EK 307: Electric Circuits

Fall 2017

Lecture 11

Oct 12, 2017

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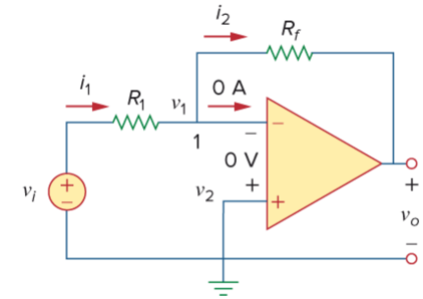
Administrivia

- Homework 3 back today (also HW1,2)

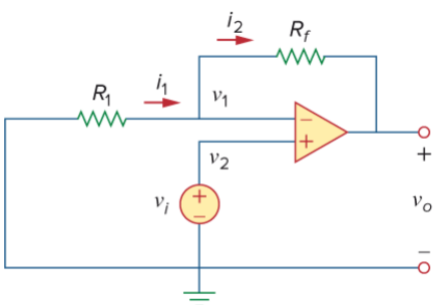
- Regrade HW2 after midterm
  - Posting corrected Problem Set and Solution online

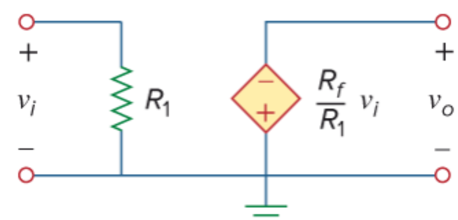
- Midterm Exam #1: in class, Tuesday, Oct 17
  - Covers Ch 1 to 4, Chap 5.1-5.5, 5.7-5.8
  - Closed book, closed notes.
  - Single page of notes (1 side), handheld calculator

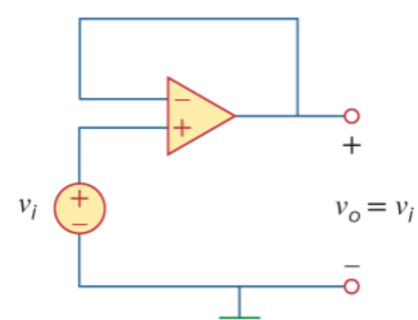
Lecture 10: Last time



1. **Review op-amp (non-ideal, ideal)**
2. **Inverting amplifier**
   a. **Trans-resistance amp**
3. **Non-inverting amplifier**
4. **Equivalent ccts**
5. **Voltage follower**
6. **Cascaded op-amp circuits**



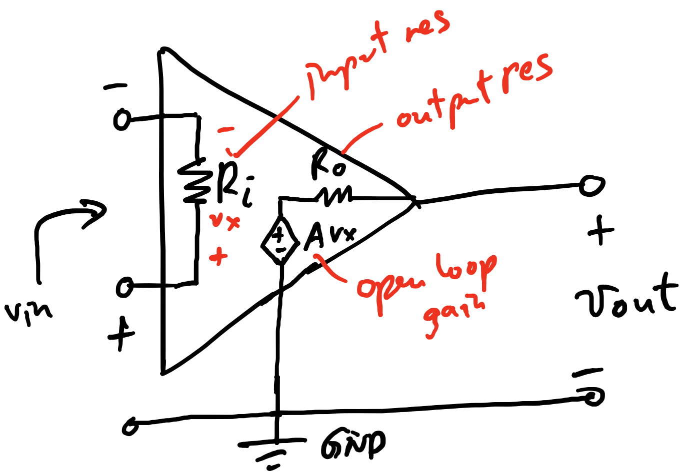




Lecture 10: Today

1. **Review opamp**
2. **DAC on the homework**
3. **Cascading Op Amps (continued)**
4. **Summing amplifiers**
5. **Differencing amplifiers**
6. **Applications: DAC, instumentation amp**

Review of the op-amps:

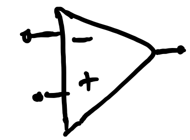


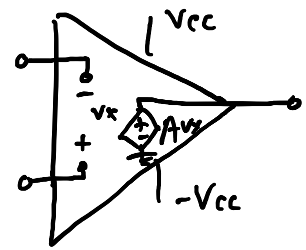
non-ideal opamp

$A = \infty$

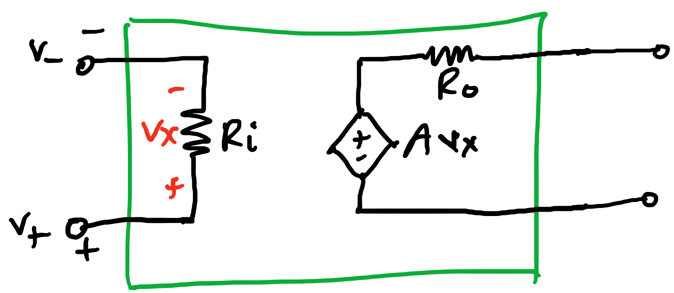
$R_i = \infty$

$R_o = 0$

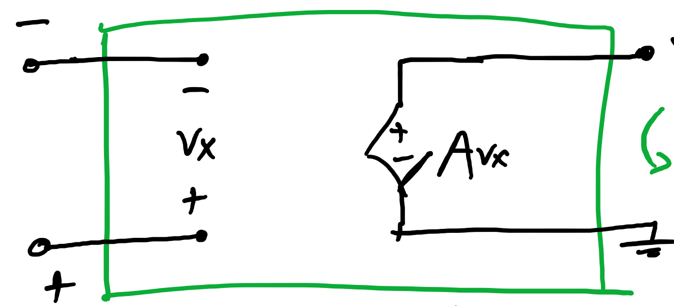




ideal

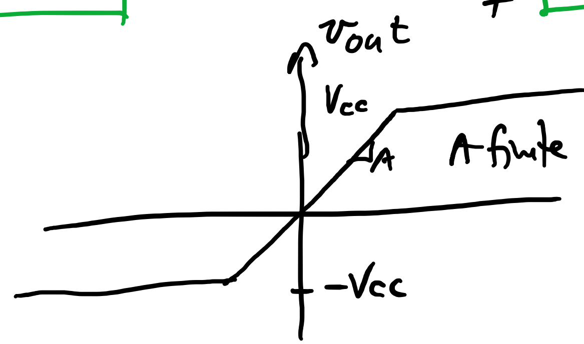


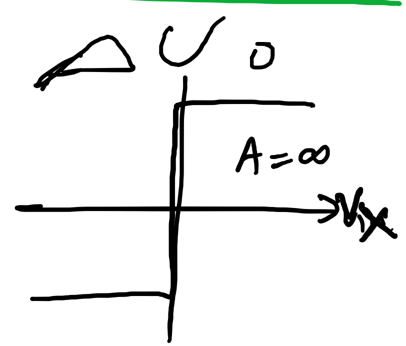
$v_x = v_+ - v_-$



$v_{out} = Av_x$

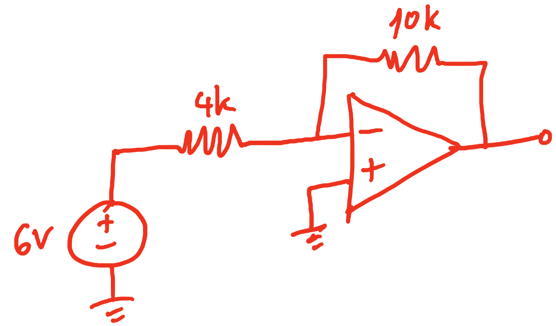
$v_x = \frac{v_{out}}{A}$





Review

Examples



$v_o = -\frac{10}{4} 6$

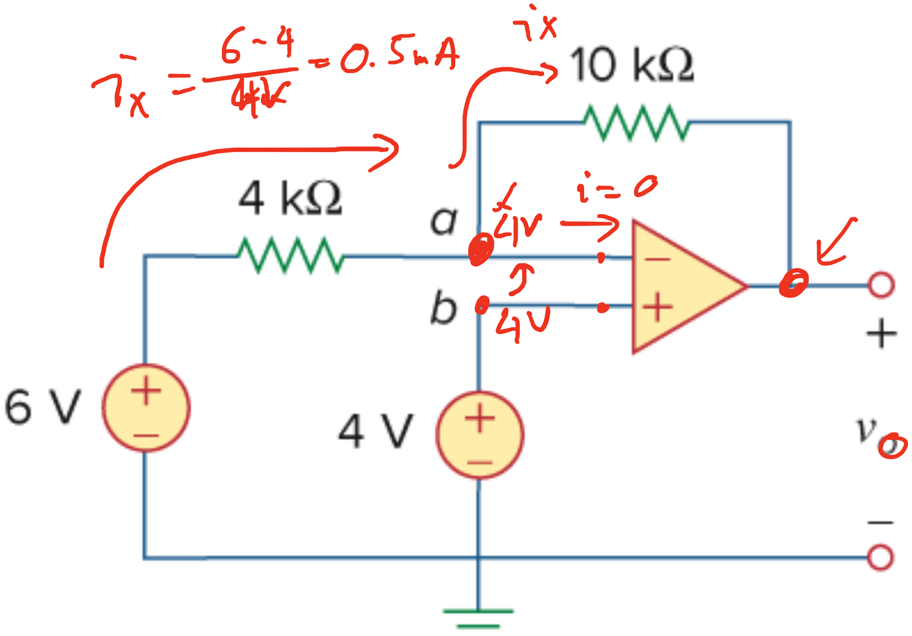
$= -15V$

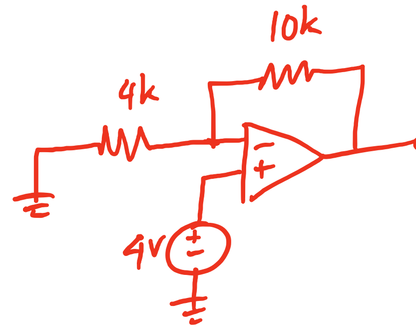
-5

6

$-1 \quad 10$

$i_x = \frac{6-4}{4k} = 0.5mA$





$v_o'' = \left(1 + \frac{10}{4}\right) 4V$

$= \frac{14}{4} 4$

$= 14V$

$\boxed{v_o = v_o' + v_o'' = -1V}$

$i_x = \frac{4V - v_o}{10k} \quad 0.5mA$

$\Rightarrow 4V - v_o = 5V$

$-1V = v_o$

Example (design 2)

