EK 307: Electric Circuits

Fall 2017

Lecture 14

Oct 26, 2017

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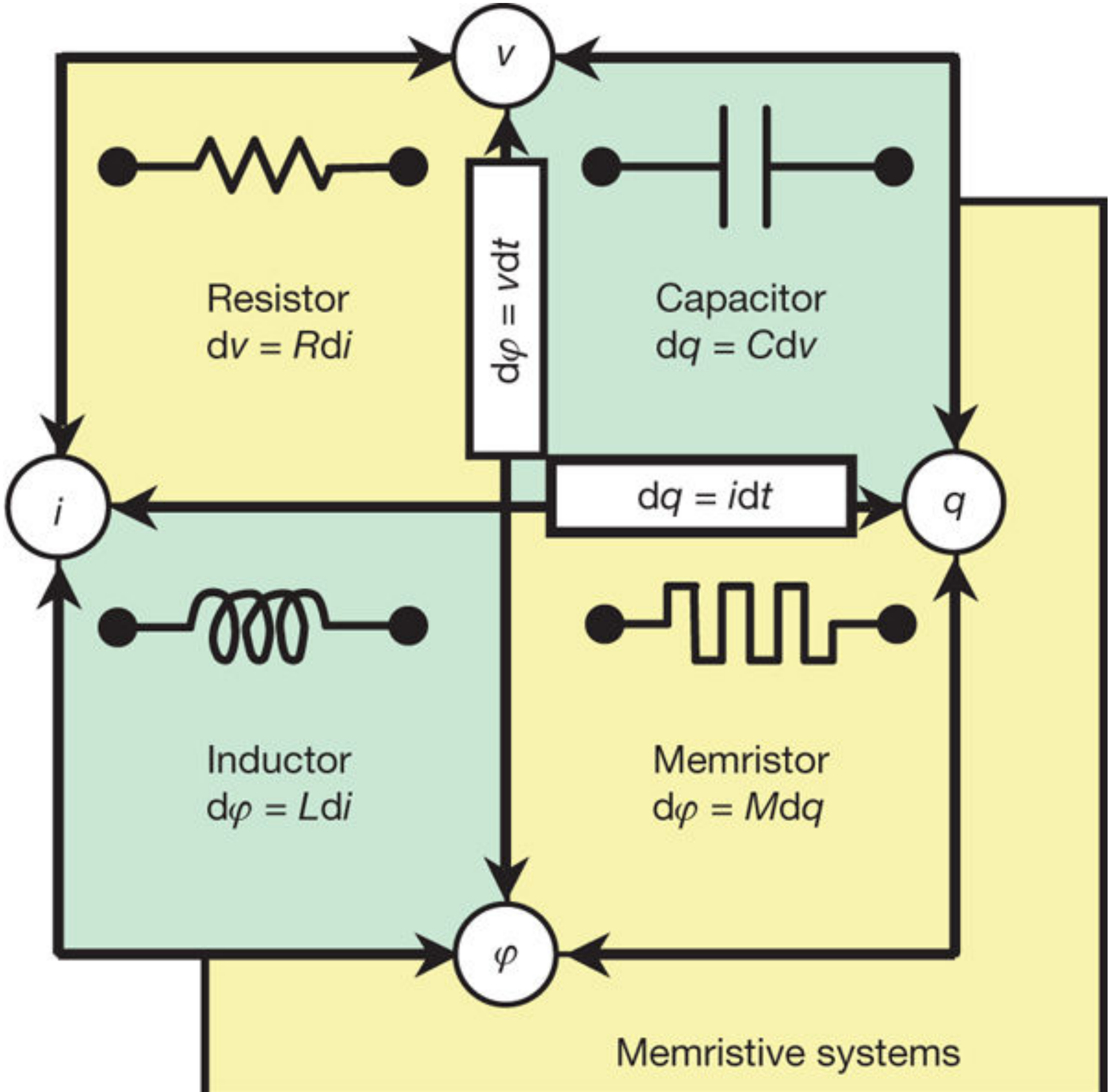
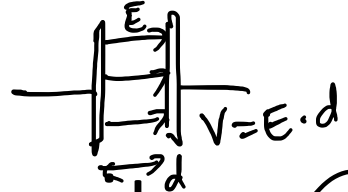
Lecture 14: Today

