BOSTON UNIVERSITY

EK 307: Electric Circuits

Fall 2017

Lecture 8

Sep 28, 2017

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Administrivia

- Homework 1 back

Lecture 7 (last time) reminder:

1. **Mesh current analysis (2 examples)**

2. **Linearity, superposition**
3. **Dependent sources, transistor model circuits**

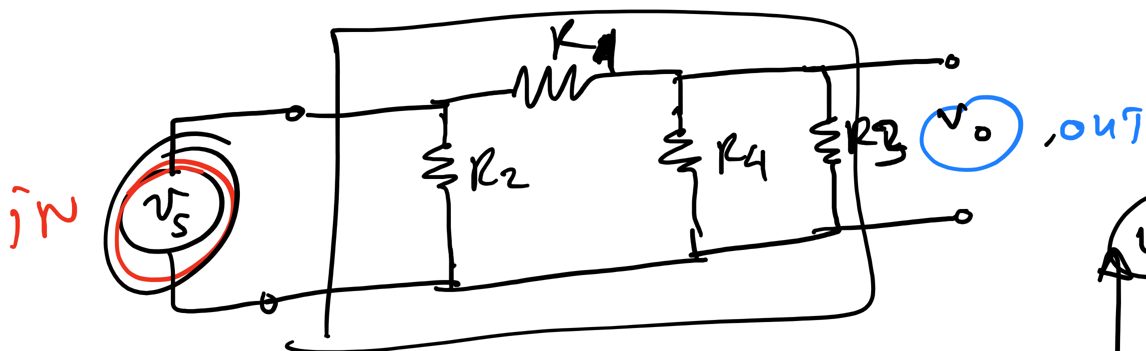
Lecture 8 (today): What you should know at end of this lecture

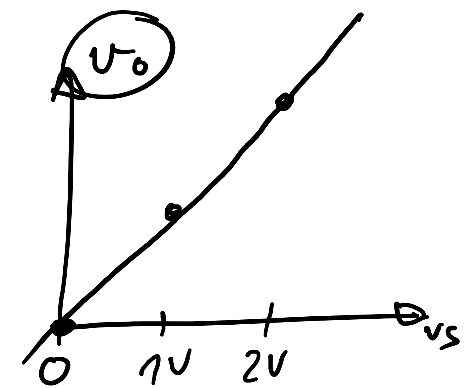
1. **Source transformations, Thevenin and Norton theorems**
2. **Equivalent resistance (max power transfer)**

Review:
Linearity, superposition, source transformation

Linearity

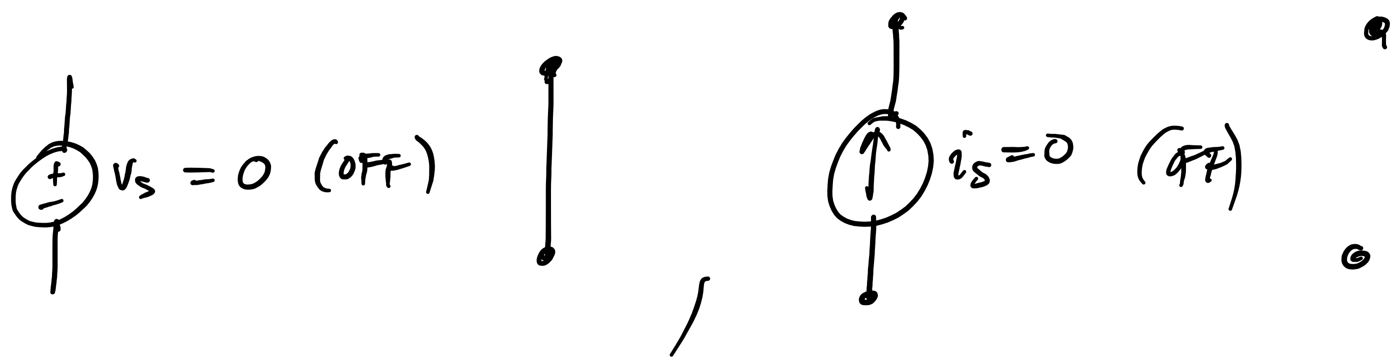
- Any output is proportional to any input





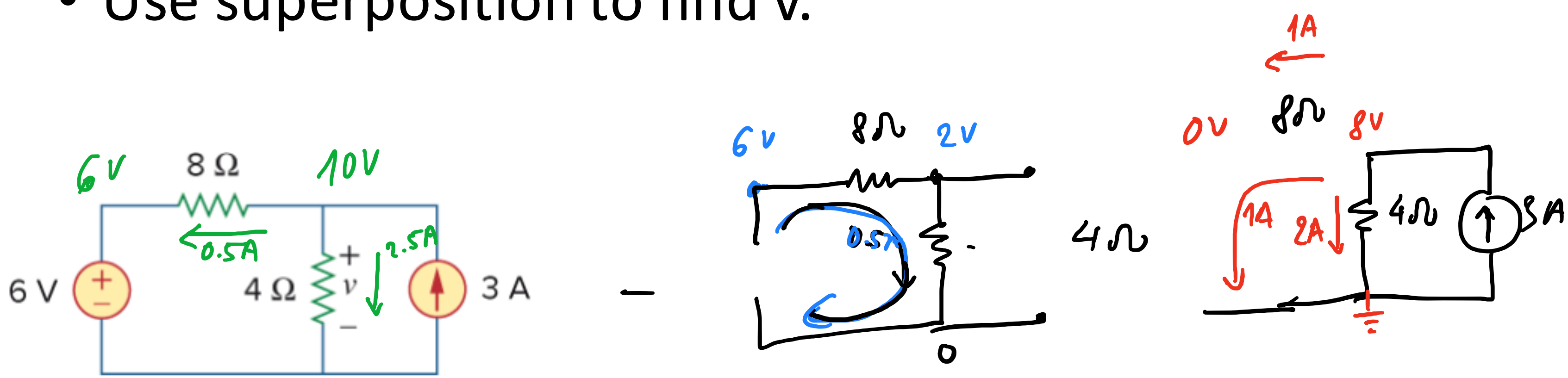
Superposition

- For multiple inputs (e.g. sources):
  - Any output is the sum of outputs due to each input turned on separately
  - “Turned off input” means V=0 for voltage sources, I=0 for current sources.
  - Dependent sources stay.

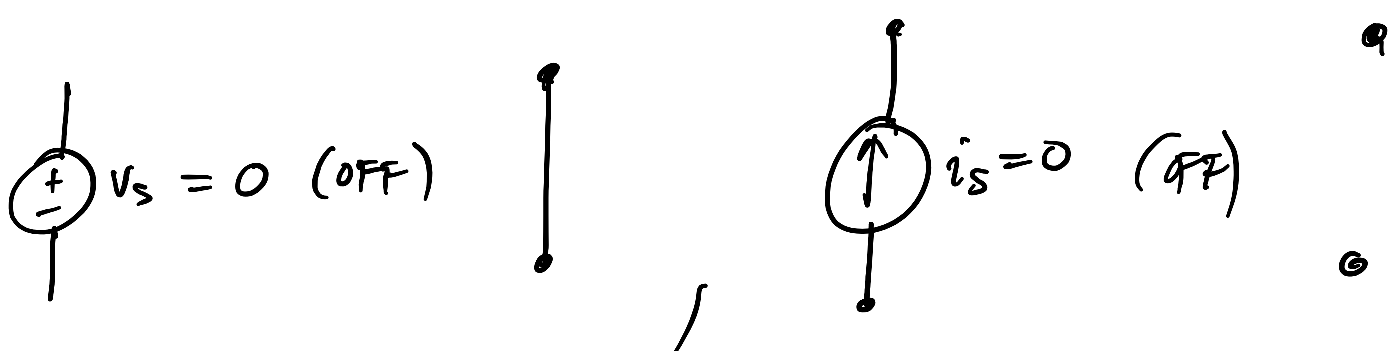


Superposition: example 1

- Use superposition to find v.



- Voltages and currents in circuit with multiple sources are sum of those due to each source applied separately.



Superposition: example 2

Use superposition to find v_x in the circuit of Fig. 4.11.

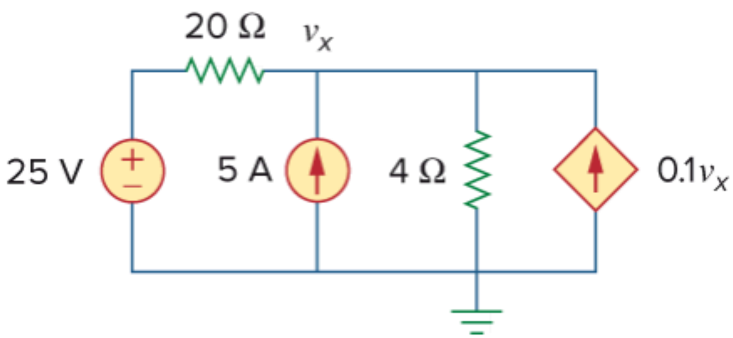
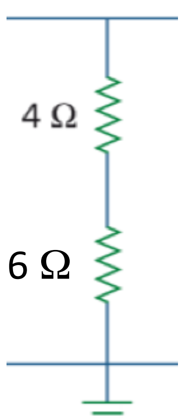


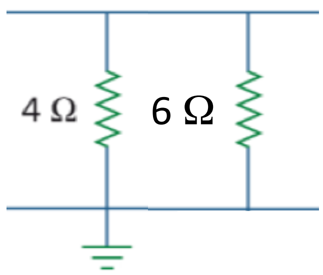
Figure 4.11 For Practice Prob. 4.4.