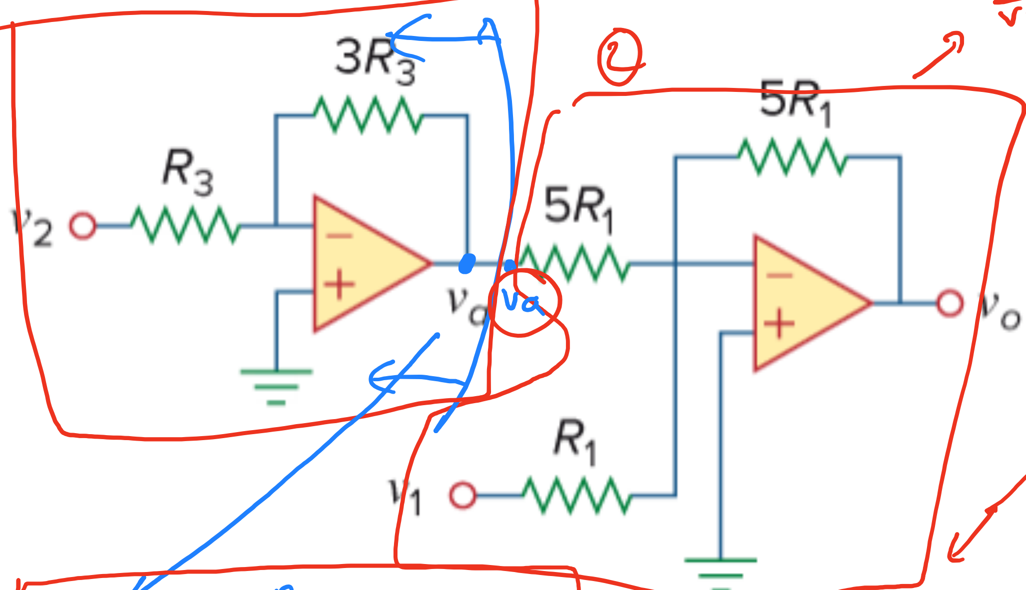
R_1 = 10 kΩ and R_3 = 20 kΩ or $R_1 = R_3$ = 10 kΩ.

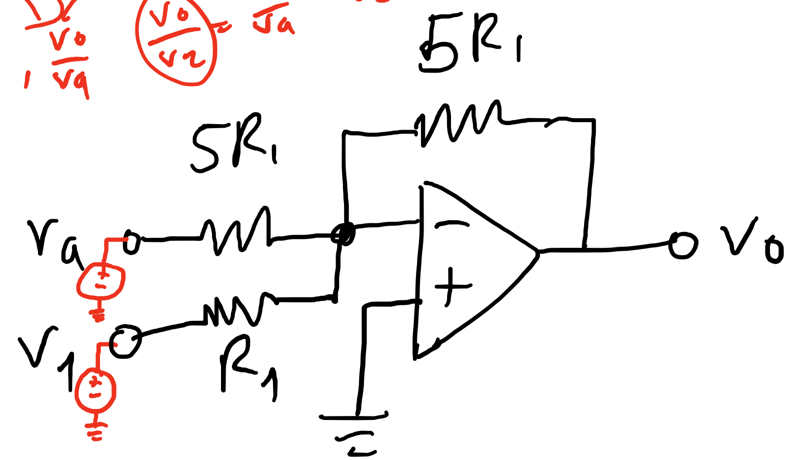
What does this do?

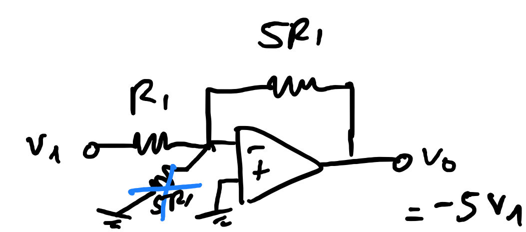
1 → $\frac{v_a}{v_2}$

2 → $\frac{v_o}{v_1}$, $\frac{v_o}{v_a}$

$\frac{v_o}{v_2} = \frac{v_o}{v_a} \cdot \frac{v_a}{v_2}$

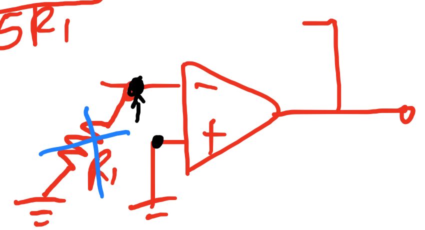






$= -5v_1$

$v_a = \frac{-3R_3}{R_3} v_2 = -3v_2$



$v_o = -v_a$

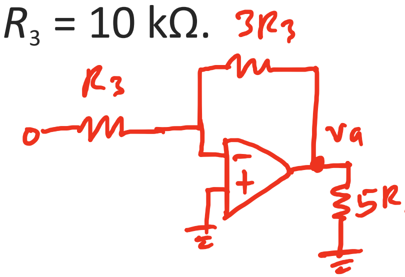
By sup:

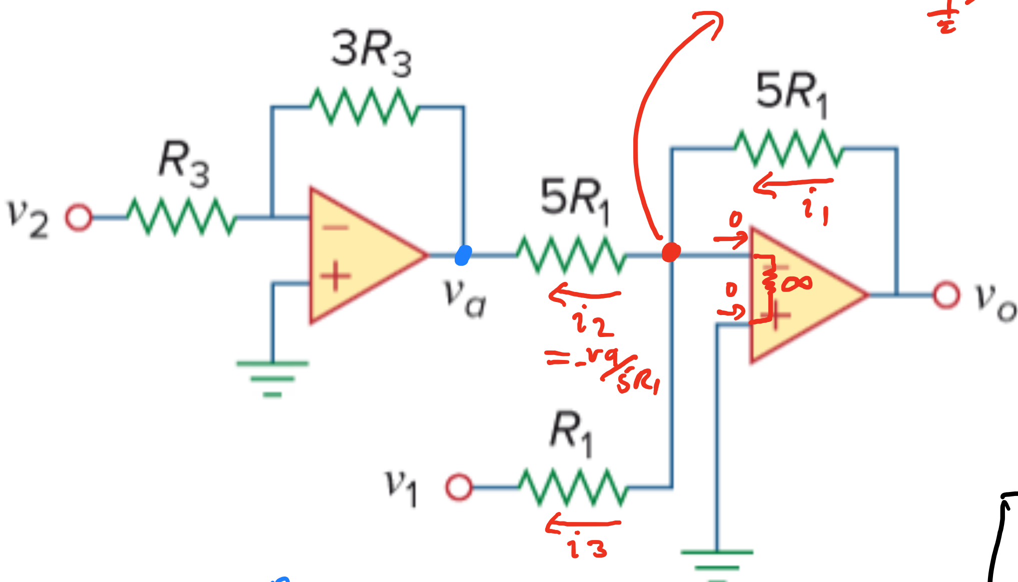
$v_o = -(5v_1 + v_a)$
$= -5v_1 + 3v_2$

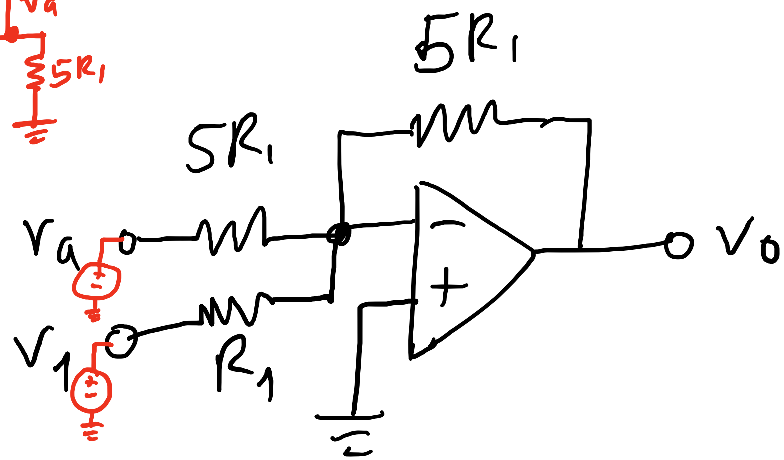
Example (design 2)

R_1 = 10 kΩ and R_3 = 20 kΩ or R_1 = R_3 = 10 kΩ.

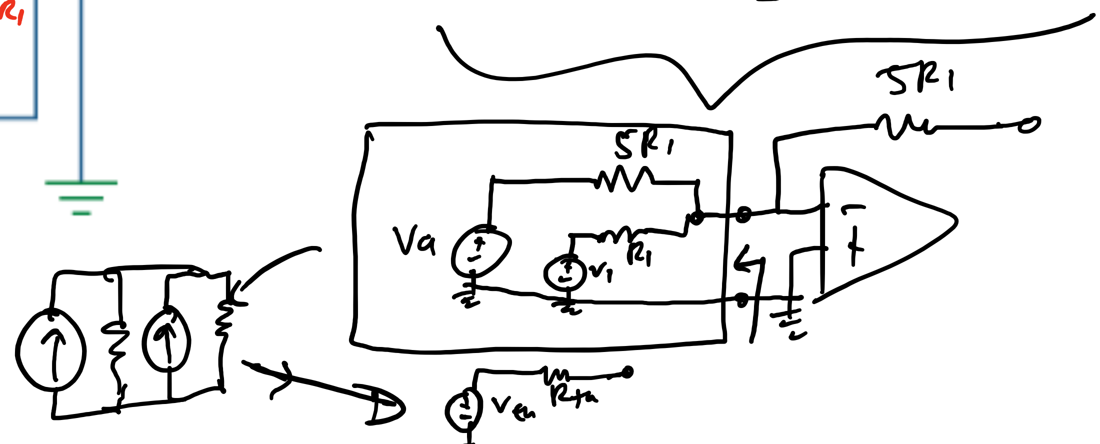
What does this do?



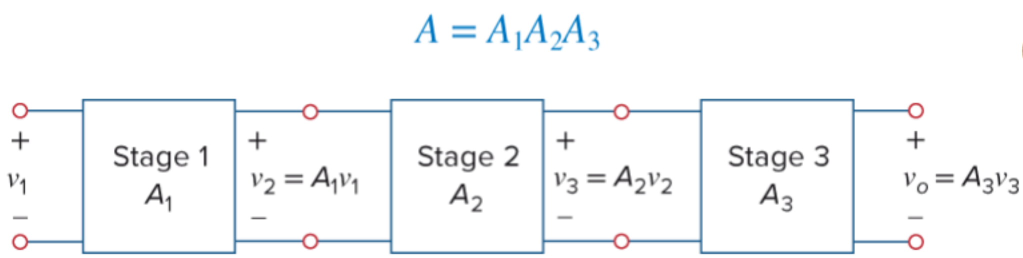




$$v_a = \frac{-3R_3}{R_3} v_2 = -3v_2$$



Cascading Op-Amp Circuits

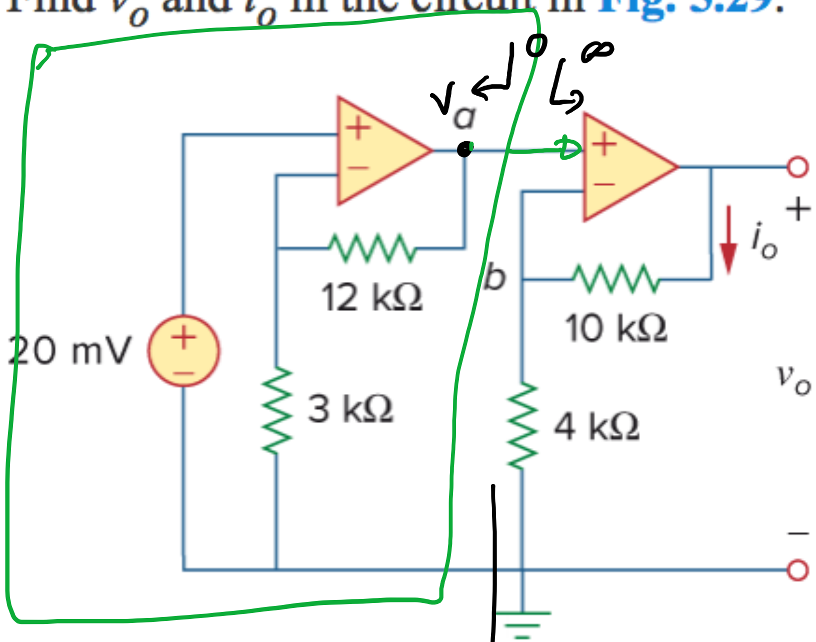


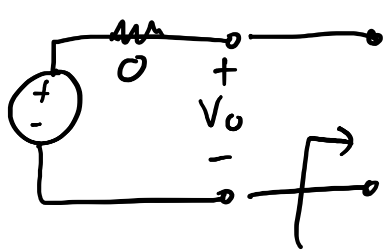
Op amp circuits have the advantage that they can be cascaded without changing their input-output relationships.