1. **Chapter 6: Capacitors**

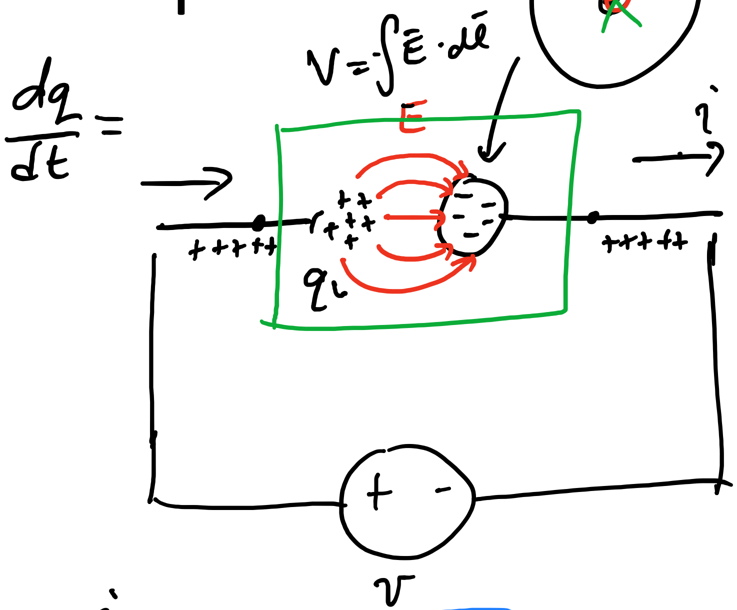


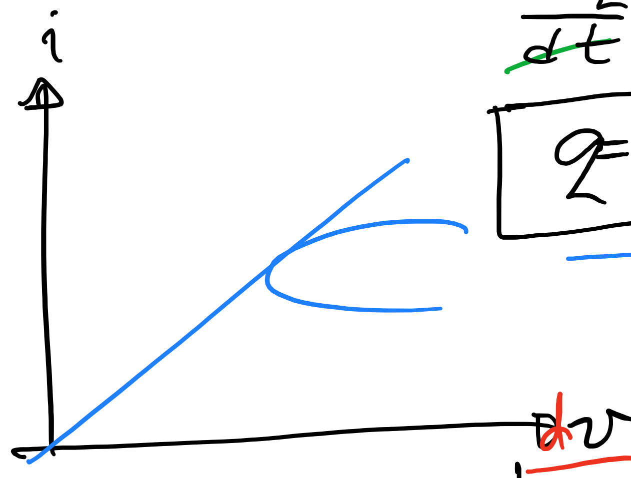
v
i
q
φ
Resistor
dv = Rdi
Capacitor
dq = Cdv
dφ = vdt
dq = idt
Inductor
dφ = Ldi
Memristor
dφ = Mdq
Memristive systems

Chapter 6: Capacitors and Inductors



Capacitor





$$\boxed{i = C\frac{dv}{dt}}$$

$$\frac{dq}{dt} = C$$

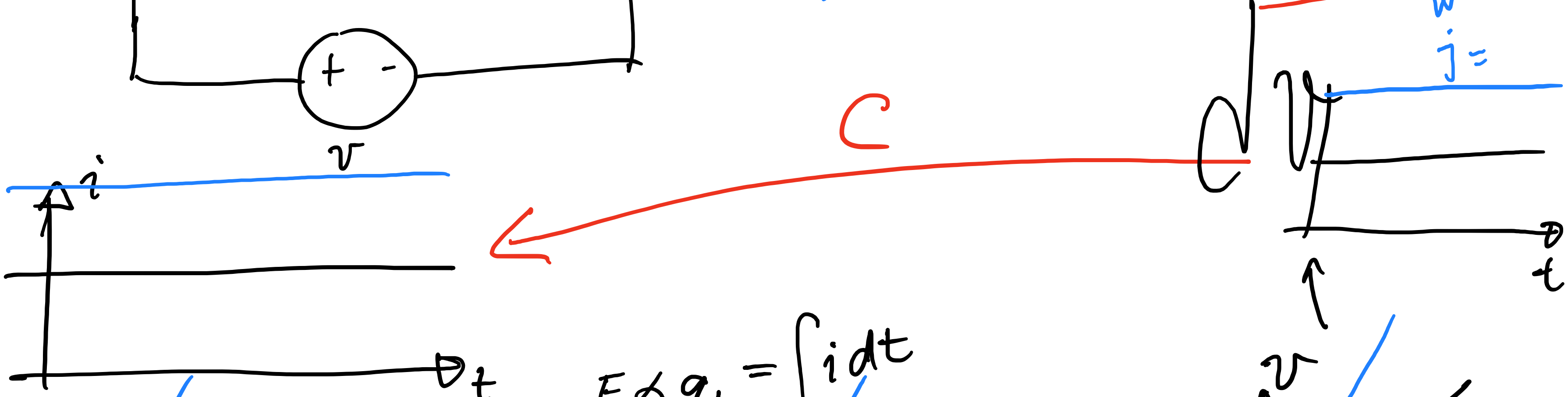
$$\boxed{q = Cv}$$

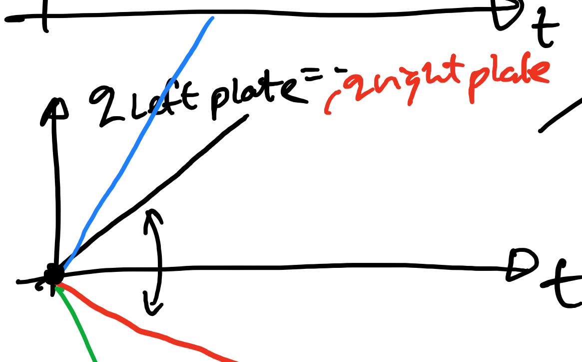
$$\frac{v}{v} = \frac{C}{V} \to \frac{C}{\;}$$

