BOSTON UNIVERSITY

EK 307: Electric Circuits

Fall 2017

Lecture 15

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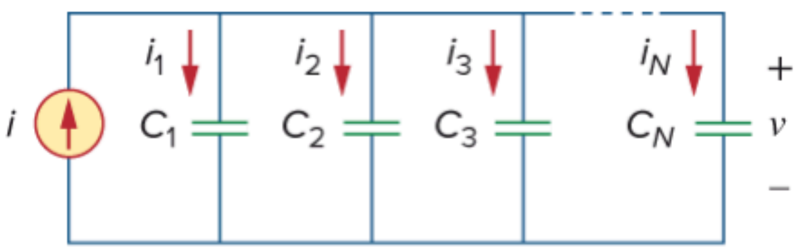
Lecture 15:

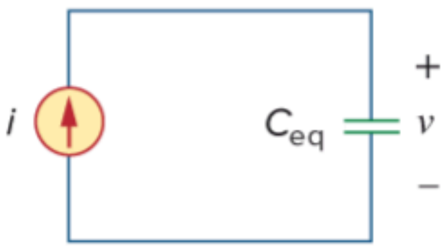
1. **Chapter 6: Capacitors/inductors**

Series and parallel

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



(a)



(b)

But $i_k = C_k\, dv/dt$. Hence,

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

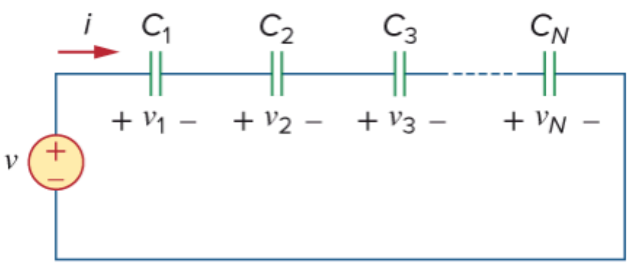
$$= \left(\sum_{k=1}^{N} C_k\right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

where

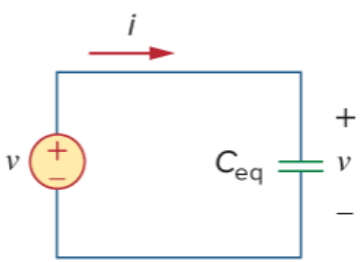
$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

Series and parallel

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$



(a)



(b)

But $v_k = \dfrac{1}{C_k}\displaystyle\int_{t_0}^{t} i(\tau)\,d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

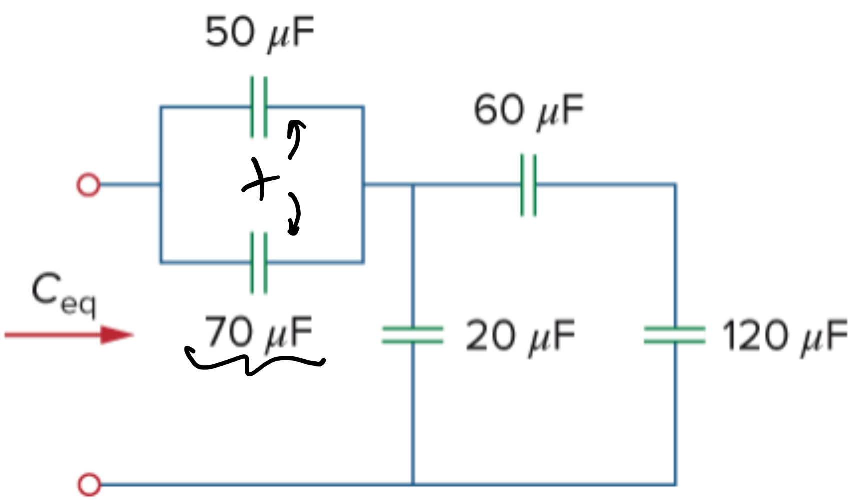
$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

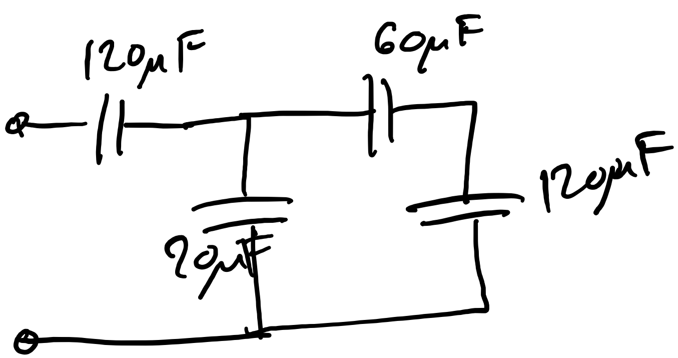
where

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

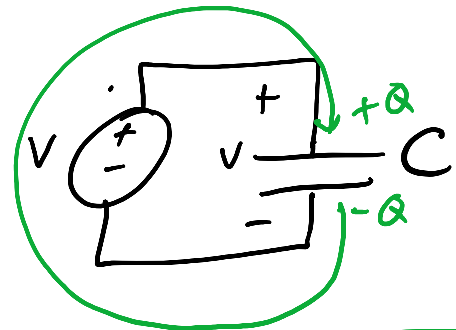
Practice Problem 6.6

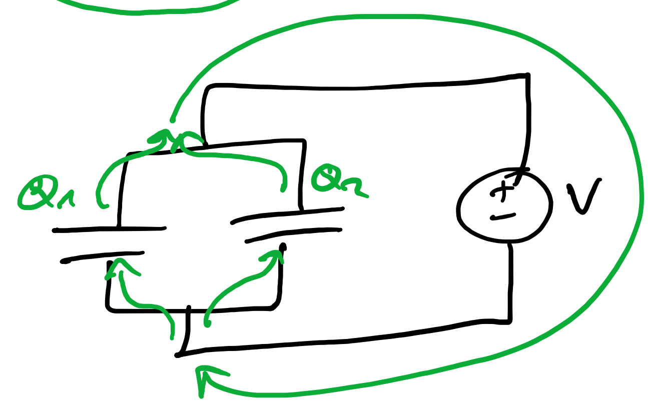
Find the equivalent capacitance seen at the terminals of the circuit in **Fig. 6.17**.





