BOSTON UNIVERSITY

EK 307: Electric Circuits

Fall 2017

Lecture 23

Dec 5, 2017

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Administrivia

- Last day of class Tue, Dec 12
- Final Exam: Tue, Dec 19, 3-5pm, PHO203

Last Lecture (22): Chapter 9

1. **Chapter 9 & 10 – Sinusoidal Steady State Circuits; Impedance/Admittance**

Today

- Chapter 10:
  - Thevenin and Norton in phasor domain
  - Active circuits (op amps)
- Chapter 11 (Sec 11.1 to 11.3):
  - Power in the phasor domain
  - Max power transfer
- Chapter 14:
  - Frequency response and intro to filter circuits

Solving AC Circuits

(Chapter 10)

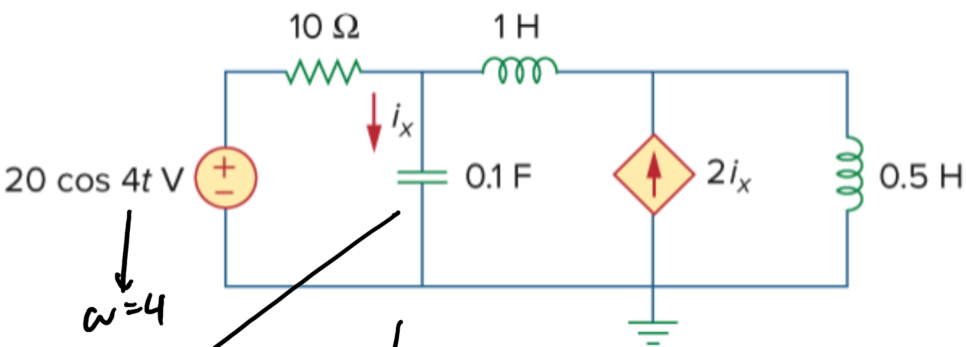
Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Node voltage method

Example 10.1

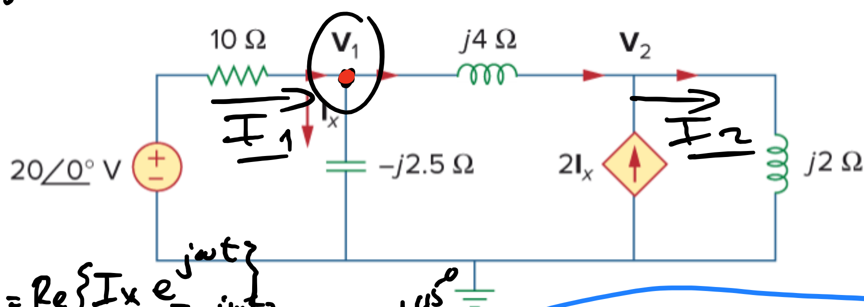
Find i_x in the circuit of **Fig. 10.1** using nodal analysis.



$\omega = 4$

$Z_C = \frac{1}{j\omega C} = -j\frac{1}{4\cdot 0.1} = -j\frac{1}{0.4} = -j2.5\,\Omega$

Find impedances



$$① : \overbrace{\frac{20\angle 0^\circ - V_1}{10}}^{I_1} = \overbrace{\frac{V_1 - V_2}{j4}}^{I_2} + \overbrace{\frac{V_1}{-j2.5}}^{I_x}$$

$$② : \frac{V_1 - V_2}{j4} + 2\frac{V_1}{-j2.5} = \frac{V_2}{j2}$$

$$I_x = \frac{V_1}{-j2.5}$$

$$\boxed{\begin{aligned} &\frac{20\angle 0^\circ - V_1}{10} = \frac{V_1 - V_2}{j4} + \frac{V_1}{.5} \\ &\frac{V_1 - V_2}{j4} + 2\frac{V_1}{-j2.5} = \frac{V_2}{j2} \end{aligned}}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

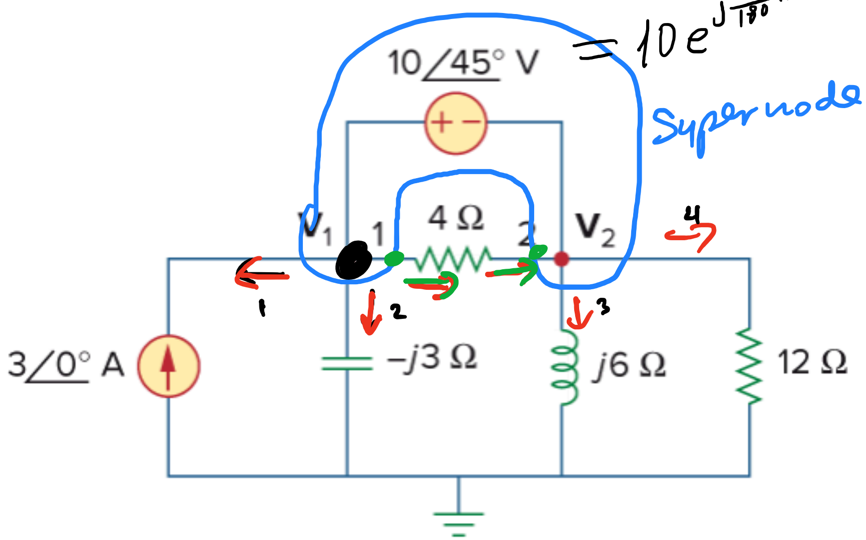
$i_x(t) = Re\{I_x e^{j\omega t}\}$
$= Re\{10 e^{-j\pi/4} e^{j\omega t}\}$ say $= 10\angle -45^\circ$

$i_x(t) = 10\cos(\omega t - 45^\circ) \leftarrow I_x = \frac{V_1}{-j2.5}$

Example 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

$= 10 e^{j\frac{45}{180}\pi} = 10 e^{j\frac{45}{180}\pi}$



$V_2 = V_1 - 10\angle 45^\circ$

$-3\angle 0^\circ + \frac{V_1}{-j3} + \frac{V_1 - 10\angle 45^\circ}{j6} + \frac{V_1 - 10\angle 45^\circ}{12} = 0$

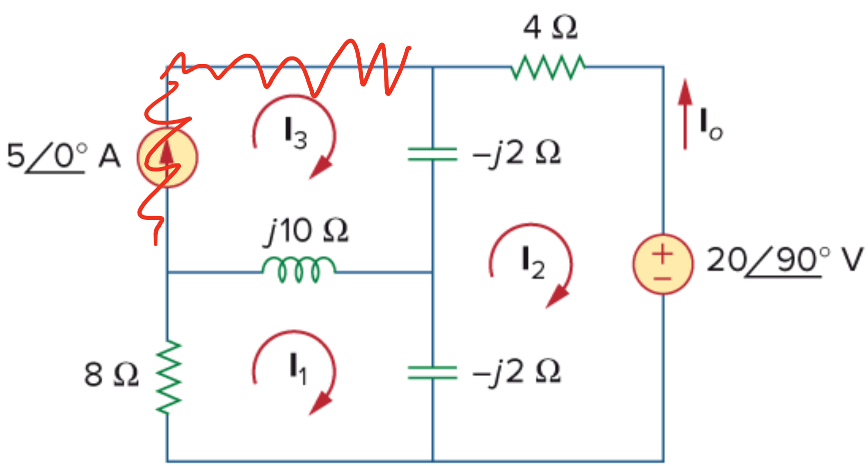
1 2 3 4

V_1

$V_2 = V_1 - 10\angle 45^\circ$

Example 10.3

Determine current $\mathbf{I}_o$ in the circuit of **Fig. 10.7** using mesh analysis.