$p = vi$

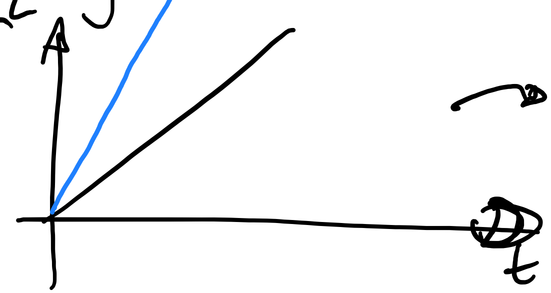
$w = v\,i\,t$

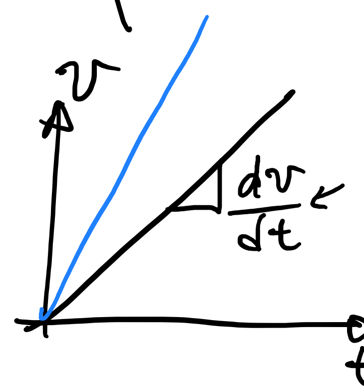
$j =$



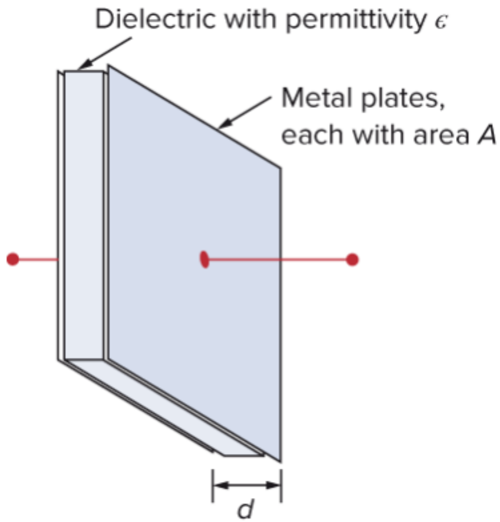


$E \propto q_L = \int i\,dt$





Capacitor

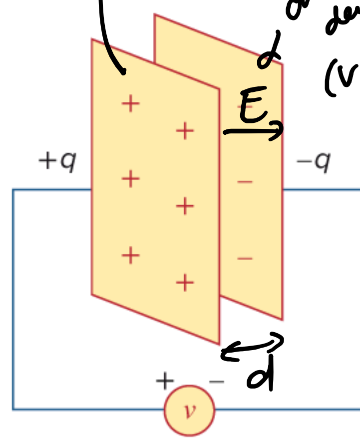


area A

area charge density σ on plates (C/m^2)

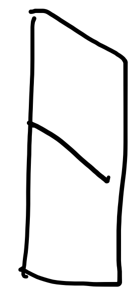
$(V = E \cdot d)$

$\boxed{q = Cv}$



E

d



$2\times A$

$\hookrightarrow 2\times$ chg dens

$\downarrow$

$2\times C.$

$$C = \frac{\epsilon A}{d}$$

1 Farad = 1 Coulomb/volt

Capacitor

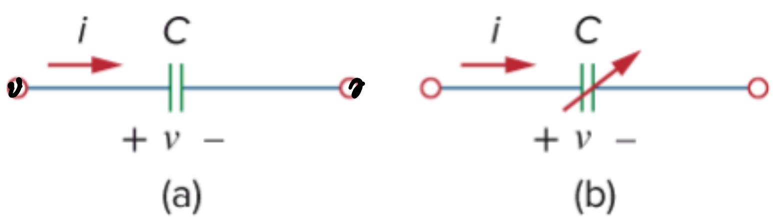
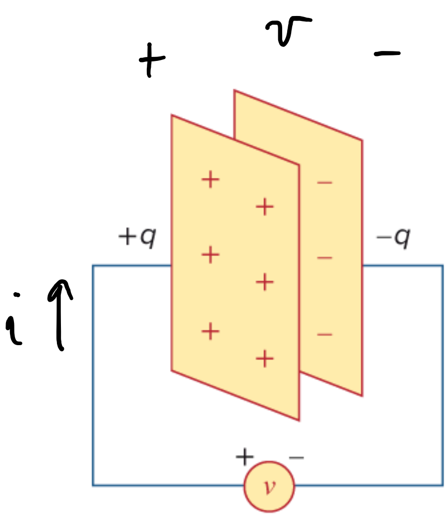


