Boston University Student Name: M. Popović's Solutions EK307 – Fall 2017

Midterm Exam 1

Prof. Miloš Popović

- ***90 min timed, closed book test.***
- ***Write your name at top of every page (or initials on later pages)***
- ***Aids: single page (single side) of notes, handheld calculator***
- ***Work in space provided (or on back of page if need more space). Turn in only these sheets.***
- ***Write clearly your method for partial credit. When possible, first do the problem without numbers plugged in.***
- ***Circle/box clearly final answers***

Grading scheme:

Multiple choice	**___/10**
Problem 1	**___/15**
Problem 2	**___/15**
Problem 3	**___/15**
Problem 4	**___/15**
Problem 5	**___/15**
Problem 6	**___/15**

Hand-ins:
4:57

4:42 – 5:10 pm.

Multiple choice questions:

[Select only all correct answers in each question for full credit.]

1. **[2 pts]** **Electric charge** is measured in units of:
   a. Coulombs [C]
   b. Amp-hours [Ah]
   c. Watt-hours [Wh]
   d. electrons (count particles) [e-]
   e. Volts per meter [V/m]
   f. Tesla [T]

2: A+B+D
1: any subset
0: C, E, F

– Partial credit 1/2 for some right but no wrong selections
– Full credit 2/2 for all right

2. **[1 point]** **Ohm's law** tells you:
   a. the I-V curve of a voltage source
   b. the I-V curve of a current source
   c. the relationship of voltage to current in a resistor
   d. the maximum speed at which electrons can travel in a wire

C

3. **[1 point]** A rectangular solid formed of a uniform resistivity material can be redesigned to have 1/3 of its original resistance by:
   a. Making it three times longer, while keeping the same width
   b. Cutting it to 1/3 the length, while keeping the same width
   c. Making it three times wider, while keeping the same length
   d. Cutting it to 1/3 its width, while keeping the same length

1: B+C
0.5: B or C

– Must get B & C for 1 point, either for 0.5 point
– A or D ∅ points

4. **[1 point]** The concept of supernode is used in node voltage method analysis of a circuit when:
   a. too many resistors are in the circuit
   b. dependent sources outnumber the independent sources
   c. the circuit contains voltage sources
   d. the circuit contains current sources

C

5. **[1 point]** The superposition principle in linear circuits can be used to:
   a. break down a circuit problem that has multiple sources into several simpler problems with a single source each
   b. determine the optimal position of a resistor in a circuit
   c. tells you that any output voltage is proportional to the input signal.
   d. only solve op-amp circuits

A

6. **[1 point]** The power, P, dissipated in a resistor with resistance R, having voltage drop v across it and current i flowing through it, is given by
   a. $P = v\,i$
   b. $P = v^2 R$
   c. $P = i^2 R$
   d. $P = i/v$
   e. $P = i^2/R$

A + C

B+C+F

7. **[1 point]** The ideal op-amp model assumes:
   a. zero input resistance
   b. zero output resistance
   c. infinite input resistance
   d. infinite output resistance
   e. unity open-loop gain
   f. infinite open-loop gain

A

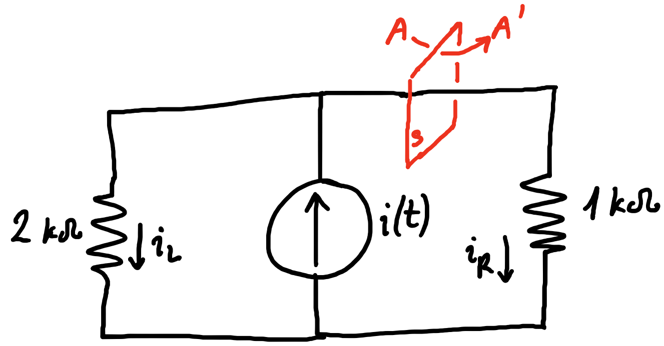
8. **[2 pts]** To deliver maximum power to be burned by a load resistor, a designer should
   a. set the load resistance equal to the Thevenin resistance of the source it's being connected to
   b. set the load resistance equal as high as possible
   c. set the load resistance to near zero to maximize the current through it
   d. connect both resistor terminals to the higher voltage terminal of the source