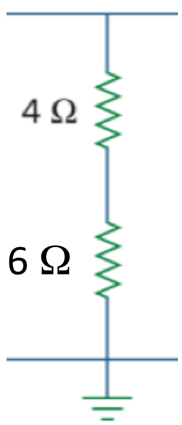
Source transformations

Series & Parallel…

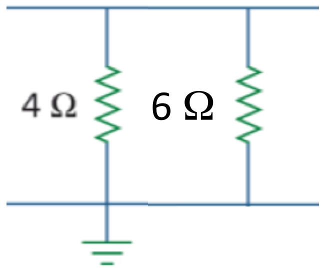


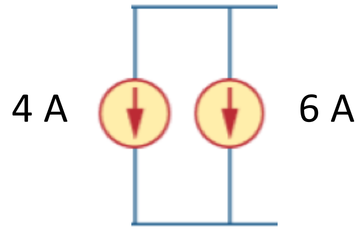
Series & Parallel…

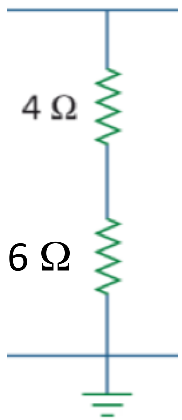




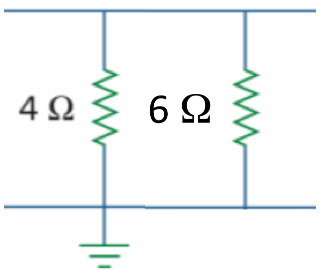
Series & Parallel…

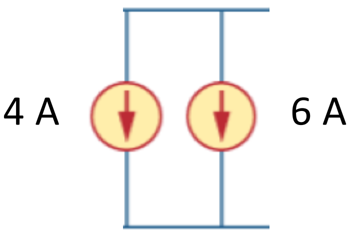


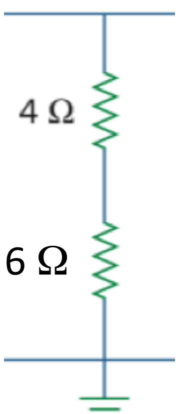


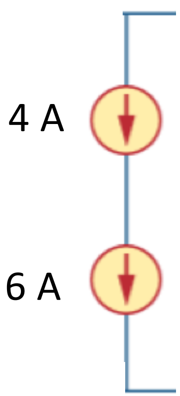


Series & Parallel…

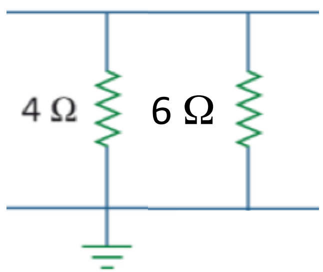


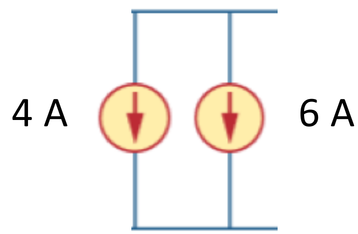


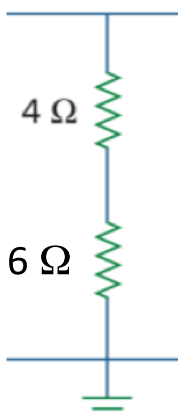


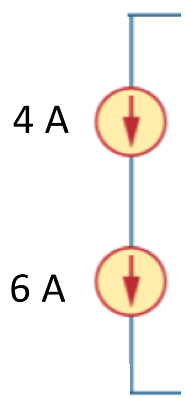


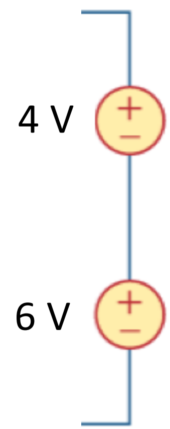
Series & Parallel…



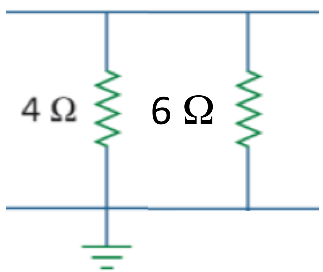


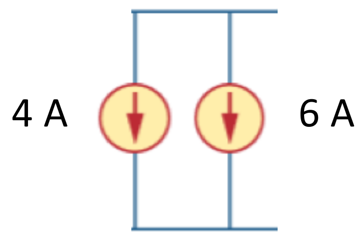


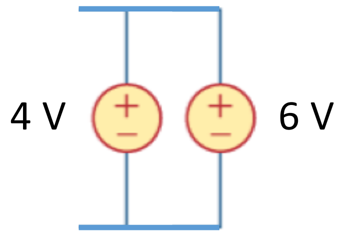


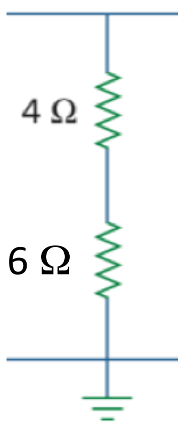


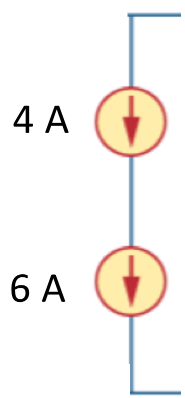
Series & Parallel…

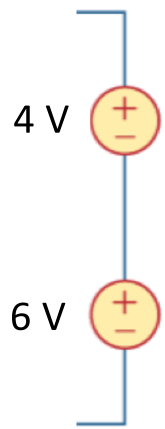




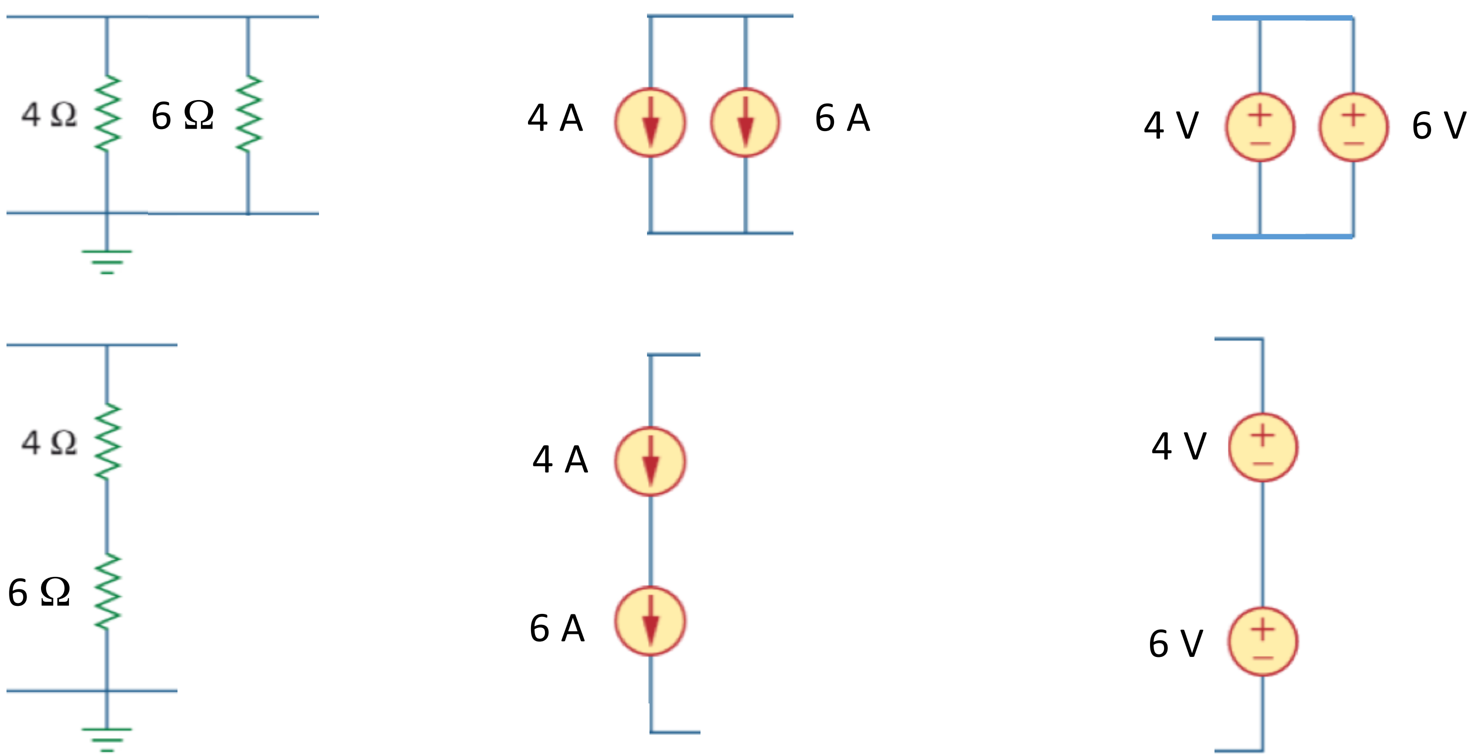






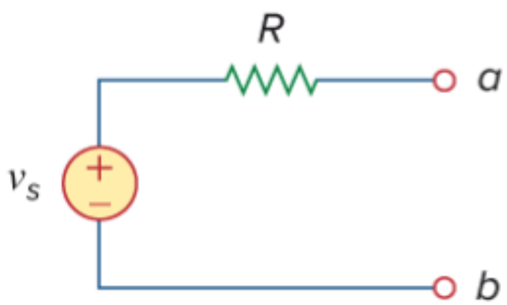


Series & Parallel…



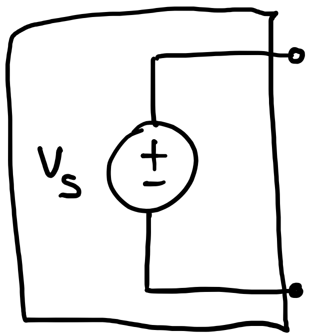
How can these be impossible? Can’t I just go in the lab and connect them up…?