2 eqns bc $\underline{I_3} = 5\angle 0°$ A, $i_3(t) = 5\cos(\omega t)$ A

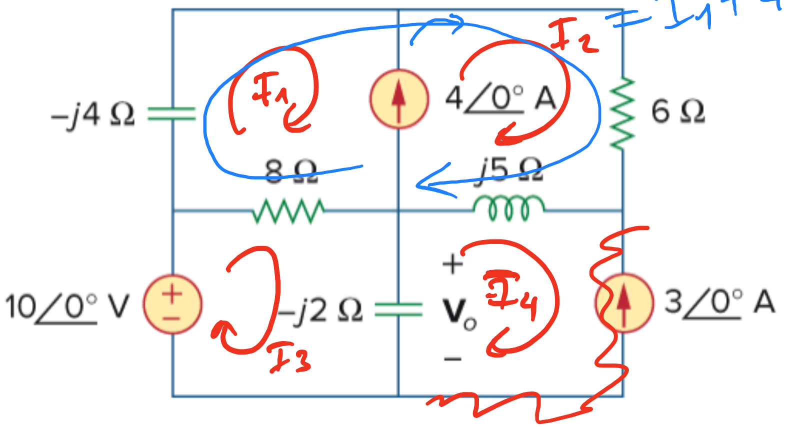
$\Rightarrow \begin{bmatrix} \ \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \underline{I_o} = -\underline{I_2}$

Example 10.4

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

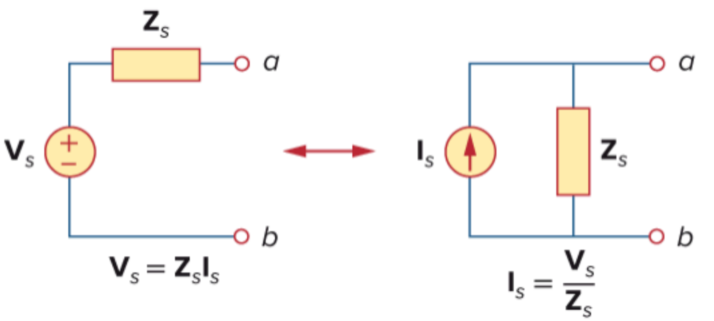
$I_2 = I_1 + 4\angle 0^\circ$ A



$I_4 = 3\angle 0^\circ$ A

Source transformations

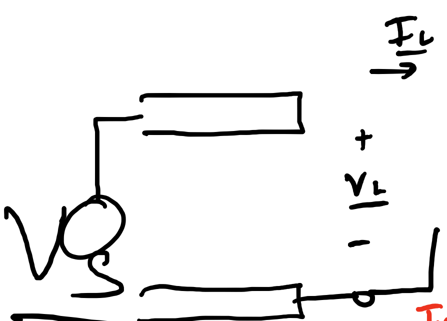
$$\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$



Same if:

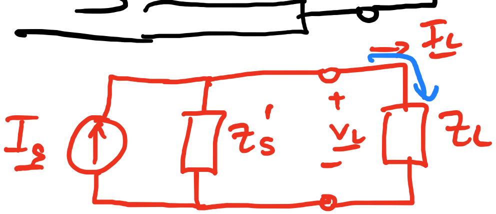
$V_s \equiv Z_s I_s$

$Z_s = Z_s'$



V_L

$\frac{1}{Z_s + Z_L} V_s$

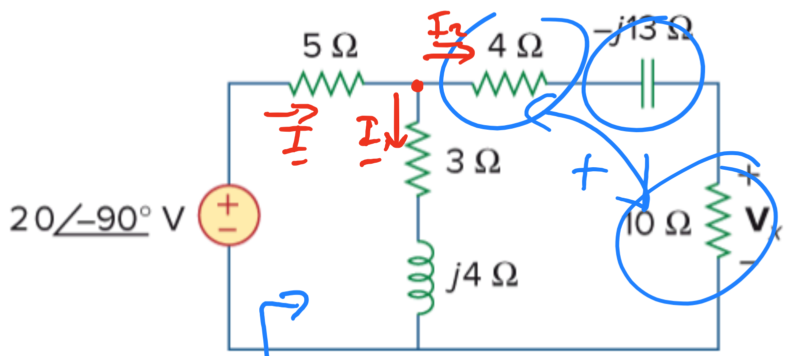


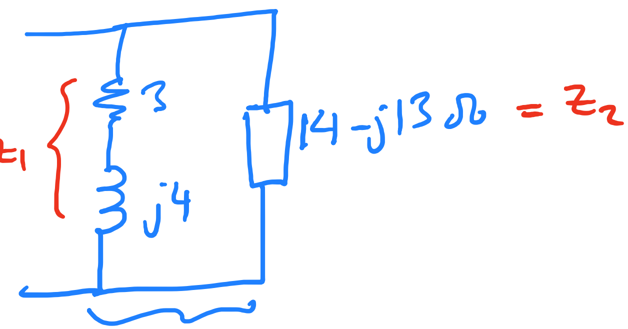
$$V_L = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_s'}} I_s = \frac{Z_L Z_s'}{Z_s' + Z_L} I_s$$

$$I_L = \frac{\frac{1}{Z_L} I_s}{\frac{1}{Z_L} + \frac{1}{Z_s'}} = \frac{Y_L}{Y_L + Y_s} I_s = \frac{Z_s'}{Z_s' + Z_L} I_s$$

Example 10.7

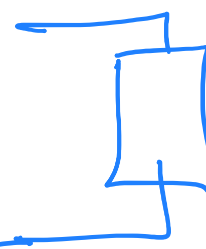
Calculate $\mathbf{V}_x$ in the circuit of **Fig. 10.17** using the method of source transformation.