Figure 6.3 Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

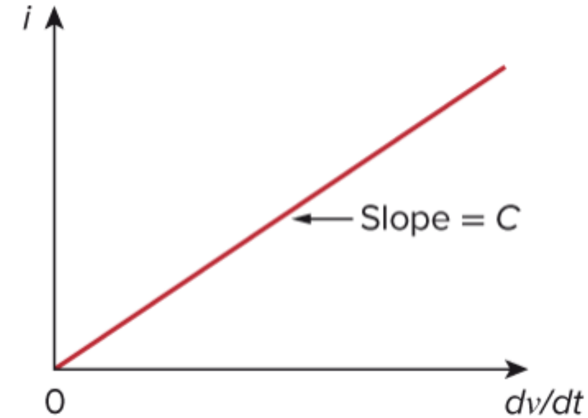
Capacitor

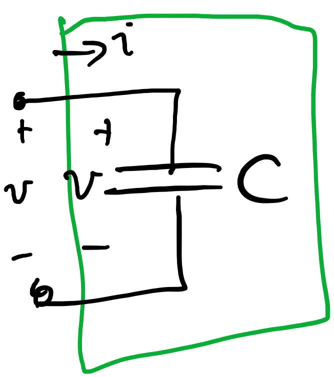


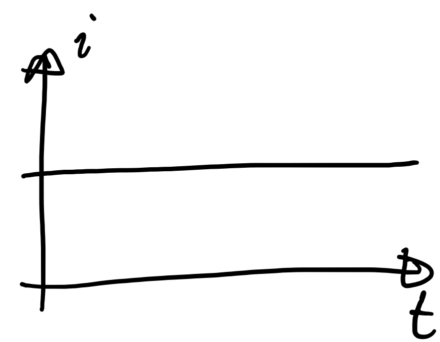
$$\boxed{q = Cv}$$

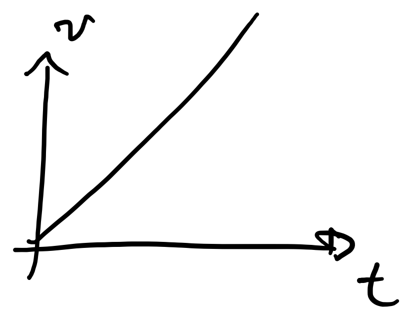
$$i = \frac{dq}{dt}$$

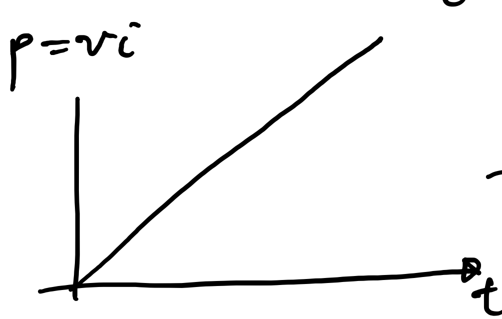
$$\boxed{i = C\frac{dv}{dt}}$$

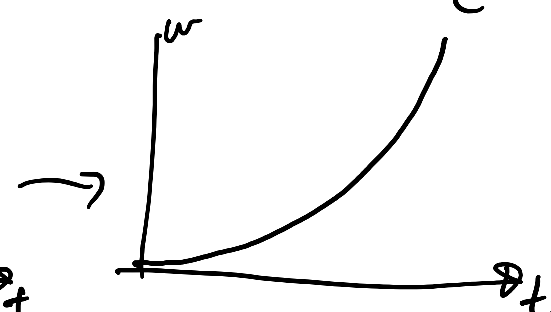


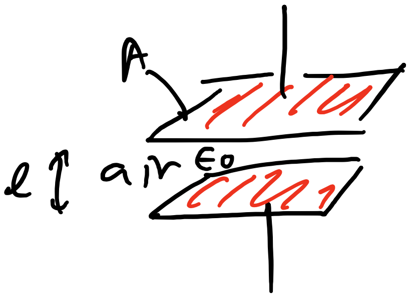




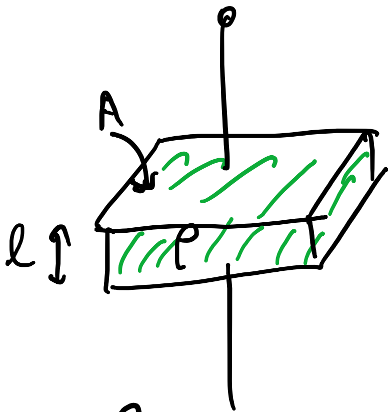




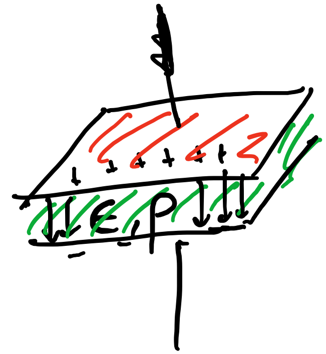




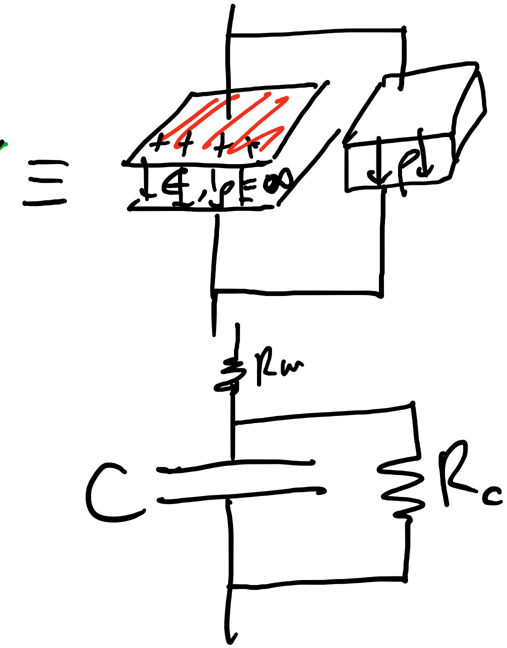
$$C_0 = \frac{\epsilon_0 A}{d}$$



$$R = \frac{\rho l}{A}$$



$$C = \frac{\epsilon A}{l}, \quad R = \rho \frac{l}{A}$$



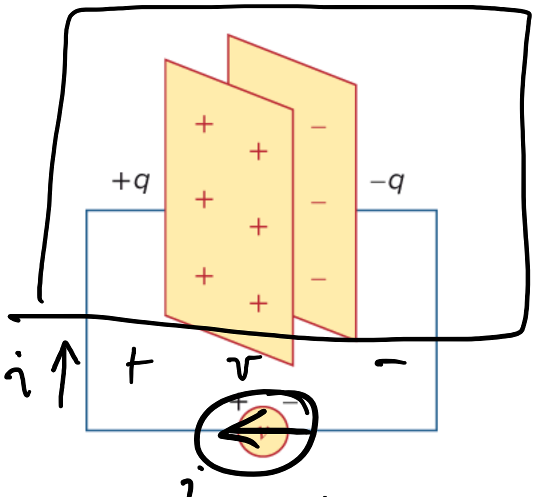
Capacitor

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,(\tau)\,d\tau$$

