Midterm Exam 2

Prof. Miloš Popović

- ***100 min timed, closed book test.***
- ***Write your name at top of every page (or initials on later pages)***
- ***Aids: single page (single side) of notes, handheld calculator***
- ***Work in the space provided (or on back of page if you need more space). Turn in only these sheets.***
- ***Write clearly your method for partial credit. When possible, first do the problem without numbers plugged in.***
- ***Circle/box clearly final answers and cross out anything incorrect. Putting down multiple inconsistent answers to the same question will not result in credit for only the (more/most) correct one. You must commit to a solution.***

Grading scheme:

Multiple choice	**___/10**
Problem 1	**___/10**
Problem 2	**___/20**
Problem 3	**___/20**
Problem 4	**___/20**
Problem 5	**___/20**

<u>Coverage</u>

The following textbook sections are the primary focus of this exam:
5.6–5.8, 5.10, 6.1–6.6, 7.1–7.7, 7.9

However, all sections covered in Midterm #1 are also fair game in general.

Multiple choice questions [10 pts]:

[Select only all correct answers in each question for full credit.]

1. **[1 pt]** Circuits using the ideal op-amp model can always be solved by assuming:
   a. currents entering both input terminals are zero
   b. current exiting the output terminal is zero
   c. voltages at the input terminals are equal
   d. voltage at the output terminal is equal to the negative input terminal

2. **[1 pt]** An N-bit digital-to-analog converter circuit implemented as an op-amp based summer contains: at least
   a. N resistors
   b. N-1 resistors
   c. N+1 resistors
   d. 2^N resistors

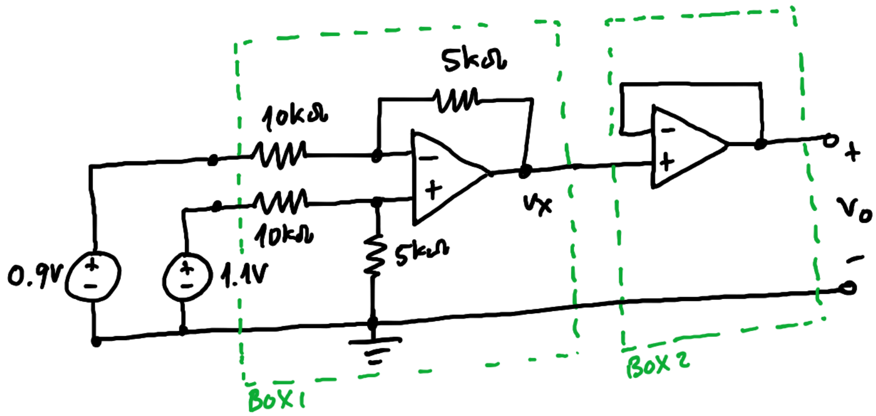
3. **[2 pts]** When designing a parallel-plate capacitor, the capacitance can be doubled by:
   a. doubling the distance between the plates
   b. doubling the area (e.g. width or length) of each of the two plates
   c. replacing the material that fills the space between the two plates with a material that has twice the value of permittivity (aka. dielectric constant), ε
   d. halving the area of each of the two plates

4. **[2 pts]** The capacitance of a single capacitor that is equivalent to (can be used to replace) a set of three capacitors connected in series, each having a capacitance of 9 mF is:
   a. 9 mF
   b. 3 mF
   c. 27 mF
   d. 1/3 mF

5. **[2 pts]** The total (equivalent) inductance of two inductors connected in series, with respective inductances of 1 nH and 2 nH is:
   a. ~0.667 nH
   b. 3 nH
   c. 1 nH
   d. 9 nH

6. **[2 pts]** The time constant, τ, of a circuit comprising an inductor and a resistor is
   a. 1/RL
   b. RL
   c. depends on whether they are connected in series or parallel
   d. L/R

Problem 1. [10] In this circuit,



(a) [4] What are the functions of the sub-circuit in Box 1, and of the subcircuit in Box 2?
(b) [6] Find v_o.