$$Q = CV$$



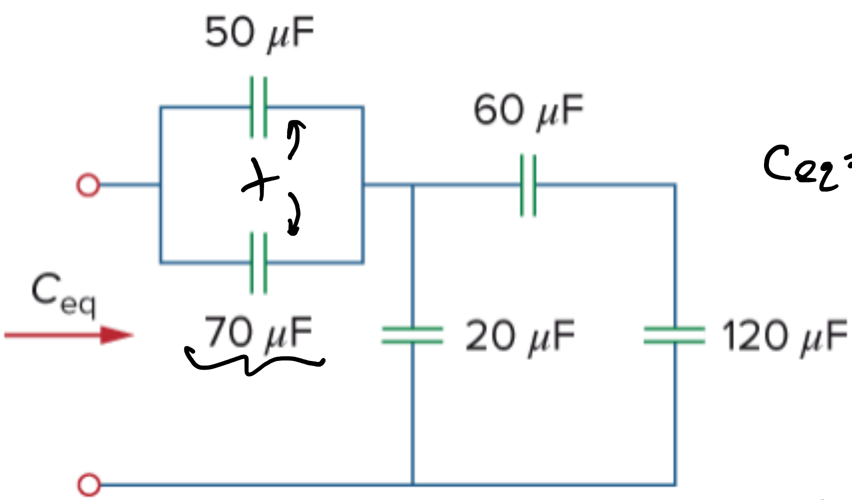


$$(Q_1 + Q_2) = C_{eq} V$$

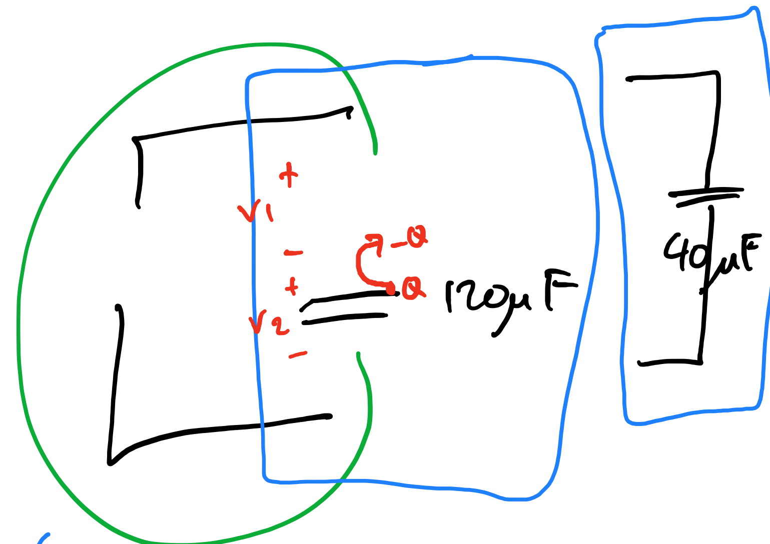
$$C_1 V + C_2 V = C_{eq} V$$

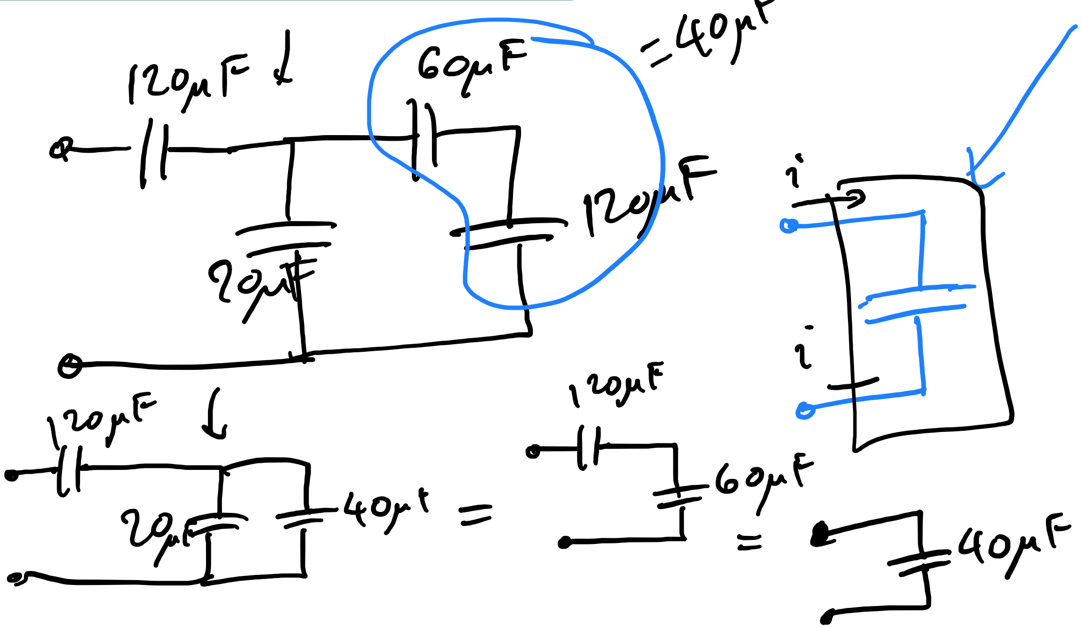
Practice Problem 6.6

Find the equivalent capacitance seen at the terminals of the circuit in **Fig. 6.17**.



$$C_{eq} = \left(\frac{1}{60} + \frac{1}{120}\right)^{-1}$$



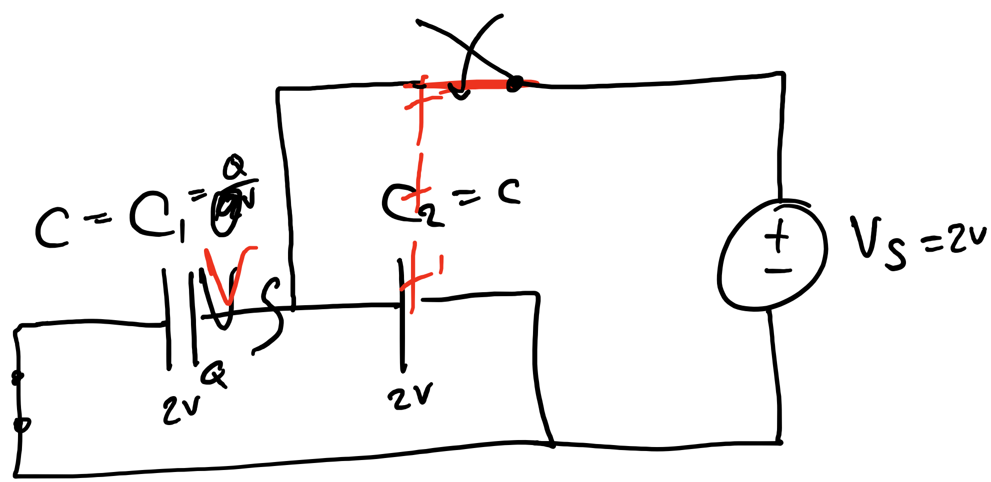


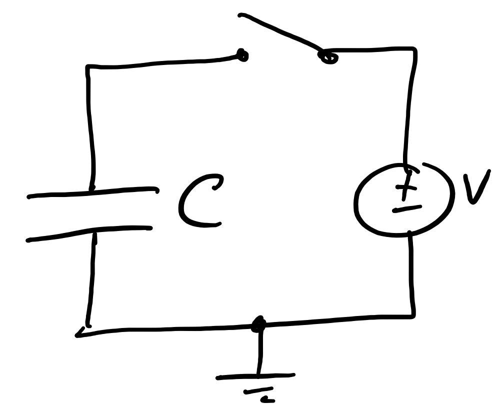
$$V_1 = \frac{Q}{C_1} = \frac{Q}{60\mu F}$$

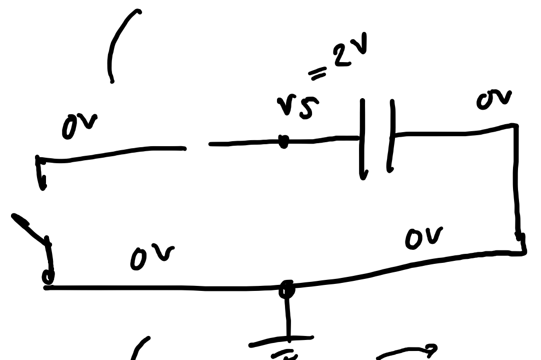
$$V_2 = \frac{Q}{C_2} = \frac{Q}{120\mu F}$$