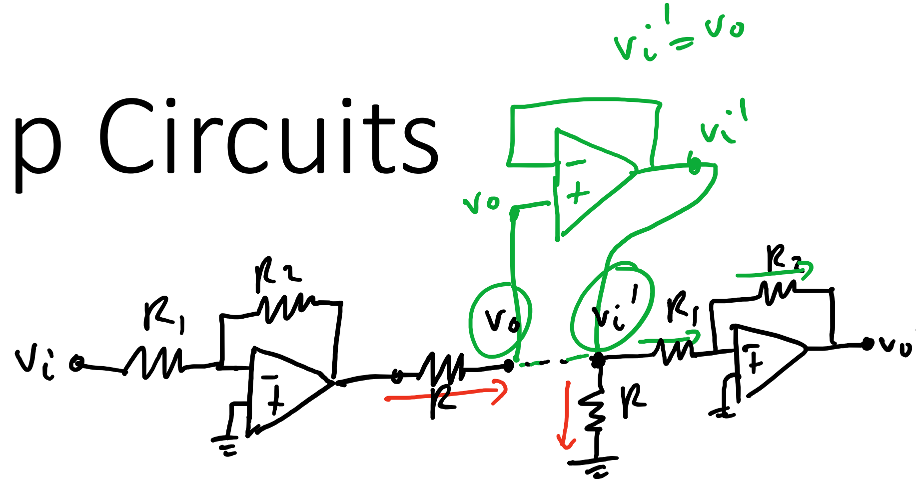
This is due to the fact that each (ideal) op amp circuit has infinite input resistance and zero output resistance.

Cascading Op-Amp Circuits

Find v_o and i_o in the circuit in **Fig. 5.29**.







$$\frac{V_o}{V_i} = \frac{-R_2}{R_1}$$

$$\frac{V_o'}{V_i'} = \frac{-R_2}{R_1}$$

$\cancel{V_i' = V_o}$

$$V_a = \left(1 + \frac{12k}{3k}\right) 20mV$$

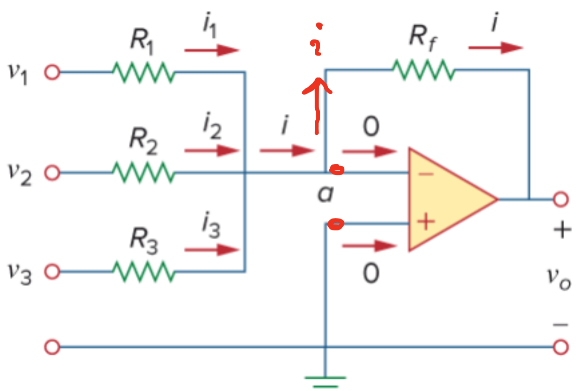
$$V_o = \left(1 + \frac{10k}{4k}\right) V_a = \left(1+\frac{10}{4}\right)\left(1+\frac{12}{3}\right) 20mV$$

$$= 3.5\,(5)\;20mV$$

$$= 0.35V$$

Summing Amplifier

Summing Amplifier



$$i = i_1 + i_2 + i_3$$

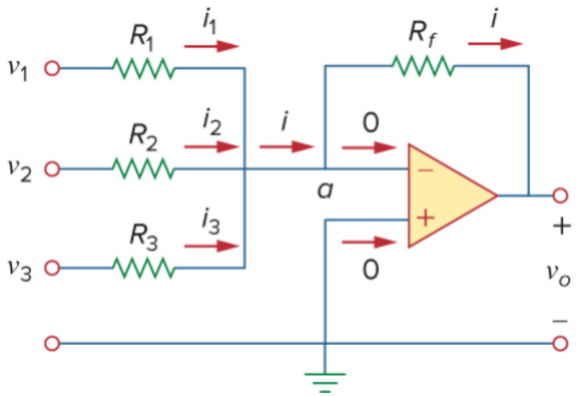
$$\left.\begin{aligned} i_3 &= \frac{v_3}{R_3} \\ i_2 &= \frac{v_2}{R_2} \\ i_1 &= \frac{v_1}{R_1} \end{aligned}\right\} \quad i = i_1 + i_2 + i_3 = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

$$(0 - v_o) = i R_F$$

$$-v_o = \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right) R_F$$

$$\boxed{v_o = -\left(\frac{R_F}{R_1} v_1 + \frac{R_F}{R_2} v_2 + \frac{R_F}{R_3} v_3\right)}$$

Summing Amplifier



$$i = i_1 + i_2 + i_3$$

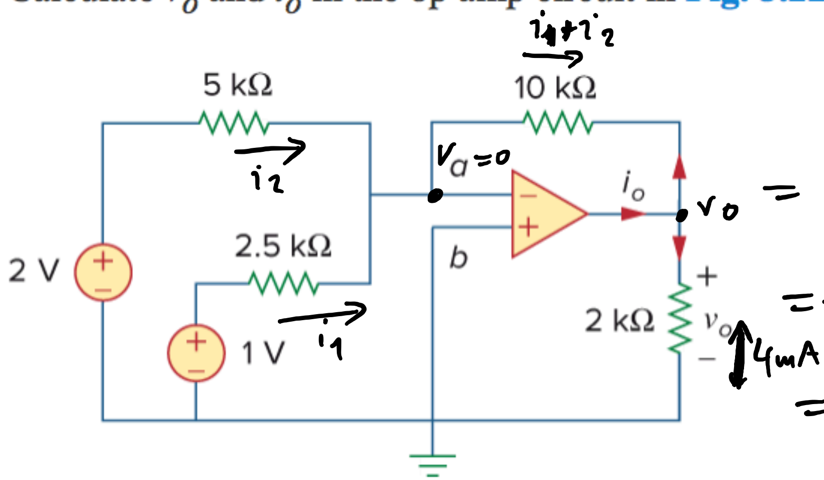
$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)}$$