$$\boxed{i = C\frac{dv}{dt}}$$

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)}$$

$$\boxed{q = Cv}$$

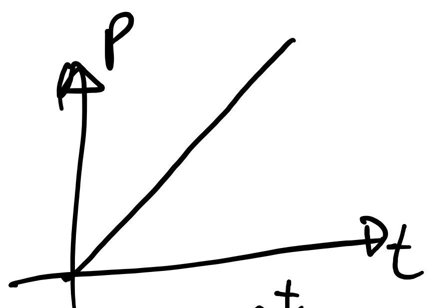


$$p = vi = Cv\frac{dv}{dt}$$

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}\,d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

$$\boxed{w = \frac{1}{2}Cv^2}$$

$$i = C\frac{dv}{dt}$$

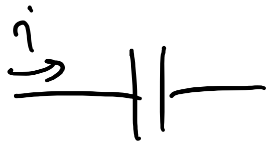


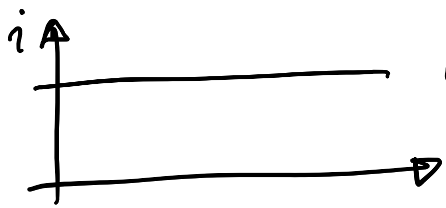
$$w = \frac{q^2}{2C}$$

$$p = vi = vC\frac{dv}{dt}$$

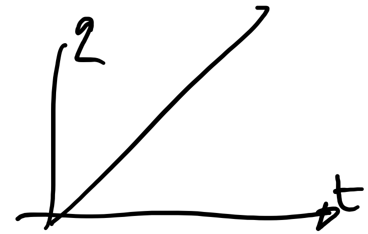
$$\frac{d}{dt}\left(v(t)^2\right) = \underbrace{2v\frac{dv}{dt}}$$

$$w_e(t) = \int_0^t p(t')\,dt' = \int_0^t vC\frac{dv}{dt}\,dt = C\int_0^t v\frac{dv}{dt}\,dt = C\frac{1}{2}\int_0\left(\frac{d}{dt}v^2\right)dt = \boxed{}$$