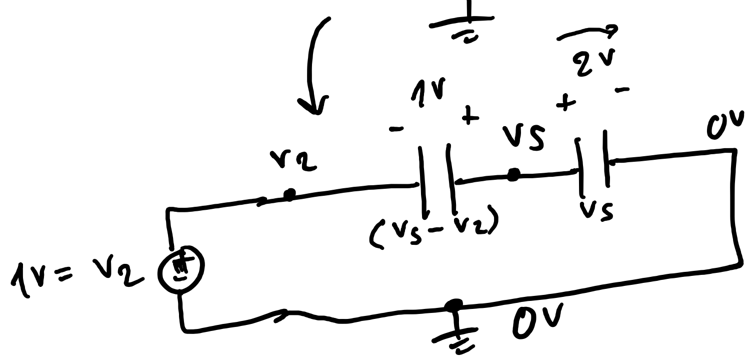
$$V = V_1 + V_2 = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$


C = C1 = Q/ΔV
C2 = C
Vs = 2v
2v
Q
2v

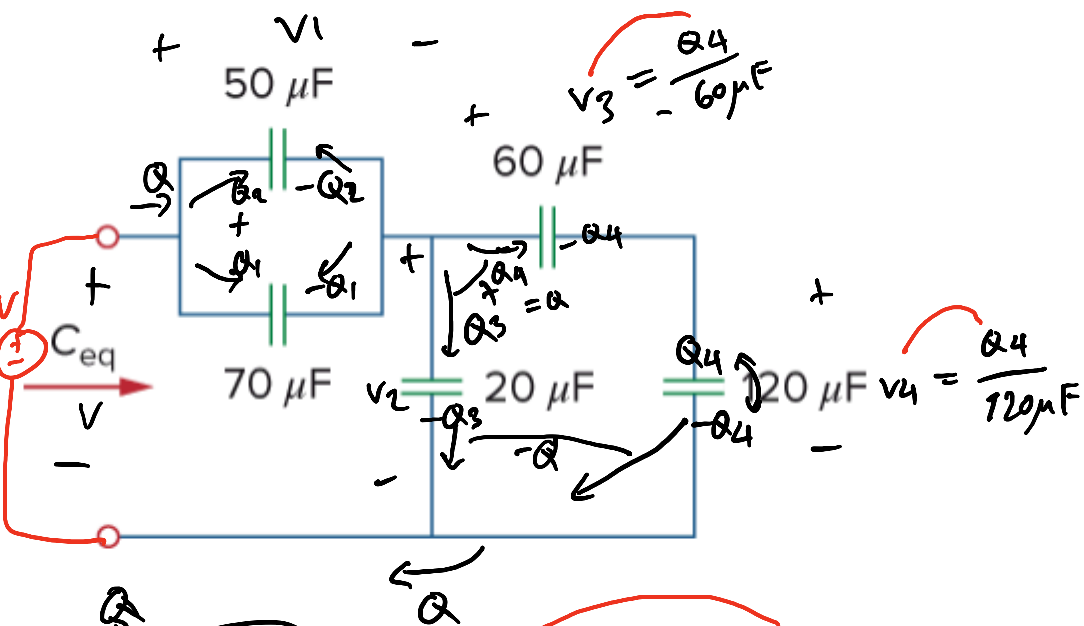

C
V

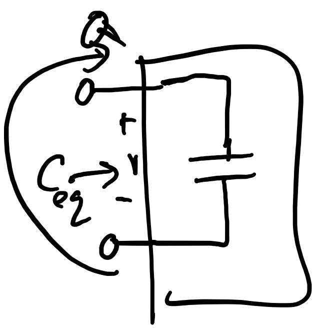

0v
vs = 2v
0v
0v
0v

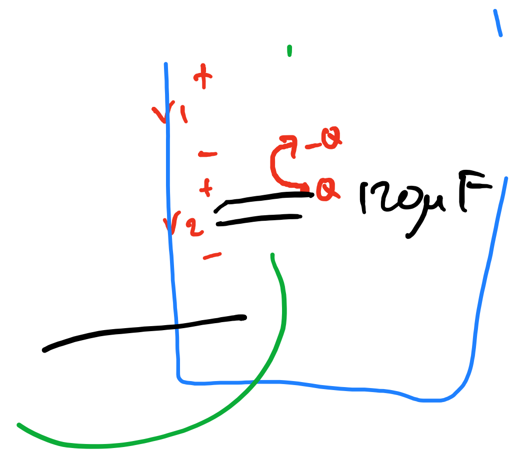

1v
2v
vs
0v
v2
(vs − v2)
vs
1v = v2
0V

Practice Problem 6.6

Find the equivalent capacitance seen at the terminals of the circuit in Fig. 6.17.







$$V_1 = \frac{Q_2}{50\mu F} = \frac{Q_1}{70\mu F}$$

$$Q = Q_1 + Q_2 = Q_3 + Q_4$$

$$V_2 = \frac{Q_3}{20\mu F} = V_3 + V_4$$

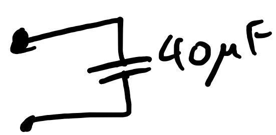
$$V = V_1 + V_2$$

$$V_1 = \frac{Q}{C_1} = \frac{Q}{60\mu F}$$

$$V_2 = \frac{Q}{C_2} = \frac{Q}{120\mu F}$$

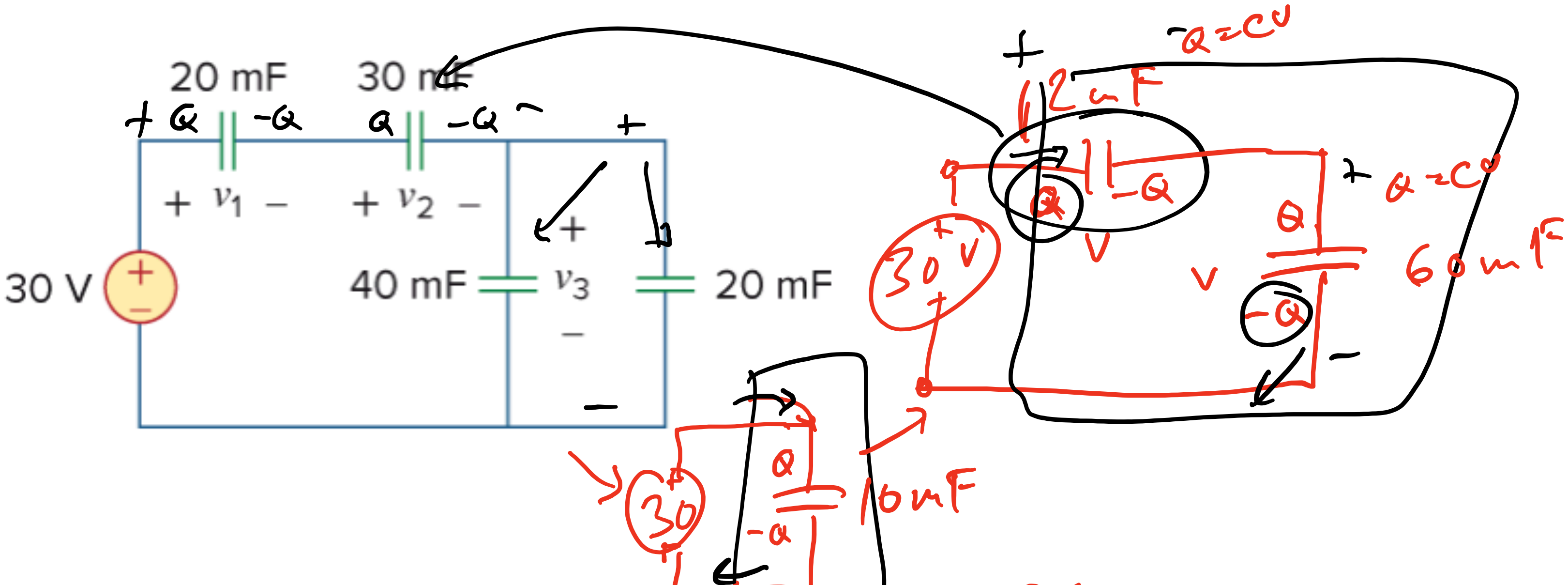
$$V = V_1 + V_2 = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$



Example 6.7

For the circuit in Fig. 6.18, find the voltage across each capacitor.