Real sources have internal resistance

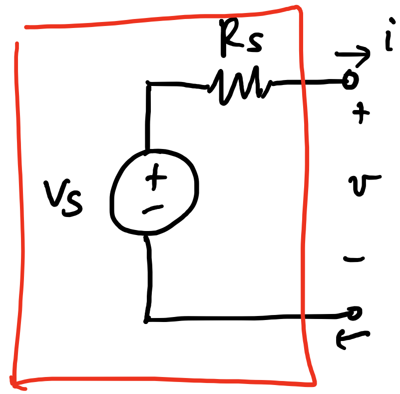


For a voltage source, it *has* to be in series. See next few slides. A parallel resistor does nothing. In the limit where R approaches 0 ohms, we recover the ideal source.

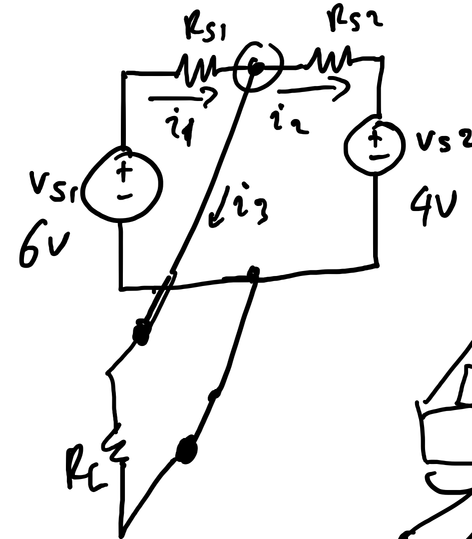
Ideal source

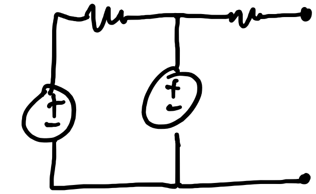


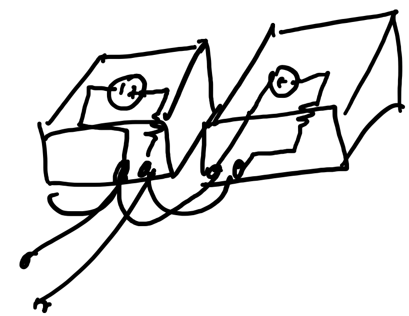
Real source model

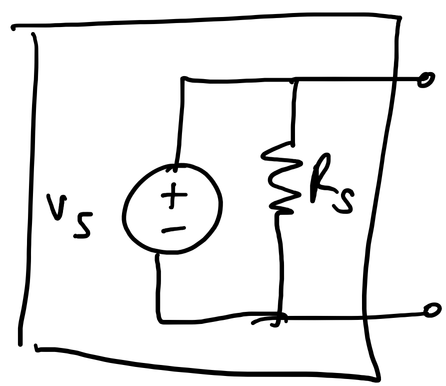


If $i_3 = 0$, $i_1 = i_2 = \frac{2V}{R_{s1}+R_{s2}}$

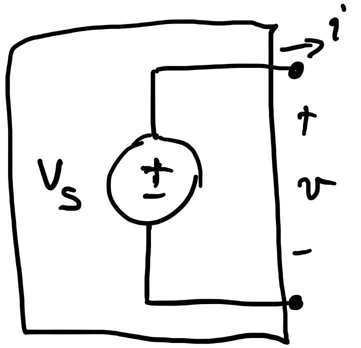




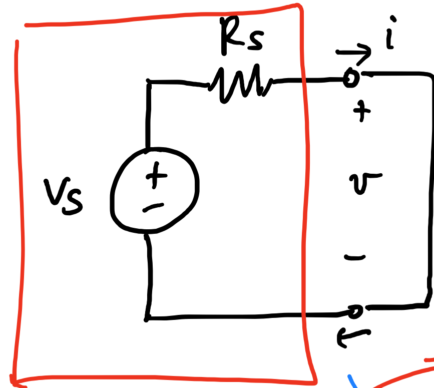


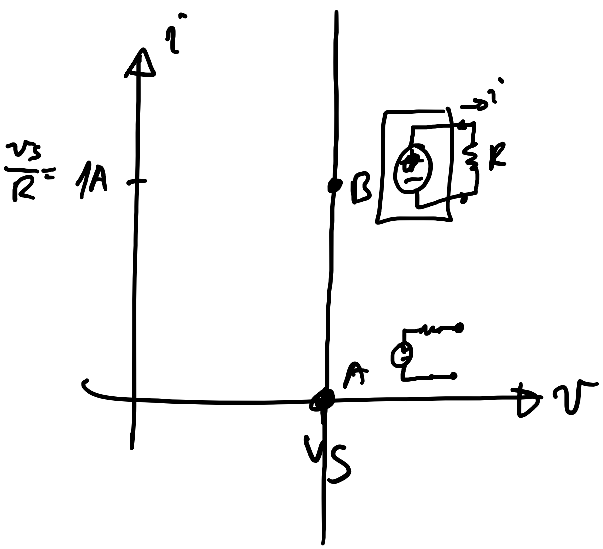


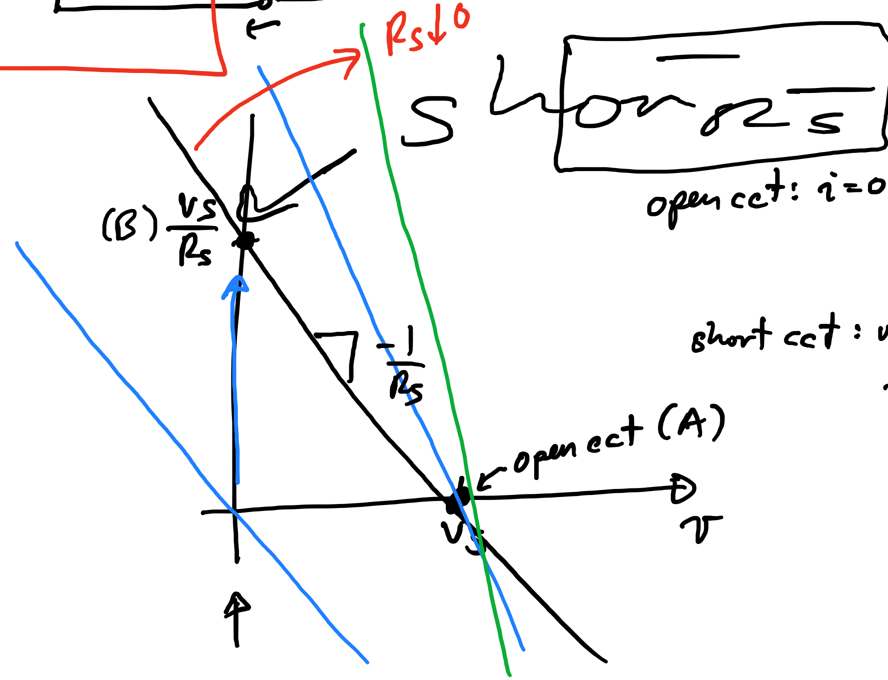
Ideal source



Real source model





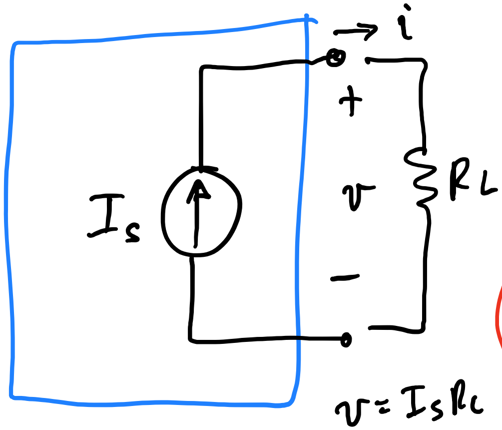


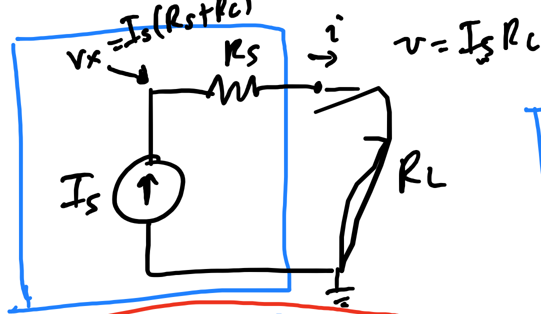
open cct: $i = 0$, $\frac{v}{R_s} = \frac{v_s}{R_s}$

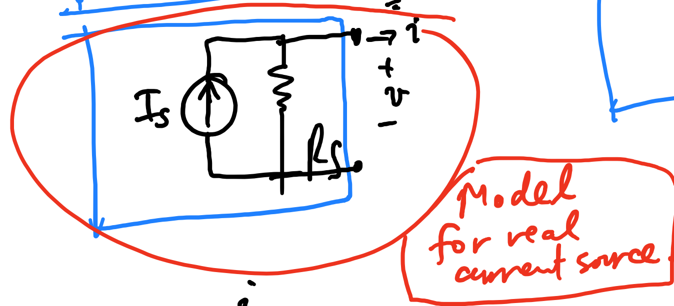
$v = v_s$ (A)