$q = Cv$



$w_e = \frac{1}{2} C v^2 = \frac{1}{2} C ($

$c \, q^2 = \frac{1}{2c}(At)^2 = \frac{A^2}{2c} t^2$

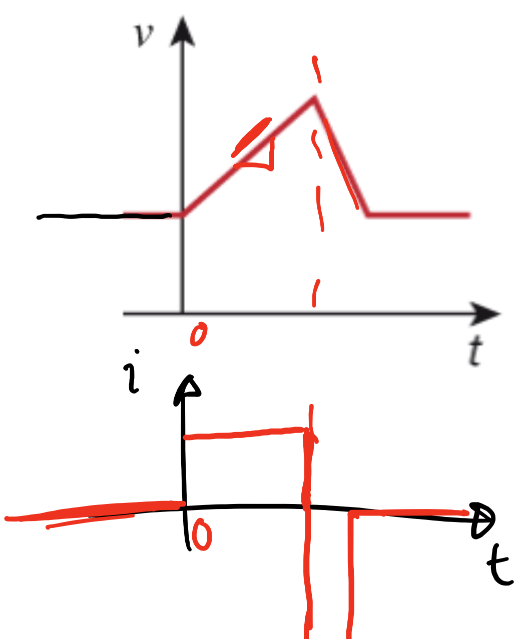
$i = \text{const} = \frac{dq}{dt}$

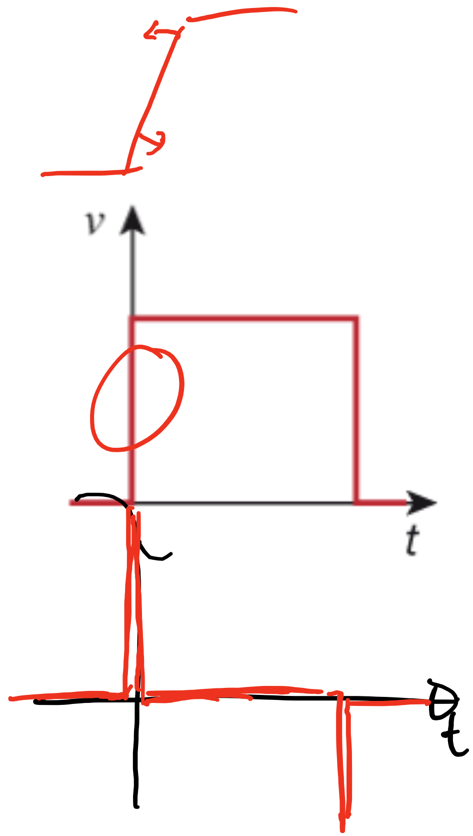
$q = At$

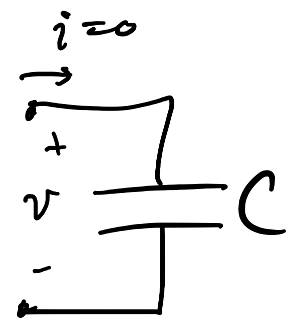
Properties of a capacitor

1. A capacitor is an open circuit to dc.
2. The voltage on a capacitor cannot change abruptly.

$i = C\frac{dv}{dt}$







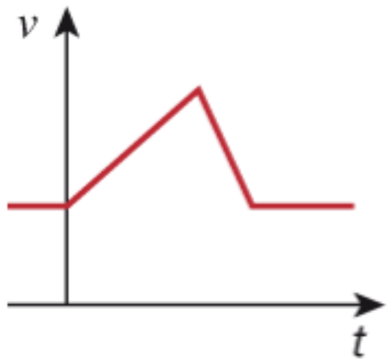
$i = C\frac{dv}{dt} \rightarrow 0$

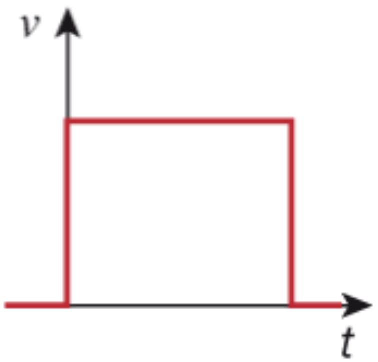
DC : steady state

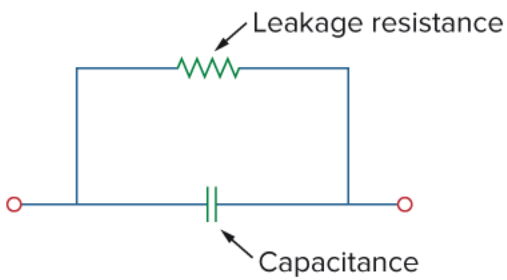
$\frac{d}{dt} \Rightarrow 0$

Properties of a capacitor

1. A capacitor is an open circuit to dc.
2. The voltage on a capacitor cannot change abruptly.
3. The ideal capacitor does not dissipate energy
4. Non-ideal capacitor may have leakage resistance

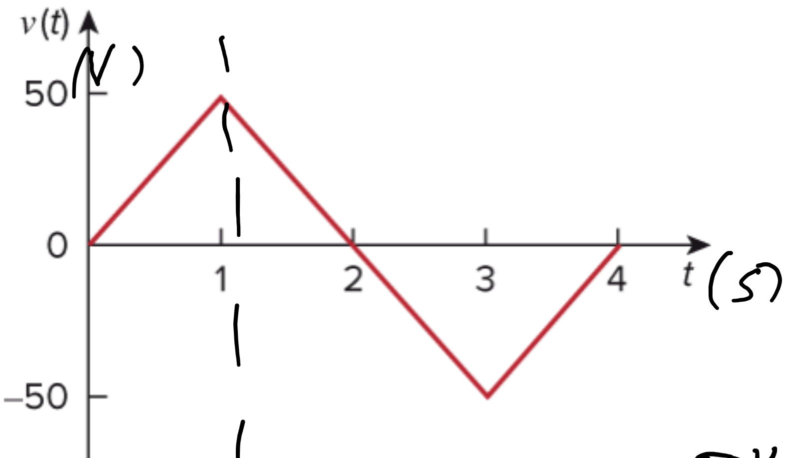






Example 6.4

Determine the current through a 200-μF capacitor whose voltage is shown in **Fig. 6.9**.