$Q = CV = 10\,\text{mF} \times 30\,\text{V}$

$= 0.3\,\text{C}$

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

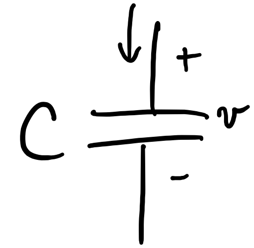
Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

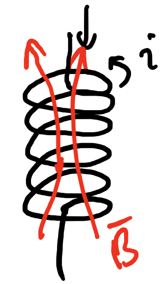
Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

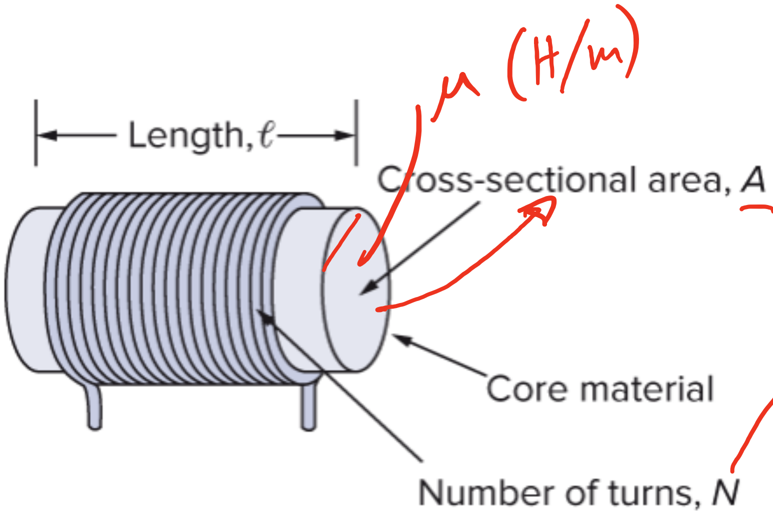
$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

Inductors (Section 6.4)



$i = C\frac{dv}{dt}$



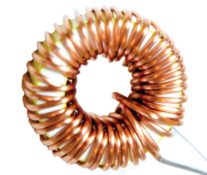


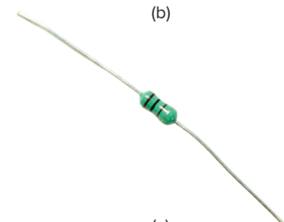
$$\boxed{v = L\frac{di}{dt}}$$

$$L = \frac{N^2\mu A}{\ell}$$

magnetic permeability of core material


(a)


(b)

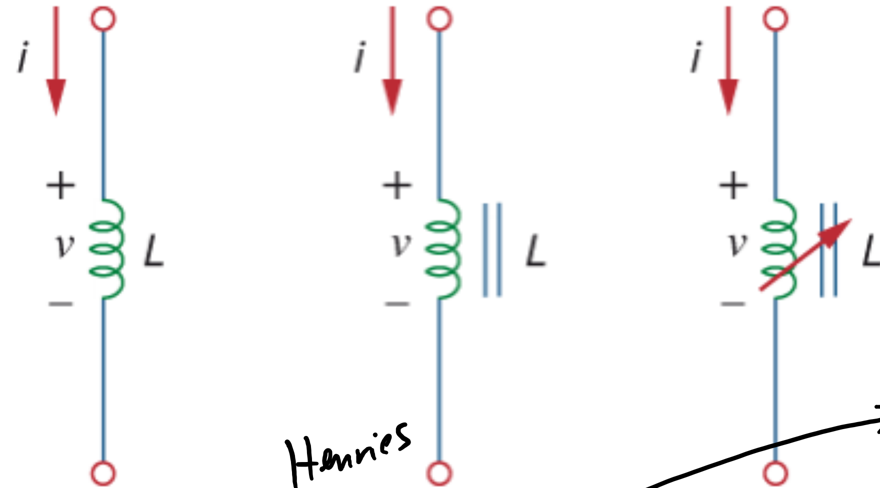

(c)

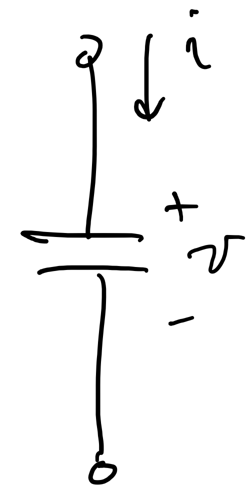
Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

Realistic inductance:

mH → discrete

nH → on-chip
or pH





Henries

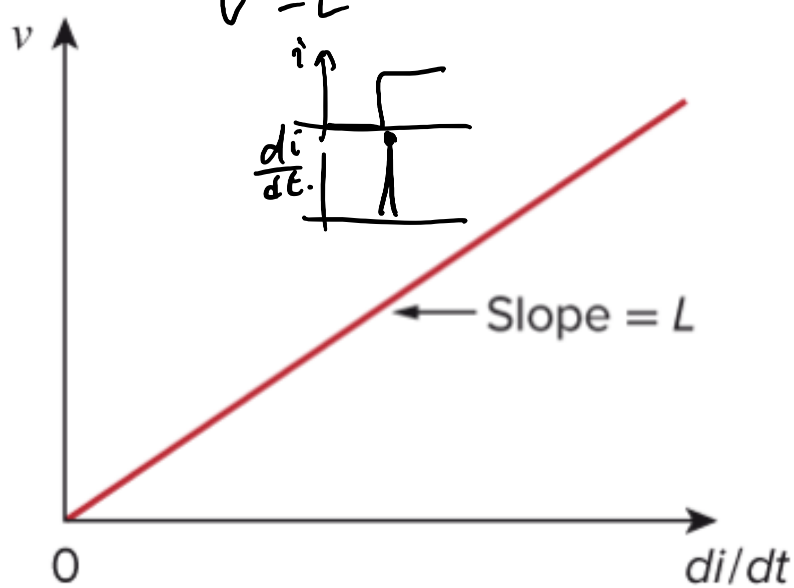
$$v = L\frac{di}{dt}$$

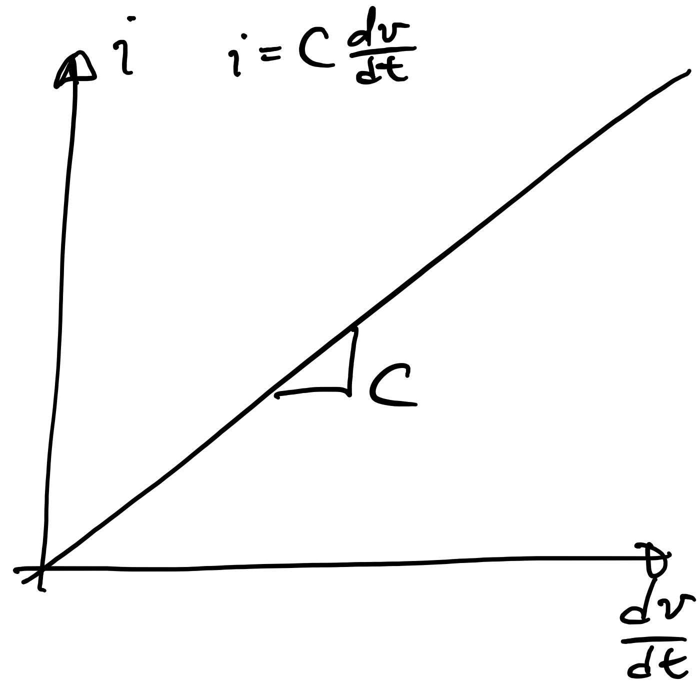
$$V = H \cdot \frac{A}{s}$$

$$1H = \frac{1Vs}{A}$$

If i is cnst

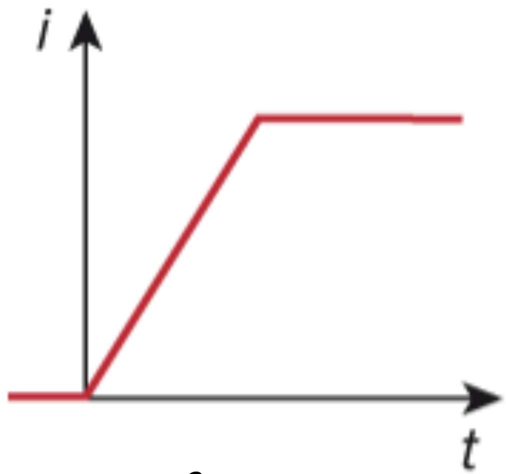
$$i = C\frac{dv}{dt}$$

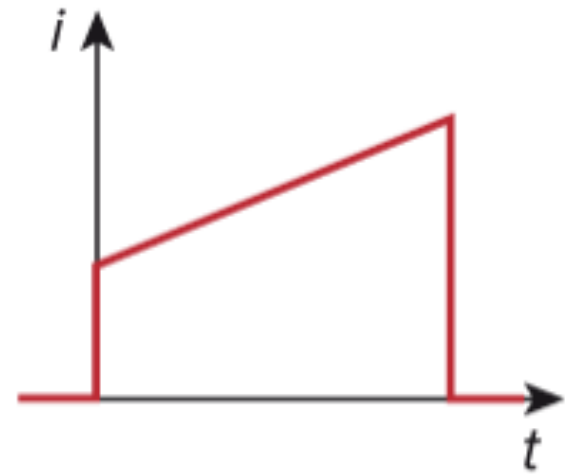


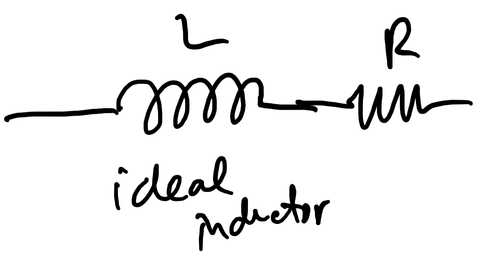


Properties

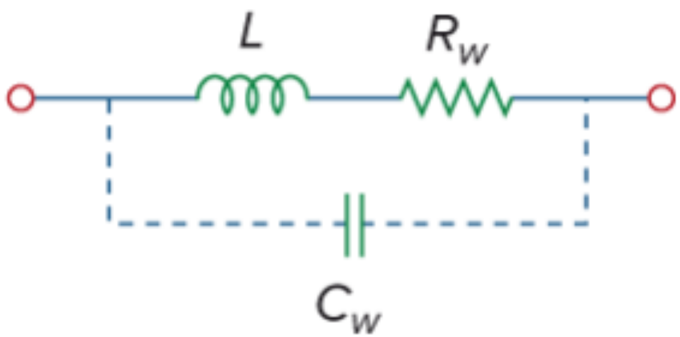
1. An inductor acts like a short circuit to dc.
2. The current through an inductor cannot change instantaneously.
3. No dissipation

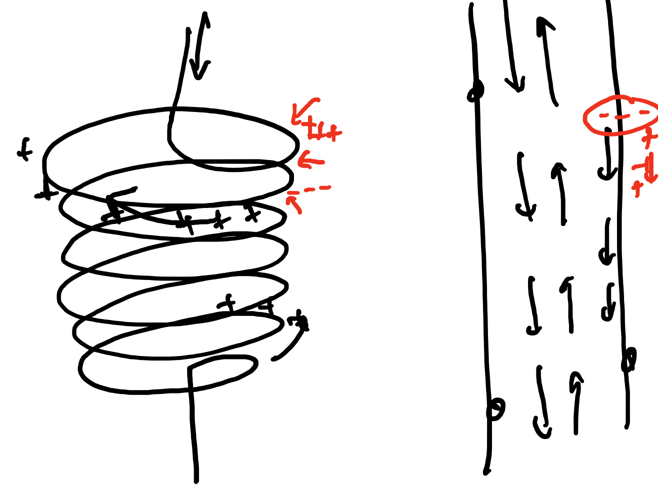


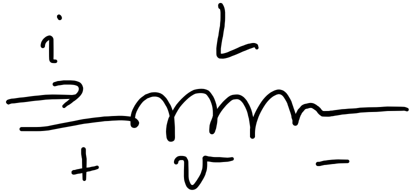




Non-ideal inductor







$$v = L\frac{di}{dt}$$

$$p = v \cdot i$$

$$= L\, i \frac{di}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{2} L i^2\right) = \frac{d}{dt} w_L$$

const $\quad i(t)$

$$\frac{d}{dt} i^2 = 2i \frac{d}{dt} i$$

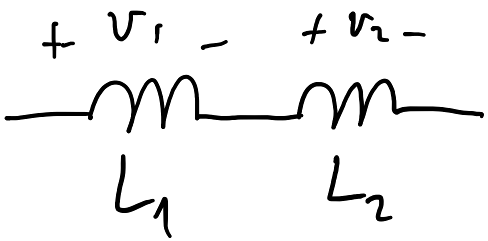
$$w_L(t) = \int^t p(t')\,dt' = \frac{1}{2} L i^2$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

$$p = vi = \left(L\frac{di}{dt}\right) i$$

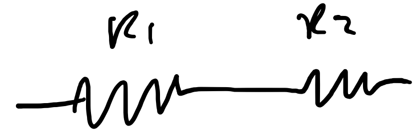
$$\boxed{w_L = \frac{1}{2} L i^2}$$

$$w_C = \frac{1}{2} C v^2$$



$v = L\frac{di}{dt}$

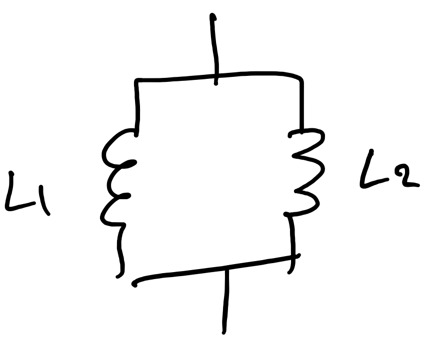
$v = iR$



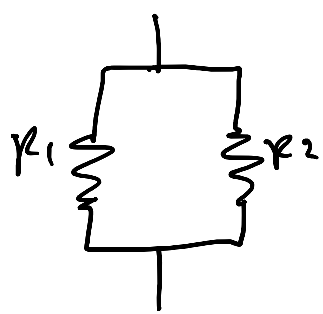
$R = R_1 + R_2$

|||

$L = L_1 + L_2$



$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$



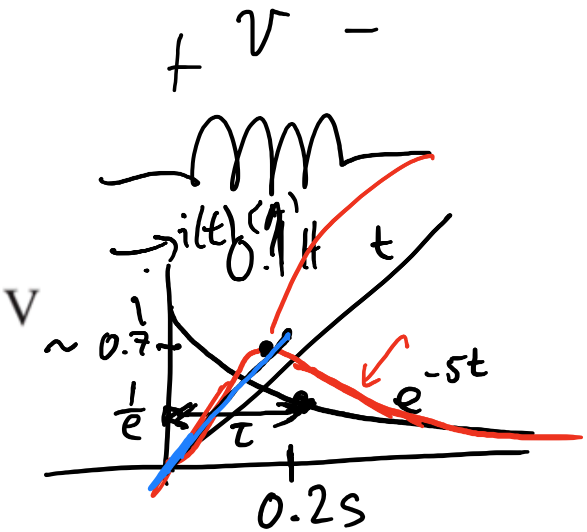
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Example 6.8