Z_{in} $\quad I = \dfrac{20\angle 90^\circ V}{A+B}$

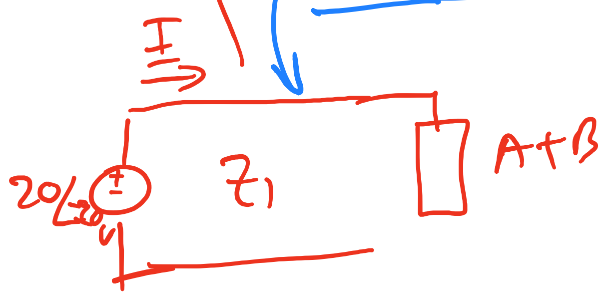
A = 5



$\left(\underbrace{\dfrac{1}{3+j4}}_{Z_1} + \underbrace{\dfrac{1}{14-j13}}_{Z}\right)^{-1} = B$

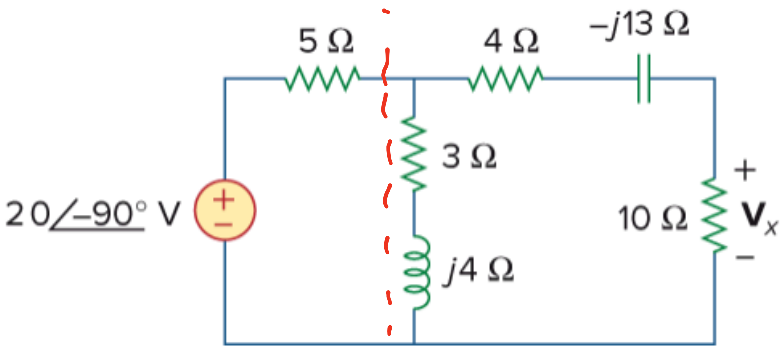
$I_2 = \dfrac{Z_1}{Z_2+Z_1} I = \dfrac{Y_2}{X_1+Y_2} I$

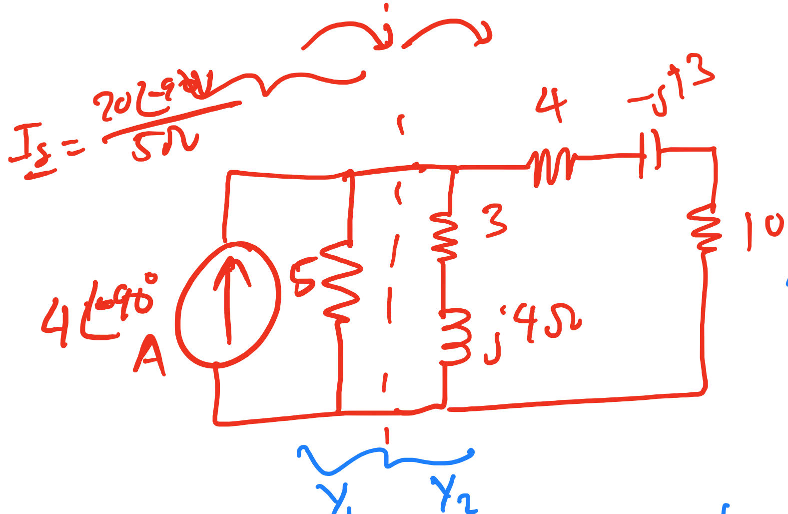
$\boxed{V_x = 10 I_2}$

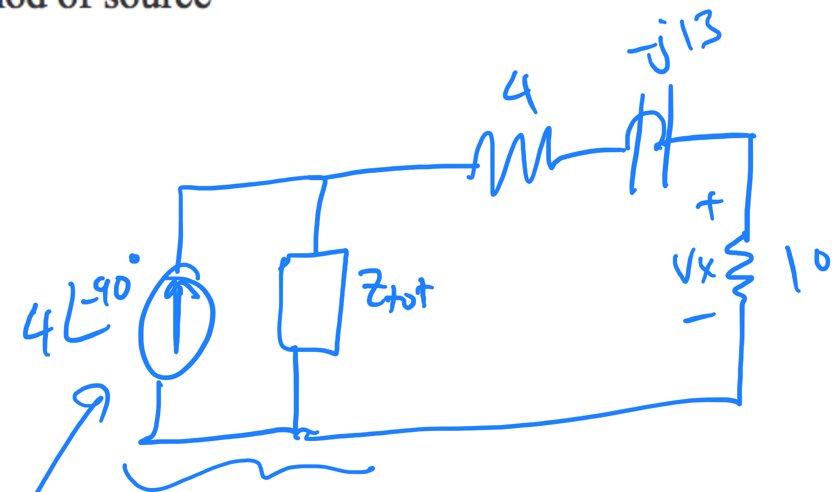


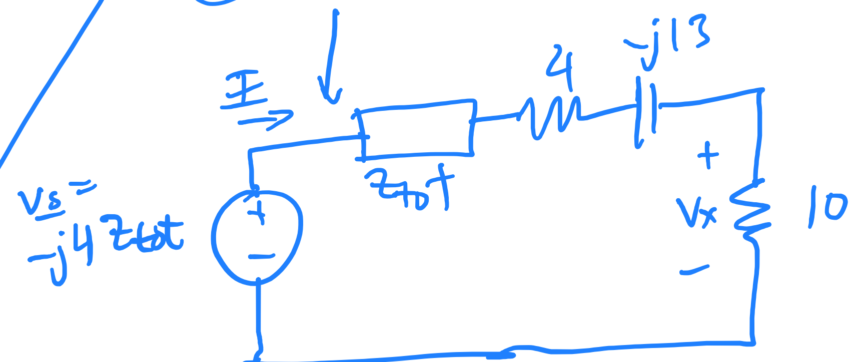
Example 10.7

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.









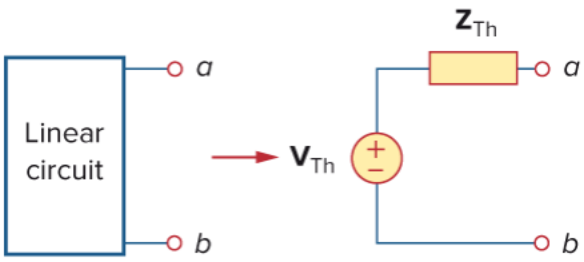
$$Y_{tot} = Y_1 + Y_2 = \frac{1}{5} + \frac{1}{3+j4}$$

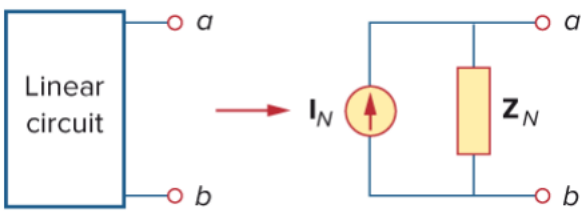
$$Z_{tot} = \frac{1}{Y_{tot}} = \left(\frac{3+j4+5}{15+j20}\right)^{-1}$$