$R_1 = R_2 = R_3 \equiv R$

$$V_0 = -\frac{R_f}{R}(v_1 + v_2 + v_3)$$

Example

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.



$$v_o = -\left(\frac{10k}{5k}\, 2V + \frac{10k}{2.5k}\, 1V\right)$$

$$= -(4V + 4V)$$

$$= -8V$$

Figure 5.22 For Example 5.6.

Example (students)

Find v_o and i_o in the op amp circuit shown in Fig. 5.23.

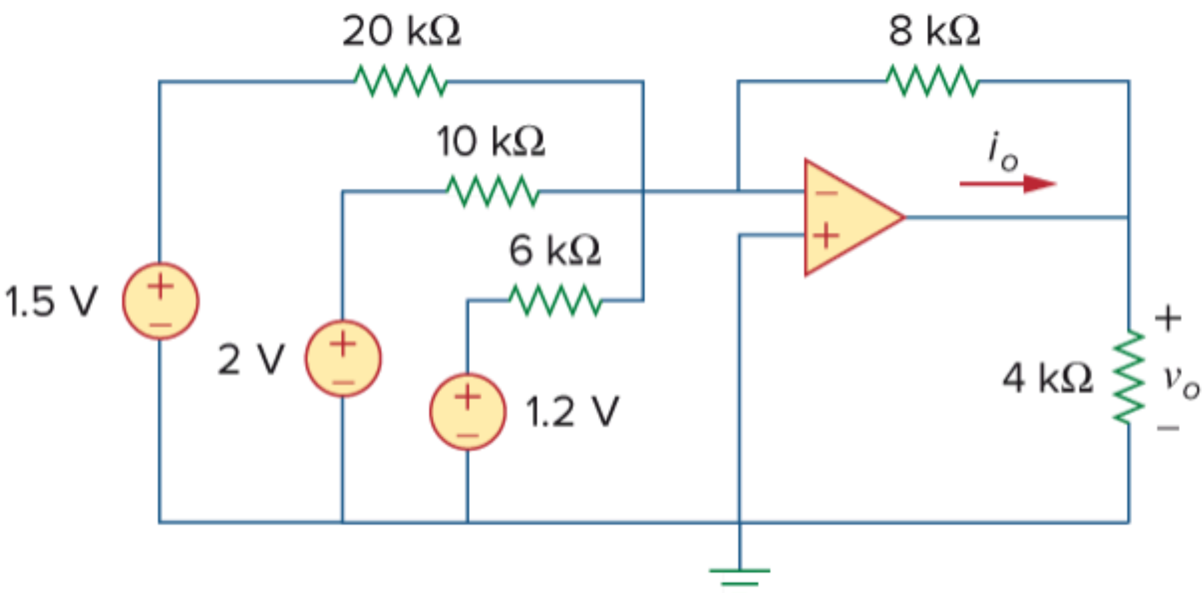
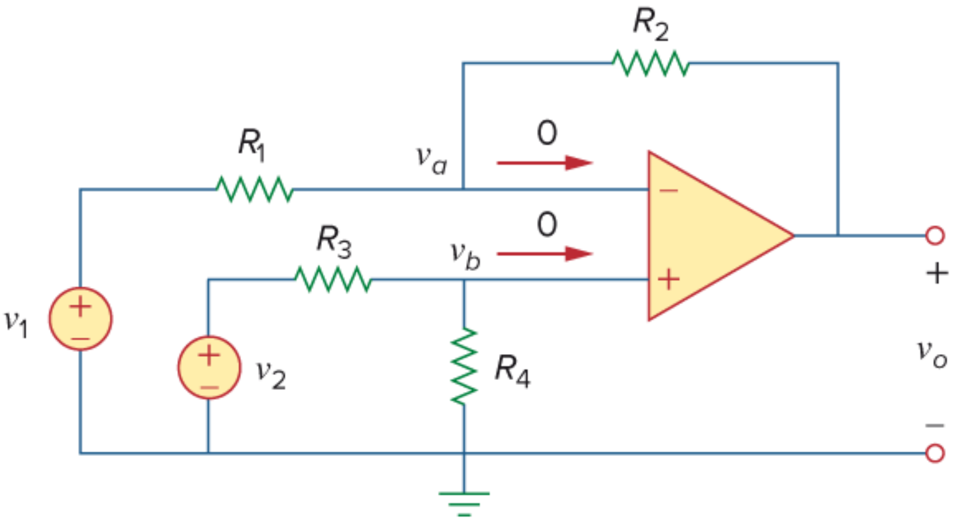


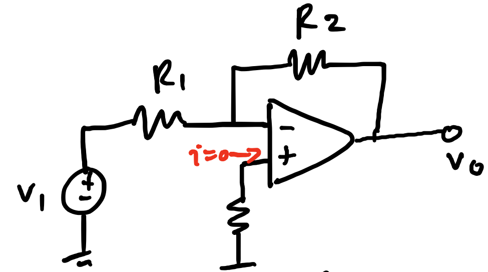
Figure 5.23 For Practice Prob. 5.6.