(a) Box 1: differencing amplifier

Box 2: unity gain buffer

(b) $v_x = +\frac{5k}{10k}(1.1V - 0.9V) = +\frac{1}{2}(0.2V) = +0.1V$

$v_o = 1 v_x = +0.1V$

Detail: $\frac{\left(v_x - \frac{5k}{15k}1.1V\right)}{5k} = \frac{\frac{5k}{15k}1.1V - 0.9V}{10k}$

$v_x - \frac{1}{3}1.1V = \frac{5}{10}\left(\frac{1}{3}1.1V - 0.9V\right)$

$v_x = \left(\frac{5}{10}+1\right)\frac{1}{3}1.1V - \frac{1}{2}0.9V$

$= \frac{3}{2}\frac{1}{3}1.1V - \frac{1}{2}0.9V$

$= \frac{1}{2}(1.1 - 0.9)V$

$= 0.1V$

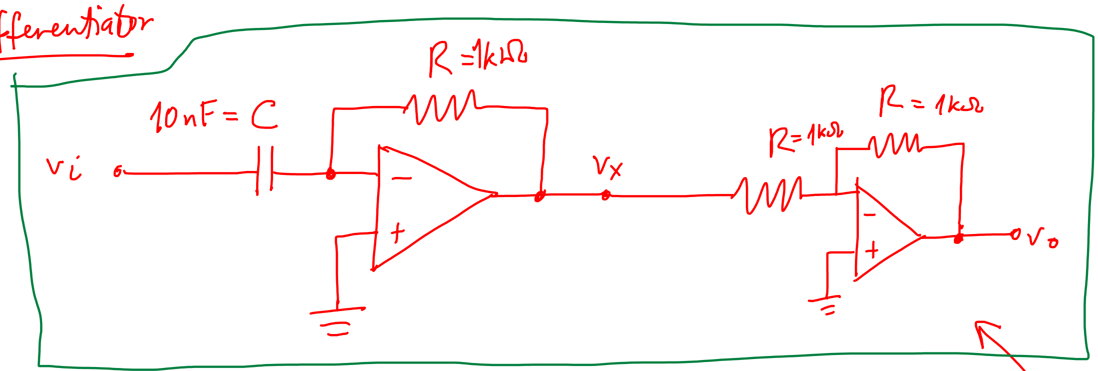
Problem 2. [20] Design an op-amp circuit that connects to its input a voltage source $v_i(t)$, and provides an output voltage $v_o(t)$ equal to

$$v_o(t) = +(10\mu\text{s})\frac{dv_i(t)}{dt}.$$

The circuit may use no more than 2 op-amps, and may also use resistors and capacitors, but you only have access to 1kΩ resistors.

Draw the circuit, and specify the values of all resistors and capacitors. Use as few circuit elements as possible.

– Differentiator



$v_x = -RC\frac{d}{dt}v_i$

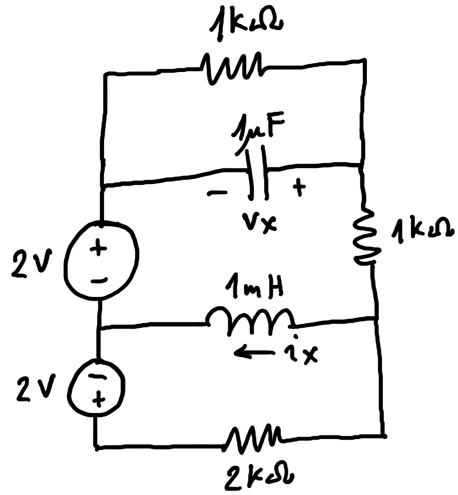
need a '+'

$RC = 10\mu s$

$C = \frac{10\mu s}{R} = \frac{10\mu s}{1k\Omega} = 10nF$

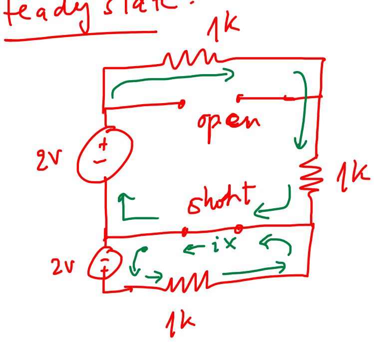
Need to cascade with inverter w/ −1 gain: R = 1k.