$$I = \frac{1}{Z_{tot}+4-j13+10}(-j4 Z_{tot})$$

$$V_x = 10 I$$

Thevenin and Norton equivalents

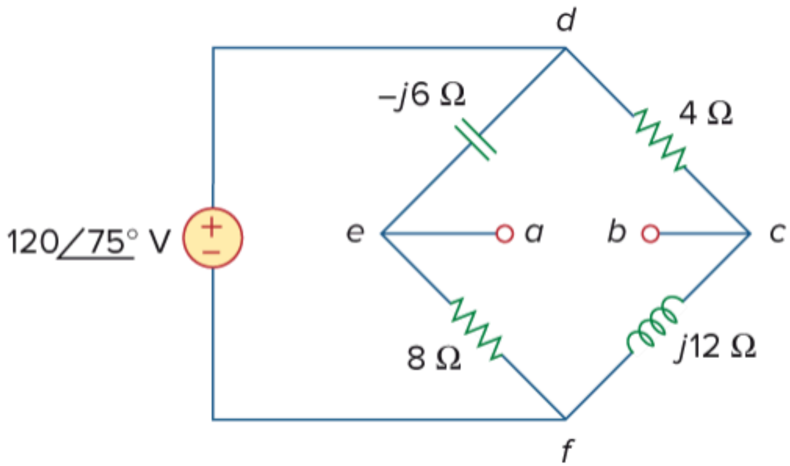


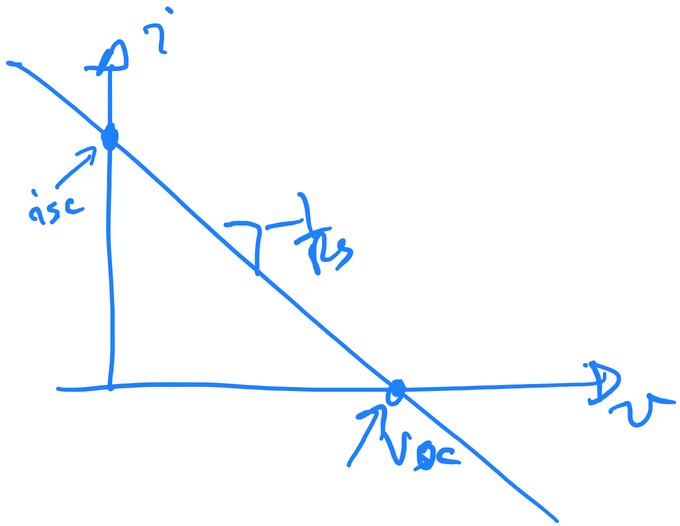


$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$$

Example 10.8

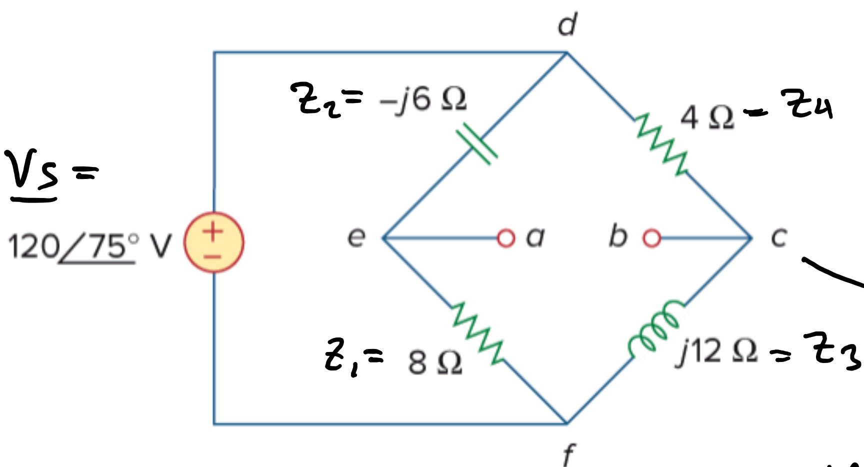
Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit in **Fig. 10.22**.





Example 10.8

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.22.

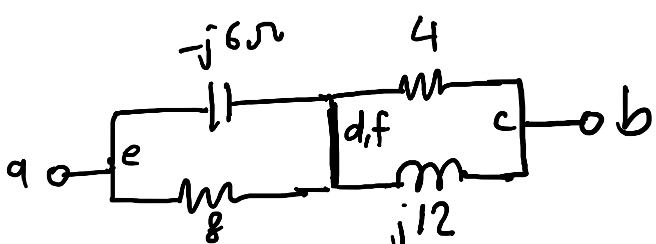


$$\underline{V_e} = \frac{8}{8-j6}\, 120\angle 75^\circ\ V = \frac{Z_1}{Z_1+Z_2}\underline{V_s}$$

$$\underline{V_c} = \frac{Z_3}{Z_3+Z_4}\underline{V_s} = \frac{j12}{j12+4}\, 120\angle 75^\circ\ V$$

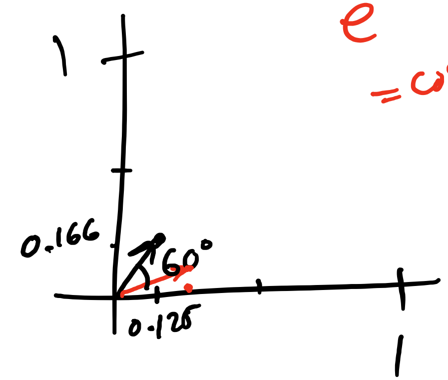
$$\underline{V_{oc}} = \underline{V_e} - \underline{V_c}$$

$$e^{-j60^\circ} = \cos 60^\circ - j\sin 60^\circ$$



$$Z_5 = \left(\frac{1}{8} + \frac{1}{-j6}\right)^{-1}$$

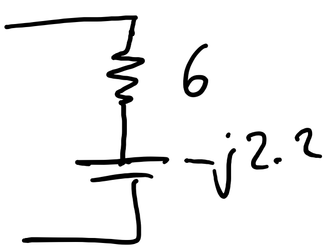
$$Z_6 = \left(\frac{1}{4} + \frac{1}{j12}\right)^{-1}$$

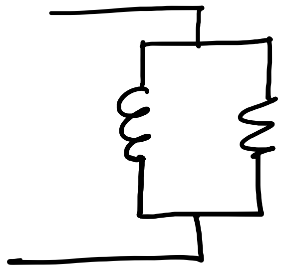


$$\boxed{Z_{eq}} = Z_5 + Z_6 = \left(\frac{1}{8} + \frac{1}{-j6}\right)^{-1} + \left(\frac{1}{4} + j\frac{1}{12}\right)^{-1}$$

$$= 5\angle -60^\circ + 4\angle -30^\circ = 5\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$$

