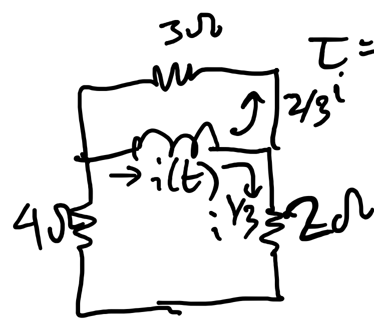






$i(t) = 16A\, e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{1H}{2\Omega} = 0.5s.$



$\rightarrow v_o = i\frac{1}{3} \cdot 2\Omega = \frac{2}{3} 16\, e^{-t/\tau}$ V

$i_o = -\frac{1}{3} i(t) = \frac{-16}{3} e^{-t/\tau}$ A

Answer:

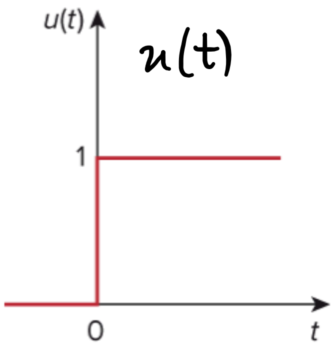
$$i = \begin{cases} 16\text{ A}, & t < 0 \\ 16e^{-2t}\text{ A}, & t \geq 0 \end{cases}, \qquad i_o = \begin{cases} 8\text{ A}, & t < 0 \\ -5.333e^{-2t}\text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32\text{ V}, & t < 0 \\ 10.667e^{-2t}\text{ V}, & t > 0 \end{cases}$$

Singularity functions

Heaviside step fcn:

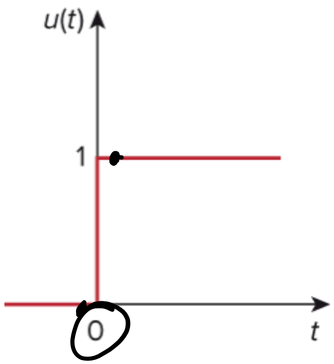
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$