$10\times \quad t \quad \to e^{-5t} = e^{-t/0.2} = e^{-t/\tau}$

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:



Since $v = L\, di/dt$ and $L = 0.1$ H,

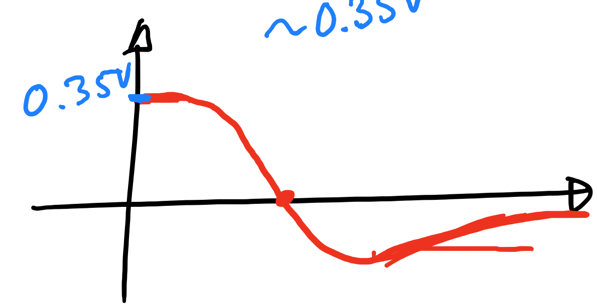
$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

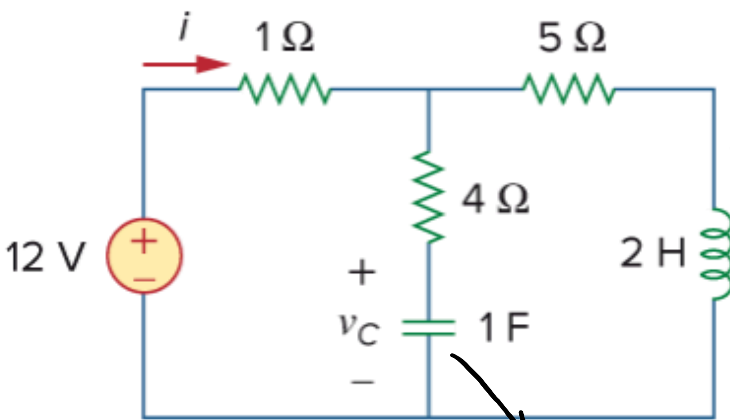
$v = L\frac{di}{dt} \to 0.1\text{H}\ \frac{0.7\text{A}}{0.2\text{s}}$

$\sim 0.35\text{V}$



Example 6.10

Consider the circuit in **Fig. 6.27(a)**. Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



$\to v_L = L\frac{di}{dt} = 0$

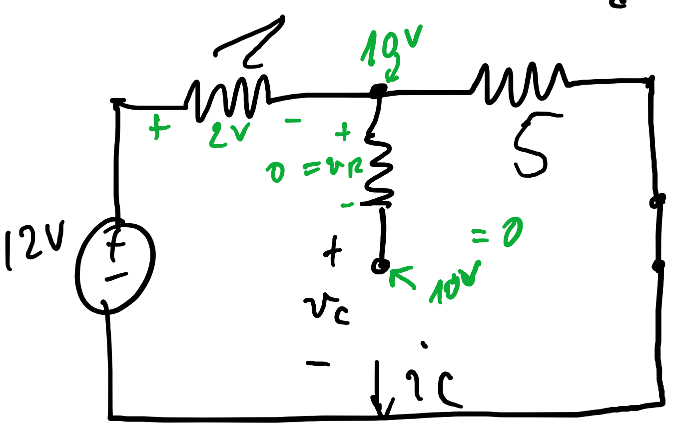
In steady state

" $\frac{d}{dt} = 0$ "

$\Downarrow$

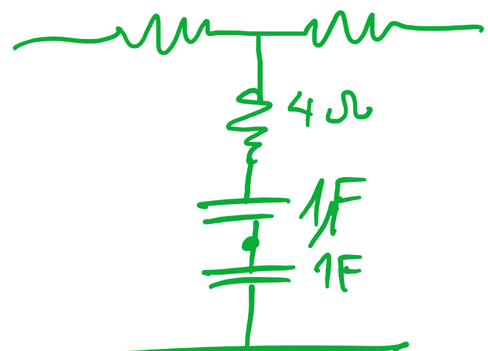
(a)

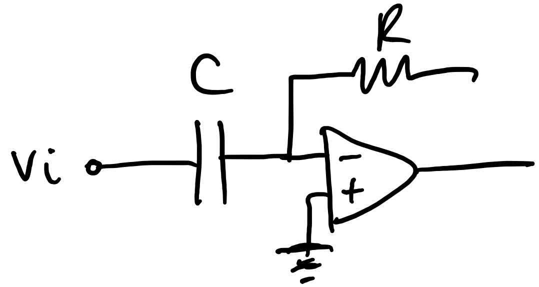
$i = C\frac{dv}{dt} =$



$\downarrow i_L = \frac{12}{5+1} = 2A$

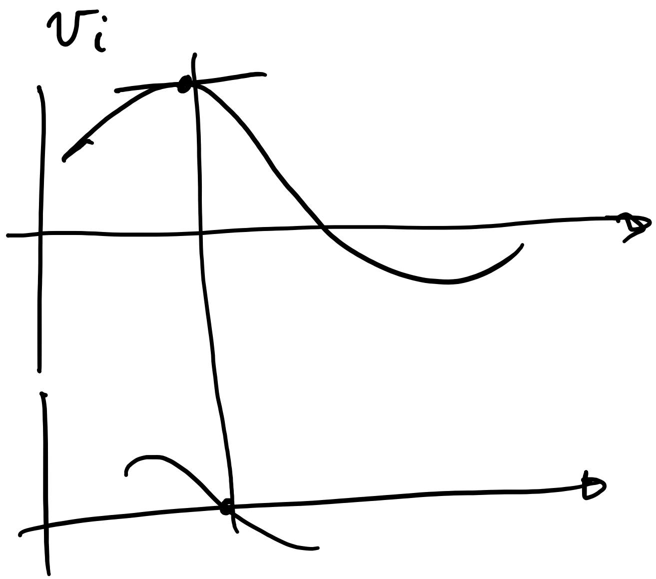
Req:

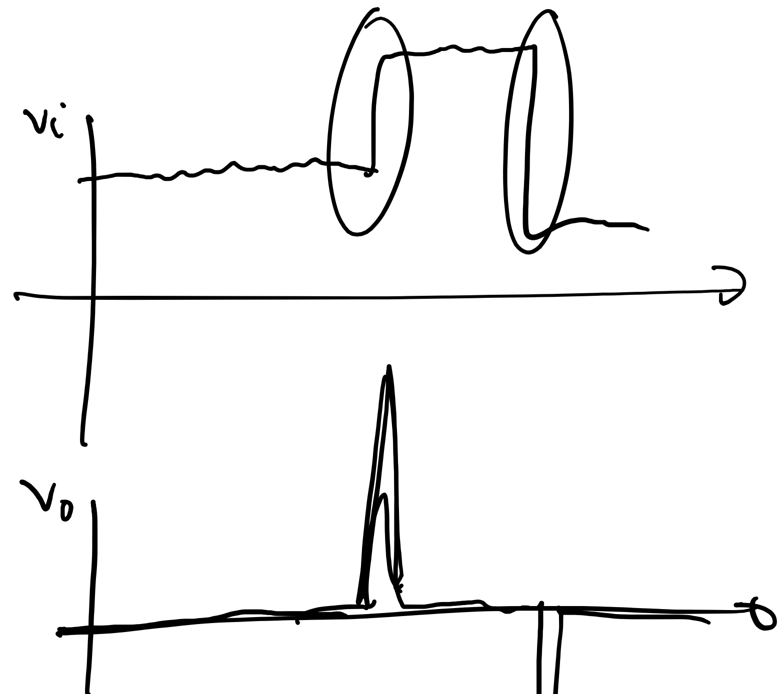


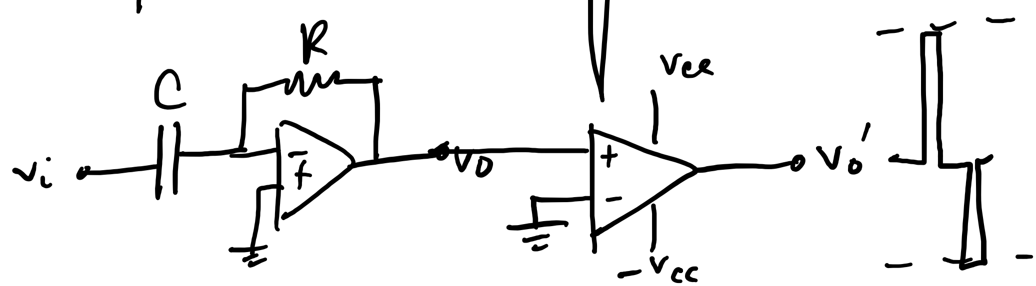


dt vi

"gain"

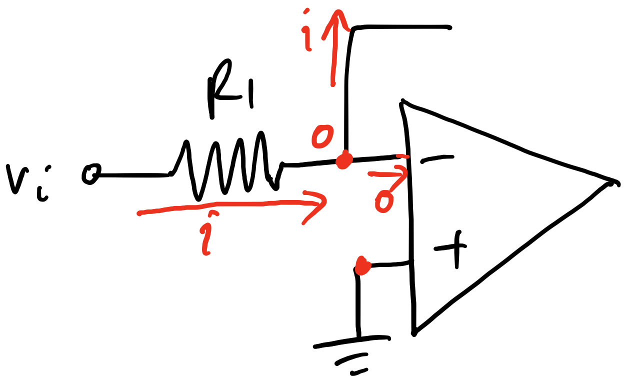


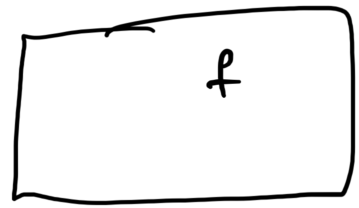




Inverting amp

R_f





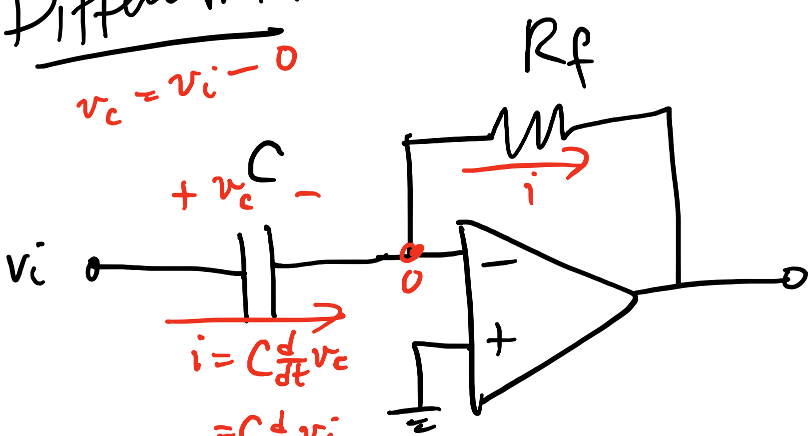
$$i = \frac{v_i}{R_1} = \frac{0 - v_o}{R_2}$$

$$\downarrow$$

$$v_o = \frac{-R_2}{R_1} v_i$$

Differentiator

$v_c = v_i - 0$



$v_o = \cancel{-\frac{R}{C} v_i}$

V Ω V F

$$i = C\frac{d}{dt}v_c$$

$$= C\frac{d}{dt}v_i$$

$$i = \frac{\cancel{0} - v_o}{R} = C\frac{d}{dt}v_i$$

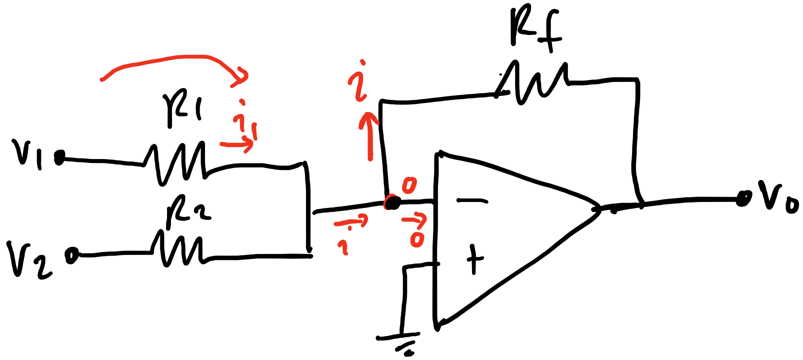
$$\boxed{v_o = -R_f C \frac{d}{dt} v_i}$$

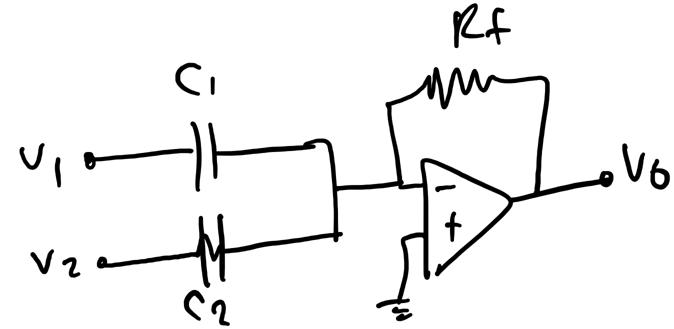
$$\frac{v}{i} = R \Rightarrow 1\Omega = \frac{1V}{1A}$$

$$C \Rightarrow 1F = \frac{1C}{1V} = \frac{q}{V}$$

$$R \cdot C = \frac{\cancel{1V}}{1A} \cdot \frac{1C}{\cancel{1V}} = \frac{\cancel{1C}}{1C/s} = 1s$$