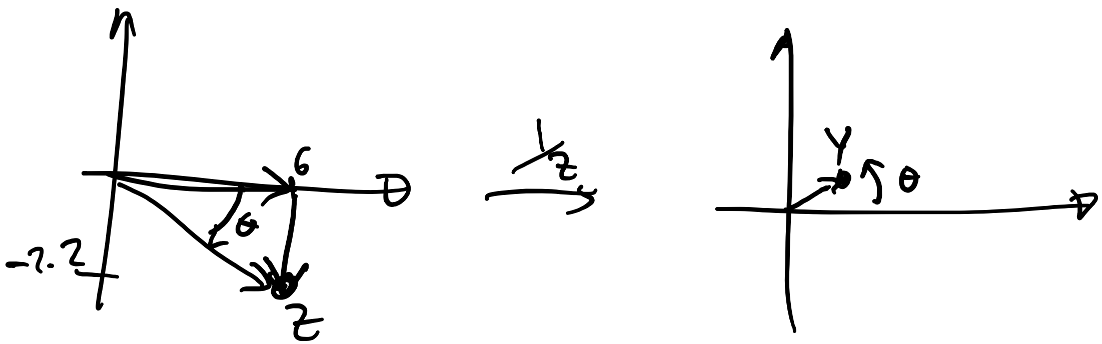
$$= \boxed{6 - j2.2}$$

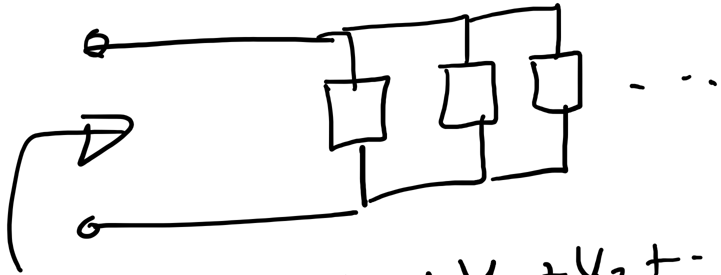




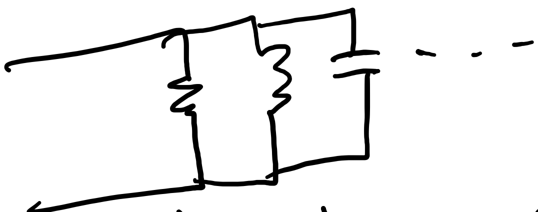
$Z = 6 - j2.2$

$$Y = \frac{1}{6 - j2.2} = \underset{>0}{G} + j\underset{>0}{B}$$





$$Y = Y_1 + Y_2 + Y_3 + \cdots$$



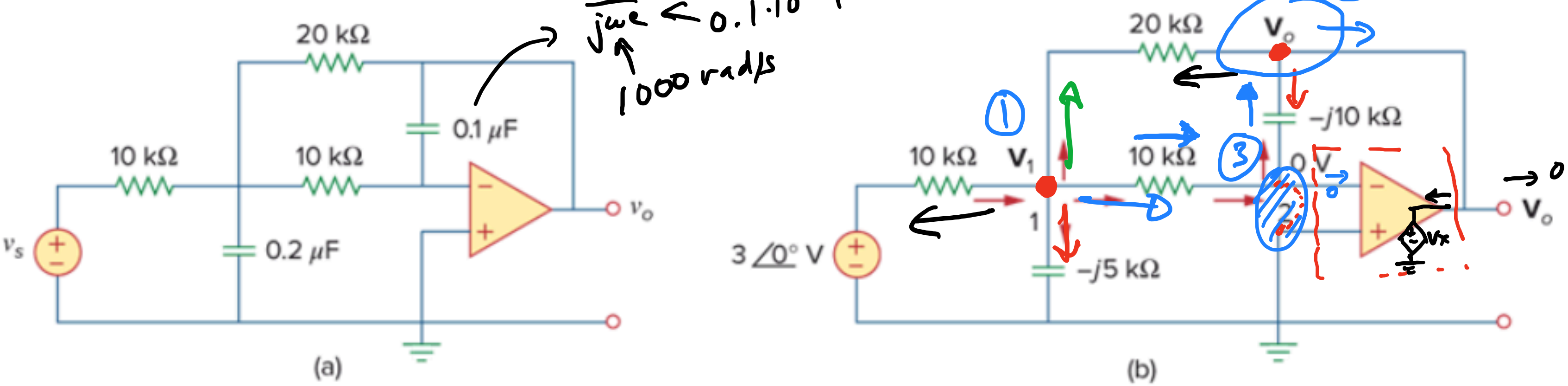
$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C + \cdots$$

Example 10.11

Determine $v_o(t)$ for the op amp circuit in **Fig. 10.31(a)** if $\boxed{v_s = 3\cos 1000t \text{ V}}$.

$\frac{1}{j\omega c} \leftarrow 0.1\cdot10^{-6}\text{ F}$

ω: 1000 rad/s



$$\underline{V_s} = 3\angle 0^\circ$$

$$①: \quad \frac{V_1 - 3\angle 0^\circ}{10k} + \frac{V_1}{-j5k} + \frac{V_1 - 0}{10k} + \frac{V_1 - V_o}{20k} = 0$$

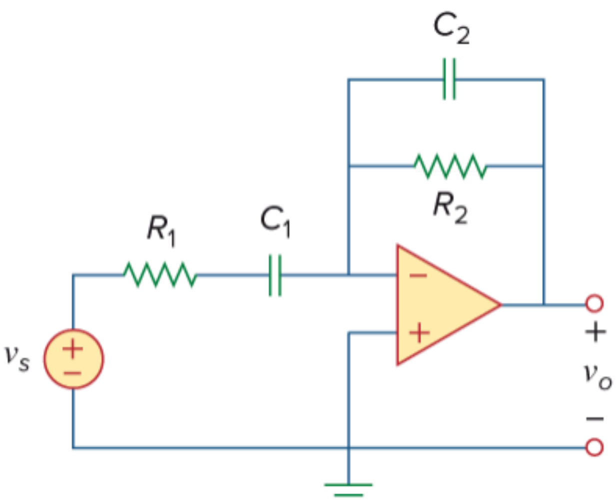
$$②: \quad \cancel{\frac{V_o - V_1}{20k} + \frac{V_o - 0}{-j10k}} +$$

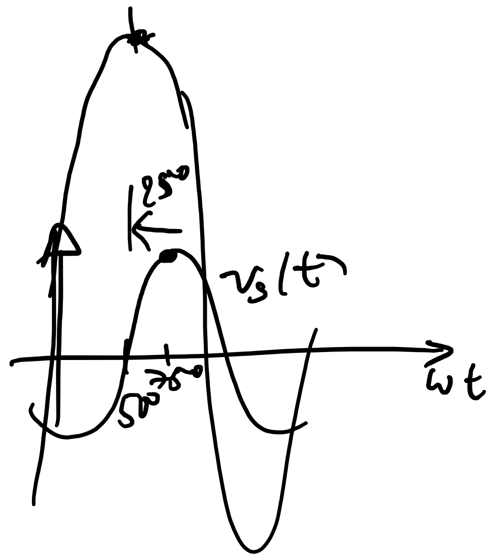
$$3: \quad \boxed{\frac{V_1 - 0}{10k\Omega} = \frac{0 - V_o}{-j10k}}$$

$$V_1 = \frac{-10k}{-j10k} V_o = -j V_o$$

Example 10.12

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.