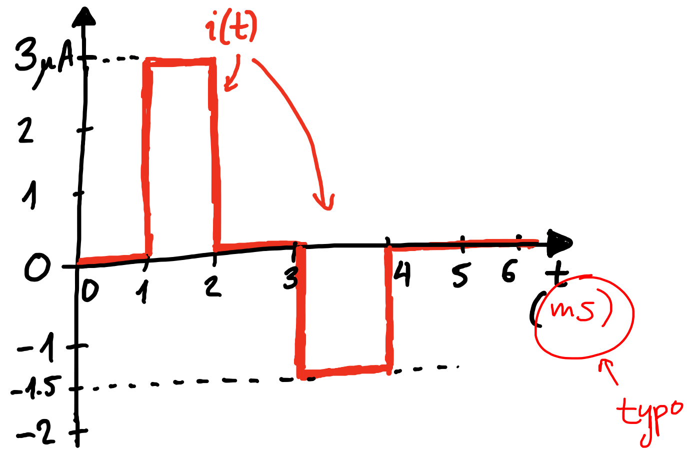
– no partial credit

— 4:43

4:43 – 4:51

Problem 1. [15] The following circuit has a single current source whose current changes with time and is only on for two short periods. The source's current vs. time is shown in the plot, and is also given mathematically below.





$$i(t) = \begin{cases} 3\mu A & 1ms < t < 2ms \\ -1.5\mu A & 3ms < t < 4ms \\ 0 & \text{at all other times } t \end{cases}$$

(a) [5] Find currents i_R and i_L for all times t shown in the plot, and sketch a plot of the two currents vs. time for t = 0 to 6 ms.

(b) [5] How much net charge crosses surface S, shown in the figure, from side A to side A', from time t = 0 to t = 6 ms? Be sure to indicate sign and units.

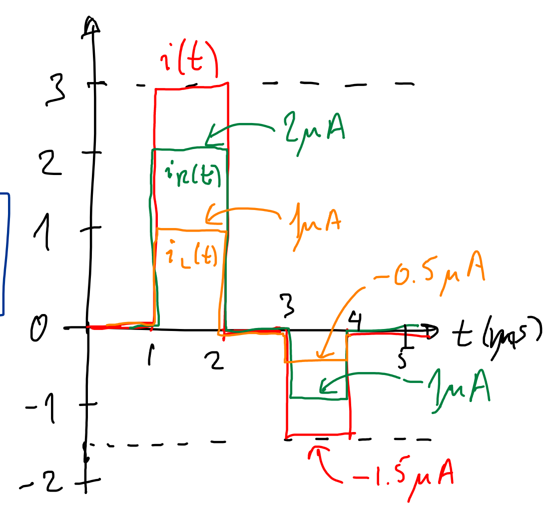
(c) [5] How much energy was burned by the right (1kΩ) resistor during time t = 0 to 6ms? Be sure to include units.

(a) $$i_R(t) = \frac{\frac{1}{1k\Omega} i(t)}{\frac{1}{1k\Omega} + \frac{1}{2k\Omega}} = \frac{2\, i(t)}{2+1} = \boxed{\frac{2}{3} i(t)}$$

$$i_L(t) = \frac{\frac{1}{2k} i(t)}{\frac{1}{1k} + \frac{1}{2k}} = \frac{1}{2+1} i(t) = \boxed{\frac{1}{3} i(t)}$$

$$i_R(t) = \begin{cases} 2\mu A & 1ms < t < 2ms \\ -1\mu A & 3ms < t < 4ms \\ 0 & \text{else} \end{cases}$$

$$i_L(t) = \begin{cases} 1\mu A & 1ms < t < 2ms \\ -0.5\mu A & 3ms < t < 4ms \\ 0 & \text{else} \end{cases}$$