$$\| \; \frac{R_f}{1/(C\frac{d}{dt})}$$



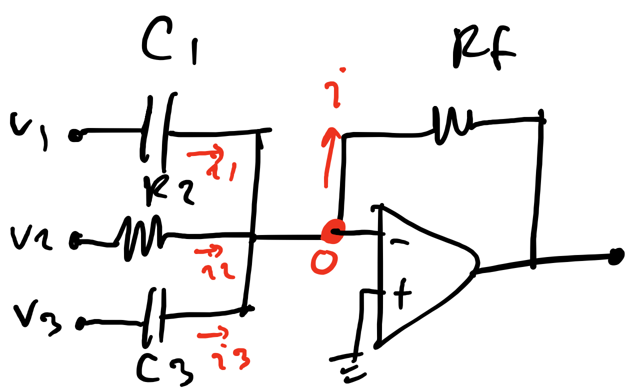


KCL: $i = i_1 + i_2$

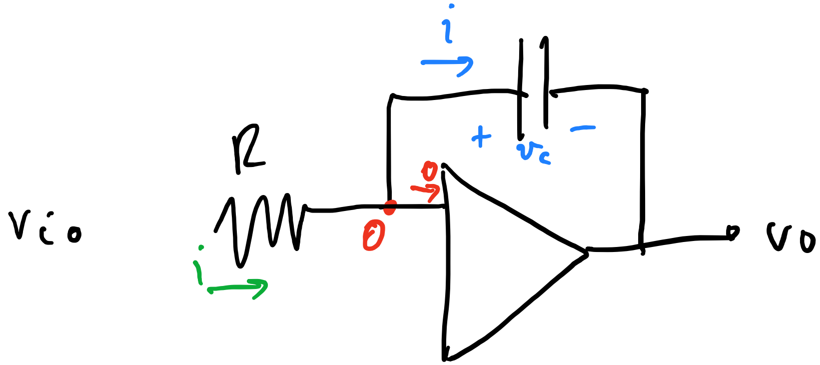
$$\frac{-V_o}{R_f} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\boxed{V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2\right)}$$

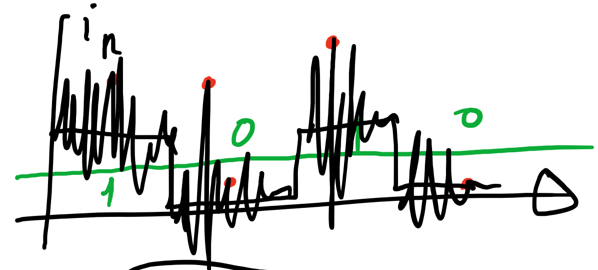
$$V_o = -\left(R_f C_1 \frac{d}{dt} v_1 + R_f C_2 \frac{d}{dt} v_2\right)$$

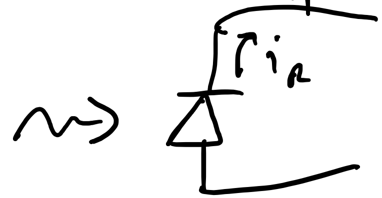


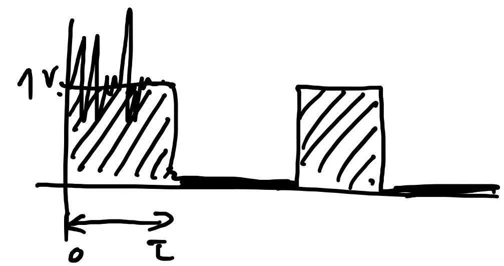
$$v_o = \left(-R_f C_1 \frac{d}{dt} v_1 - \frac{R_f}{R_2} v_2 - R_f C_3 \frac{d}{dt} v_3\right)$$



Applications:







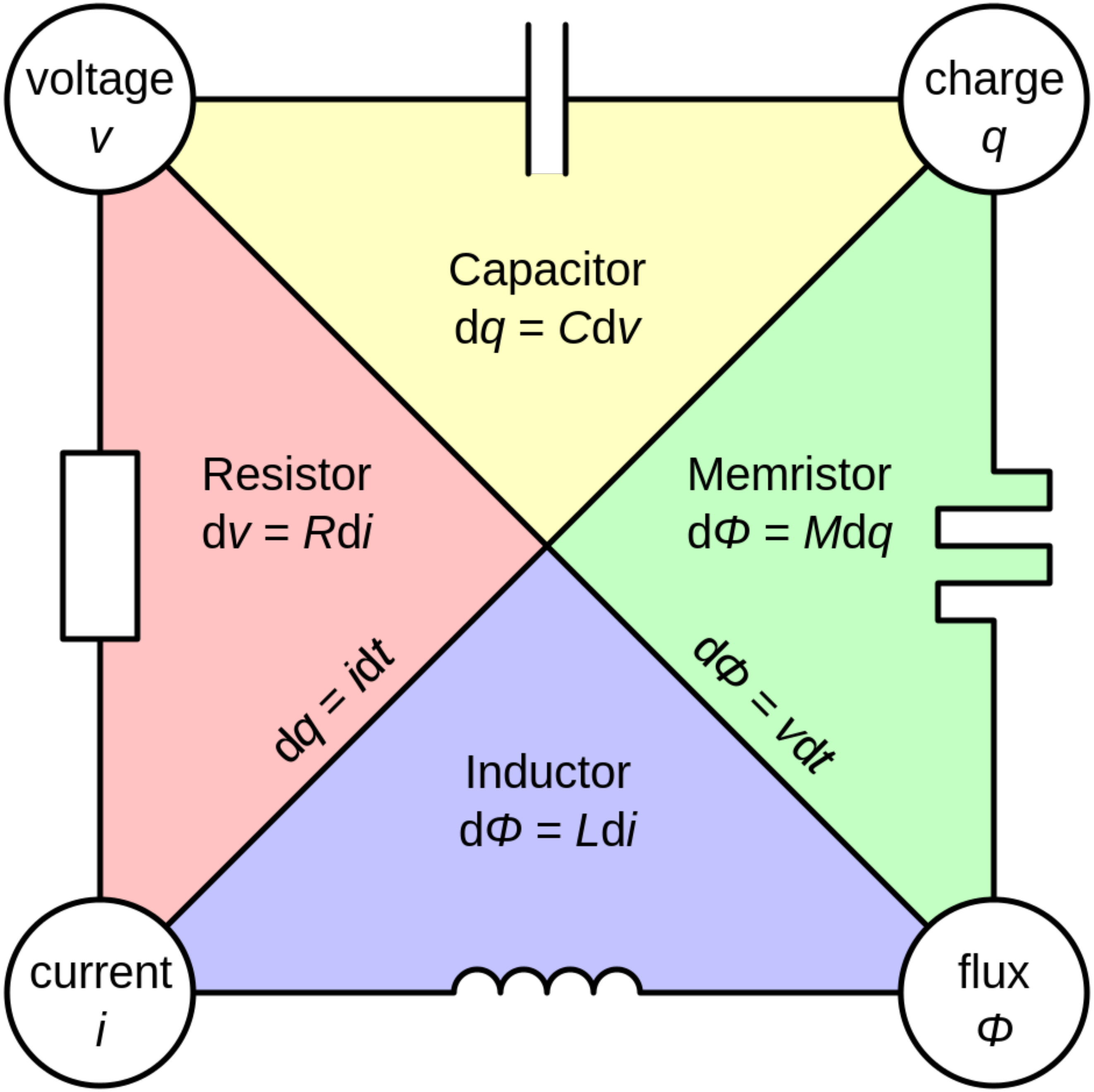
$$i = \frac{v_i - 0}{R} = C\frac{d}{dt}(0 - v_o)$$

$$\frac{v_i}{R} = -C\frac{d}{dt}v_o$$

$$\frac{d}{dt}v_o(t) = -\frac{1}{RC}v_i(t)$$

$$\int_0^t dt'\, v_o(t') = \int_0^t -\frac{1}{RC}v_i(t')\,dt'$$

$$\boxed{v_o(t) - \underbrace{v_o(0)}_{0 \text{ if no voltage at time } 0.} = -\frac{1}{RC}\int_0^t v_i(t')\,dt'}$$


voltage
v
charge
q
Capacitor
dq = Cdv
Resistor
dv = Rdi
Memristor
dΦ = Mdq
dq = idt
dΦ = vdt
Inductor
dΦ = Ldi
current
i
flux
Φ