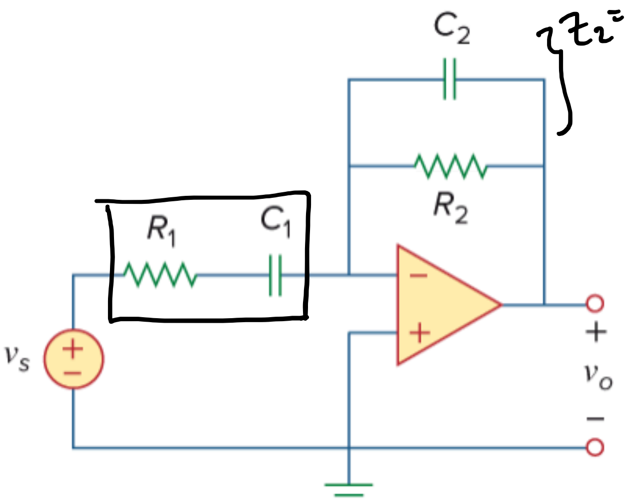
$$\frac{v_o}{v_s} = \frac{1000\cos(\omega t - 50^\circ)}{3\cos(\omega t - 75^\circ)}$$

$$\frac{V_o}{V_s} = \frac{1000\, e^{-j50^\circ}}{3\, e^{-j75^\circ}} \cancel{= e^{-j50}}$$

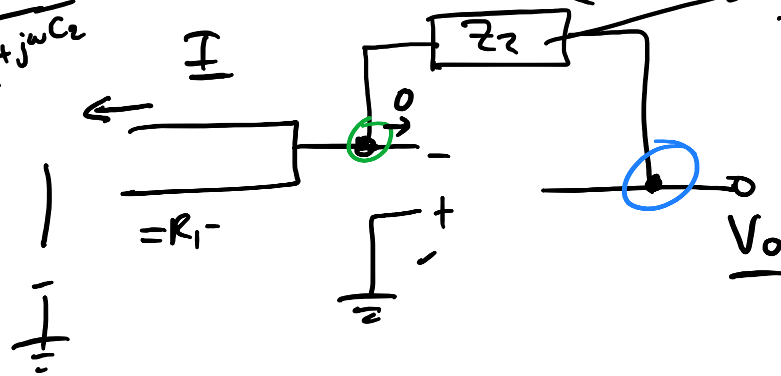
$$\underline{G} = \frac{V_o}{V_s} = 333\, e^{+j25^\circ}$$

Example 10.12

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.



$Z_2 = \frac{1}{\frac{1}{R_2} + j\omega C_2}$



BUG!

$Z_2 = \frac{1}{\frac{1}{R_2} + j\omega C_2} = \frac{1}{0.1m - j5k}$

$= j0.2m$

$\approx 3k\angle 30^\circ$

$$\frac{V_o}{V_s} = -\frac{Z_2}{Z_1} \quad \& \quad \begin{cases} I = \frac{V_o - 0}{Z_2} \\ I = \frac{0 - V_s}{Z_1} \end{cases}$$

$$= \frac{\cancel{j0.2m}\ 3k\angle 30^\circ}{10k - j2.5k}$$

$$= \frac{\cancel{0.2m\angle 90^\circ}\ 3k\angle 30^\circ}{10k\angle -25^\circ} \quad 0.3\angle 55^\circ$$

For 200rad/s ≈30Hz →

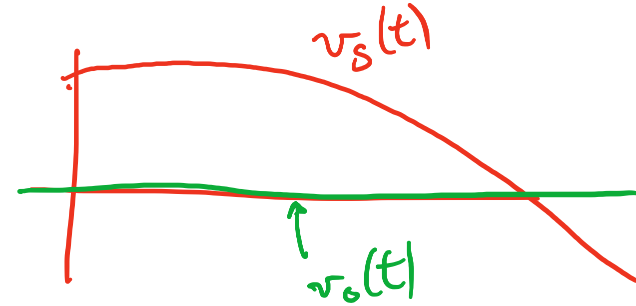
$$\boxed{\frac{V_o}{V_s} = 0.02\frac{10^{-3}}{10^{3}}\angle 115^\circ = 10^{-8}\angle 115^\circ}$$

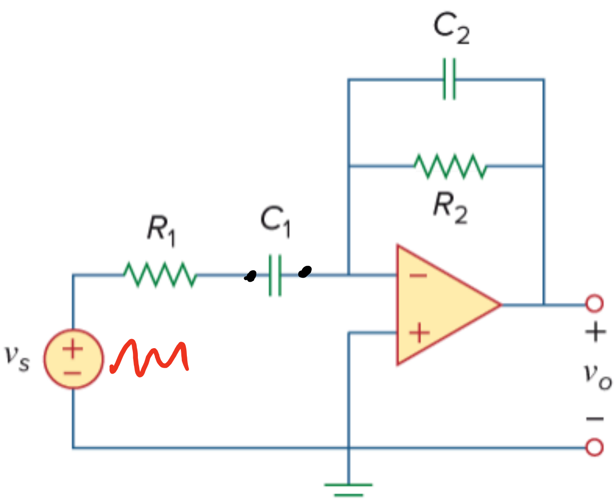
$v_s = 1\cos(200t)$

$v_o = 10^{-8}\cos(200t + 115^\circ)$

Example 10.12

Compute the closed-loop gain and phase shift for the circuit in **Fig. 10.33**. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.