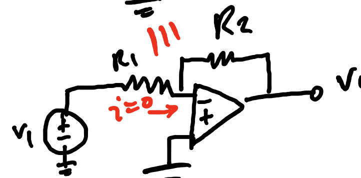
$-3.8V$ ✓

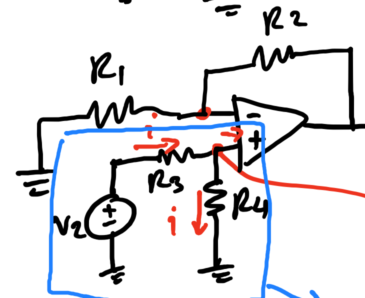
$v_o = -\frac{19}{5} V$ ✓

Difference amplifier

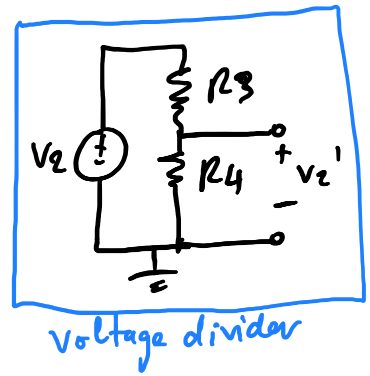






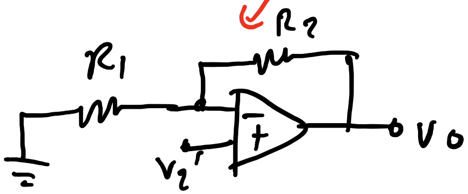


$$v_2' = \frac{R_4}{R_3+R_4}v_2$$



Goal: $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

$$\frac{\cancel{R_1+R_2}}{R_1}\cdot\frac{R_2}{\cancel{R_3+R_4}} \quad \cancel{R_1+R_2}$$



Together:

$$V_o = -\frac{R_2}{R_1}v_1 + \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_2$$

$$= -\frac{R_2}{R_1}v_1 + \frac{R_2}{R_1}v_2$$

$$V_o = \left(1+\frac{R_2}{R_1}\right)v_2'$$

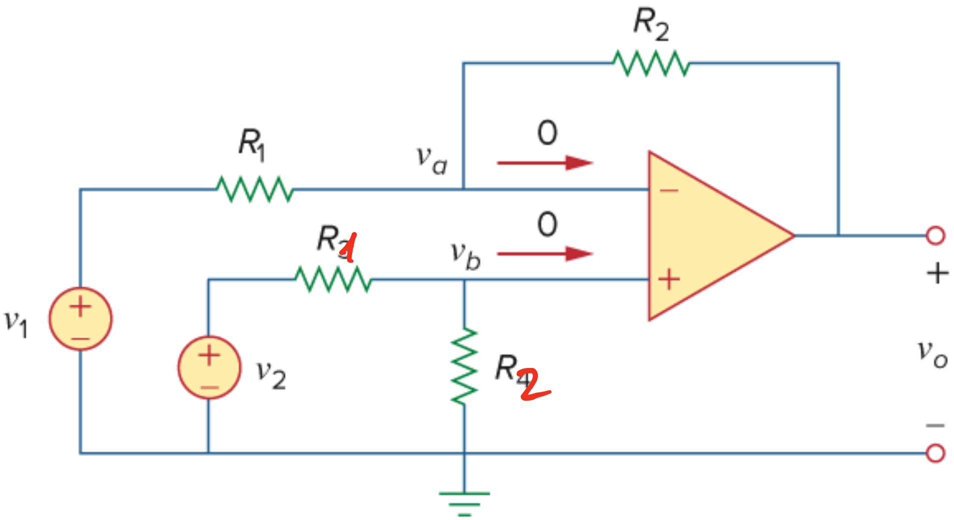
$$= \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_2$$

$$\frac{R_1+R_2}{\cancel{R_1}}\cdot\frac{R_4}{R_3+R_4} = \frac{R_2}{\cancel{R_1}} \rightarrow (R_1+R_2)R_4 = (R_3+R_4)R_2$$

$R_3 = R_1$

$R_4 = R_2$

Difference amplifier



Goal: $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$

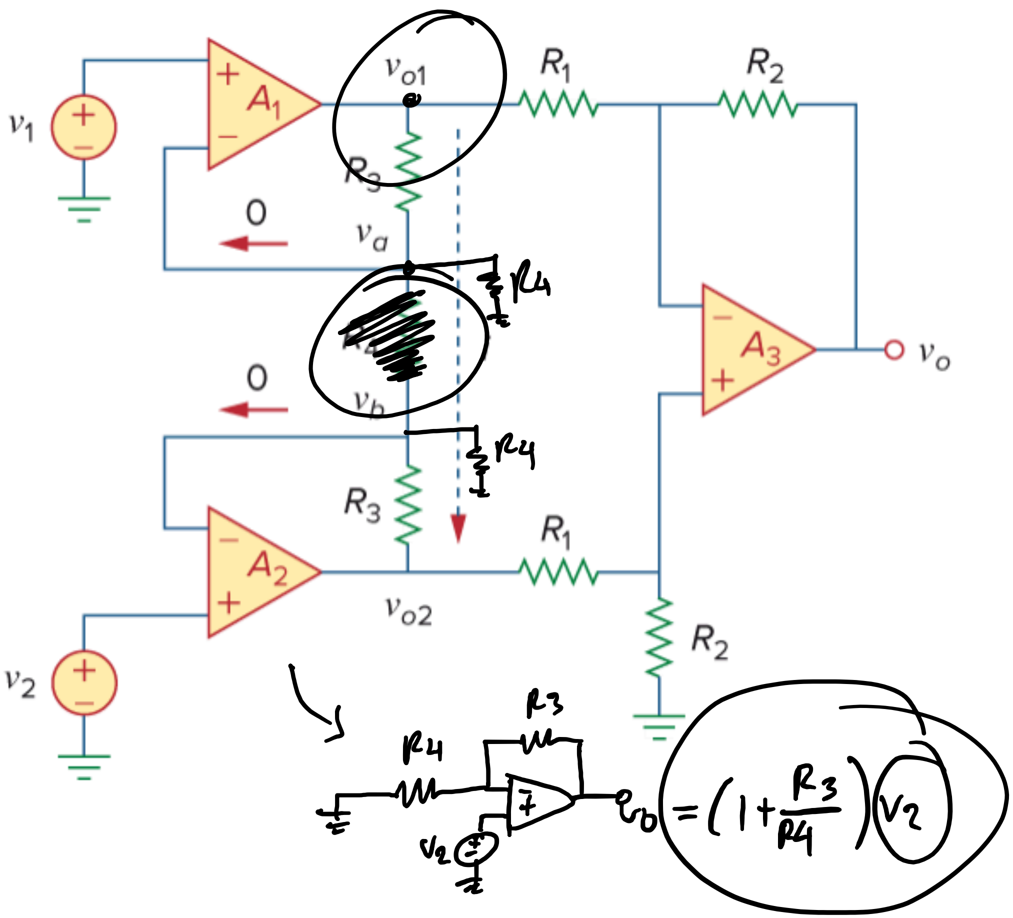
Example (students)