$$Q = C\,V$$

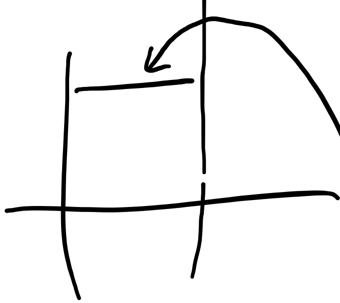
$$\underbrace{[1C] = [1F][1V]}_{F = \frac{C}{V}}$$

$$i = C\frac{dv}{dt} = C\frac{\Delta v}{\Delta t} = \frac{50V}{1s}\,200\mu F = 10000\mu\frac{VF}{s}$$

$$10000 \cdot 10^{-6}$$

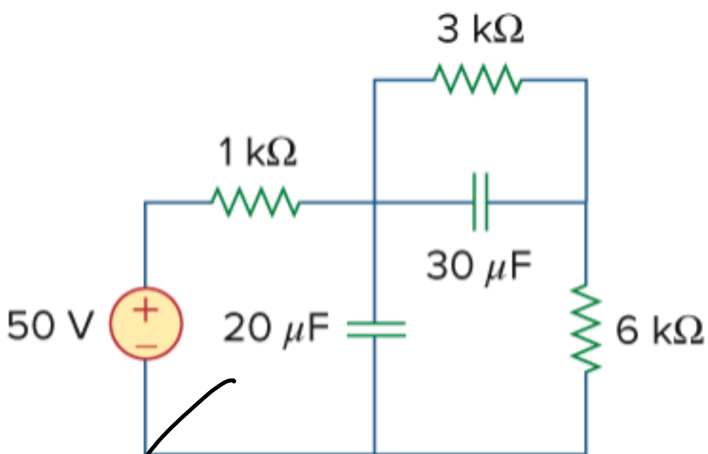
$$= 10^{-2}\,\frac{VF}{s} = 10^{-2}\,\frac{\cancel{V}}{s}\frac{C}{\cancel{V}} = 10^{-2} A$$

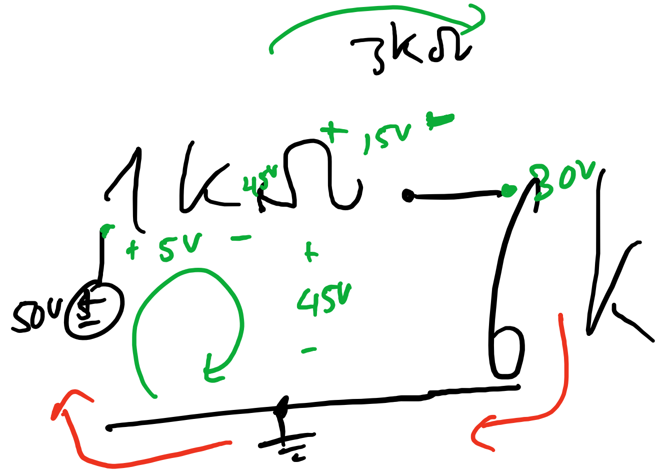
$$= 10\,mA$$



Practice Problem 6.5

Under dc conditions, find the energy stored in the capacitors in **Fig. 6.13**.





$i = \frac{50V}{10k} = 5mA$

10^3 10^{-3}

F
mF
μF
nF
pF
("puff")

$w_e = \frac{1}{2} C v^2$

$= \frac{1}{2} 20\mu F \cdot (45V)^2$

$= 10 \cdot 10^{-6} F \cdot \tilde{2000}\ V^2$

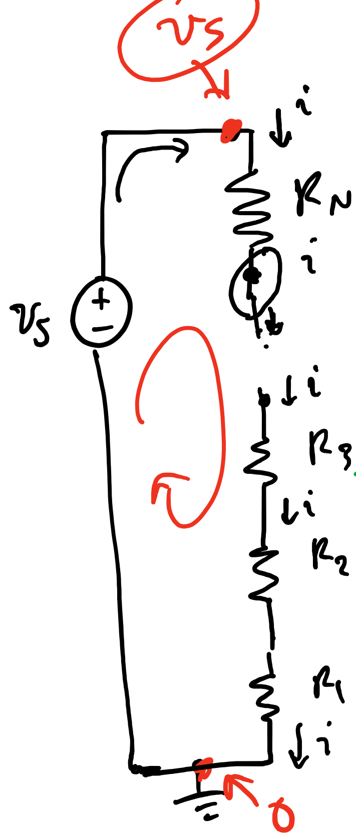
$= 2 \cdot 10^4 \cdot 10^{-6} \cdot FV^2$

$= 0.02 \cdot FV^2 \longrightarrow F \cdot V \cdot V = \frac{\cancel{C}}{\cancel{V}} \cancel{V} \cdot \frac{J}{\cancel{C}}$