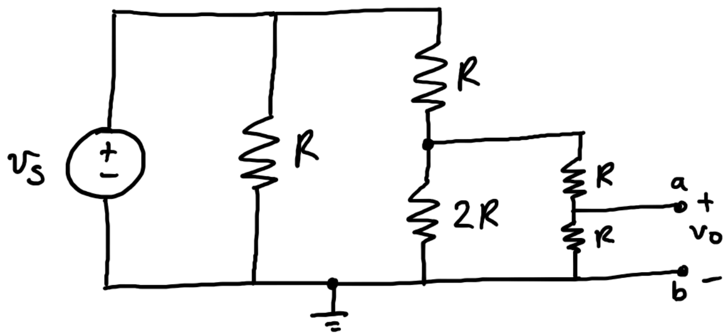


(b) $$q = \int_0^{6ms} i_R(t)\,dt = 2\mu A \cdot 1ms + (-1\mu A) \cdot 1ms$$

$$\boxed{q = 1nC}$$

(c) $$P_{1k} = i_R^2(t) \cdot 1k\Omega \Rightarrow W_{1k} = \int_0^{6ms} P_{1k}\,dt = \left((2\mu A)^2 1k\Omega + (-1\mu A)^2 1k\Omega\right) 1ms$$
$$= (4nW + 1nW) 1ms = 5nW \cdot 1ms = \boxed{5pJ}$$

Problem 2. [15] For the following circuit

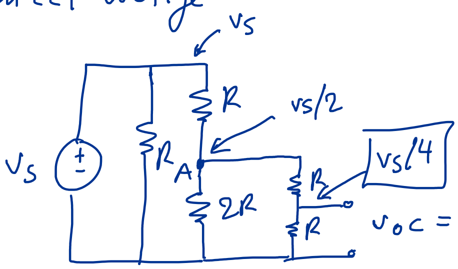


(a) [7] Find the Thevenin equivalent circuit with respect to terminals *a*,*b*.

(b) [6] What load resistor R_L should be connected to terminals a,b for the source to dissipate the maximum possible amount of power, if you can only choose resistance R_L?

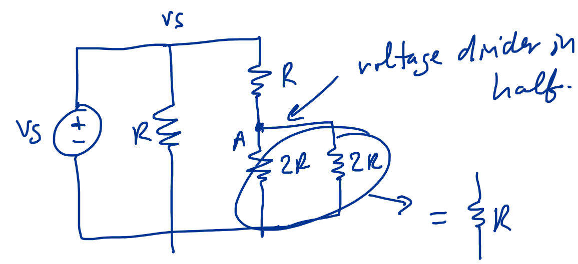
(c) [2] Consider the open circuit version of the circuit and your Thevenin equivalent. Do they burn the same amount of power in resistors without a load connected?

(a) Open cct voltage:

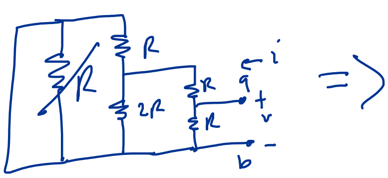


$v_{oc} = v_s/4$

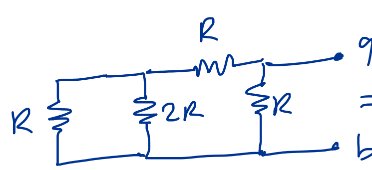
since



Input resistance: $v_s = 0$:

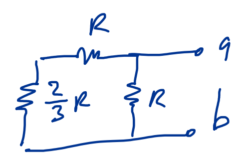


⇒



$$\frac{1}{\frac{1}{R}+\frac{1}{2R}} = \frac{2R^2}{R+2R} = \frac{2}{3}R$$

⇒



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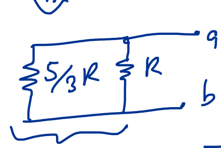
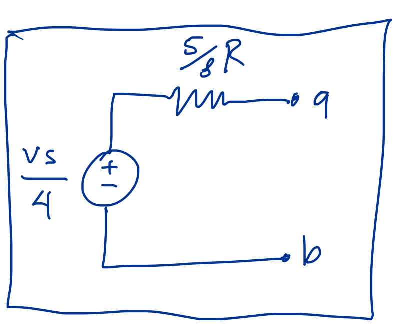


$$\frac{1}{\frac{3}{5R}+\frac{1}{R}} = \frac{1}{\frac{8}{5R}} = \boxed{\tfrac{5}{8}R}$$