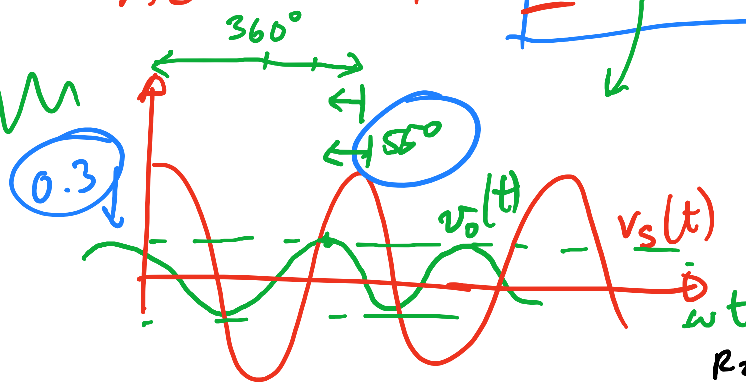


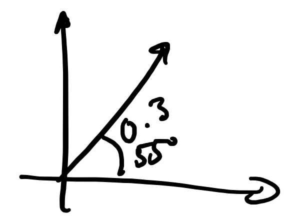


At $\omega = 0$: $\underline{G} = 0$

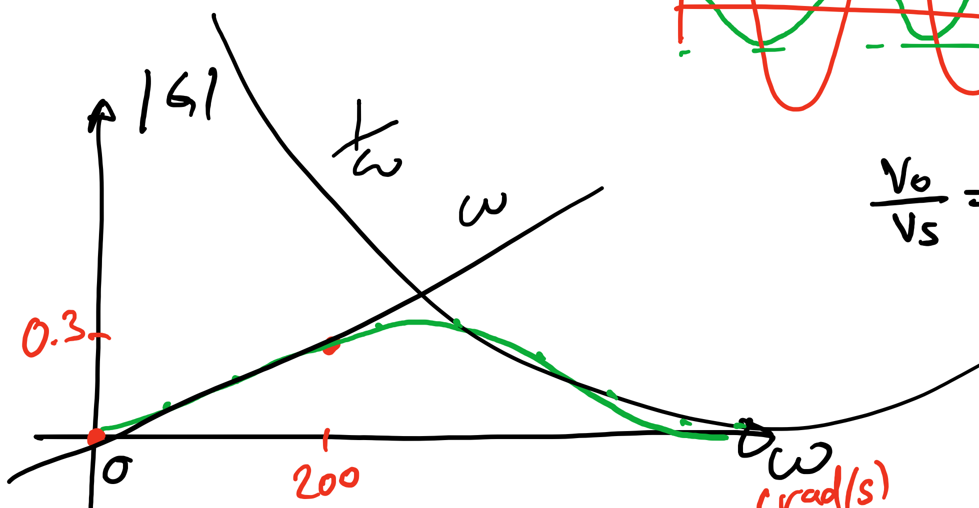
At $\omega = 200$ rad/s : $\boxed{\underline{G} = 0.3\angle 55^\circ}$

$|\underline{G}| = 0.3$

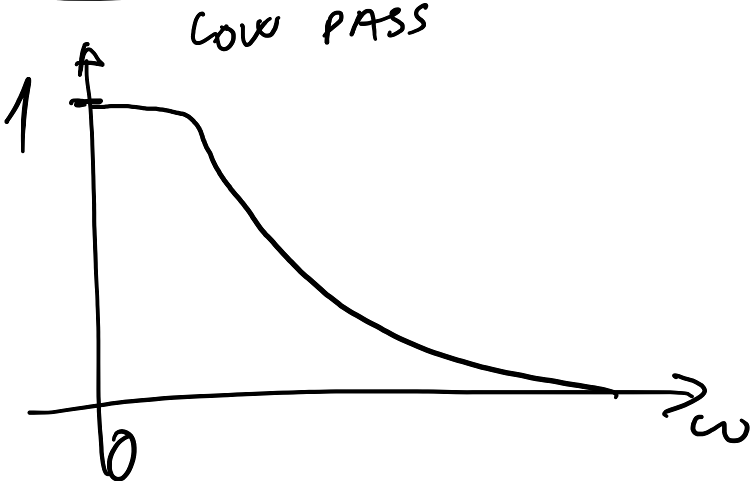


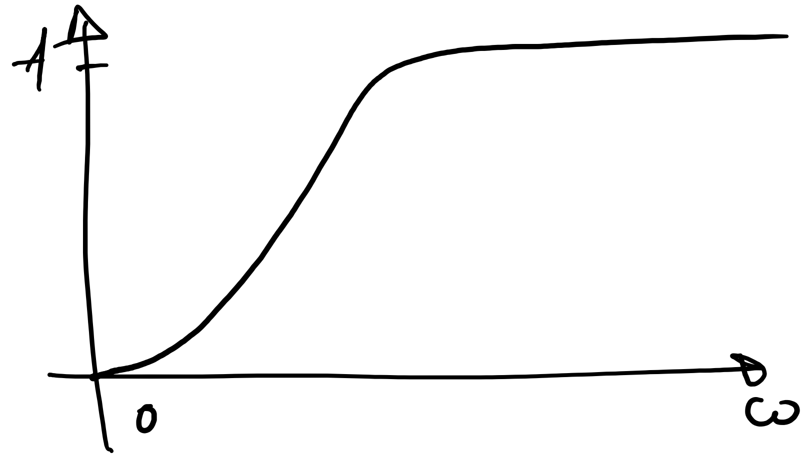


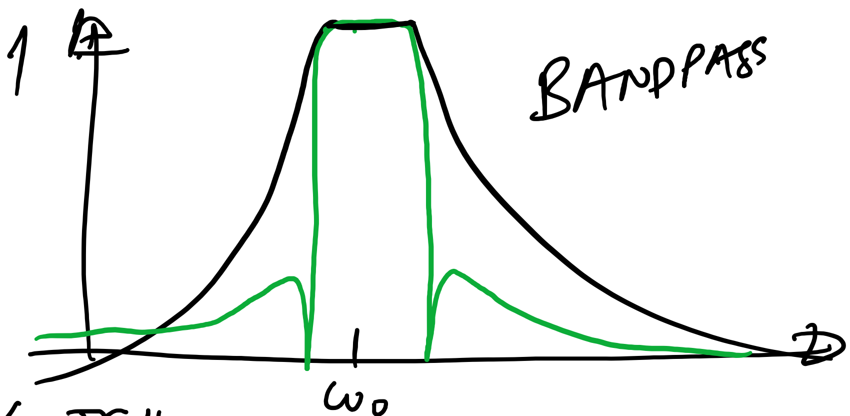
$$\frac{V_o}{V_s} = \frac{-Z_2}{Z_1} = \frac{\dfrac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}}{R_1 + \frac{1}{j\omega C_1}}$$