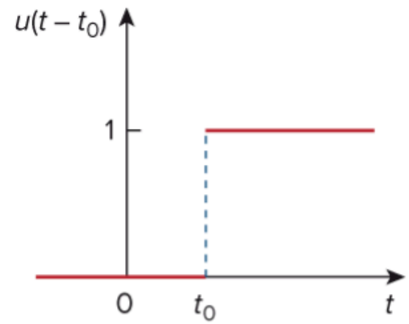
Singularity functions

$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

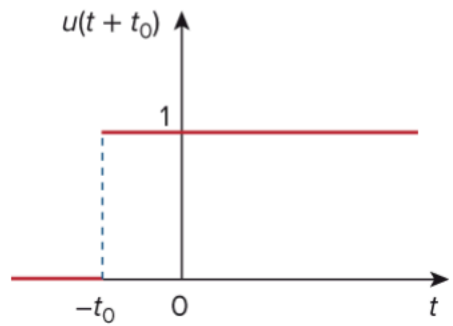


$u(t)$

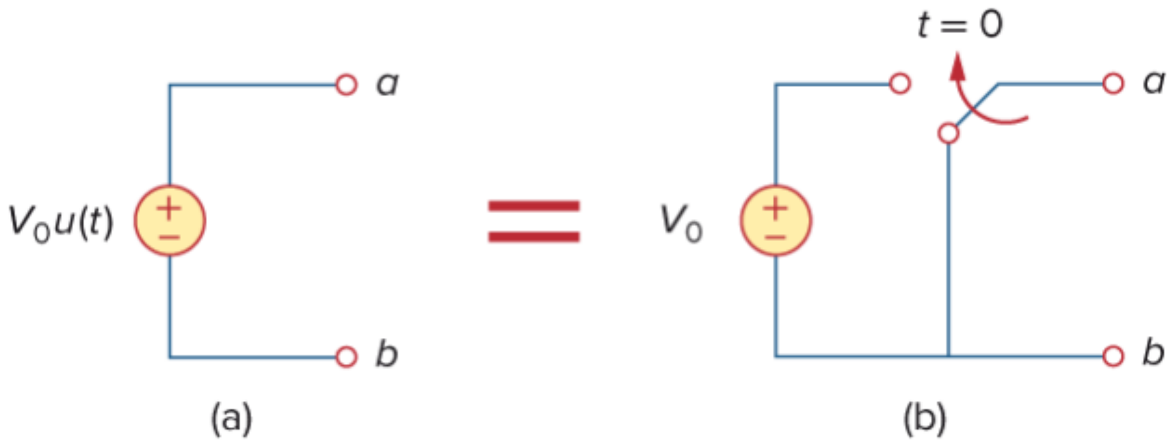


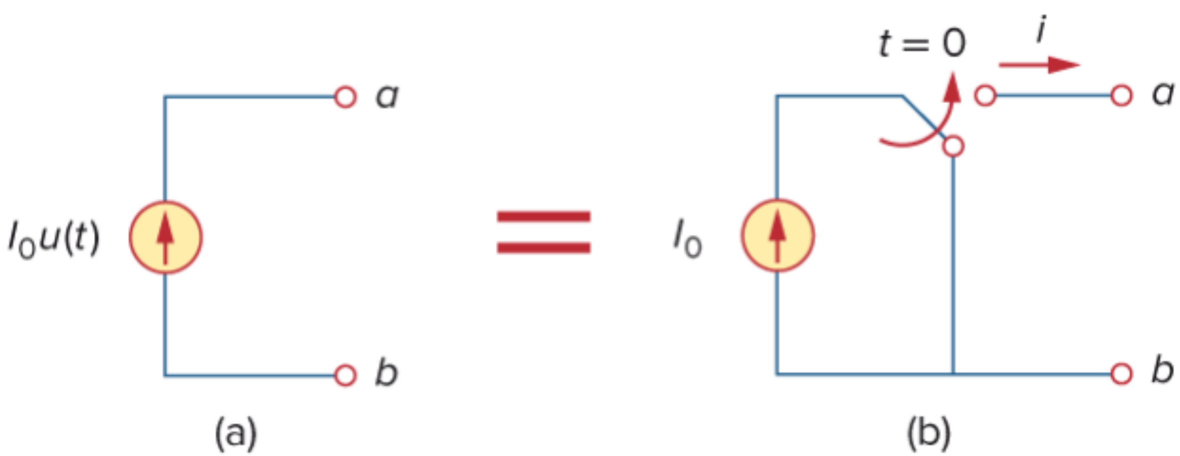
(a)