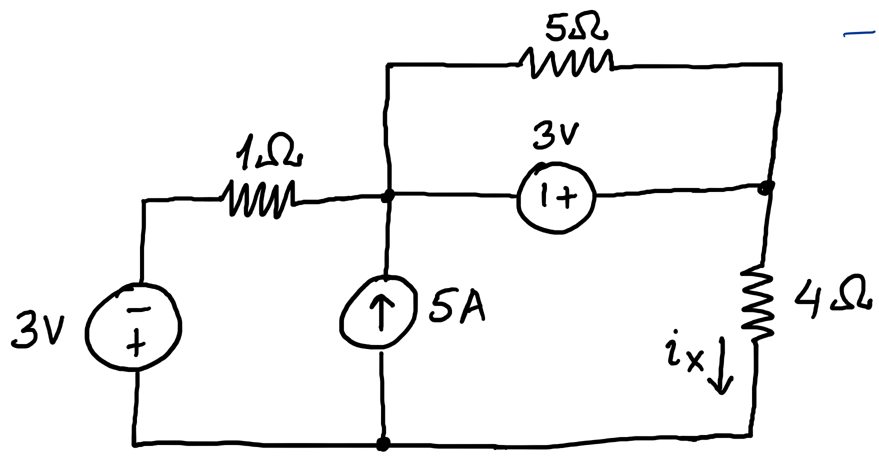


(b) $R_L = \frac{5}{8}R$

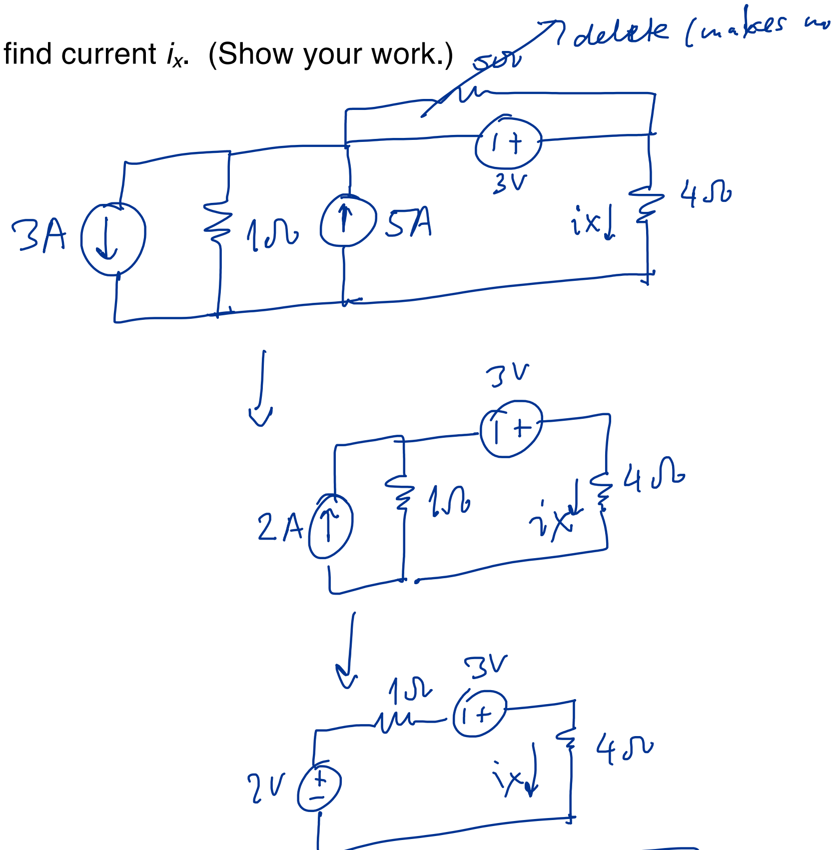
(c) No! Thevenin burns no power when open cct, but original cct does!