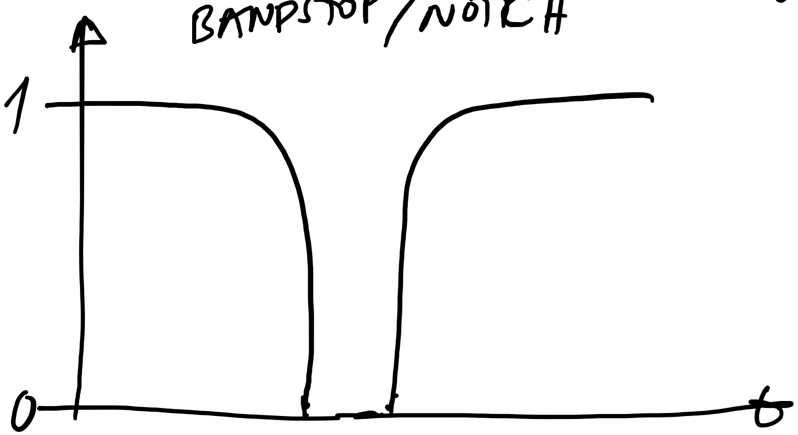


FILTERS









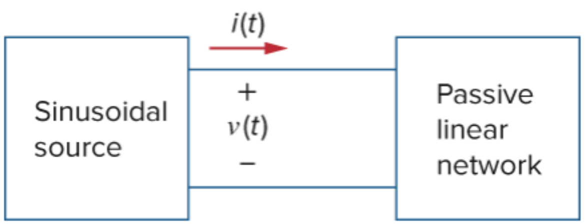
Chapter 11: Power in phasor domain

Instantaneous and average power

$$\boxed{p(t) = v(t)i(t)}$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

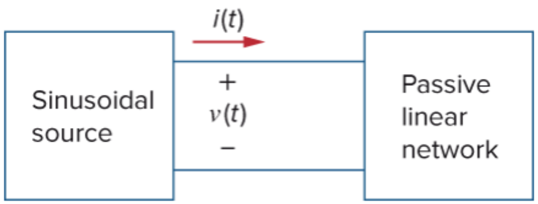


Instantaneous and average power

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

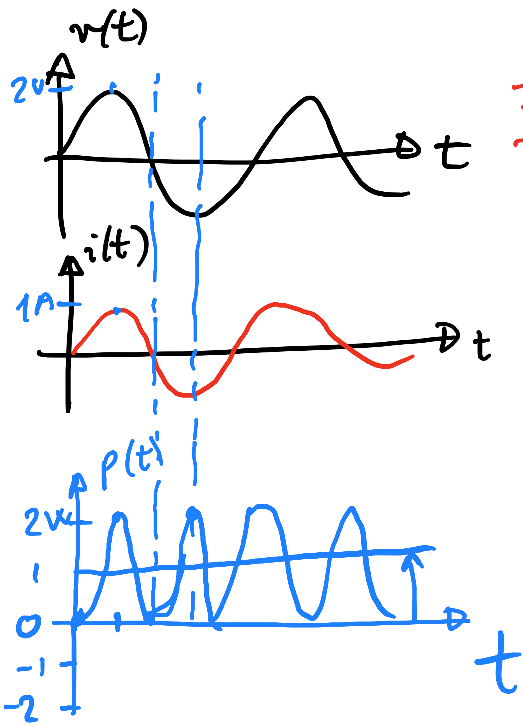
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

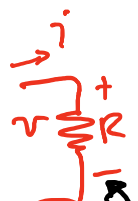


$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

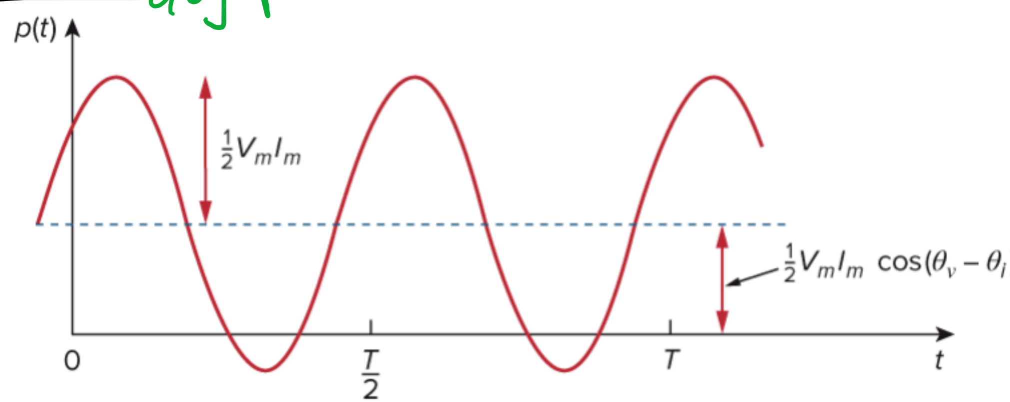
Instantaneous power:

$$\boxed{p(t) = \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}_{\text{avg power}} + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$





p(t) burned in resistor W (J/s)

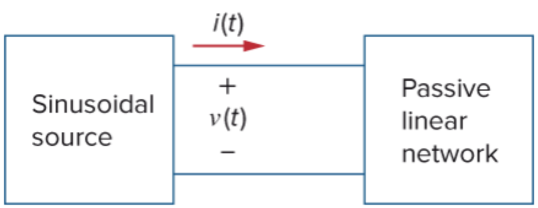


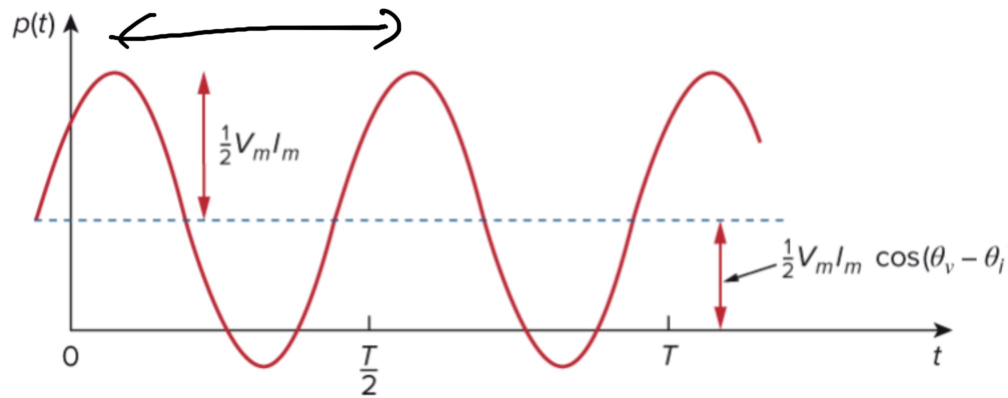
Instantaneous and average power

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$





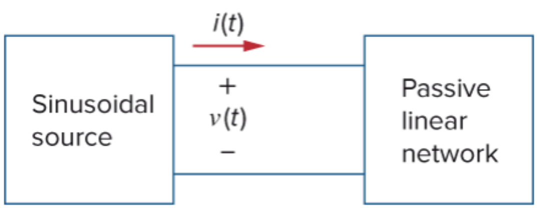
$$\boxed{P = \frac{1}{T} \int_0^T p(t)\, dt}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

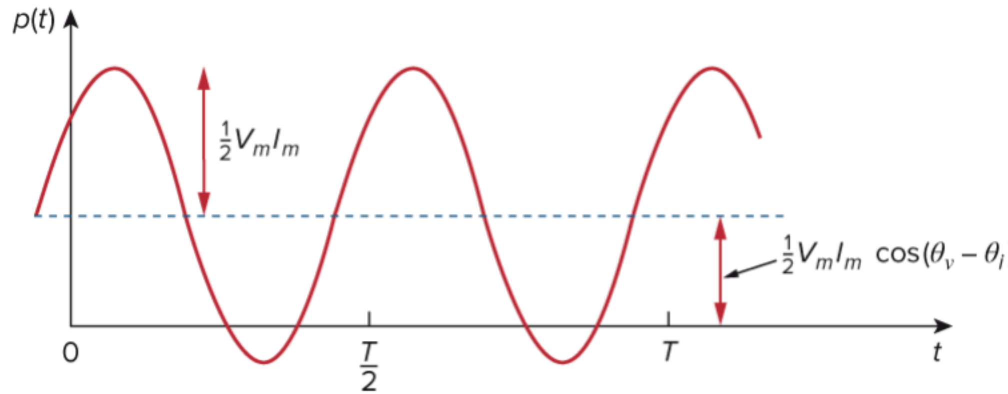
Instantaneous and average power

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$



$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\frac{1}{2}\mathbf{VI}^* = \frac{1}{2} V_m I_m \underline{/\theta_v - \theta_i}$$

$$= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$$

$$\boxed{P = \frac{1}{2}\mathrm{Re}[\mathbf{VI}^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$