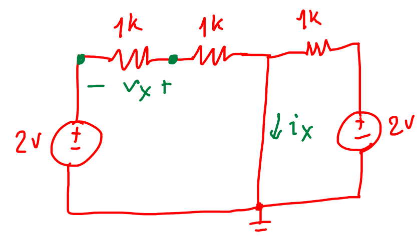
Problem 3. [20] The following circuit is operating in steady state.



(a) [12] Find the steady state voltage v_x and current i_x.
(b) [8] Find the energy stored in each of the capacitor and in the inductor. Give the expressions that depend on the circuit variables, and label them, for partial credit in case your result in part (a) is incorrect.

(a) Steady state:





$$i_x = \frac{2V}{2k} + \frac{2V}{1k} = 3mA$$

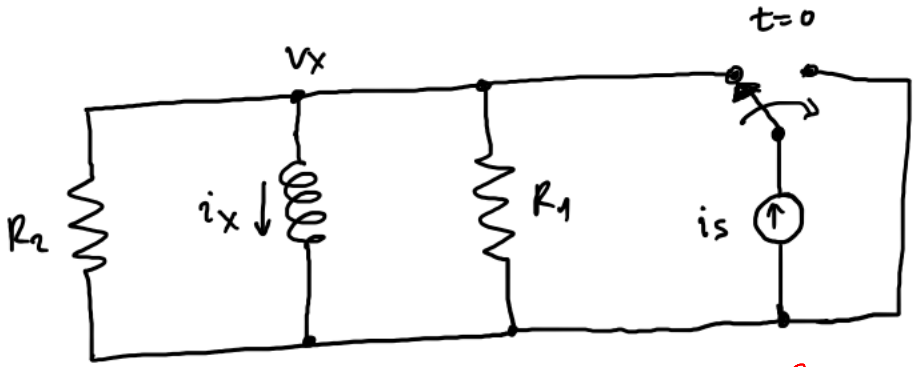
$$v_x = -\left(\frac{2V}{2k}\right)\cdot 1k = -1V$$

(b)

$$w_C = \frac{1}{2}Cv_x^2 = \frac{1}{2}(1\mu F)(-1V)^2 = 0.5\mu J$$

$$w_L = \frac{1}{2}Li_x^2 = \frac{1}{2}(1mH)(3mA)^2 = 4.5mJ$$

Problem 4. [20] In the following circuit,



a switch is toggled at time t = 0 as shown. Here, i_s = 1mA, R_1 = 2kΩ, R_2 = 2kΩ, and the inductor inductance value is L = 1μH.

(a) [5] For t<0, the circuit has reached a steady state. Find v_x, i_x and the energy stored in the inductor for t<0.

(b) [5] When the switch is toggled at t=0, the circuit undergoes a transient. What is the time constant with which this transient occurs?

(c) [10] Solve for the current i_x and voltage v_x as a function of time, and give the equations for them, as well as sketches of the waveforms with labeled axes.