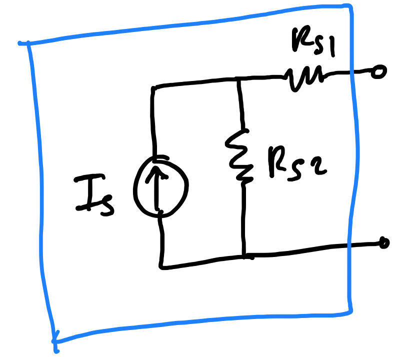
short cct: $v = 0$

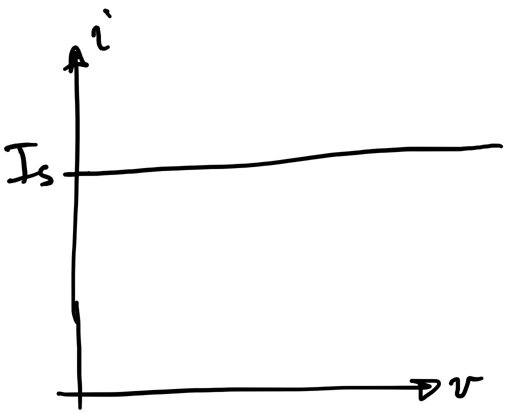
$i = \frac{v_s}{R_s}$ (B)

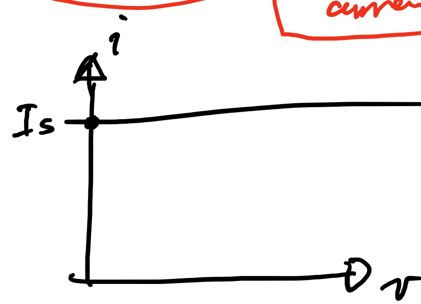

i
+
v
−
I_s
R_L
$v = I_s R_L$

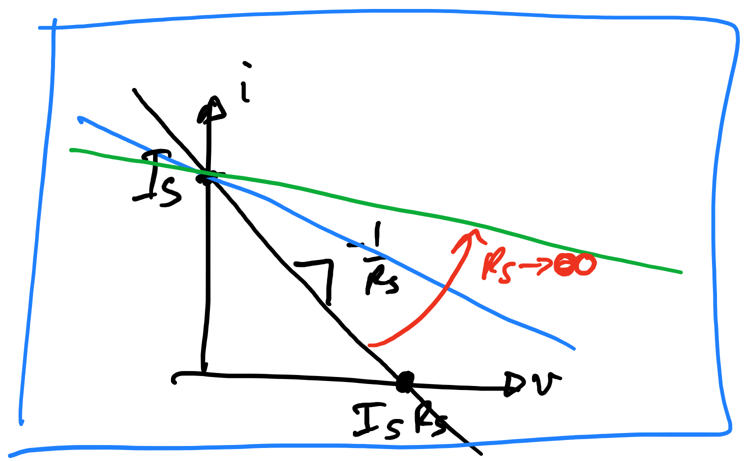

$v_x = I_s(R_s + R_L)$
R_s
i
$v = I_s R_L$
I_s
R_L

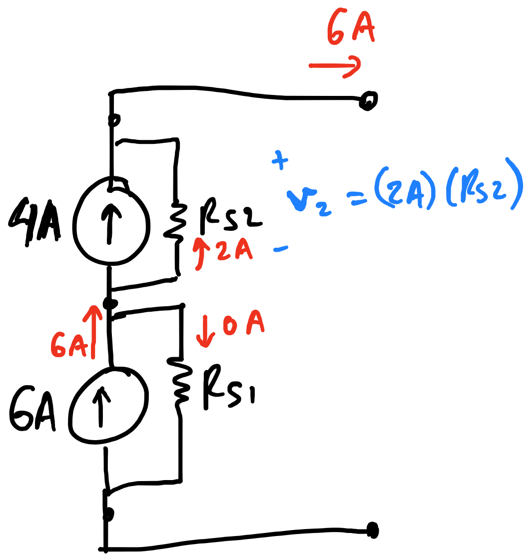

i
+
v
−
I_s
R_s
Model for real current source


R_{s1}
R_{s2}
I_s


i
I_s
v

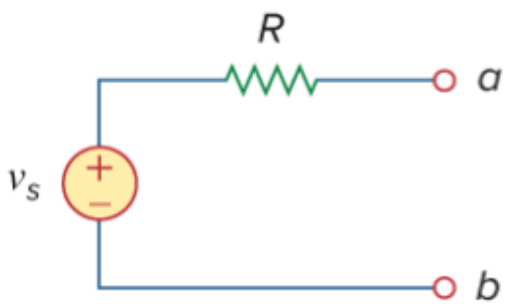

i
I_s
v


i
I_s
$-\frac{1}{R_s}$
$R_s \to \infty$
v
$I_s R_s$

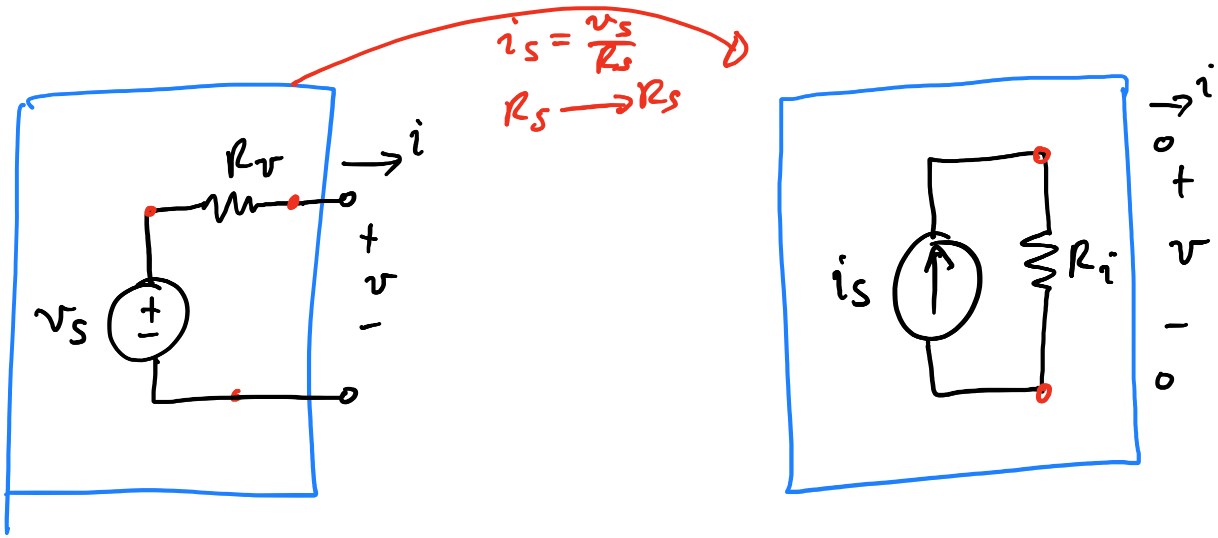


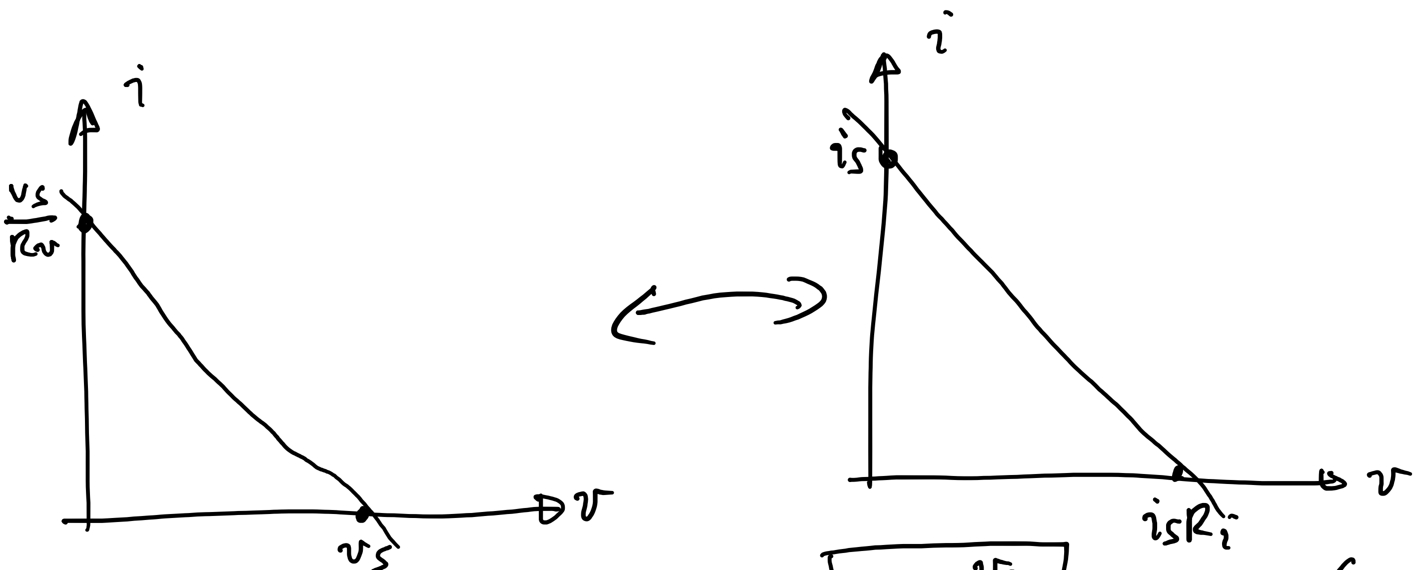
6A
4A
Rs2
2A
+
v2 = (2A)(Rs2)
-
6A
0A
6A
Rs1

Source transformations: IV curve



A voltage source can be replaced by a current source, with the connected load circuit unable to tell the difference between them! See next slide.