Problem 3. For the following circuit

–5:02

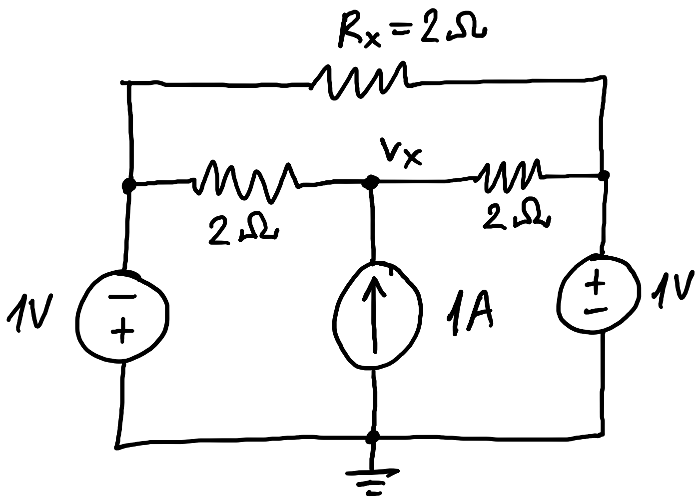


find current i_x. (Show your work.)



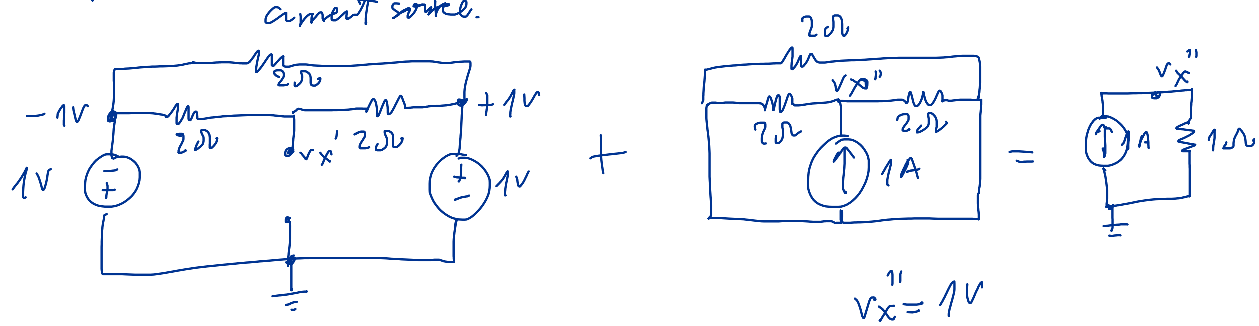
$$i_x = \frac{5V}{5\Omega} = 1A$$

Problem 4. [15] For the following circuit



(a) **[13]** Given that $R_x = 2\Omega$, find node voltage v_x.
(b) **[2]** If resistor R_x value is unknown, what is node voltage v_x now? Explain why.

(a) Superposition: Let me do 2 voltage sources together, and separately current source.