$u(\underbrace{t - t_0}_{x>0})$

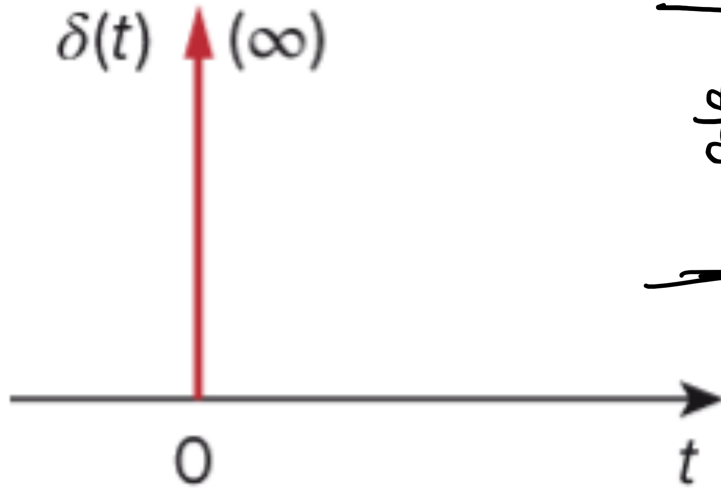


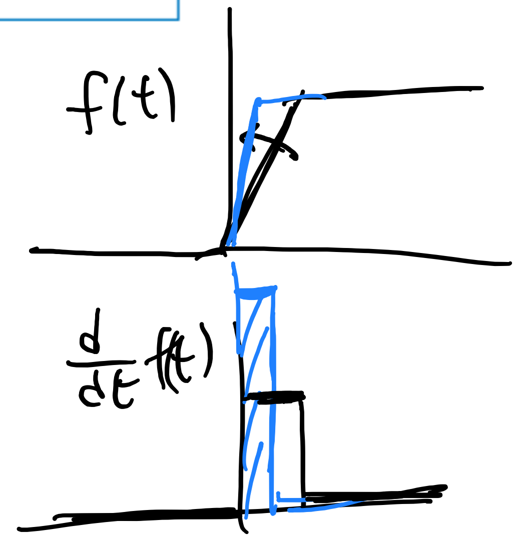
(b)


a
$V_0u(t)$
b
(a)
$t = 0$
a
V_0
b
(b)


a
$I_0u(t)$
b
(a)
$t = 0$
i
a
I_0
b
(b)

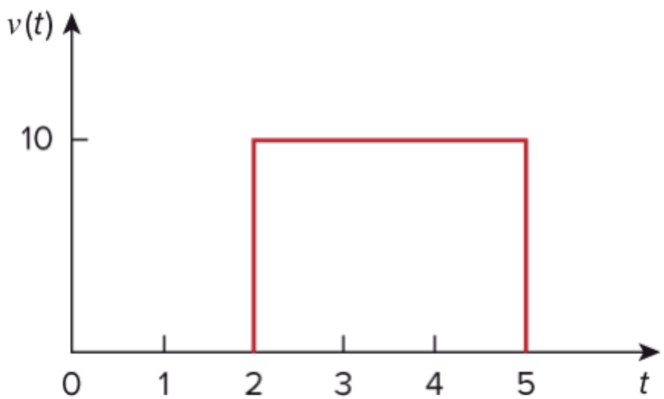
$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$$



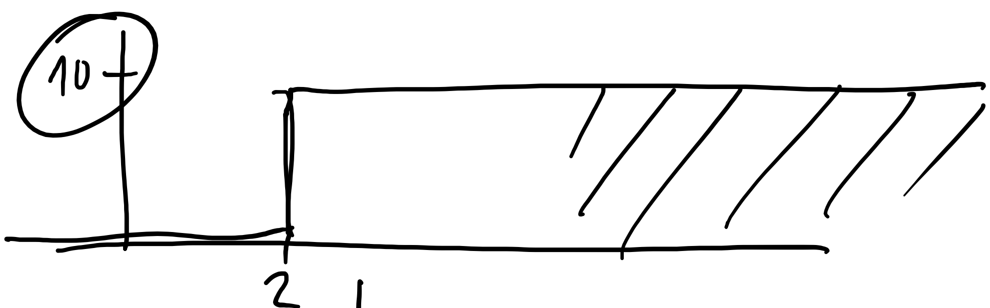


Example 7.6

Express the voltage pulse in **Fig. 7.31** in terms of the unit step. Calculate its derivative and sketch it.