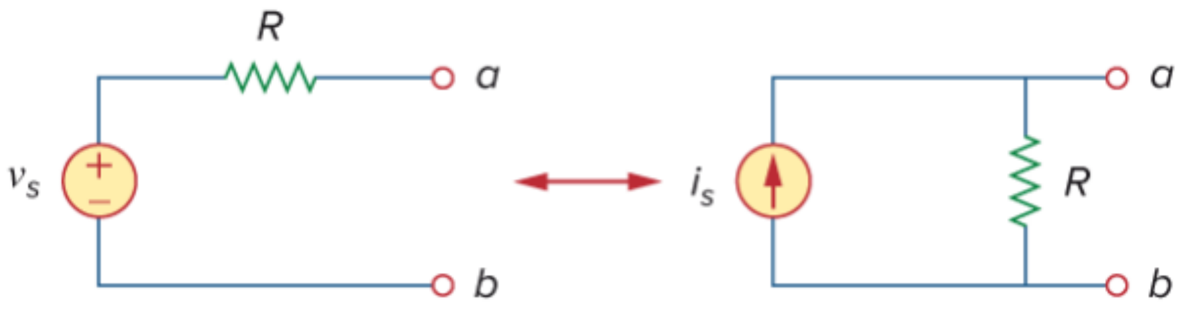




v_s , R_v $\longrightarrow$ $\boxed{i_s = \frac{v_s}{R_v}}$

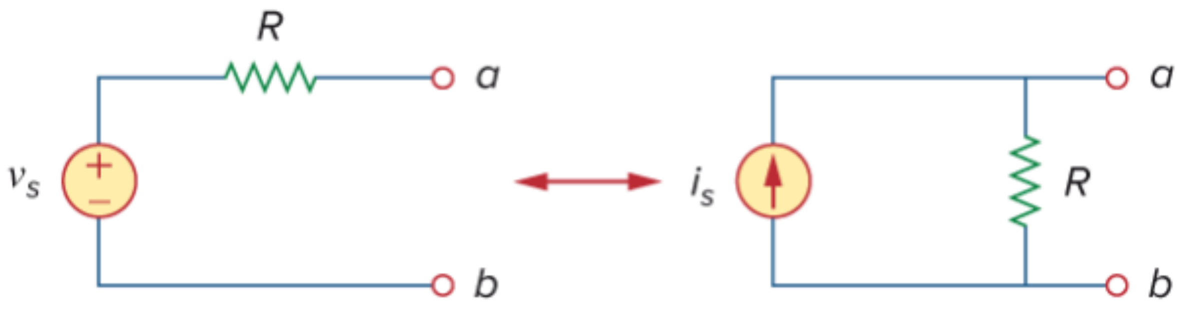
$v_s = i_s R_i \rightarrow i_s = \frac{v_s}{R_i}$ $\rightarrow$ $\boxed{R_i = R_v \equiv R_s}$

Source transformations

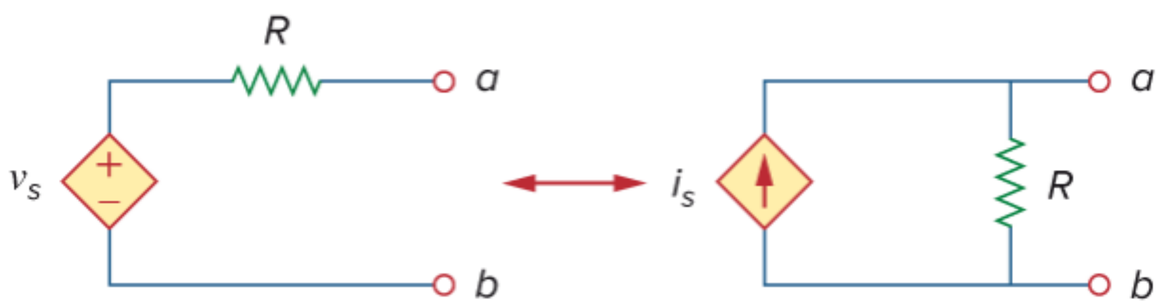


$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

Source transformations

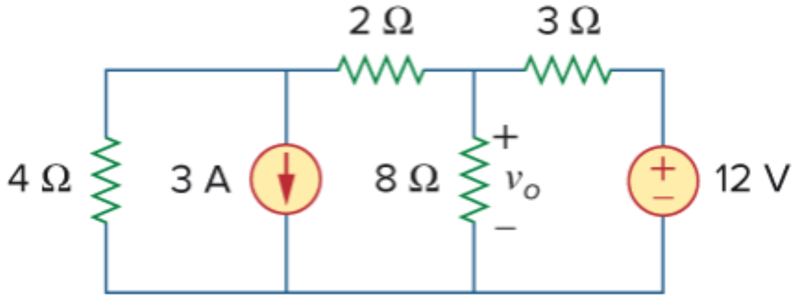


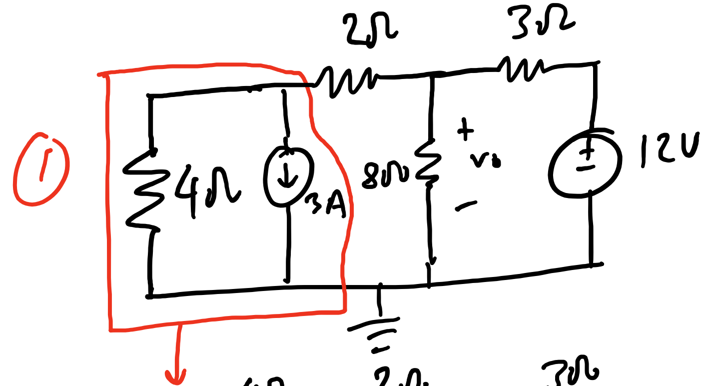
$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

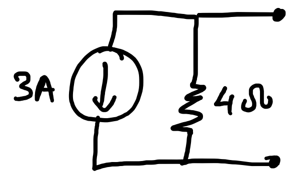


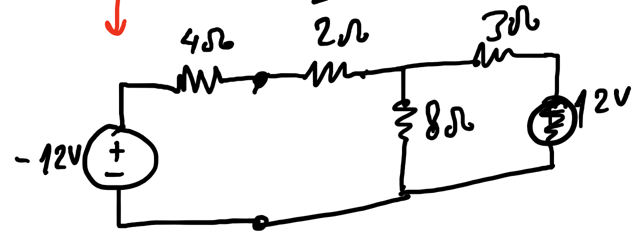
Source transformations: example

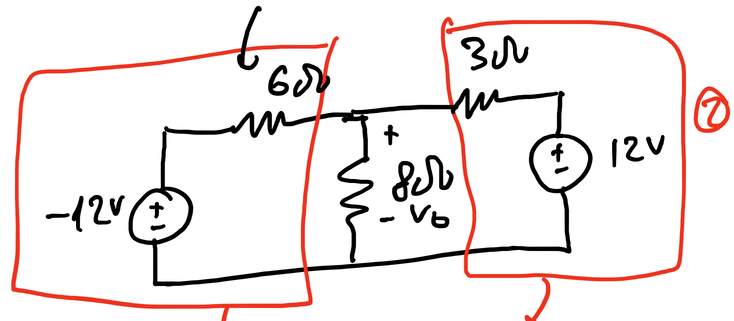
Find v_o.

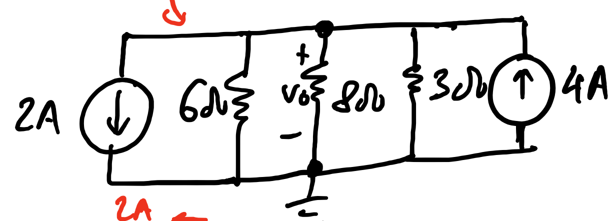


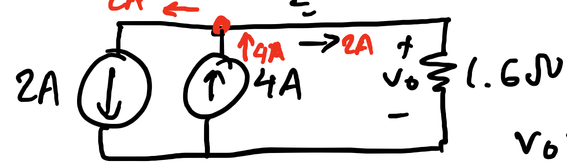








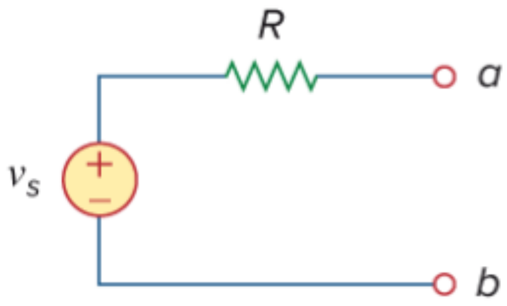


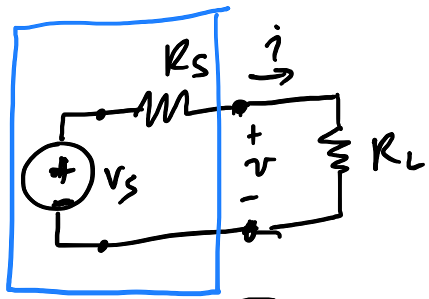


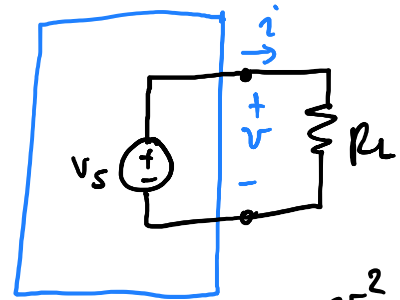
$v_o = (1.6\Omega)\ 2A$

$= 3.2V$

Maximum power transfer



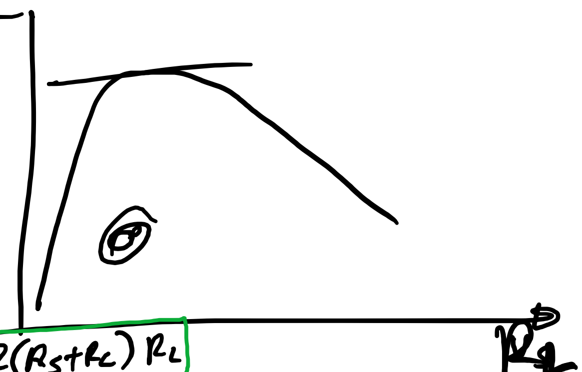




$$P_{R_L} = \frac{v^2}{R} = \frac{v_S^2}{}$$

$$= \frac{v_S}{} \quad R_L$$

$$\boxed{P_{R_L} = \frac{R_L}{(R_S+R_L)^2} v_S^2}$$



$$\frac{d}{dR_L} P_{R_L} = 0 = v_S^2 \frac{1(R_S+R_L)^2 - 2(R_S+R_L)R_L}{(R_S+R_L)^4}$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'g - g'f}{g^2}$$

$$(R_S+R_L)^2 = 2(R_S+R_L)R_L$$

$$R_S + R_L = 2R_L \rightarrow \boxed{R_L = R_S}$$

Let's see what happens... (let's plot power burned in RL vs load resistance RL in Mathematica).