(a) $i_x = 1mA, \quad v_x = 0$

(b) $\tau = \frac{L}{R} = \frac{L}{R_1 \| R_2} = \frac{1\mu H}{1k\Omega} = 1ns$

(c) $i_x(t) = i_x(\infty) + [i_x(0) - i_x(\infty)]e^{-t/\tau}$

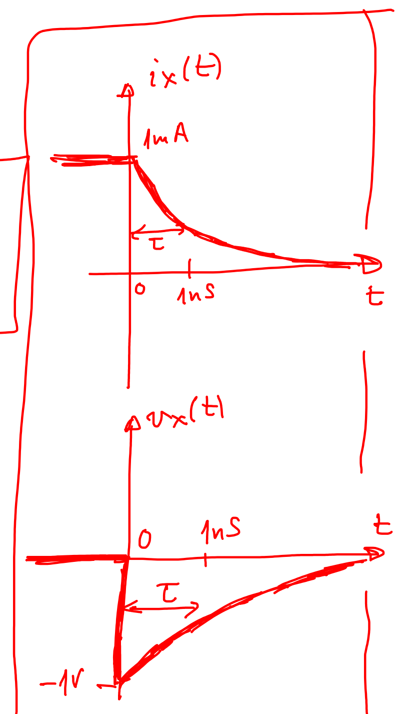
$i_x(\infty) = 0$

$i_x(0^+) = 1mA$

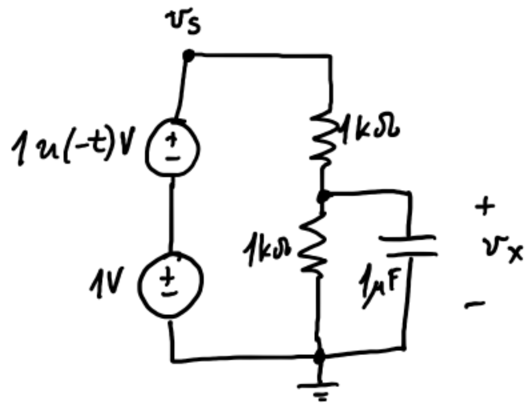
$\therefore \quad i_x(t) = \begin{cases} 1mA\, e^{-t/\tau}, \ \tau = 1ns & \text{for } t \geq 0 \\ 1mA & \text{for } t < 0 \end{cases}$

$v_x = -(R_1 \| R_2)\, i_x = -1k\Omega\, i_x(t)$

$v_x = \begin{cases} -1V\, e^{-t/\tau}, \ \tau = 1ns, & t \geq 0 \\ 0, & t < 0 \end{cases}$

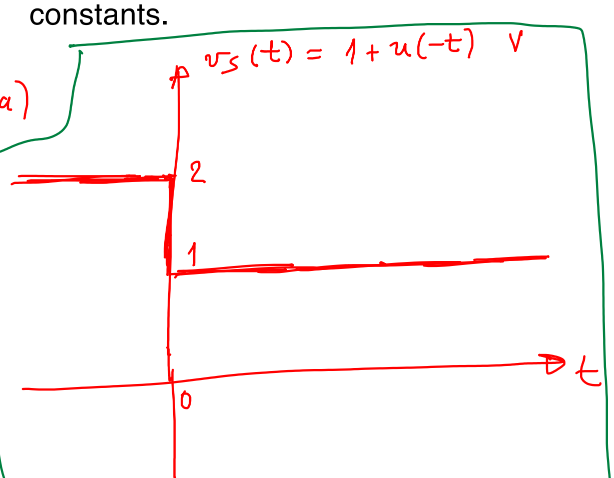


Problem 5. [20 pts] In the circuit shown, there is one fixed and one time-dependent voltage source, with their time-dependent voltages provided in the circuit diagram.



(a) [4] Plot the node voltage $v_s(t)$ as a function of time.
(b) [4] What is $v_x(t)$ at time t<0? What is $v_x(t)$ when time *t* approaches +infinity?
(c) [12] Solve for $v_x(t)$ for all times, give the equation, sketch it and provide any time constants.

(a)



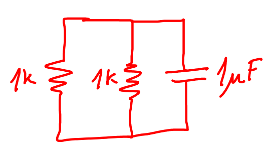
(b) At $t<0$, $v_x(t) = \frac{1k}{1k+1k} 2V = \frac{1}{2} 2V = 1V$

For $t \to \infty$, $v_x(t) = \frac{1}{2} 1V = 0.5V$

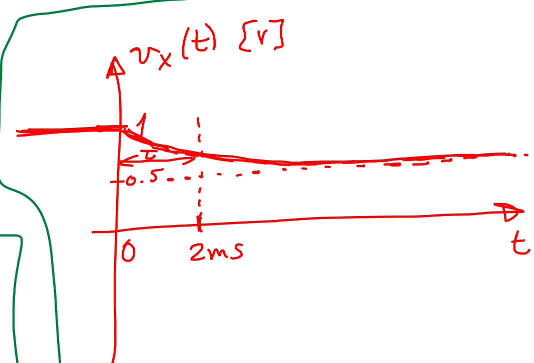
(c) For all times:

$$v_x(t) = v_x(\infty) + [v_x(0) - v_x(\infty)] e^{-t/\tau}$$

$\tau = RC = 2k\Omega \cdot 1\mu F$



$\tau = 2\ ms$



$$\therefore\ v_x(t) = \begin{cases} 0.5 + 0.5e^{-t/\tau} & [V],\ t \geq 0 \\ 1 & [V],\ t < 0 \end{cases}$$