Design a differencing amplifier with a gain of 20.

Instrumentation Amplifier

Show that: $$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Discussion: what does this do?



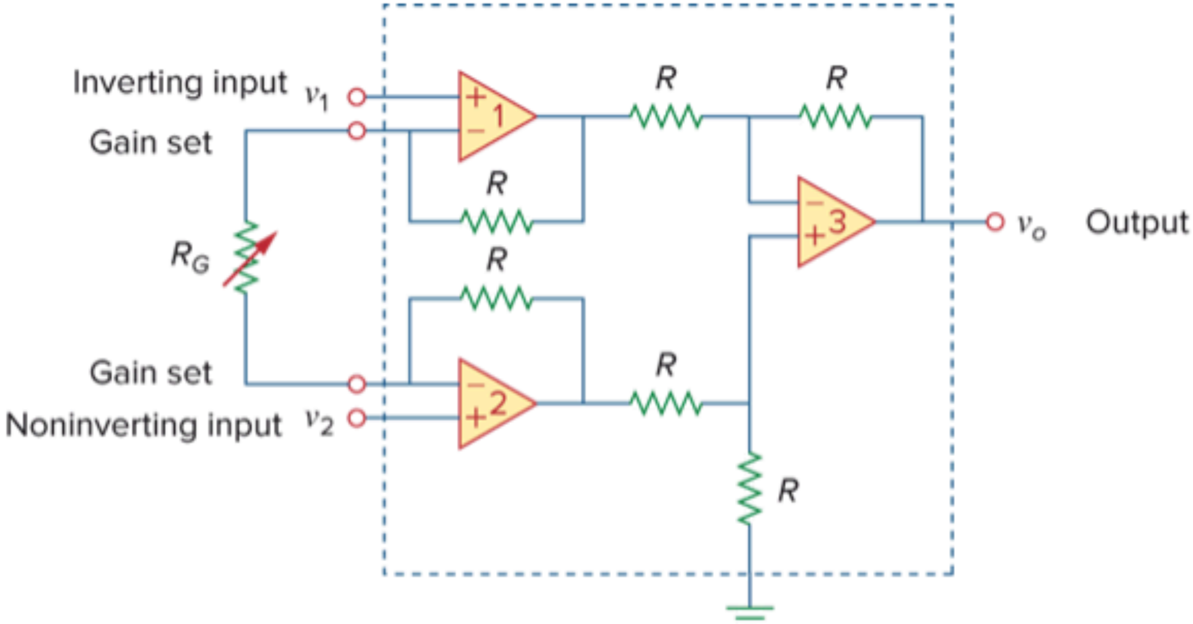
Instrumentation Amplifier

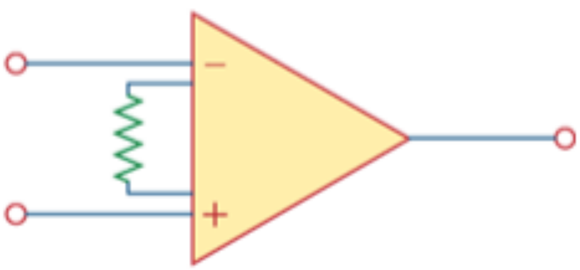
Now that we talked about it, check out:
https://en.wikipedia.org/wiki/Instrumentation_amplifier

Instrumentation Amplifiers

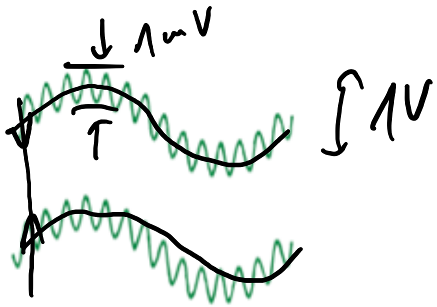
$$v_o = A_v(v_2 - v_1)$$

$$A_v = 1 + \frac{2R}{R_G}$$

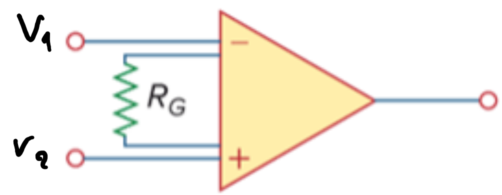




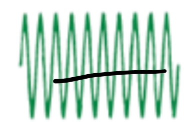
Instrumentation Amplifiers



Small differential signals riding on larger common-mode signals



Instrumentation amplifier



Amplified differential signal, no common-mode signal