At low load resistances RL, the voltage is near zero and the current is higher.
At high load resistances RL, the voltage is equal to the source voltage, but the current approaches zero.
In both cases, the power P=vi approaches zero.
In between at some arbitrary RL, both voltage and current can be non-zero, so it seems there is a sweet spot: an ideal choice of resistance to dissipate the most power.

In[23]:=
```
Rs = 1; vs = 10;
ii = vs/(Rs + Rl);
vv = Rl/(Rs + Rl) vs;
PP = vv ii;
Plot[vv, {Rl, 0, 3}, PlotStyle → Thick]
Plot[ii, {Rl, 0, 3}, PlotStyle → Thick]
Plot[PP, {Rl, 0, 3}, PlotStyle → Thick]
```

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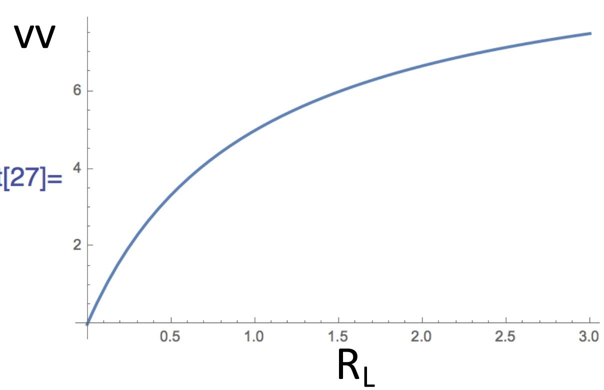

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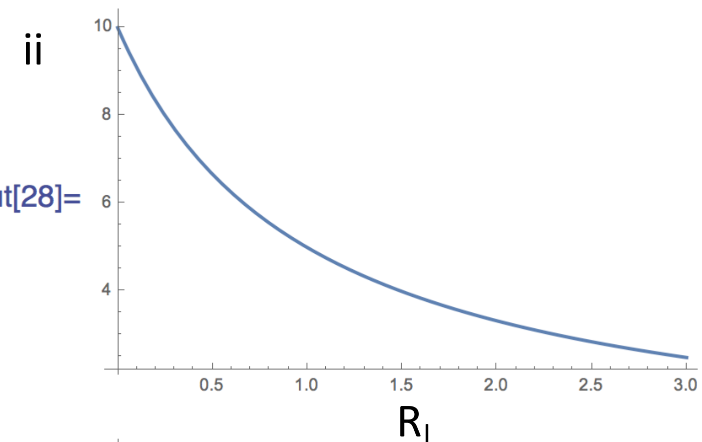

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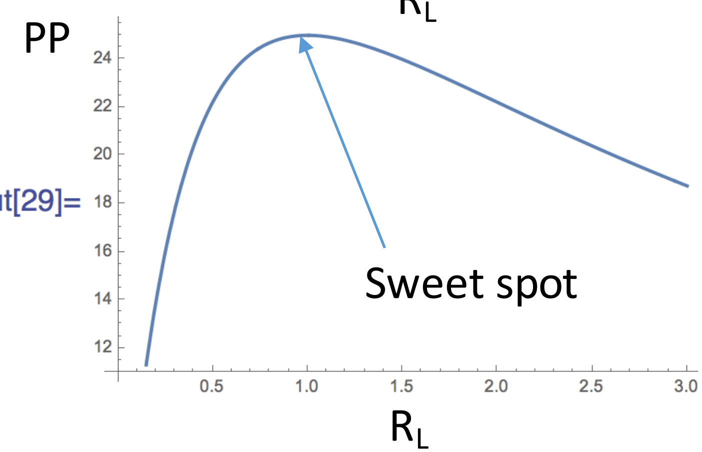
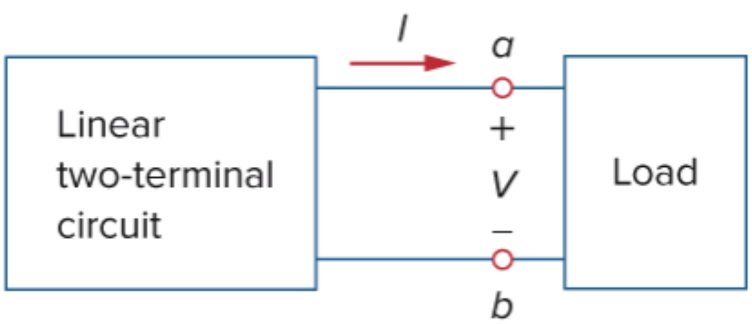
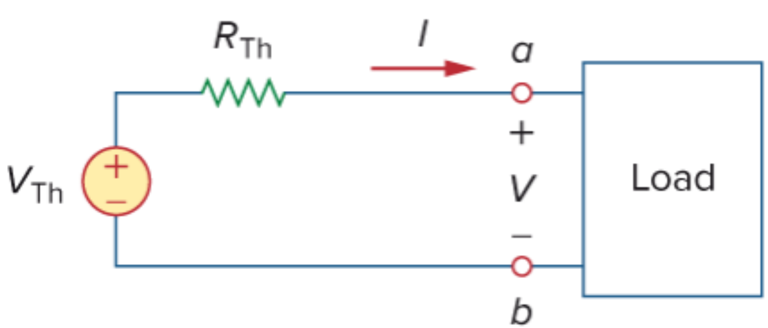