$v_x'' = 1V$

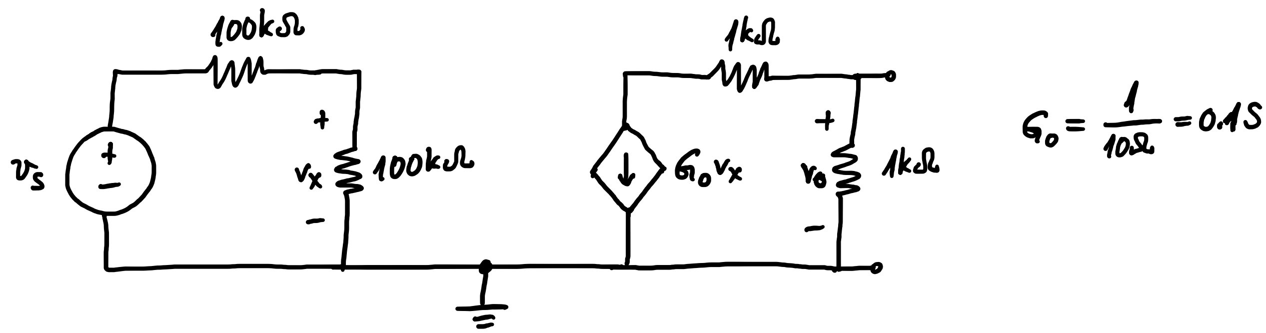
$v_x' = 0V$

$v_x = v_x' + v_x'' = 1V$

(b) For any R_x, $v_x = 1V$ since it doesn't affect voltage in left or right cct.

Problem 5. [15] For the circuit below,

−5:07



[15] Find the gain v_o/v_s.

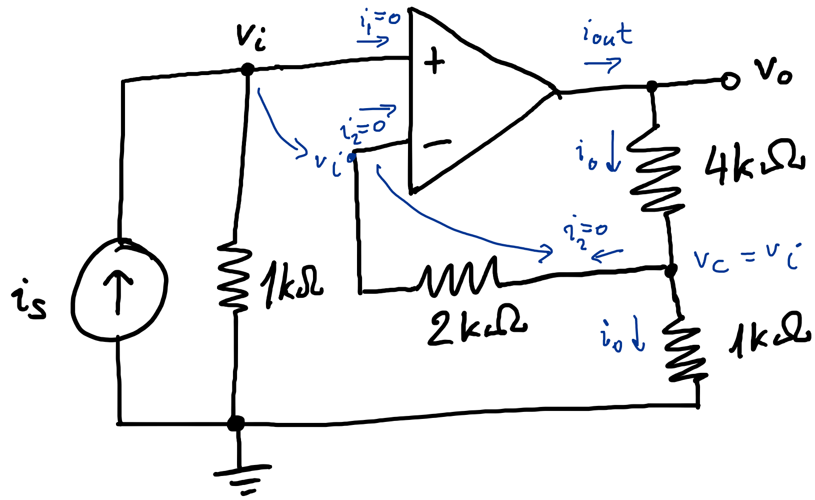
$$v_x = \frac{100k}{100k+100k} v_s = \frac{1}{2} v_s$$

$$v_o = -G_o v_s (1k) = -\frac{1}{10\Omega}\left(\frac{1}{2}v_s\right)(1k) = -\frac{1000}{10}\frac{1}{2}v_s = -50 v_s$$

$$\boxed{\frac{v_o}{v_s} = -50}\ (V/V)$$

(c) **Problem 6. [15]** We're designing a type of op-amp based transresistance amplifier, that converts input current i_s to output voltage v_o. We have labeled another node in the circuit to have unknown voltage v_i (which is neither an input nor an output).

Done: 5:10 pm



(a) [8] Find the voltage gain v_o/v_i. Be clear about any units for the gain.

(b) [6] Find the transresistance gain v_o/i_s, and give v_o if the current i_s = 1mA. Be clear about units for the gain.

(c) [1] What is the power delivered by the op-amp to the circuit?

(a) Since no current thru 2kΩ, $v_c = v_i$

$$i_0 = \frac{v_o - v_c}{4k} = \frac{v_c}{1k} \Rightarrow \frac{v_o - v_i}{4} = \frac{v_i}{1}$$

$$v_o = (1+4)v_i = 5v_i$$

$$\boxed{\frac{v_o}{v_i} = 5} \ (V/V)$$

(b) $v_i = i_s \cdot 1k\Omega$

$$\boxed{\frac{v_o}{i_s} = 5k\Omega}$$

For $i_s = 1mA$, $v_o = 5V$

(c) $P_{opamp} = v_o \cdot i_{out} = v_o \cdot i_o = (5k\Omega\, i_s)\, i_s = 5k\Omega\, i_s^2$ = 5mW if $i_s = 1mA$.

Inputs deliver no power since $i_2 = 0$.