$Q = CV$

J

$= 20 mJ$



$v_N = i R_N$

$v_3 = i R_3$

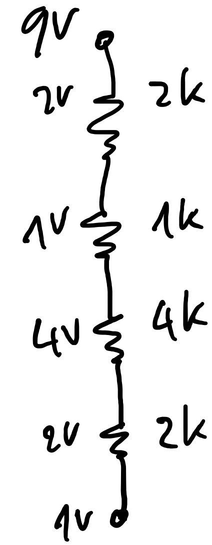
$v_1 = i R_1$

$-v_S + v_N + v_{N-1} \cdots + v_3 + v_2 + v_1 = 0$

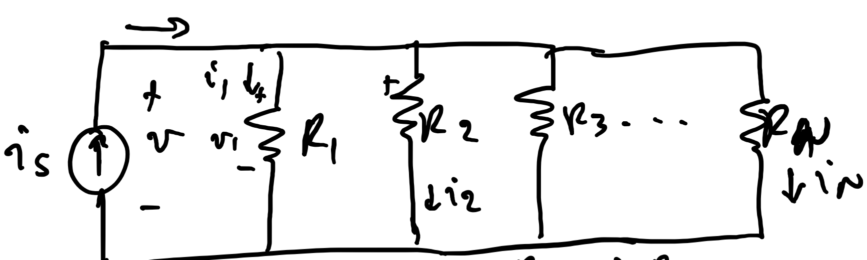
$$\sum_{n=1}^{N} v_n = v_S$$

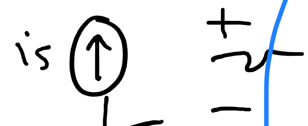
$$\boxed{i(\underbrace{R_1 + R_2 + \cdots R_N}_{R_{eq}}) = v_S}$$

$$v_3 = i R_3 = \frac{v_S}{R_1 + R_2 \cdots R_N} \cdot R_3 = \frac{v_S}{R_{series}}$$



$$i_2 = i_S \frac{G_2}{G_1 + G_2 + \cdots G_N}$$





Ω

conductance
mhos
Siemens

$$\frac{1}{R_{eq}} = G_{eq}$$

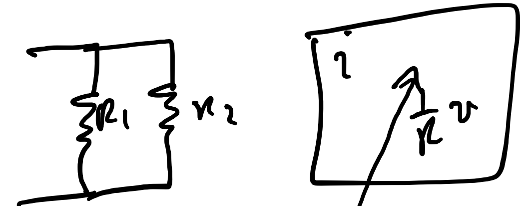
$v_1 = i_1 R_1 = i_2 R_2 = i_3 R_3 \cdots$

$v(\underbrace{G_1 + G_2 + G_3 \cdots}_{})$

$v_1 G_1 + v_2 G_2 \cdots = i_S = i_1 + i_2 + \cdots i_N = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \cdots$

$$v = \frac{i_S}{\sum_n G_n}$$

Series and parallel

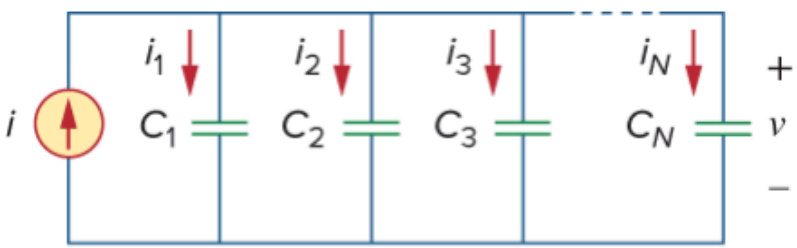


$$i_1 = v G_1$$

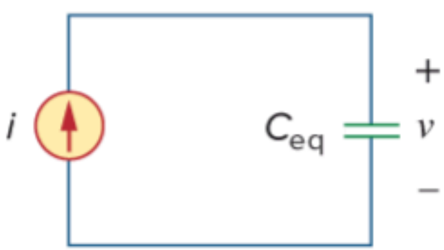
$$i_2 = v G_2$$

$$i = C \frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



(a)



(b)

But $i_k = C_k\, dv/dt$. Hence,

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

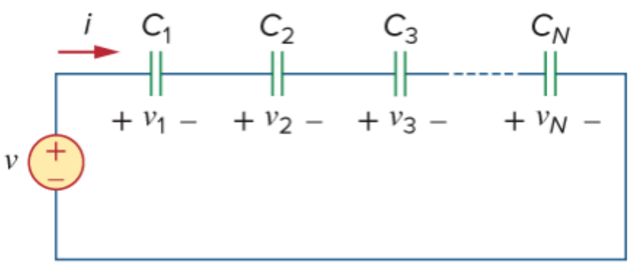
$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

where

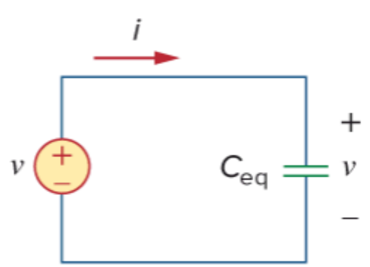
$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

Series and parallel

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$



(a)



(b)

But $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)\,d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{\mathrm{eq}}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

where

$$\boxed{\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$