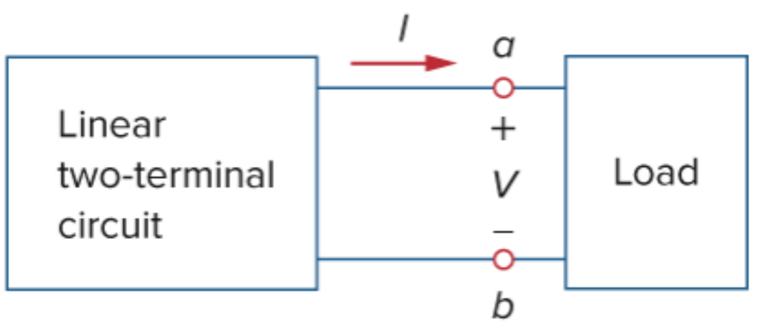
Thevenin and Norton Theorems

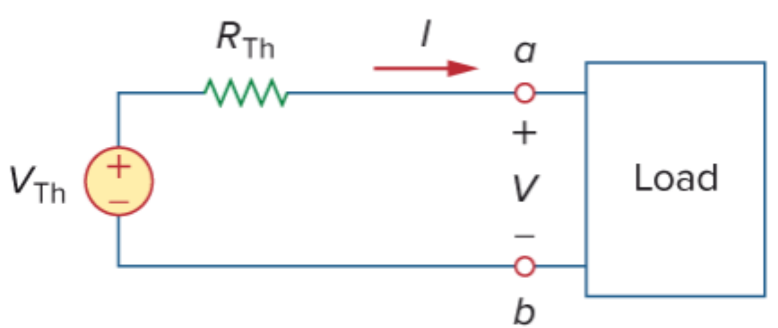
Thevenin Theorem





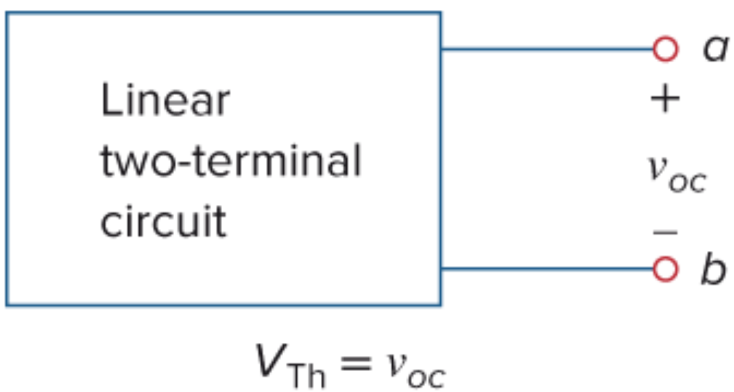
Thevenin Theorem

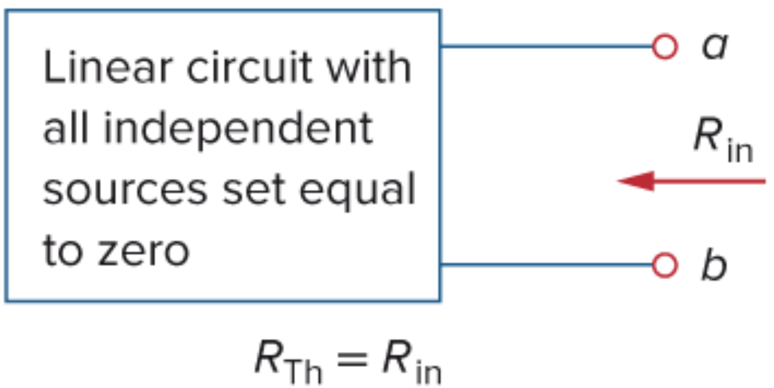




Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

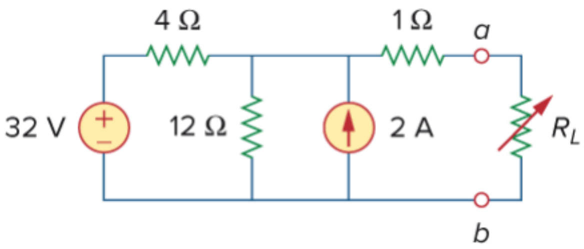
Thevenin Theorem

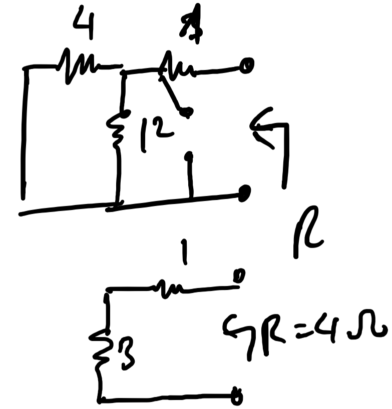




Thevenin example

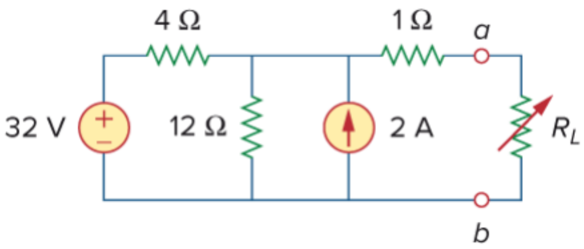
Find the Thevenin equivalent of the circuit to the left of the terminals *a*-*b*. Then find the current through R_L = 6, 16, and 36 Ω.

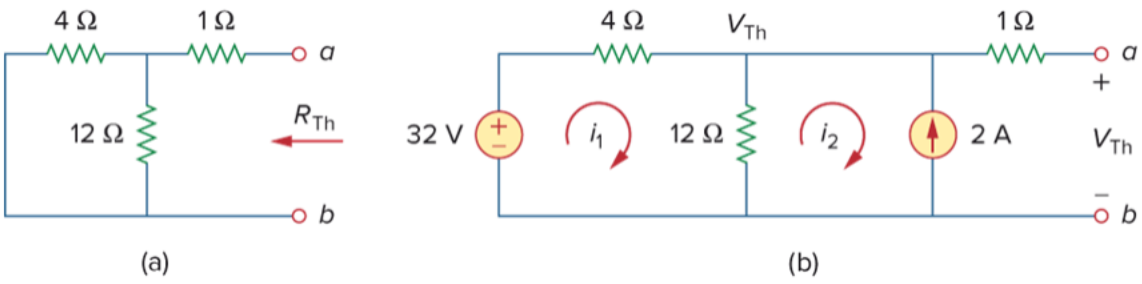



4
12
R
1
3
R = 4 Ω

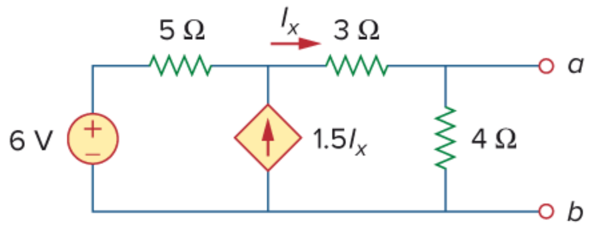
Thevenin example

Find the Thevenin equivalent of the circuit to the left of the terminals *a*-*b*. Then find the current through R_L = 6, 16, and 36 Ω.

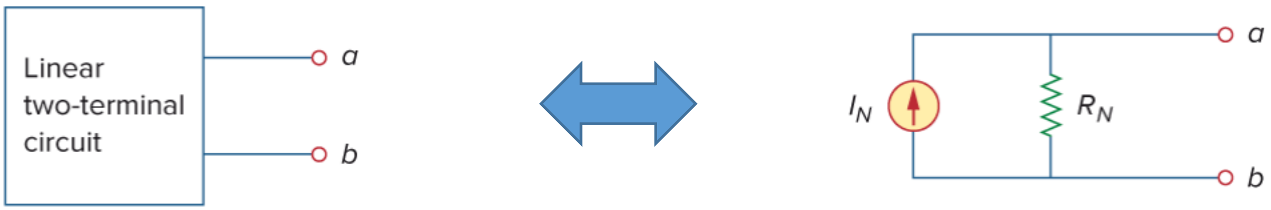




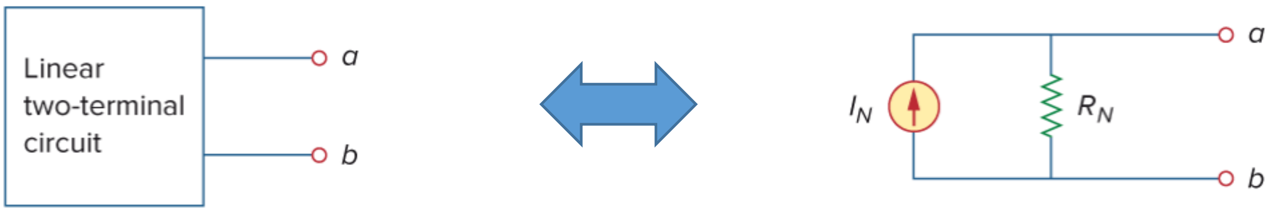
Thevenin example 2



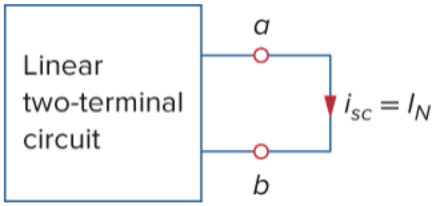
Norton Theorem



Norton Theorem

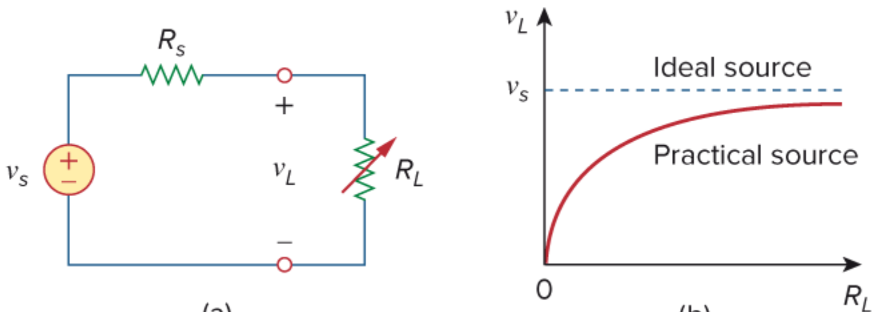


Find Thevenin resistance and short-circuit current.



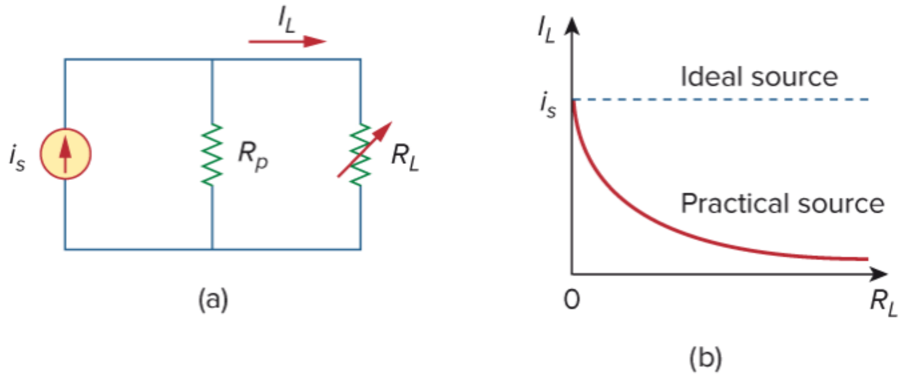
Source modeling, bridge circuits, interface ccts

Models of real sources



(a) (b)

Models of real sources

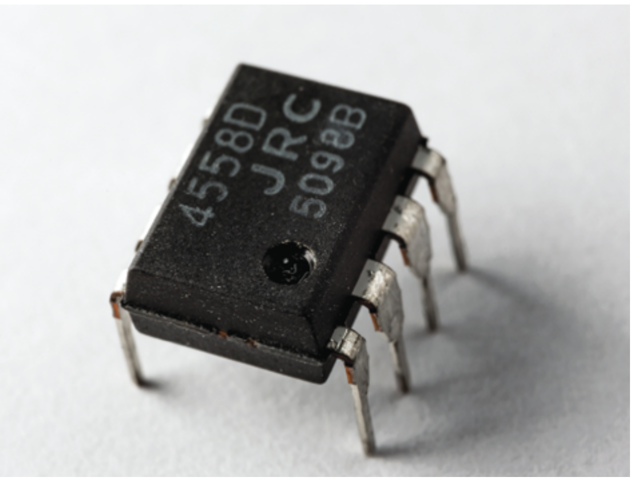


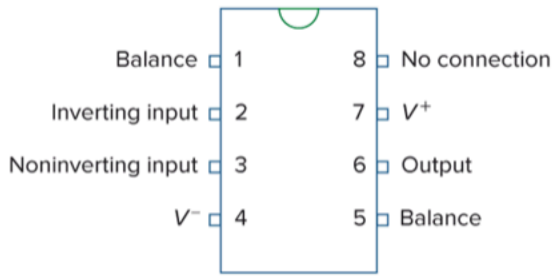
Example

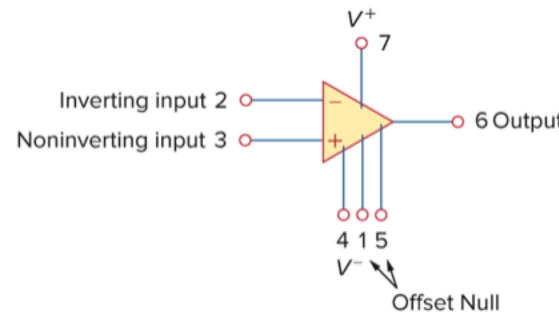
The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V. (a) Calculate the source voltage v_s and internal resistance R_s. (b) Determine the voltage when an 8-Ω load is connected to the source.