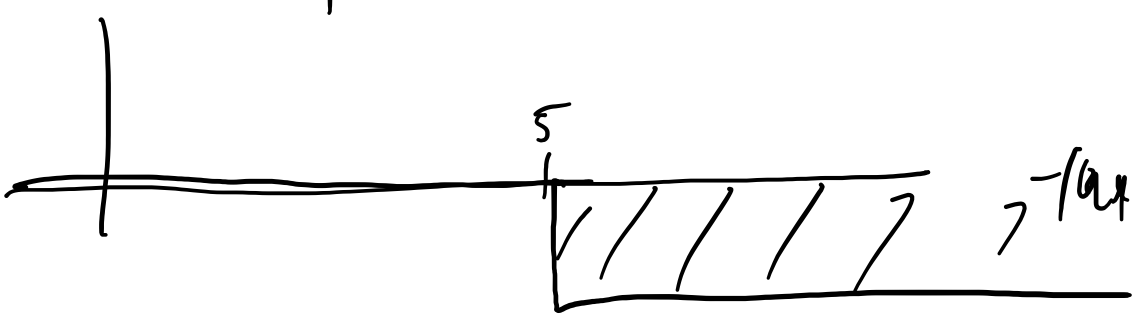


$$v(t) = 10[u(t-2) - u(t-5)]$$



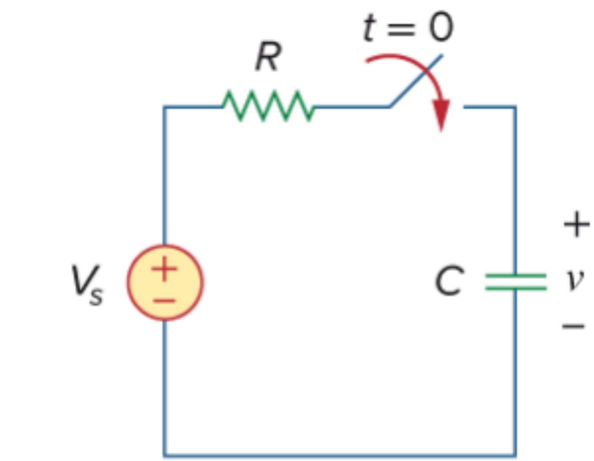
$10u(t-2)$

$+$

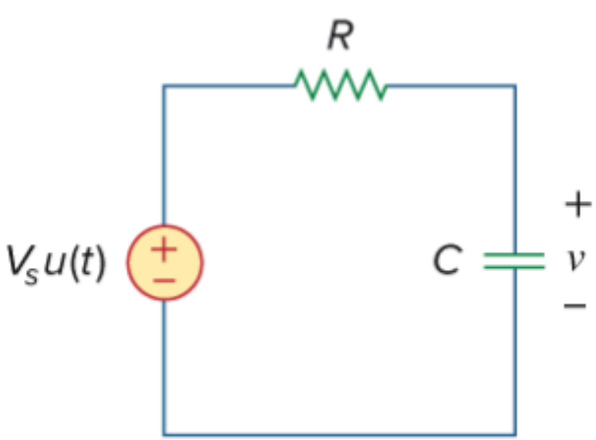


$-10u(t-5)$

$$v(0^-) = v(0^+) = V_0$$



(a)



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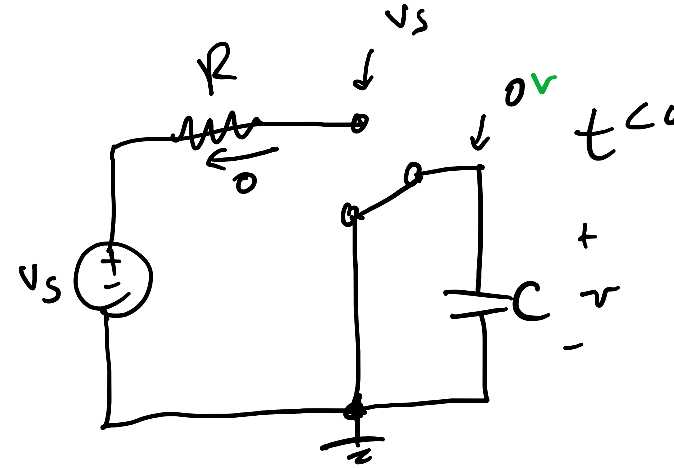

$$i = C\frac{dv}{dt}$$