Operational Amplifiers (Op-Amps)

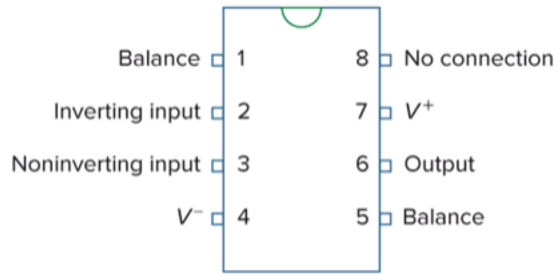
Op-Amps

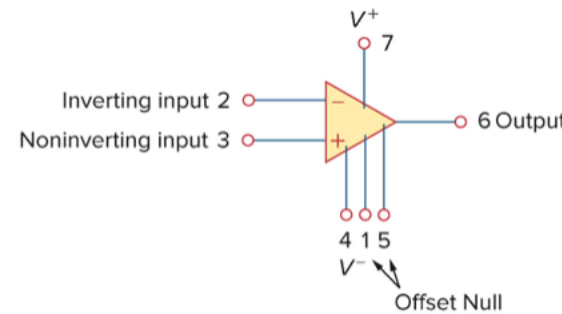

4558D
JRC
509BB


Balance 1
Inverting input 2
Noninverting input 3
V⁻ 4
8 No connection
7 V⁺
6 Output
5 Balance

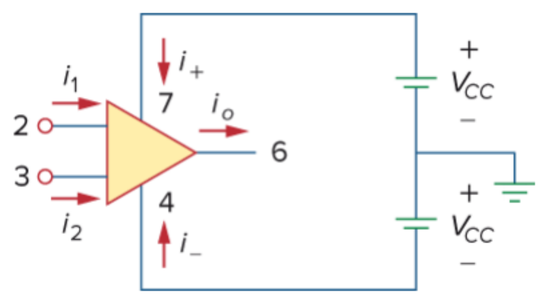

V⁺
7
Inverting input 2
Noninverting input 3
−
+
6 Output
4 1 5
V⁻
Offset Null

Op-Amps

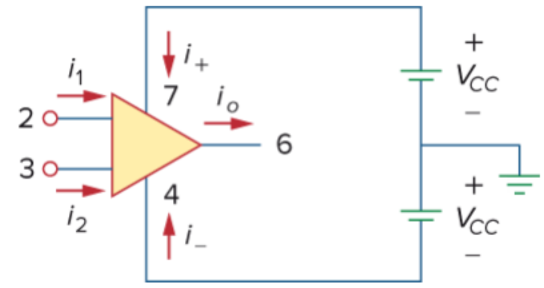


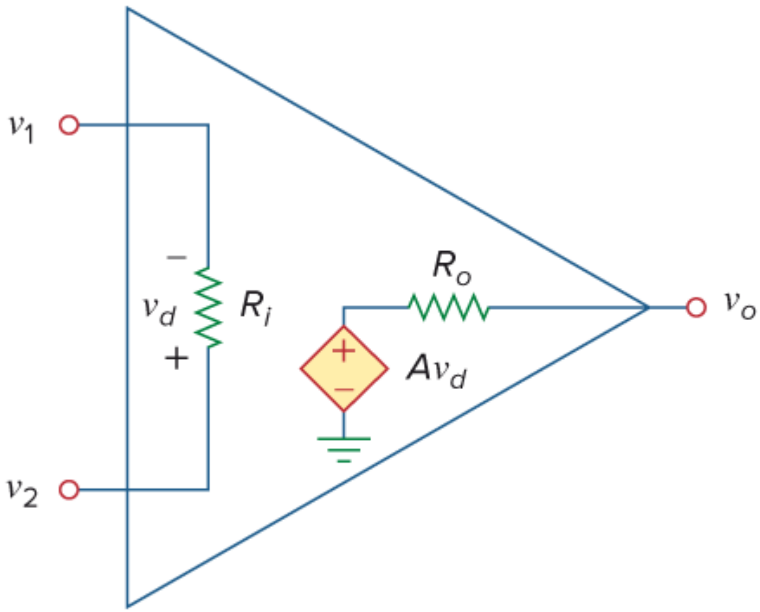


$$i_o = i_1 + i_2 + i_+ + i_-$$



Op-Amps

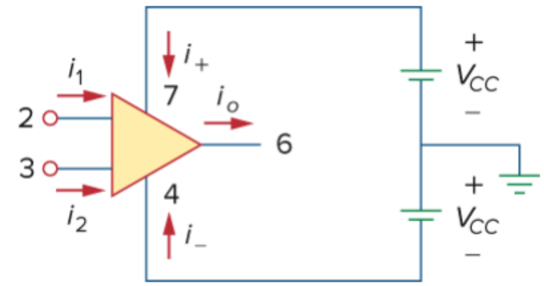


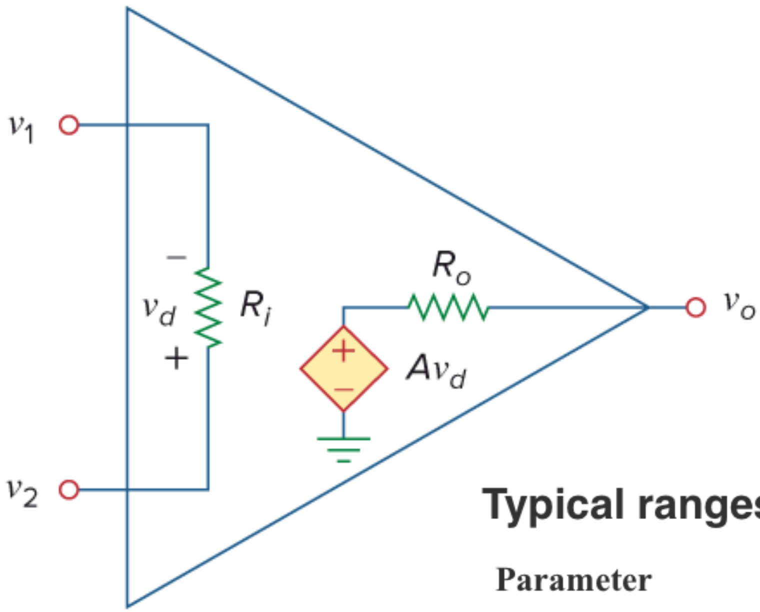


$$v_d = v_2 - v_1$$

$$v_o = Av_d = A(v_2 - v_1)$$

Op-Amps





$$v_d = v_2 - v_1$$

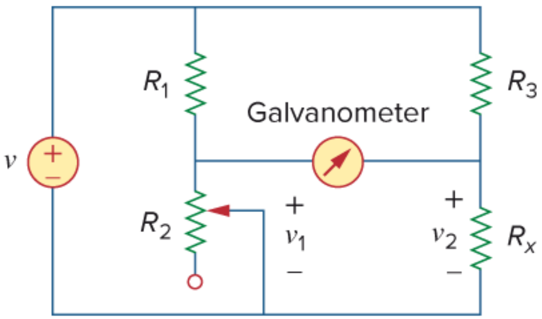
$$v_o = Av_d = A(v_2 - v_1)$$

Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	`
Input resistance, R_i	10^5 to 10^{13} Ω	`Ω
Output resistance, R_o	10 to 100 Ω	0Ω
Supply voltage, V_{CC}	5 to 24 V	

We skipped in this lecture…

Bridge circuits: Wheatstone bridge

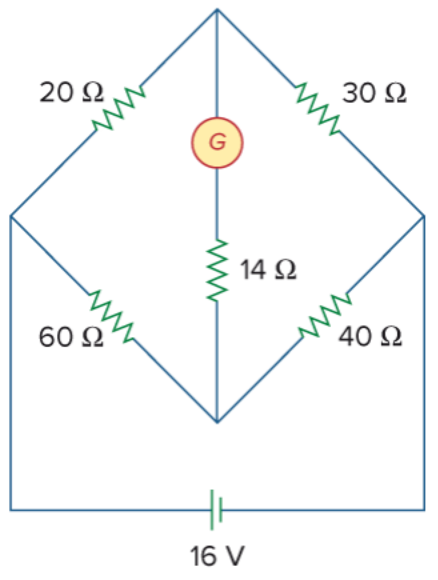


$$v_1 = \frac{R_2}{R_1 + R_2} v = v_2 = \frac{R_x}{R_3 + R_x} v$$

$$\frac{R_2}{R_1 + R_2} = \frac{R_x}{R_3 + R_x} \quad \Rightarrow \quad R_2 R_3 = R_1 R_x$$

$$\boxed{R_x = \frac{R_3}{R_1} R_2}$$

Example: unbalanced bridge



Find the current through the galvanometer.